2.7 Hw Solutions

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Question 1:

Solution: 😄

a.)

If:

$$m_{tan} = \lim_{x \to -3} \frac{f(x) - f(a)}{x - a}.$$

And **a = -3, f(a) = -18, then:**

$$\begin{aligned} m_{tan} &= \lim_{x \to -3} \frac{x^2 + 9x - (-18)}{x - (-3)} \\ &= \lim_{x \to -3} \frac{x^2 + 9x + 18}{x + 3}. \end{aligned}$$

numerator factors into:

$$\lim_{x \to -3} \frac{(x+3)(x+6)}{x+3}.$$

Cancel out common factor:

$$\lim_{x \to -3} x + 6.$$

Plug in -3 for x

$$\begin{aligned} m_{tan} &= -3 + 6 \\ &= 3. \end{aligned}$$

b.)

If:

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

And a = -3 and f(a) = -18, and we plug in (a+h) for x:

$$m_{tan} = \lim_{h \to 0} \frac{(-3+h)^2 + 9(-3+h) - (-18)}{h}$$
$$.$$

And we distribute out the terms:

$$\begin{aligned} m_{tan} = \lim_{h \to 0} \frac{h^2 - 6h + 9 - 27 + 9h - 18}{h} \\ = \frac{h^2 + 3h}{h} \\ = \frac{h(h+3)}{h} \\ = h + 3. \end{aligned}$$

Now if we plug in zero:

$$\begin{aligned} m_{tan} &= 0+3 \\ &= 3. \end{aligned}$$

c.) The equation of the tangent line if:

$$\begin{aligned} y - y_1 &= m\left(x - x_1\right) \\ &. \end{aligned}$$

Then:

$$\begin{aligned} y - (-18) &= 3\left(x - (-3)\right) \\ y + 18 &= 3\left(x+3\right) \\ y + 18 &= 3x + 9 \\ y &= 3x - 9. \end{aligned}$$

Question 2:

Solution:

We know that:

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

And a = 6 and f(a) = 19:

$$\begin{aligned} &\lim_{x \to 6} \frac{2x^2 - 9x - 1 - 19}{x - 19} \\ &= \lim_{x \to 6} \frac{2x^2 - 9x - 18}{x - 19}. \end{aligned}$$

factor using the x method:

$$\begin{aligned} &\lim_{x \to 6} \frac{(2x+3)\left(x-6\right)}{x-6} \\ &= \lim_{x \to 6} 2x + 3 \\ &= 2\left(6\right) + 3 \\ &= 15. \end{aligned}$$

Plug $m_{tan} = 15$ into ***Point slope form equation*** to get equation of tangent line:

$$\begin{aligned} y - 19 &= 15\left(x - 6\right) \\ y - 19 &= 15x - 90 \\ y &= 15x - 71. \end{aligned}$$

Question 3:

Solution: 😄

a.)

If:

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Then:

$$m_{tan} = \lim_{h \to 0} \frac{4 + 5\left(a+h\right)^2 - 2\left(a+h\right)^3 - \left(4 + 5a^2 - 2a^3\right)}{h}.$$

Distribute -1 to each term in $4 + 5a^2 - 2a^3$

$$= -4 - 5a^2 + 2a^3.$$

Foil out $-2\left(a+h\right)^3$

$$= -2a^3 - 2h^3 - 6a^2h - 6ah^2.$$

Foil out $5\left(a+h\right)^2$

$$= 5a^2 + 5h^2 + 10ah.$$

And we also have the 4 in the beginning, so combine like terms

$$-2h^3 - 6a^2h - 6ah^2 + 5h^2 + 10ah.$$

Add to equation:

$$\lim_{h \to 0} \frac{-2h^3 - 6a^2h - 6ah^2 + 5h^2 + 10ah}{h}.$$

factor out a ***h***, and cancel out common term ***h***

$$\begin{aligned} \lim_{h \to 0} \frac{h\left(-2h^2 - 6a^2 - 6ah + 5h + 10a\right)}{h} \\ = -2h^2 - 6a^2 - 6ah + 5h + 10a. \end{aligned}$$

