Calculus 1 Notes

Nathan Warner

December 17, 2023

Chapter 2

Contents

2.1: The Tangent and Velocity Problems

The Tangent Problem:

Question 1

Can we find an equation of the tangent line to $y = x^2$ at the point P(1,1)?

1.png

Explanation: .
$y = x^2$: Red parabola
Tangent line: Blue line
Secent Line: Pink line with points q and p

We are asked to get the equation of the tangent line to $y = x^2$ at the point P(1,1), However to find the equation of this line we know we need **2 things,**

- Point
- Slope

Since we only have one point, we cannot find slope. Therefore, we must use another point as an approximation and create a secent line instead. **This secent line is the pink line in the above graphic.**

So, lets use the point Q(0,0) as our second point. Now we can find slope with P(1,1), and Q(0,0).

$$\textbf{If Slope} = \frac{y2-y1}{x2-x1}, \text{ Then M of PQ} \longrightarrow \frac{1-0}{1-0} = 1$$

Lets get a better approximation by using a point closer to the tangent line Lets use Q(0.9, 0.81)

$$\textbf{So} \text{ M of PQ} \longrightarrow \frac{1-0.81}{1-0.9} = 1.9$$

Now, lets get an even closer approximation by using the point Q(0.99, 0.9801)

$$\textbf{So}, \text{ M of PQ} \longrightarrow \frac{1-0.9801}{1-0.99} = 1.99$$

Notice, as the point Q gets closer to P, the slope of PQ is getting closer to 2

We write,

$$\lim_{Q \to P} \text{M of PQ} = \text{m}$$

Where **m** on the right of equation is slope of tangent line at **P**, And **M of PQ** is slope of the secent line

Now,

We will use our approximation of $m \approx 2$ to write the equation of the tangent line, using the orginial point P(1,1).

$$\begin{aligned} y - 1 &= 2\,(x - 1) \\ y - 1 &= 2x - 2 \\ y &= 2x - 1. \end{aligned}$$

The Velocity Problem:

- Average Velocity: $\frac{distance\ traveled}{time\ elapsed}$, which is represented by the slope of the secent line.
- Instantaneous Velocity = Velocity at a given instant of time, which is represented by the slope of the tangent line

Example 0.0.1

If a rock is thrown upward on the planet Mars, with a Velocity of 10 m/s, It's height in meters t seconds later is given by $y = 10t - 1.86t^2$

Question 2

Find the average Velocity over the given time intervals:

(i) [1,2] $\rightarrow$ 1 and 2 represent values of t

Substitute values into equation above

$$\begin{aligned} y\left(1\right) &= 10\left(1\right) - 1.86\left(1\right)^2 \\ &= 8.14. \end{aligned}$$

$$\begin{aligned} y\left(2\right) &= 10\left(2\right) - 1.86\left(2\right)^2 \\ &= 12.56. \end{aligned}$$

If Average Velocity = $\frac{distance\ traveled}{time\ elapsed}$ Or better yet $\frac{Change\ in\ height}{change\ in\ time}$

And we have the points (1,8.14) and (2,12.56)

Then,

$$\begin{aligned} Average\ Velocity &= \frac{12.56 - 8.14}{2 - 1} \\ &= 4.42m\diagdown s. \end{aligned}$$

(ii) [1,1.5]

Substitute values into equation above

$$\begin{aligned} y\,(1) &= 10\,(1) - 1.86\,(1)^2 \\ &= 8.14. \end{aligned}$$

$$\begin{aligned} y\,(1.5) &= 10\,(1.5) - 1.86\,(1.5)^2 \\ &= 10.815. \end{aligned}$$

After solving theses equations we have the points (1,8.14) and (1.5,10.815)

So,

$$\begin{aligned} Average\ Velocity &= \frac{10.815 - 8.14}{1.5 - 1} \\ &= 5.35 m \diagdown s. \end{aligned}$$