Quiz 2 Solutions

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1.a: Differentiate *<u>without</u>* using the product or quotient rule

$$f(x) = \frac{x^3 - 3x}{2x^2}.$$

Solution: In order to bypass using the quotient rule we will simplify the function: 😄

So:

$$\begin{aligned} \frac{x^3 - 3x}{2x^2} &\longrightarrow \frac{x^3}{2x^2} - \frac{3x}{2x^2} \\ &= \frac{x}{2} - \frac{3}{2x} \\ &= \frac{1}{2}x - \frac{3}{2}x^{-1} \end{aligned}$$

Now this is the function we will differentiate:

$$\begin{aligned} f'(x) &= \frac{1}{2}(1) - (-1)\frac{3}{2}x^{-1-1} \\ &= \frac{1}{2} + \frac{3}{2}x^{-2} \\ &= \frac{1}{2} + \frac{3}{2x^2} \end{aligned}$$

Now we find a common denominator:

$$\begin{aligned} f'(x) &= \frac{3}{2x^2} + \frac{1}{2}(\frac{x^2}{x^2}) \\ &= \frac{3}{2x^2} + \frac{x^2}{2x^2} \\ &= \frac{3 + x^2}{2x^2}. \end{aligned}$$

Therefore our answer is:

$$f'(x) = \frac{3 + x^2}{2x^2}.$$

1.b

$$G(x) = \frac{3x}{x^2+1}.$$

Solution: To find the deriviative we will use the quotient rule:: 😄

If:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

And:

$$\begin{aligned} f(x) &= 3x \\ f'(x) &= 3. \end{aligned}$$

$$\begin{aligned} g(x) &= x^2+1 \\ g'(x) &= 2x. \end{aligned}$$

Then:

$$\begin{aligned} G'(x) &= \frac{(x^2+1)(3)-(3x)(2x)}{(x^2+1)^2} \\ &= \frac{(x^2+1)(3)-6x^2}{(x^2+1)^2} \\ &= \frac{3x^2+3-6x^2}{(x^2+1)^2} \\ &= \frac{-3x^2+3}{(x^2+1)^2}. \end{aligned}$$

1.c

$$f(x) = -x + \tan x.$$

Solution: Derive: 😄

$$\begin{aligned} f'(x) &= -1 + \sec^2 x \\ &= \sec^2 x - 1. \end{aligned}$$

By the pathagorean identity:

$$\sec^2 x - 1 = \tan^2 x.$$

Then:

$$f'(x) = \tan^2 x.$$

1.d

$$g(t) = (1 + \sin t)^3.$$

Solution: Using the chain rule we can differentiate:: 😄

$$\begin{aligned} g'(t) &= 3(1 + \sin t)^2 \cdot (0 + \cos t) \\ &= 3\cos t(1 + \sin t)^2. \end{aligned}$$

Question 2: Find $\frac{dy}{dx}$:

$$2x^2 + xy + 3y^2 = 0.$$

Solution: By using implicit differentiate we can solve:: 😄

$$\begin{aligned} 4x + x\frac{dy}{dx} + y + 6y\frac{dy}{dx} &= 0 \\ = x\frac{dy}{dx} + 6y\frac{dy}{dx} &= -4x - y \\ = \frac{dy}{dx}(x + 6y) &= -4x - y \\ = \frac{dy}{dx} &= \frac{-4x - y}{x + 6y} \\ = \frac{dy}{dx} &= \frac{-(4x + y)}{x + 6y} \\ = \frac{dy}{dx} &= -\frac{4x + y}{x + 6y} \\ &. \end{aligned}$$

Question 3: Use logarithmic differentation to find the deriviative:

$$y = \frac{\sqrt{(x^2+1)^3}}{\sqrt[3]{(x^3+1)^7}}.$$

We will begin by rewriting the equation:

$$\begin{aligned} y &= \frac{\left[(x^2+1)^3\right]^{\frac{1}{2}}}{\left[(x^3+1)^7\right]^{\frac{1}{3}}} \\ &= \frac{(x^2+1)^{\frac{3}{2}}}{(x^3+1)^{\frac{7}{3}}}. \end{aligned}$$

Now we can differentiate by using logarithmic differentiate:

$$\ln y = \ln \frac{(x^2+1)^{\frac{3}{2}}}{(x^3+1)^{\frac{7}{3}}}.$$

From here we can use properties of natural logarithms to rewrite the equation:

