

## 3.4 Hw Solutions

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### Question 1:

*Solution:*



$$\begin{aligned}y' &= 9(6 - x^3)^8 \cdot (-3x^2) \\ &= -27(6 - x^3)^8.\end{aligned}$$

### Question 2:

*Solution:*



$$\begin{aligned}y' &= \sec^2 x^5 \cdot 5x^4 \\ &= 5x^4 \sec^2 x^5.\end{aligned}$$

### Question 3:

*Solution:*



$$\begin{aligned}e^{5x^{\frac{1}{2}}} \cdot \frac{5}{2}x^{-\frac{1}{2}} \\ \frac{5}{2}x^{-\frac{1}{2}}e^{5x^{\frac{1}{2}}} \\ = \frac{5e^{5x^{\frac{1}{2}}}}{2x^{\frac{1}{2}}}.\end{aligned}$$

### Question 4:

*Solution:*



*Part 1:*

$$g'(x) = 6x^2 - 16x.$$

*Part 2:*

$$\begin{aligned}&6(2x^3 - 8x^2 + 9)^5 \cdot (6x^2 - 16x) \\ &= 6(2x^3 - 8x^2 + 9)^5 \cdot 2(3x^2 - 8x) \\ &= 12(2x^3 - 8x^2 + 9)^5 \cdot (3x^2 - 8x) \\ &\quad .\end{aligned}$$

### Question 5:

*Solution:*



$$f(x) = 1$$

$$f'(x) = 0.$$

$$g(x) = (2x^2 - 5)^{\frac{1}{3}}$$

$$g'(x) = \frac{1}{3}(2x^2 - 5)^{-\frac{2}{3}} \cdot 4x.$$

$$F'(x) = \frac{(2x^2 - 5)^{\frac{1}{3}}(0) - (1)\left[\frac{1}{3}(2x^2 - 5)^{-\frac{2}{3}}(4x)\right]}{(2x^2 - 5)^{\frac{2}{3}}}$$

$$= \frac{(2x^2 - 5)^{\frac{1}{3}}(0) - (1)\left[\frac{1}{3}(2x^2 - 5)^{-\frac{2}{3}}(4x)\right]}{(2x^2 - 5)^{\frac{2}{3}}}$$

$$= \frac{-\frac{4}{3}x(2x^2 - 5)^{-\frac{2}{3}}}{(2x^2 - 5)^{\frac{2}{3}}}$$

$$= \frac{-\frac{4}{3}x}{(2x^2 - 5)^{\frac{4}{3}}}$$

$$= -\frac{4x}{3(2x^2 - 5)^{\frac{4}{3}}}.$$

### Question 6:

*Solution:*



$$F(t) = (9t + 1)^{-5}.$$

$$F'(t) = -5(9t + 1)^{-6} \cdot 9$$

$$= -45(9t + 1)^{-6}$$

$$= -\frac{45}{(9t + 1)^6}.$$

### Question 7:

*Solution:*



Move the constant outside:

$$7(\cos^3 \theta).$$

$$\frac{d}{d\theta} = 7(\cos \theta)^3$$

$$= 7 \cdot 3(\cos \theta)^2 \cdot -\sin \theta$$

$$-21 \cos^2 \theta \sin \theta.$$

### Question 8:

*Solution:*

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$$e^{x^4-5x}(4x^3-5).$$

### Question 9:

*Solution:*

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$$\begin{aligned}f(x) &= (4x+5)^3 \\f'(x) &= 3(4x+5)^2(4).\end{aligned}$$

$$\begin{aligned}g(x) &= (x^2-8x+5)^4 \\g'(x) &= 4(x^2-8x+5)^3(2x-8).\end{aligned}$$

$$\begin{aligned}F'(x) &= (4x+5)^3[4(x^2-8x+5)^3(2x-8)] + (x^2-8x+5)^4[3(4x+5)^2 \cdot 4] \\&= (4x+5)^2(x^2+8x+5)^3[4(4x+5)(2x-8) + 12(x^2-8x+5)] \\&= (4x+5)^2(x^2+8x+5)^3[4(8x^2-22x-40) + 12x^2-96x+60] \\&= (4x+5)^2(x^2+8x+5)^3[32x^2-88x-160+12x^2-96x+60] \\&= (4x+5)^2(x^2+8x+5)^3[44x^2-184x-100] \\&= (4x+5)^2(x^2+8x+5)^3[4(11x^2-46x-25)] \\&= 4(4x+5)^2(x^2+8x+5)^3(11x^2-46x-25).\end{aligned}$$

