Calculus 1: Chapter 3

Nathan Warner

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Chapter 3

Contents

3.1

Differential Rule:

Diffential Fomulas:

- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(x^n) = n \cdot x^{n-1} \rightarrow$ ***Power Rule***
- $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

Example 0.0.1

Differentiate the following functions:

1.) $f(t) = \frac{1}{2}t^6 - 3t^4 + 1$

For the first term, we will use the ***Third and Fourth*** Rule:

$$\frac{1}{2} \cdot 6t^{t-1}.$$

For the second term, $-3t^4$, We will use the ***Third and Fifth*** Rule:

$$-3 \cdot 4t^{4-1}.$$

The last term is a constant, so according to the first rule, the Derivative of a constant is ***Zero***:

So our full equation is:

$$\begin{aligned} f\prime(x) &= \frac{1}{2} \cdot 6t^{6-1} - 3 \cdot 4t^{4-1} + 0 \\ &= 3t^5 - 12t^3. \end{aligned}$$

2.) $h(x) = (x-2)(2x+3)$

First we need to distribute out the terms:

$$\begin{aligned} h(x) &= 2x^2 + 3x - 4x - 6 \\ &= 2x^2 - x - 6. \end{aligned}$$

Now this is the function we want to diferentiate.

So $\rightarrow$

$$h\prime(x) = 2 \cdot 2x^{2-1} - 1 - 0$$
$$h\prime(x) = 4x - 1.$$

3.) $y = \frac{x^2 - 2\sqrt{x}}{x}$

So:

$$y = \frac{x^2 - 2x^{\frac{1}{2}}}{x}.$$

Since the numerator only has ***one term****, we can split the equation like:*

$$y = \frac{x^2}{x} - \frac{2x^{\frac{1}{2}}}{x}$$
$$y = x - 2x^{-\frac{1}{2}}.$$

Now:

$$\frac{dy}{dx} = 1 - 2 \cdot (-\frac{1}{2})x^{-\frac{1}{2}-1}$$
$$\frac{dy}{dx} = 1 + x^{-\frac{3}{2}}.$$

And we can even rewrite it as:

$$\frac{dy}{dx} = 1 + \frac{1}{3^{\frac{3}{2}}}.$$

4.) $V = (\sqrt{x} + \frac{1}{\sqrt[3]{x}})^2$

So:

$$V = (x^{\frac{1}{2}} + x^{\frac{-1}{3}})^2$$
$$= (x^{\frac{1}{2}})^2 + 2(x^{\frac{1}{2}})(x^{-\frac{1}{3}}) + (x^{-\frac{1}{3}})^2$$
$$= x + 2x^{\frac{1}{6}} + x^{-\frac{2}{3}}.$$

Now we find the Derivative:

$$V\prime = 1 + 2 \cdot \frac{1}{6}x^{\frac{1}{6}-1} + (\frac{-2}{3})x^{-\frac{2}{3}-1}$$
$$v\prime = 1 + \frac{1}{3}x^{-\frac{5}{6}} - \frac{2}{3}x^{-\frac{5}{3}}$$
$$v\prime = 1 + \frac{1}{3x^{\frac{5}{6}}} - \frac{2}{3x^{\frac{5}{3}}}.$$

Exponential Functions:

Recall: $(1+\frac{1}{n})^n \to e \approx 2.71828...asn \to \infty$

Definition 0.0.1: Definiton of e:

$\lim\limits_{h\to 0} \frac{e^h-1}{h} = 1$

Note:-

We'll use the above definiton to derive $\frac{d}{dx}(e^x)$

$\to$ Let $f(x) = e^x$

$$f\prime(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$