软件理论基础第二次作业

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1. 设 $A \downarrow B$ 表示 $\neg(A \vee B)$, 证明连接符 $\{\downarrow\}$ 是命题逻辑连接符的充足集。

   证明:

   由数学归纳法: 若 $|F| = 1$, 即 F 为原子公式。则必为一个原子命题或其否定形式, 记原子命题为 p, 则 $F = p$ 或 $F = \neg P$ 成立, 则有:

   (1)
   $$F = p = \neg\neg(p \vee p) = \neg(p \downarrow p) = \neg((p \downarrow \mathrm{p}) \vee (p \downarrow p)) = (\mathrm{p} \downarrow p) \downarrow (p \downarrow p)$$

   (2)
   $$F = \neg p = \neg(p \vee p) = p \downarrow p$$

   假设对于所有的公式 F, 若 $|F| < n$ 则 F 能用'$\downarrow$' 表示，现考虑 $|F| = n$

   (1) 若 $F = \neg A$ (A 为 F 的子公式), 可得:
   $$F = \neg A = \neg(A \vee A) = A \downarrow A.$$

   (2) 若 $F = A \vee B$ (A,B 为 F 的子公式), 可得:
   $$F = A \vee B = \neg(\neg(A \vee B)) = \neg(A \downarrow B) = (A \downarrow B) \downarrow (A \downarrow B)$$

   因此, 无论何种情况 A、B 必能由'$\downarrow$' 表示, 所以由数学归纳法可得所有公式 F 均能用'$\downarrow$' 表示, 即 $\{\downarrow\}$ 是充足集。

2. (1) 证明
   $$\vdash (B \to C) \to ((A \to B) \to (B \to C))$$

   证明: 由演绎定理, 只需证明
   $$\{(B \to C)\} \vdash ((A \to B) \to (B \to C))$$

(1)

$$B \to C \qquad \mathit{\Gamma}$$

(2)

$$(B \to C) \to ((A \to B) \to (B \to C)) \qquad \text{L2}$$

(3)

$$(A \to B) \to (B \to C) \qquad \text{MP(1,2)}$$

(2) 证明

$$\vdash (A \to (A \to B)) \to (A \to B)$$

证明: 由演绎定理, 只需证明

$$\{A \to (A \to B), A\} \vdash B$$

构造推演序列如下：

(1)

$$A \qquad \mathit{\Gamma}$$

(2)

$$A \to (A \to B) \qquad \mathit{\Gamma}$$

(3)

$$A \to B \qquad \text{MP(1,2)}$$

(4)

$$B \qquad \text{MP(1,3)}$$

3. 试证:

(1) 证明

$$(A \to (B \to C)) \approx (B \to (A \to C))$$

证明

1.

$$\vdash (A \to (B \to C)) \to (B \to (A \to C))$$

由演绎定理, 只需证明

$$\{A \to (B \to C), B\} \vdash (A \to C)$$

构造推演序列如下：

(1)

$$A \to (B \to C) \qquad \mathit{\Gamma}$$

(2)

$$(A \to (B \to C)) \to ((A \to B) \to (A \to C)) \qquad \text{L2}$$

(3)

$$(A \to B) \to (A \to C) \qquad \text{MP(1,2)}$$

(4)

$$B \qquad \mathit{\Gamma}$$

(5)

$$B \to (A \to B) \qquad \text{L1}$$

(6)

$$A \to B \qquad \text{MP(4,5)}$$

(7)

$$A \to C \qquad \text{MP(3,6)}$$

故 $\{A \to (B \to C), B\} \vdash (A \to C)$, 即 $\vdash (A \to (B \to C)) \to (B \to (A \to C))$

2.

$$\vdash (B \to (A \to C)) \to (A \to (B \to C))$$

由演绎定理, 只需证明

$$\{B \to (A \to C), A\} \vdash (B \to C)$$

构造推演序列如下：

(1)

$$B \to (A \to C) \qquad \mathit{\Gamma}$$

(2)

$$(B \to (A \to C)) \to ((B \to A) \to (B \to C)) \qquad \text{L2}$$

(3)

$$(B \to A) \to (B \to C) \qquad \text{MP(1,2)}$$

(4)

$$A \qquad \mathit{\Gamma}$$

(5)

$$A \to (B \to A) \qquad \text{L1}$$

(6)

$$B \to A \qquad \text{MP(4,5)}$$

(7)

$$B \to C \qquad \text{MP(3,6)}$$

故 $\{B \to (A \to C), A\} \vdash (B \to C)$, 即 $\vdash (B \to (A \to C)) \to (A \to (B \to C))$

因此,

$$(A \to (B \to C)) \approx (B \to (A \to C))$$

(2) 证明

$$(A \to (A \to B)) \approx (A \to B)$$

证明

1.

$$\{A \to (A \to B)\} \vdash (A \to B)$$

由演绎定理, 只需证明

$$\{A \to (A \to B), A\} \vdash (B)$$

(1)

$$A \qquad \mathit{\Gamma}$$

(2)

$$A \to (A \to B) \qquad \mathit{\Gamma}$$

(3)

$$A \to B \qquad \text{MP(1,2)}$$

(4)

$$B \qquad \text{MP(1,3)}$$

故

$$\{A \to (A \to B), A\} \vdash (B)$$

即

$$\vdash (A \to (A \to B)) \to (A \to B)$$

2.

$$\{A \to B\} \vdash (A \to (A \to B))$$

(1)

$$A \to B \qquad \Gamma$$

(2)

$$(A \to B) \to (A \to (A \to B)) \qquad \text{L1}$$

(3)

$$A \to (A \to B) \qquad \text{MP(1,2)}$$

故

$$\{A \to B\} \vdash (A \to (A \to B))$$

即

$$\vdash (A \to B) \to (A \to (A \to B))$$

因此，

$$(A \to (A \to B)) \approx (A \to B)$$