Plug in zero for each h

$$\begin{aligned} -2\left(0\right)^2 - 6a^2 - 6a\left(0\right) + 5\left(0\right) + 10a \\ = -6a^2 + 10a. \end{aligned}$$

b.) Plug in 1 for x,

$$\begin{aligned} m = -6\left(1\right)^2 + 10\left(1\right) \\ = 4. \end{aligned}$$

Plug into point slope form equation

$$\begin{aligned} y - 7 = 4\left(x - 1\right) \\ y = 4x + 3. \end{aligned}$$

Question 4:

Solution:

Part b.)

$$
\begin{aligned}
16t^2 &= 36 \\
t^2 &= \frac{36}{16} \\
t &= \frac{\sqrt{36}}{\sqrt{16}} \\
t &= \frac{6}{4} \\
t &= \frac{3}{2} \\
t &= 1.5s.
\end{aligned}
$$

Part d.)

$$
\begin{aligned}
Formula = v_{inst} &= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \\
&= \lim_{h\to 0} \frac{16(1.5+h)^2 - 16(1.5)^2}{h} \\
&= \lim_{h\to 0} \frac{16(h^2+3h+2.25)-36}{h} \\
&= \lim_{h\to 0} \frac{16h^2+48h+36-36}{h} \\
&= \lim_{d\to 0} \frac{16h^2+48h}{h} \\
&= \lim_{h\to 0} \frac{h(16h+48)}{h} \\
&= \lim_{h\to 0} 16h+48 \\
&= 16(0)+48 \\
&= 48.
\end{aligned}
$$

Question 5:

Solution:

Part a.)

$$t = a.$$

$$v_{inst} = \lim_{t\to a} \frac{\frac{6}{t^2} - \frac{6}{a^2}}{t-a}$$
.

Multiply by lcd a^2t^2

$$\lim_{t\to a}\frac{\frac{6a^2t^2}{t^2}-\frac{6a^2t^2}{a^2}}{a^2t^2(t-a)}$$
$$=\lim_{t\to a}\frac{6a^2-6t^2}{a^2t^2(t-a)}$$
$$\lim_{t\to a}\frac{6(a^2-t^2)}{a^2t^2(t-a)}$$
$$=\lim_{t\to a}\frac{6(a-t)(a+t)}{a^2t^2(t-a)}$$
$$=\lim_{t\to a}\frac{-6(t-a)(t+a)}{a^2t^2(t-a)}$$
$$=\lim_{t\to a}\frac{-6(t+a)}{a^2t^2}$$
.

Plug in a for t

$$\frac{-6(a+a)}{a^2a^2}$$
$$=\frac{-6a-6a}{a^4}$$
$$=\frac{-12a}{a^4}$$
$$=\frac{-12}{a^3}.$$

We can use this equation to get parts b-d, but instead here is work if we didnt have the equation above (:

Part b.)

$$t=1.$$

$$v_{inst}=\lim_{t\to 1}\frac{f(t)-f(a)}{t-a}$$
$$=\lim_{t\to 1}\frac{\frac{6}{t^2}-\frac{6}{(-1)^2}}{t-1}$$
$$=\lim_{t\to 1}\frac{\frac{6}{t^2}-6}{t-1}.$$

Clear out fraction in numerator by Multiplying by lcd

$$\lim_{t\to 1}\frac{(\frac{6}{t^2}\cdot\frac{t^2}{1})-(\frac{6}{1}\cdot\frac{t^2}{1})}{t^2(t-1)}$$
$$=\lim_{t\to 1}\frac{6-6t^2}{t^2(t-1)}$$
$$=\lim_{t\to 1}\frac{-6(t^2-1)}{t^2(t-1)}$$
$$=\lim_{t\to 1}\frac{-6(t-1)(t+1)}{t^2(t-1)}$$
$$=\lim_{t\to 1}-6(t+1)$$
$$=-6(1+1)$$
$$=-12.$$

c.)

