

软件理论基础第二次作业

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1. 设 $A \downarrow B$ 表示 $\neg(A \vee B)$, 证明连接符 $\{\downarrow\}$ 是命题逻辑连接符的充足集。

2. (1) 证明

$$\vdash (B \rightarrow C) \rightarrow ((A \rightarrow B) \rightarrow (B \rightarrow C))$$

证明: 由演绎定理, 只需证明

$$\{(B \rightarrow C)\} \vdash ((A \rightarrow B) \rightarrow (B \rightarrow C))$$

(1)

$$B \rightarrow C \quad \Gamma$$

(2)

$$(B \rightarrow C) \rightarrow ((A \rightarrow B) \rightarrow (B \rightarrow C)) \quad \text{L2}$$

(3)

$$(A \rightarrow B) \rightarrow (B \rightarrow C) \quad \text{MP(1,2)}$$

(2) 证明

$$\vdash (A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$$

证明: 由演绎定理, 只需证明

$$\{A \rightarrow (A \rightarrow B), A\} \vdash B$$

构造推演序列如下:

(1)

$$A \quad \Gamma$$

(2)	$A \rightarrow (A \rightarrow B)$	Γ
(3)	$A \rightarrow B$	MP(1,2)
(4)	B	MP(1,3)

3. 试证:

(1) 证明

$$(A \rightarrow (B \rightarrow C)) \approx (B \rightarrow (A \rightarrow C))$$

证明

1.

$$\vdash (A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$$

由演绎定理, 只需证明

$$\{A \rightarrow (B \rightarrow C), B\} \vdash (A \rightarrow C)$$

构造推演序列如下:

(1)	$A \rightarrow (B \rightarrow C)$	Γ
(2)	$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$	L2
(3)	$(A \rightarrow B) \rightarrow (A \rightarrow C)$	MP(1,2)
(4)	B	Γ
(5)	$B \rightarrow (A \rightarrow B)$	L1
(6)	$A \rightarrow B$	MP(4,5)

(7)

$$A \rightarrow C \quad \text{MP(3,6)}$$

故 $\{A \rightarrow (B \rightarrow C), B\} \vdash (A \rightarrow C)$, 即 $\vdash (A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$

2.

$$\vdash (B \rightarrow (A \rightarrow C)) \rightarrow (A \rightarrow (B \rightarrow C))$$

由演绎定理, 只需证明

$$\{B \rightarrow (A \rightarrow C), A\} \vdash (B \rightarrow C)$$

构造推演序列如下:

(1)

$$B \rightarrow (A \rightarrow C) \quad \Gamma$$

(2)

$$(B \rightarrow (A \rightarrow C)) \rightarrow ((B \rightarrow A) \rightarrow (B \rightarrow C)) \quad \text{L2}$$

(3)

$$(B \rightarrow A) \rightarrow (B \rightarrow C) \quad \text{MP(1,2)}$$

(4)

$$A \quad \Gamma$$

(5)

$$A \rightarrow (B \rightarrow A) \quad \text{L1}$$

(6)

$$B \rightarrow A \quad \text{MP(4,5)}$$

(7)

$$B \rightarrow C \quad \text{MP(3,6)}$$

故 $\{B \rightarrow (A \rightarrow C), A\} \vdash (B \rightarrow C)$, 即 $\vdash (B \rightarrow (A \rightarrow C)) \rightarrow (A \rightarrow (B \rightarrow C))$

因此,

$$(A \rightarrow (B \rightarrow C)) \approx (B \rightarrow (A \rightarrow C))$$

(2) 证明

$$(A \rightarrow (A \rightarrow B)) \approx (A \rightarrow B)$$

证明

1.

$$\{A \rightarrow (A \rightarrow B)\} \vdash (A \rightarrow B)$$

由演绎定理, 只需证明

$$\{A \rightarrow (A \rightarrow B), A\} \vdash (B)$$

(1)

$$A \quad \Gamma$$

(2)

$$A \rightarrow (A \rightarrow B) \quad \Gamma$$

(3)

$$A \rightarrow B \quad \text{MP}(1,2)$$

(4)

$$B \quad \text{MP}(1,3)$$

故

$$\{A \rightarrow (A \rightarrow B), A\} \vdash (B)$$

即

$$\vdash (A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$$

2.

$$\{A \rightarrow B\} \vdash (A \rightarrow (A \rightarrow B))$$

(1)

$$A \rightarrow B \quad \Gamma$$

(2)

$$(A \rightarrow B) \rightarrow (A \rightarrow (A \rightarrow B)) \quad \text{L1}$$

(3)

$$A \rightarrow (A \rightarrow B) \quad \text{MP}(1,2)$$

故

$$\{A \rightarrow B\} \vdash (A \rightarrow (A \rightarrow B))$$

即

$$\vdash (A \rightarrow B) \rightarrow (A \rightarrow (A \rightarrow B))$$

因此，

$$(A \rightarrow (A \rightarrow B)) \approx (A \rightarrow B)$$