3.5 HW Solutions

Nathan Warner

Feb 28, 2023

Question 1:

Solution: 😊

Part 1:

$$\begin{aligned} 45x^8 + 7y^6 \cdot \frac{dy}{dx} &= 4 \\ = 7y^6 \frac{dy}{dx} &= -45x^8 + 4 \\ = \frac{dy}{dx} &= \frac{-45x^8 + 4}{7y^6}. \end{aligned}$$

Part 2: (solve for y)

$$\begin{aligned} y^7 &= -5x^9 + 4x \\ y &= (-5x^9 + 4x)^{\frac{1}{7}}. \end{aligned}$$

$$\begin{aligned} y' = \frac{1}{7}(-5x^9 + 4x)^{-\frac{6}{7}} \cdot (-45x^8 + 4) \\ = \frac{-45x^8 + 4}{7(-5x^9 + 4x)^{\frac{6}{7}}}. \end{aligned}$$

Part 3: Plug in what we got when we solved for y in part b into part a's y in the denominator:

$$\begin{aligned} = \frac{dy}{dx} &= \frac{-45x^8 + 4}{7[(-5x^9 + 4x)^{\frac{1}{7}}]^6} \\ = \frac{dy}{dx} &= \frac{-45x^8 + 4}{7(-5x^9 + 4x)^{\frac{6}{7}}} \end{aligned}$$

.

Question 2:

Solution: 😊

Part 1:

$$\begin{aligned} \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} &= 0 \\ = \frac{1}{2\sqrt{y}} \frac{dy}{dx} &= -\frac{1}{2\sqrt{x}} \\ = \frac{dy}{dx} &= -\frac{2\sqrt{y}}{2\sqrt{x}} \\ = \frac{dy}{dx} &= -\frac{\sqrt{y}}{\sqrt{x}}. \end{aligned}$$

Part 2: Solve for y and differentiate

$$y = (-\sqrt{x} + 3)^2.$$

$$\begin{aligned} y' &= 2(-\sqrt{x} + 3) \cdot (-\frac{1}{2}x^{-\frac{1}{2}}) \\ &= \frac{-2(-x^{\frac{1}{2}} + 3)}{2x^{\frac{1}{2}}} \\ &= \frac{-(-x^{\frac{1}{2}} + 3)}{x^{\frac{1}{2}}} \\ &= \frac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} \end{aligned}$$

.

Part 3: Plug what we got when we solved for y in part b into the y from y' in part 1

$$\begin{aligned} y' &= \frac{-\sqrt{(-x^{\frac{1}{2}} + 3)^2}}{x^{\frac{1}{2}}} \\ y' &= \frac{-(-x^{\frac{1}{2}} + 3)}{x^{\frac{1}{2}}} \\ y' &= \frac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} \end{aligned}$$

.

Question 3:

Solution: 😊

Part 1:

$$\begin{aligned} y' &= 5x^{-2} + y^{-2} = 0 \\ &= -\frac{5}{x^2} + \frac{1}{y^2} = 0 \\ &= \frac{1}{y^2} = \frac{5}{x^2} \\ &= \frac{5y^2}{x^2} \end{aligned}$$

.

Part 2: Solve for y:

$$\begin{aligned}
y = \\
-\frac{1}{y} &= -\frac{5}{x} + 4 \\
-\frac{1}{y} &= -\frac{5}{x} + 4 \\
-\frac{1}{y} \cdot xy &= -\frac{5}{x} \cdot xy + 4 \cdot xy \\
-\frac{xy}{y} &= -\frac{5xy}{x} + 4xy \\
-x &= -5y + 4xy \\
-x &= y(-5 + 4x) \\
\frac{-x}{-5+4x} &= y \\
y &= \frac{-x}{-5+4x}.
\end{aligned}$$

Now plug in y into the y in y' from part 1:

$$\begin{aligned}
y' &= \frac{5(-\frac{x}{4x-5})^2}{x^2} \\
&= \frac{5(-1)^2 \frac{x^2}{(-5+4x)^2}}{x^2} \\
&= \frac{\frac{5x^2}{(-5+4x)^2}}{x^2} \\
&= \frac{5x^2}{(-5+4x)^2} \cdot \frac{1}{x^2} \\
&= \frac{5x^2}{(-5+4x)^2 x^2} \\
&= \frac{5}{(-5+4x)^2}.
\end{aligned}$$

Question 4:

Solution: 😊

$$\begin{aligned}
2x - 10x\frac{dy}{dx} - 10y + 2y\frac{dy}{dx} &= 0 \\
= \frac{dy}{dx}(-10x + 2y) &= -2x + 10y \\
\frac{dy}{dx} &= \frac{-2x + 10y}{-10x + 2y} \\
\frac{dy}{dx} &= \frac{2(-x + 5y)}{2(-5x + y)} \\
\frac{dy}{dx} &= \frac{-x + 5y}{-5x + y}
\end{aligned}$$

.

Question 5:

Solution: 😊

$$\begin{aligned} -y\sin x + \frac{dy}{dx}\cos x &= 6x + 10y\frac{dy}{dx} \\ = \frac{dy}{dx}(cosx - 10y) &= 6x + y\sin x \\ = \frac{dy}{dx} &= \frac{6x + y\sin x}{\cos x - 10y}. \end{aligned}$$

Question 6:

Solution: 😊

$$\begin{aligned} e^{\frac{x}{y}} \cdot \frac{y}{x} &= 2 - \frac{dy}{dx} \\ e^{\frac{x}{y}}(\frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2}) &= 2 - \frac{dy}{dx} \\ e^{\frac{x}{y}}(\frac{y - x \cdot \frac{dy}{dx}}{y^2}) &= 2 - \frac{dy}{dx} \\ e^{\frac{x}{y}}(\frac{y}{y^2} - \frac{x}{y^2}\frac{dy}{dx}) &= 2 - \frac{dy}{dx} \\ e^{\frac{x}{y}}(\frac{y}{y^2} - \frac{x}{y^2}\frac{dy}{dx}) &= 2 - \frac{dy}{dx} \\ e^{\frac{x}{y}}(\frac{1}{y} - \frac{x}{y^2}\frac{dy}{dx}) &= 2 - \frac{dy}{dx} \\ \frac{1}{y}e^{\frac{x}{y}} - \frac{x}{y^2}e^{\frac{x}{y}}\frac{dy}{dx} &= 2 - \frac{dy}{dx} \\ \frac{dy}{dx} - \frac{x}{y^2}e^{\frac{x}{y}}\frac{dy}{dx} &= 2 - \frac{1}{y}e^{\frac{x}{y}} \\ \frac{dy}{dx}(1 - \frac{x}{y^2}e^{\frac{x}{y}}) &= 2 - \frac{1}{y}e^{\frac{x}{y}} \\ \frac{dy}{dx} &= \frac{2 - \frac{1}{y}e^{\frac{x}{y}}}{(1 - \frac{x}{y^2}e^{\frac{x}{y}})} \end{aligned}$$

.

From here multiply by common denominator y^2 to clear out the fractions:

$$\begin{aligned} & \frac{2y^2 - ye^{\frac{x}{y}}}{y^2 - xe^{\frac{x}{y}}} \\ \frac{dy}{dx} &= \frac{y(2y - e^{\frac{x}{y}})}{y^2 - xe^{\frac{x}{y}}}. \end{aligned}$$

Question 7:

Solution: 😊

Question 8:

Solution: 😊

Question 9:

Solution: 😊

Question 10:

Solution: 😊

Question 11:

Solution: 😊