### Question 10:

*Solution:*

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$$\begin{aligned}f(x) &= (u^3-3)^9 \\f'(x) &= 9(u^3-3)^8(3u^2).\end{aligned}$$

$$\begin{aligned}g(x) &= (u^3+3)^9 \\g'(x) &= 9(u^3+3)^8(3u^2).\end{aligned}$$

$$\begin{aligned}
G'(x) &= \frac{(u^3 + 3)^9[9(u^3 - 3)^8(3u^2)] - (u^3 - 3)^9[9(u^3 + 3)^8(3u^2)]}{(u^3 + 3)^{18}} \\
&= \frac{(u^3 - 3)^8(u^3 + 3)^8(3u^2)[9(u^3 + 3)^8] - [9(u^3 - 3)]}{(u^3 + 3)^{18}} \\
&= \frac{(u^3 - 3)^8(3u^2)[9(u^3 + 3) - [9(u^3 - 3)]]}{(u^3 + 3)^{10}} \\
&= \frac{(u^3 - 3)^8(3u^2)[9(u^3 + 3) - (9u^3 - 27)]}{(u^3 + 3)^{10}} \\
&= \frac{(u^3 - 3)^8(3u^2)[9u^3 + 27 - (9u^3 - 27)]}{(u^3 + 3)^{10}} \\
&= \frac{(u^3 - 3)^8(3u^2)(9u^3 + 27 - 9u^3 + 27)}{(u^3 + 3)^{10}} \\
&= \frac{(u^3 - 3)^8(3u^2)(27 + 27)}{(u^3 + 3)^{10}} \\
&= \frac{(u^3 - 3)^8(3u^2)(54)}{(u^3 + 3)^{10}} \\
&= \frac{(u^3 - 3)^8 162u^2}{(u^3 + 3)^{10}} \\
&= \frac{162u^2(u^3 - 3)^8}{(u^3 + 3)^{10}}.
\end{aligned}$$

### Question 11:

*Solution:*

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$$\ln 7 \cdot \ln 8 \cdot \ln 9 \cdot 7^{8^{9^x}} \cdot 8^{9^x} \cdot 9^x.$$

### Question 12:

*Solution:*

⊖

Part 1:

$$\begin{aligned}
& -\sin(\sin 9\theta) \cdot 9 \cos 9\theta \\
& = -9 \cos 9\theta \sin(\sin 9\theta).
\end{aligned}$$

Part 2:

$$\begin{aligned}
f(x) &= \cos 9\theta \\
f'(x) &= -9 \sin 9\theta.
\end{aligned}$$

$$\begin{aligned}
g(x) &= \sin \sin 9\theta \\
g'(x) &= \cos(\sin 9\theta) \cdot 9 \cos 9\theta.
\end{aligned}$$

$$\begin{aligned}
& -9(\cos 9\theta \cdot \cos(\sin 9\theta) \cdot 9 \cos 9\theta + \sin(\sin 9\theta) \cdot -9 \sin 9\theta) \\
& -9(9 \cos^2 9\theta \cdot \cos(\sin 9\theta) + \sin(\sin 9\theta) \cdot -9 \sin 9\theta) \\
& -9(9 \cos^2 9\theta \cdot \cos(\sin 9\theta) - 9 \sin(\sin 9\theta) \cdot \sin 9\theta) \\
& -81 \cos^2 9\theta \cdot \cos(\sin 9\theta) + 81 \sin(\sin 9\theta) \cdot \sin 9\theta
\end{aligned}$$

### Question 13:

*Solution:*

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By the shortcut:

$$\begin{aligned} & \frac{d}{dx} \sqrt{5+x^3} \\ &= \frac{3x^2}{2(5+x^3)^{\frac{1}{2}}}. \end{aligned}$$

Part 2:

$$\begin{aligned} & \frac{3(-1)^2}{2(5+(-1)^3)^{\frac{1}{2}}} \\ &= \frac{3}{4}. \end{aligned}$$

$$\begin{aligned} y-2 &= \frac{3}{4}(x-(-1)) \\ y-2 &= \frac{3}{4}(x+1) \\ y-2 &= \frac{3}{4}x + \frac{3}{4} \\ y &= \frac{3}{4}x + \frac{11}{4} \end{aligned}$$

### Question 14:

*Solution:*

⊗

By the shortcut:

$$y' = \frac{1}{(-6+2x)^{\frac{1}{2}}}.$$

Part 2:

$$\text{If } 8x + 4y = 1$$

And

$$m = \frac{-a}{b}.$$

Then:

$$m = -\frac{8}{4}$$

$$m = -2.$$

$$\sqrt{-6+2x} = 2$$

$$-6+2x = 4$$

$$2x = 10$$

$$x = 5$$

$$\text{so } (5, 2).$$

### Question 15:

*Solution:*



$$\begin{aligned}f'(g(x)) \cdot g'(x) \\&= 9 \cdot 2 \\&= 18.\end{aligned}$$

### Question 16:

*Solution:*



$$\begin{aligned}F'(2) &= f'(2) \cdot f'(f(2)) \\&= 5 \cdot f'(1) \\&= 5 \cdot 4 \\&= 20.\end{aligned}$$

$$\begin{aligned}G'(3) &= g'(3) \cdot g'(g(3)) \\&= 9 \cdot g'(1) \\&= 9 \cdot 6 \\&= 54.\end{aligned}$$

### Question 17:

*Solution:*

