2.8 Hw Solutions

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Question 1:

Solution: 😄

See hw:

Question 2:

Solution: 😄

See hw:

Question 3:

Solution: 😄

If:

$$f\prime(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Then:

$$\begin{aligned} f\prime(t) &= \lim_{h\to 0} \frac{5.5(t+h)^2 + 7(t+h) - (5.5t^2 + 7t)}{h} \\ &= \lim_{h\to 0} \frac{5.5t^2 + 5.5h^2 + 11th + 7t + 7h - 5.5t^2 - 7t}{h} \\ &= \lim_{h\to 0} \frac{5.5h^2 + 11th + 7h}{h} \\ &= \lim_{h\to 0} \frac{h(5.5h + 11t + 7)}{h} \\ &= \lim_{h\to 0} 5.5h + 11t + 7 \\ &= 5.5(0) + 11t + 7 \\ &= 11t + 7. \end{aligned}$$

Domain for both is $\mathbb{R}$

Question 4:

Solution: 😄

If:

$$f\prime(x) = \lim_{h\to 0} \frac{f(x+h) - f(a)}{h}.$$

Then:

$$\begin{aligned}
f\prime(x) &= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2-36} - \frac{1}{x^2-36}}{h} \\
&= \lim_{h\to 0} \frac{(x^2-36)-((x+h^2)-36)}{h(x^2-36)((x+h)^2-36)} \\
&= \lim_{h\to 0} \frac{x^2-36-(x^2+h^2+2xh-36)}{h(x^2-36)((x+h)^2-36)} \\
&= \lim_{h\to 0} \frac{x^2-36-x^2-h^2-2xh+36}{h(x^2-36)((x+h)^2-36)} \\
&= \lim_{h\to 0} \frac{-h^2-2xh}{h(x^2-36)((x+h)^2-36)} \\
&= \lim_{h\to 0} \frac{h(-h-2x)}{h(x^2-36)((x+h)^2-36)} \\
&= \lim_{h\to 0} \frac{-h-2x}{(x^2-36)((x+h)^2-36)} \\
&= \frac{-(0)-2x}{(x^2-36)(x+0)^2-36} \\
&= \frac{-2x}{(x^2-36)^2}.
\end{aligned}$$

Domain: f(x)

$$\begin{aligned}
x^2-36 &= 0 \\
x^2 &= 36 \\
x &= \pm 6 \\
(-\infty,-6)\cup(-6,6)\cup(6,\infty).
\end{aligned}$$

Domain: $f\prime(x)$

$$\begin{aligned}
(x^2-36)^2 &= 0 \\
x^2-36 &= 0x^2 = 36 \\
x &= \pm 6 \\
(-\infty,-6)\cup(-6,6)\cup(6,\infty).
\end{aligned}$$

Question 5:

Solution: 😄

If:

$$f\prime(x) = \lim_{h\to 0} \frac{f(x+h)-f(a)}{h}.$$

Then:

$$\begin{aligned}
f\prime(x) &= \lim_{h\to 0} \frac{\frac{1}{9+\sqrt{x+h}} - \frac{1}{9+\sqrt{x}}}{h} \\
&= \lim_{h\to 0} \frac{(9+\sqrt{x}) - (9+\sqrt{x+h})}{h(9+\sqrt{x})(9+\sqrt{x+h})} \\
&= \lim_{h\to 0} \frac{9+\sqrt{x} - 9 - \sqrt{x+h}}{h(9+\sqrt{x})(9+\sqrt{x+h})} \\
&= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h(9+\sqrt{x})(9+\sqrt{x+h})}
\end{aligned}$$

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Multiply both halfs by the conjugate: $\sqrt{x} + \sqrt{x+h}$

$$\begin{aligned}
&\lim_{h\to 0} \frac{x-(x+h)}{h(9+\sqrt{x})(9+\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})} \\
&= \lim_{h\to 0} \frac{x-x-h}{h(9+\sqrt{x})(9+\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})} \\
&= \lim_{h\to 0} \frac{-h}{h(9+\sqrt{x})(9+\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})} \\
&= \lim_{h\to 0} \frac{-1}{(9+\sqrt{x})(9+\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})} \\
&= \frac{-1}{(9+\sqrt{x})(9+\sqrt{x+0})(\sqrt{x}+\sqrt{x+0})} \\
&= \frac{-1}{(9+\sqrt{x})(9+\sqrt{x})(\sqrt{x}+\sqrt{x})} \\
&= -\frac{1}{2\sqrt{x}(9+\sqrt{x})^2}
\end{aligned}$$

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Domain: f(x)

$$\begin{aligned}
x &\geqslant 0 \\
&\textit{and} \\
9+\sqrt{x} &= 0 \\
\sqrt{x} &= -9 \\
x = \sqrt{-9} &= undefined \\
&= [0,\infty).
\end{aligned}$$

Domain: $f\prime(x)$

$$\begin{aligned}
x &\geqslant 0 \\
&\textit{and} \\
2\sqrt{x} &= 0 \\
\sqrt{x} &= 0 \\
x &= \sqrt{0} \\
&= 0 \\
therefore\ x\ cannot\ be\ zero \\
&= (0,\infty).
\end{aligned}$$

Question 6:

Solution: 😄

F cannot be differentiable at:

- a corner (-4)
- a discontinuity (0)
- a vertical tangent (2)

Question 7:

Solution: 😄

Question 9:

Solution: 😄

Question 9:

Solution: 😄