$$t = 2.$$

$$\begin{aligned}\lim_{t\to 2} \frac{f(t) - f(a)}{t-1} \\ \lim_{t\to 2} \frac{\frac{6}{t^2} - \frac{6}{4}}{t-2} \\ \lim_{t\to 2} \frac{\frac{6}{t^2} - \frac{3}{2}}{t-2} \\ .\end{aligned}$$

Multiply by lcd of $2t^2$

$$\begin{aligned}\lim_{t\to 2} \frac{12 - 3t^2}{2t^2(t-2)} \\ = \lim_{t\to 2} \frac{-3(t^2-4)}{2t^2(t-2)} \\ = \lim_{t\to 2} \frac{-3(t-2)(t+2)}{2t^2(t-2)} \\ = \lim_{t\to 2} \frac{-3(t+2)}{2t^2} \\ \frac{-2(2+2)}{2(2)^2} \\ = -1.5.\end{aligned}$$

Question 6:

Solution: 😄

Part a.) (10,400), (60,750)

$$\begin{aligned}m_{pq} &= \frac{750-400}{60-10} \\ &= \frac{350}{50} \\ &= 7.\end{aligned}$$

Part c.)

$$\begin{aligned}\frac{200-600}{40-0} \\ = \frac{-400}{40} \\ = -10.\end{aligned}$$

d.)

$$f\prime(50).$$

So drawn at tangent line at point (50,f(50)), and calculate the slope, we can see we have another point on the tangent line at (60, f(60))

$$\frac{(60, f(60)) - (50, f(50))}{60 - 50}$$
$$= \frac{600 - 400}{60 - 50}$$
$$= 20.$$

Question 7:

Solution: 😄

Formula:

$$f\prime(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

If a = 7

$$\lim_{x \to 7} \frac{\sqrt{6x+7} - (\sqrt{6(7)+7})}{x - 7}$$
$$\lim_{x \to 7} \frac{\sqrt{6x+7} - 7}{x - 7}.$$

Multiply by the conjugate:

$$\lim_{x \to 7} \frac{\sqrt{6x+7} - 7}{x - 7} \cdot \frac{\sqrt{6x+7+7}}{\sqrt{6x+7+7}}$$
$$= \lim_{x \to 7} \frac{6x - 42}{(x-7)(\sqrt{6x+7}+7)}$$
$$= \lim_{x \to 7} \frac{6(x-7)}{(x-7)(\sqrt{6x+7}+7)}$$
$$= \lim_{x \to 7} \frac{6}{\sqrt{6x+7}+7}.$$

Plug in 7 for x:

$$\frac{6}{\sqrt{6(7)+7}+7}$$
$$= \frac{3}{7}.$$

Question 8:

Solution: 😄

$$f\prime(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

So:

$$\begin{aligned}
&\lim_{h\to 0}\frac{2(a+h)^2-3(a+h)+3-[2a^2-3a+3]}{h}\\
&=\lim_{h\to 0}\frac{2a^2+2h^2+4ah+3-2a^2+3a-3-3a-3h}{h}\\
&=\lim_{h\to 0}\frac{2h^2-3h+4ah}{h}\\
&=\lim_{h\to 0}\frac{h(h-3+4a)}{h}\\
&=\lim_{h\to 0}h+4a-3\\
&=0+4a-3\\
&=4a-3.
\end{aligned}$$

Question 9:

Solution: 😃

Review from on HW

Question 10:

Solution: 😃

Formula:

$$m_{tan}=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}.$$

Therefore:

$$\begin{aligned}
&\lim_{x\to 8}\frac{2x-5-[2(8)-5]}{x-8}\\
&\lim_{x\to 8}\frac{2x-5-11}{x-8}\\
&\lim_{x\to 8}\frac{2x-16}{x-8}\\
&\lim_{x\to 8}\frac{2(x-8)}{x-8}\\
&=2.
\end{aligned}$$

Since we have no more x value to plug 8 into, our m is just ***2***