2.1 HW Solutions

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0.1 Question 1

Question 1

A tank holds 3,000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

A). If P is the point (15, 765) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with t = 5, 10, 20, 25, and 30. (Round your answers to one decimal place.)

Solution:: 😄

1. **(5,2070)**

$$\frac{2070 - 765}{5 - 15} = -130.5.$$

2. **(10,1329)**

$$\frac{1329 - 765}{10 - 15} = -112.8.$$

3. **(20,348)**

$$\frac{348 - 765}{20 - 15} = -83.4.$$

4. **(25,81)**

$$\frac{81 - 765}{25 - 15} = -68.4.$$

5. **(30,0)**

$$\frac{0 - 765}{30 - 15} = -51.$$

B). Estimate the slope of the tangent line at P by averaging the slopes of the two adjacent secant lines corresponding to the two points closest to P. (Round your answer to one decimal place.)

solution:: 😄

The two points closest to P are (10,1329) and (20,348), we take the average of both their slopes and get **-98.1**

0.2 Question 2

Question 2

The point P(6,-4) lies on the curve, $y = \frac{4}{5-x}$

A). if Q is the point $\left(x, \frac{4}{5-x}\right)$ find the slope of the secant line PQ (correct to six decimal places) for the following values of x.

i.) x = 5.9

$$\frac{-4 - \frac{4}{5-5.9}}{6 - 5.9} = 4.\overline{4}.$$

B). Using the results of part (a), guess the value of the slope of the tangent line to the curve at P(6,-4)

Solution:: 😄

The numbers appear to be get closer and closer to 4, so the slope of the tangent line at P(6,-4) is likely 4

C). Using the slope from part (b), find an equation of the tangent line to the curve at P(6,-4)

Solution:: 😄

If the equation for a line is

$$y - y_1 = m\left(x - x_1'\right).$$

And the slope of the tangent line is 4, then the equation would be,

$$\begin{gathered} y - (-4) = 4(x - 6) \\ y + 4 = 4x - 24 \\ y = 4x - 28 \end{gathered}$$

0.3 Question 3

Question 3

If a rock is thrown upward on the planet Mars with a velocity 18 m/s, its height in meters t seconds later is given by $y = 18t - 1.86t^2$. (Round your answers to two decimal places.)

A). Find the average velocity in m/s over the given time interval

(i) [1,2]

$$y\left(1\right) = 18\left(1\right) - 1.86\left(1\right)^2$$
$$y\left(1\right) = 16.14.$$

$$y\left(2\right) = 18\left(2\right) - 1.86\left(2\right)^2$$
$$y\left(2\right) = 28.56.$$

So we have the points (1,16.14) and (2,28.56) so we can plug into average velocity equation

$$Average\ velocity = \frac{28.56 - 16.14}{2 - 1}$$
$$= 12.42m\backslash s.$$

0.4 Question 4

Question 4

The table shows the position of a motorcyclist after accelerating from rest.

t (seconds)	0	1	2	3	4	5	6
s (feet)	0	4.9	10.6	23.2	50.3	109.4	237.9

(a) Find the average velocity (in ft/s) for each time period.

(i) [2,4]

So, we use the average velocity formula with values at t = 2 and t = 4

$$\begin{aligned} \textit{Average velocity} &= \frac{50.3 - 10.6}{4 - 2} \\ &= 19.85. \end{aligned}$$

(b) Plot the points in the table to create a graph of s as a function of t to estimate the instantaneous velocity (in ft/s) when t = 3. (Round your answer to one decimal place.)

solution:: 😊