软件理论基础第一次作业

2022 年 9 月 27 日

1-(1) 题目：证重言式

$$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$$

解：列出真值表 由真值表可得公式为永真式

A	B	C	$(A \to (B \to C)) \to ((A \to B) \to (A \to C))$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

1-(2) 题目：证重言式

$$(\neg A \to \neg B) \to (B \to A)$$

解：

$$\begin{aligned}
&(\neg A \to \neg B) \to (B \to A) \\
&= (\neg\neg A \vee \neg B) \to (B \to A) \\
&= (A \vee \neg B) \to (B \to A) \\
&= (\neg B \vee A) \to (B \to A) \\
&= (B \to A) \to (B \to A) \\
&= \neg(B \to A) \vee (B \to A) \\
&= 1
\end{aligned}$$

因此此公式为永真式

2-(1) 题目:

$$(A \wedge B) \to C = (A \to C) \vee (B \to C)$$

解:

$$\begin{aligned} leftform &= (A \wedge B) \to C \\ &= \neg(A \wedge B) \vee C \\ &= \neg A \vee \neg B \vee C \\ &= \neg A \vee C \vee \neg B \vee C \\ &= \neg A \vee C \vee \neg B \vee C \\ &= (A \to C) \vee (B \to C) \\ &= rightform \end{aligned}$$

由上述推导，左式等于右式, 因此公式为永真式

2-(2) 题目：

$$(A \to (B \to C) = B \to (A \to C)$$

解：

$$\begin{aligned} leftfrom &= (A \to (B \to C) \\ &= A \to (\neg B \vee C) \\ &= \neg A \vee (\neg B \vee C) \\ &= \neg B \vee (\neg A \vee C) \\ &= B \to (\neg A \vee C) \\ &= B \to (A \to C) \\ &= rightform \end{aligned}$$

由上述推导，左式等于右式, 因此公式为永真式

3 $(\neg p_1 \rightarrow p_2) \rightarrow p_3$ 的析取范式和合取范式

解：

p_1	p_2	p_3	$(\neg p_1 \rightarrow p_2) \rightarrow p_3$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

由真值表易得

主析取范式为：

$$(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge \neg p_2 \wedge p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge p_2 \wedge p_3)$$

主合取范式为:

$$(p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$$

4-(1) 题目：

$$(p_1 \vee p_2) \rightarrow p_3$$

解：

$$\tau((p_1 \vee p_2) \rightarrow p_3) = \frac{5}{2^3} = \frac{5}{8}$$

4-(2) 题目：

$$(p_1 \rightarrow p_2) \vee (p_3 \rightarrow p_4)$$

解：

$$\tau((p_1 \rightarrow p_2) \vee (p_3 \rightarrow p_4)) = \frac{15}{2^4} = \frac{15}{16}$$

4-(3) 题目：

$$(\neg p_1 \rightarrow p_2) \rightarrow p_3$$

解：

$$\tau((\neg p_1 \rightarrow p_2) \rightarrow p_3) = \frac{5}{2^3} = \frac{5}{8}$$