2.7 Hw Solutions

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Question 1:

Solution: 😄

a.)

If:

$$m_{tan} = \lim_{x \to -3} \frac{f(x) - f(a)}{x - a}.$$

And **a = -3, f(a) = -18, then:**

$$\begin{aligned} m_{tan} &= \lim_{x \to -3} \frac{x^2 + 9x - (-18)}{x - (-3)} \\ &= \lim_{x \to -3} \frac{x^2 + 9x + 18}{x + 3}. \end{aligned}$$

numerator factors into:

$$\lim_{x \to -3} \frac{(x+3)(x+6)}{x+3}.$$

Cancel out common factor:

$$\lim_{x \to -3} x + 6.$$

Plug in -3 for x

$$\begin{aligned} m_{tan} &= -3 + 6 \\ &= 3. \end{aligned}$$

b.)

If:

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

And a = -3 and f(a) = -18, and we plug in (a+h) for x:

$$m_{tan} = \lim_{h \to 0} \frac{(-3+h)^2 + 9(-3+h) - (-18)}{h}$$
.

And we distribute out the terms:

$$\begin{aligned} m_{tan} = \lim_{h \to 0} \frac{h^2 - 6h + 9 - 27 + 9h - 18}{h} \\ = \frac{h^2 + 3h}{h} \\ = \frac{h(h+3)}{h} \\ = h + 3. \end{aligned}$$

Now if we plug in zero:

$$\begin{aligned} m_{tan} &= 0 + 3 \\ &= 3. \end{aligned}$$

c.) The equation of the tangent line if:

$$y - y_1 = m\left(x - x_1\right)$$
$$.$$

Then:

$$\begin{aligned} y - (-18) &= 3\left(x - (-3)\right) \\ y + 18 &= 3\left(x + 3\right) \\ y + 18 &= 3x + 9 \\ y &= 3x - 9. \end{aligned}$$

Question 2:

Solution: ☺

We know that:

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

And a = 6 and f(a) = 19:

$$\begin{aligned} &\lim_{x \to 6} \frac{2x^2 - 9x - 1 - 19}{x - 19} \\ &= \lim_{x \to 6} \frac{2x^2 - 9x - 18}{x - 19}. \end{aligned}$$

factor using the x method:

$$\begin{aligned} &\lim_{x \to 6} \frac{(2x + 3)(x - 6)}{x - 6} \\ &= \lim_{x \to 6} 2x + 3 \\ &= 2(6) + 3 \\ &= 15. \end{aligned}$$

Plug $m_{tan} = 15$ into ***Point slope form equation*** to get equation of tangent line:

$$\begin{aligned} y - 19 &= 15\left(x - 6\right) \\ y - 19 &= 15x - 90 \\ y &= 15x - 71. \end{aligned}$$

Question 3:

Solution: 😄

a.)

If:

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Then:

$$m_{tan} = \lim_{h \to 0} \frac{4 + 5\left(a+h\right)^2 - 2\left(a+h\right)^3 - \left(4 + 5a^2 - 2a^3\right)}{h}.$$

Distribute -1 to each term in $4 + 5a^2 - 2a^3$

$$= -4 - 5a^2 + 2a^3.$$

Foil out $-2\left(a+h\right)^3$

$$= -2a^3 - 2h^3 - 6a^2h - 6ah^2.$$

Foil out $5\left(a+h\right)^2$

$$= 5a^2 + 5h^2 + 10ah.$$

And we also have the 4 in the beginning, so combine like terms

$$-2h^3 - 6a^2h - 6ah^2 + 5h^2 + 10ah.$$

Add to equation:

$$\lim_{h \to 0} \frac{-2h^3 - 6a^2h - 6ah^2 + 5h^2 + 10ah}{h}.$$

factor out a $\boldsymbol{h}$, and cancel out common term $\boldsymbol{h}$

$$\begin{aligned} &\lim_{h \to 0} \frac{h\left(-2h^2 - 6a^2 - 6ah + 5h + 10a\right)}{h} \\ &\quad = -2h^2 - 6a^2 - 6ah + 5h + 10a. \end{aligned}$$

Plug in zero for each h

$$\begin{aligned} -2\left(0\right)^2 - 6a^2 - 6a\left(0\right) + 5\left(0\right) + 10a \\ = -6a^2 + 10a. \end{aligned}$$

b.) Plug in 1 for x,

$$\begin{aligned} m = -6\left(1\right)^2 + 10\left(1\right) \\ = 4. \end{aligned}$$

Plug into point slope form equation

$$\begin{aligned} y - 7 &= 4\left(x - 1\right) \\ y &= 4x + 3. \end{aligned}$$

Question 4:

Solution: 😄

Part b.)

$$
\begin{aligned}
16t^2 &= 36 \\
t^2 &= \frac{36}{16} \\
t &= \frac{\sqrt{36}}{\sqrt{16}} \\
t &= \frac{6}{4} \\
t &= \frac{3}{2} \\
t &= 1.5s.
\end{aligned}
$$

Part d.)

$$
\begin{aligned}
&\lim_{h\to 0} \frac{16(a+h)^2 - 16a^2}{h} \\
&\lim_{h\to 0} \frac{16a^2 + 16h^2 + 36ah - 16a^2}{h} \\
&\lim_{h\to 0} \frac{16h^2 + 36ah}{h} \\
&\lim_{h\to 0} \frac{h(16h + 36a)}{h} \\
&\lim_{h\to 0} 16h + 36a \\
&16(0) + 36(1.5) \\
&= 48.
\end{aligned}
$$

Question 5:

Solution: 😄

Question 6:

Solution: 😄

Question 7:

Solution: 😄

Question 8:

Solution: 😄

Question 9:

Solution: 😄

Question 10:

Solution: 😄