3.2 Hw Solutions

Nathan Warner

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Question 1:

Solution: 😄

If:

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

And:

$$\begin{aligned} f(x) = 5x^2 + 4 &\longrightarrow f'(x) = 10x \\ g(x) = 7x + 9 &\longrightarrow g'(x) = 7. \end{aligned}$$

Then:

$$\begin{aligned} y' &= (5x^2 + 4)(7) + (7x + 9)(10x) \\ &= 105x^2 + 90x + 28. \end{aligned}$$

Question 2:

Solution: 😄

If:

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

And:

$$\begin{aligned} f(x) = x^4 &\longrightarrow f'(x) = 4x^3 \\ g(x) = e^x &\longrightarrow g'(x) = e^x. \end{aligned}$$

Then:

$$\begin{aligned} y' &= (x^4)(e^x) + (e^x)(4x^3) \\ &= x^4 e^x + 4x^3 e^x \\ &= x^3 e^x (x + 4). \end{aligned}$$

Question 3:

Solution: 😄

If:

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

And:

$$\begin{aligned} f(x) = (7x^2 - 4x) &\longrightarrow f'(x) = (14x - 4) \\ g(x) = e^x &\longrightarrow g'(x) = e^x. \end{aligned}$$

Then:

$$\begin{aligned}(7x^2-4x)(e^x)+(e^x)(14x-4)\\ =7x^2e^x-4xe^x+14xe^x-4e^x\\ =7x^2e^x+10xe^x-4e^x\\ =e^x(7x^2+10x-4).\end{aligned}$$

Question 4:

Solution: 😄

If:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

And:

$$\begin{aligned}f(x)=e^x\longrightarrow f'(x)=e^x\\ g(x)=(4-e^x)\longrightarrow g'(x)=-e^x.\end{aligned}$$

Then:

$$\begin{aligned}y'=\frac{(4-e^x)(e^x)-(e^x)(-e^x)}{(4-e^x)^2}\\ =\frac{4e^x-e^{2x}+e^{2x}}{(4-e^x)^2}\\ =\frac{4e^x}{(4-e^x)^2}.\end{aligned}$$

Question 5:

Solution: 😄

$$\begin{aligned}f'(u)=12u^3-8\\ h'(u)=1.\end{aligned}$$

If:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

Then:

$$\begin{aligned}G'(u)=\frac{(u+1)(12u^3-8)-(3u^4-8u)(1)}{(u+1)^2}\\ =\frac{12u^4-8u+12u^3-8-(3u^4-8u)}{(u+1)^2}\\ =\frac{12u^4-8u+12u^3-8-3u^4+8u}{(u+1)^2}\\ =\frac{9u^4+12u^3-8}{(u+1)^2}.\end{aligned}$$

Question 6:

Solution: ☺

If:

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

And:

$$\begin{aligned} f(x) &= (4u^{-1} + 4u{-}2) \\ f'(x) &= (-4u{-}2 - 8u^{-3}). \end{aligned}$$

$$\begin{aligned} g(x) &= (2u + 2u^{-1}) \\ g'(x) &= (2 - 2u^{-2}). \end{aligned}$$

Then:

$$\begin{aligned} J'(u) = (4u^{-1} + 4u^{-2})(2 - 2u^{-2}) + (2u + 2u{-}1)(-4u^{-2} - 8u^{-3}) \\ = (8u^{-1} - 8u^{-3} + 8u{-}2 - 8u^{-4}) + (-8u^{-1} - 16u^{-2} - 8u^{-3}16u^{-4}) \\ = -8u^{-3} + 8u{-}2 - 8u^{-4} - 16u^{-2} - 8u^{-3}16u^{-4} \\ = -16u^{-3} - 8u^{-2} - 24u{-}4 \\ = -\frac{16}{u^3} - \frac{8}{u^2} - \frac{24}{u^4}. \end{aligned}$$

Question 7:

Solution: ☺

In this problem we need to use both formulas, product rule in the numerator to find $f'(x)$. And then the quotient rule once we found $f'(x)$.

So $f'(x)$*: If* $f(x) = x^7$ *and* $g(x) = e^x$

$$\begin{aligned} f'(x) &= (x^7)(e^x) + (e^x)(7x^6) \\ &= x^7e^x + 7x^6e^x. \end{aligned}$$

Now:

$$\begin{aligned} \frac{(x^7 + e^x)(x^7e^x + 7x^6e^x) - (x^7e^x)(7x^6e^x)}{(x^7 + e^x)^2} \\ = \frac{(x^{14}e^x + 7x^{13}e^x + x^7e^{2x} + 7x^6e^{2x}) - (7x^{13}e^x + x^7e^{2x})}{(x^7 + e^x)^2} \\ = \frac{x^{14}e^x + 7x^{13}e^x + x^7e^{2x} + 7x^6e^{2x} - 7x^{13}e^x - x^7e^{2x}}{(x^7 + e^x)^2} \\ = \frac{x^{14}e^x + 7x^6e^{2x}}{(x^7 + e^x)^2} \\ = \frac{x^6(x^8e^x + 7e^{2x})}{(x^7 + e^x)^2} \\ . \end{aligned}$$

Question 8:

Solution: 😄

Question 9:

Solution: 😄

Question 10:

Solution: 😄

Question 11:

Solution: 😄

Question 12:

Solution: 😄

Question 13:

Solution: 😄