

# Calculus 1: Chapter 3

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# Chapter 3

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## 3.1

### Differential Rule:

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#### *Differential Formulas:*

- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(x^n) = n \cdot x^{n-1} \rightarrow \textbf{Power Rule}$
- $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$
- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

#### Example 0.0.1

Differentiate the following functions:

1.)  $f(t) = \frac{1}{2}t^6 - 3t^4 + 1$

For the first term, we will use the *Third and Fourth* Rule:

$$\frac{1}{2} \cdot 6t^{t-1}.$$

For the second term,  $-3t^4$ , We will use the *Third and Fifth* Rule:

$$-3 \cdot 4t^{4-1}.$$

The last term is a constant, so according to the first rule, the Derivative of a constant is *Zero*:

*So our full equation is:*

$$\begin{aligned} f'(x) &= \frac{1}{2} \cdot 6t^{6-1} - 3 \cdot 4t^{4-1} + 0 \\ &= 3t^5 - 12t^3. \end{aligned}$$

2.)  $h(x) = (x - 2)(2x + 3)$

First we need to distribute out the terms:

$$\begin{aligned} h(x) &= 2x^2 + 3x - 4x - 6 \\ &= 2x^2 - x - 6. \end{aligned}$$

Now this is the function we want to differentiate.

So  $\rightarrow$

$$h'(x) = 2 \cdot 2x^{2-1} - 1 - 0$$

$$h'(x) = 4x - 1.$$

**3.)**  $y = \frac{x^2 - 2\sqrt{x}}{x}$

So:

$$y = \frac{x^2 - 2x^{\frac{1}{2}}}{x}.$$

Since the numerator only has **one term**, we can split the equation like:

$$y = \frac{x^2}{x} - \frac{2x^{\frac{1}{2}}}{x}$$

$$y = x - 2x^{-\frac{1}{2}}.$$

Now:

$$\frac{dy}{dx} = 1 - 2 \cdot \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$

$$\frac{dy}{dx} = 1 + x^{-\frac{3}{2}}.$$

And we can even rewrite it as:

$$\frac{dy}{dx} = 1 + \frac{1}{x^{\frac{3}{2}}}.$$

**4.)**  $V = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2$

So:

$$V = \left(x^{\frac{1}{2}} + x^{-\frac{1}{3}}\right)^2$$

$$= (x^{\frac{1}{2}})^2 + 2(x^{\frac{1}{2}})(x^{-\frac{1}{3}}) + (x^{-\frac{1}{3}})^2$$

$$= x + 2x^{\frac{1}{6}} + x^{-\frac{2}{3}}.$$

Now we find the Derivative:

$$V' = 1 + 2 \cdot \frac{1}{6}x^{\frac{1}{6}-1} + \left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1}$$

$$v' = 1 + \frac{1}{3}x^{-\frac{5}{6}} - \frac{2}{3}x^{-\frac{5}{3}}$$

$$v' = 1 + \frac{1}{3x^{\frac{5}{6}}} - \frac{2}{3x^{\frac{5}{3}}}.$$

## Exponential Functions:

**Recall:**  $(1 + \frac{1}{n})^n \rightarrow e \approx 2.71828...$  as  $n \rightarrow \infty$

**Definition 0.0.1: Definiton of e:**

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

**Note:-**

*We'll use the above definiton to derive  $\frac{d}{dx}(e^x)$*

→ Let  $f(x) = e^x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$