软件理论基础第三次作业

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1. 利用 L 的完备性定理证明以下各式成立

(1) $\vdash \neg(A \to B) \to (B \to A)$ 证：由逻辑等价：

$$\begin{aligned}
&\neg(A \to B) \to (B \to A) \\
&= \neg\neg(A \to B) \vee (B \to A) \\
&= (\neg A \vee B) \vee (\neg B \vee A) \\
&= \neg A \vee A \vee B \vee \neg B \\
&= 1
\end{aligned}$$

任取 $v \in \Omega$, 有 $v(\neg(A \to B) \to (B \to A)) = 1$

因此 $\models \neg(A \to B) \to (B \to A)$

由完备性定理 $\vdash \neg(A \to B) \to (B \to A)$

(2) $((A \vee B) \to C) \approx (A \to C) \wedge (B \to C)$

证：由逻辑等价：

$$\begin{aligned}
&(A \vee B) \to C \\
&= \neg(A \vee B) \vee C \\
&= (\neg A \wedge \neg B) \vee C \\
&= (\neg A \vee C) \wedge (\neg B \vee C) \\
&= (A \to C) \wedge (B \to C)
\end{aligned}$$

任取 $v \in \Omega$, 有 $v((A \vee B) \to C) = v((A \to C) \wedge (B \to C))$

因此 $\models ((A \vee B) \to C) \to (A \to C) \wedge (B \to C)$

由完备性定理 $\vdash ((A \vee B) \to C) \to (A \to C) \wedge (B \to C)$

同理可得 $\vdash ((A \to C) \wedge (B \to C)) \to ((A \vee B) \to C)$

因此可得 $((A \vee B) \to C) \approx (A \to C) \wedge (B \to C)$ 证毕

(3) $((A \wedge B) \to C) \approx (A \to C) \vee (B \to C)$

证：由逻辑等价：

$$
\begin{aligned}
&(A \wedge B) \to C \\
&= (\neg(A \wedge B) \vee C) \\
&= \neg A \vee \neg B \vee C \\
&= \neg A \vee \neg B \vee C \vee C \\
&= \neg A \vee C \vee \neg B \vee C \\
&= (A \to C) \vee (B \to C)
\end{aligned}
$$

任取 $v \in \Omega$, 有 $v((A \wedge B) \to C) = v((A \to C) \vee (B \to C))$

因此 $\models ((A \wedge B) \to C) \to ((A \to C) \vee (B \to C))$

由完备性定理 $\vdash ((A \wedge B) \to C) \to ((A \to C) \vee (B \to C))$

同理可得 $\vdash ((A \to C) \vee (B \to C)) \to ((A \wedge B) \to C)$

因此可得 $((A \wedge B) \to C) \approx (A \to C) \vee (B \to C)$ 证毕

2.