If:

$$\ln \frac{x}{y} = \ln x - \ln y.$$

Then we can rewrite the equation as:

$$\ln y = \ln{(x^2+1)^{\frac{3}{2}}} - \ln{(x^3+1)^{\frac{7}{3}}}.$$

Now we can the power rule of natural logarithms:

If:

$$\ln x^y = y \cdot \ln x.$$

Then we can rewrite as:

$$\ln y = \frac{3}{2} \ln{(x^2+1)} - \frac{7}{3} \ln{(x^3+1)}.$$

Now we can differentiate:

$$\begin{aligned} \frac{1}{y}\frac{dy}{dx} &= \frac{3}{2} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{7}{3} \cdot \frac{1}{x^3+1} \cdot 3x^2 \\ &= \frac{6x}{2} \cdot \frac{1}{x^2+1} - \frac{21x^2}{3} \cdot \frac{1}{x^3+1} \\ &= 3x \cdot \frac{1}{x^2+1} - 7x^2 \cdot \frac{1}{x^3+1} \\ &= \frac{3x}{x^2+1} - \frac{7x^2}{x^3+1}. \end{aligned}$$

From here we find a common denominator:

$$\begin{aligned}
\frac{1}{y}\frac{dy}{dx} &= \frac{3x}{x^2+1}\cdot\frac{x^3+1}{x^3+1} - \frac{7x^2}{x^3+1}\cdot\frac{x^2+1}{x^2+1} \\
&= \frac{3x(x^3+1)}{(x^2+1)(x^3+1)} - \frac{7x^2(x^2+1)}{(x^3+1)(x^2+1)} \\
&= \frac{3x(x^3+1) - 7x^2(x^2+1)}{(x^2+1)(x^3+1)} \\
&= \frac{3x^4+3x-(7x^4+7x^2)}{(x^2+1)(x^3+1)} \\
&= \frac{3x^4+3x-7x^4-7x^2}{(x^2+1)(x^3+1)} \\
&= \frac{-4x^4-7x^2+3x}{(x^2+1)(x^3+1)} \\
&.
\end{aligned}$$

Now we can mulitply both sides by y to get $\frac{dy}{dx}$ alone and then plug our original equation into y and solve:

$$\begin{aligned}
y\cdot\frac{1}{y}\frac{dy}{dx} &= \frac{-4x^4-7x^2+3x}{(x^2+1)(x^3+1)}\cdot y \\
=\frac{dy}{dx} &= \frac{-4x^4-7x^2+3x}{(x^2+1)(x^3+1)}\cdot y \\
=\frac{dy}{dx} &= \frac{x(-4x^3-7x+3)}{(x^2+1)(x^3+1)}\cdot y \\
=\frac{dy}{dx} &= \frac{x(-4x^3-7x+3)}{(x^2+1)(x^3+1)}\cdot\frac{(x^2+1)^{\frac{3}{2}}}{(x^3+1)^{\frac{7}{4}}} \\
=\frac{dy}{dx} &= \frac{x(-4x^3-7x+3)(x^2+1)^{\frac{3}{2}}}{(x^2+1)(x^3+1)(x^3+1)^{\frac{7}{4}}} \\
=\frac{dy}{dx} &= \frac{x(-4x^3-7x+3)(x^2+1)^{\frac{3}{2}}}{(x^2+1)(x^3+1)(x^3+1)^{\frac{7}{4}}} \\
=\frac{dy}{dx} &= \frac{x(-4x^3-7x+3)(x^2+1)^{\frac{1}{2}}}{(x^3+1)(x^3+1)^{\frac{7}{4}}} \\
&.
\end{aligned}$$

Lastly we can rewrite (x^3+1) as a ***sum of cubes****, because (x^3+1) is the same as (x^3+1^3),*

If:

$$(a^3+b^3) = (a+b)(a^2-ab+b^2).$$

Where:

$$\begin{aligned} a &= x \\ b &= 1. \end{aligned}$$

Therefore:

$$\frac{dy}{dx} = \frac{x(-4x^3 - 7x + 3)(x^2+1)^{\frac{1}{2}}}{(x+1)(x^2 - x - x^2)(x^3+1)^{\frac{7}{4}}}$$

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