Quantum Approximate Optimization Algorithm (QAOA) with Zero Noise Extrapolation (ZNE) applied to carbon credit portfolio optimization in the Brazilian Cerrado biome

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Abstract

Quantum optimization has emerged as a promising framework for addressing complex decision-making problems, particularly those formulated as Quadratic Unconstrained Binary Optimization (QUBO). In this work, we apply the Quantum Approximate Optimization Algorithm (QAOA) to the construction of carbon-credit portfolios in Goias (Brazil), a task that combines environmental planning, economic constraints andcombinatorial structure. Because current quantum hardware operates in the Noisy Intermediate-Scale Quantum (NISQ) regime, we incorporate Zero Noise Extrapolation (ZNE) to mitigate decoherence and gate-induced errors. Our formulation maps project selection, carbon-credit valuations andcompatibility constraints into a QUBO Hamiltonian. We evaluate QAOA with and without ZNE on two IBM Quantum backends (ibm_torino and ibm_fez), comparing its performance against classical baselines including simulated annealing and random search. Across multiple runs, ZNE improves QAOA's score by 29.9% on average, consistently yielding higher-quality solutions and reducing variability in the presence of hardware noise. These results demonstrate that QAOA—when combined with lightweight mitigation techniques such as ZNE—can provide measurable advantages even on current NISQ devices. Beyond the specific application to carbon-credit portfolios, our findings highlight the potential of variational quantum algorithms for real-world environmental decision-making and optimization tasks under noise-limited quantum conditions.

Keywords: quantum approximate optimization algorithm, zero noise extrapolation, carbon credit portfolio, environmental optimization, NISQ era, Cerrado conservation

1. Introduction

Brazil faces significant challenges in implementing effective climate mitigation strategies due to both its continental scale and the ecological diversity of its biomes. The policies discussed in global climate forums only become meaningful when converted into concrete territorial decisions with measurable impacts.
In the Cerrado biome, however, this translation meets a critical mathematical barrier. Allocating carbon-credit portfolios is not a simple logistical task, but a highly complex combinatorial problem in which environmental, social andbiological variables interact in nonlinear ways. As anthropogenic pressure intensifies across the Goias region, the limitations of classical optimization algorithms become increasingly evident and motivate the exploration of more powerful computational tools.

It is at this intersection—between conservation urgency and computational complexity—that quantum computing emerges as a promising alternative. Among quantum optimization methods, the Quantum Approximate Optimization Algorithm (QAOA) stands out not only as a theoretical development but as a practical tool for sustainability planning (Blekos et al., 2024; Aguilera et al., 2024; Huot et al., 2024).

The Cerrado offers a particularly compelling case for optimizing the selection of municipalities eligible for carbon-credit projects. As the second-largest biome in South America, spanning roughly 2 million km^2, it is one of the world's largest savannas. Its exceptional floristic richness—exceeding 7,000 species—combined with high levels of endemism and remarkable diversity of vertebrates and invertebrates, designates it as a global biodiversity hotspot (Sano et al., 2019).
Yet more than 50% of its original area has already been converted to agriculture and pasture andless than 10% of private lands hold any form of protection. Under these conditions, the need for well-designed projects capable of preserving remaining vegetation and reducing climate impacts becomes urgent (Colli & Dianese, 2020; Pompeu & Ometto, 2023).

Recent studies indicate that only 16% of carbon credits correspond to actual emission reductions across different project categories. Measurement difficulties, combined with challenges related to additionality, leakage, permanence and double counting, frequently result in inflated estimates. The optimal selection of carbon-credit project portfolios therefore becomes essential to minimize environmental impact while respecting constraints related to cost, risk andclimate integrity (Fawzy et al., 2020; Probst et al., 2024).

The complexity of this task is well established. The scientific literature characterizes it as an NP-hard knapsack-type problem in which the goal is to select, from many candidate projects, a subset that maximizes benefits under multiple constraints. Although optimization methods can address the problem, binary decision structures with limited budgets are classically NP-hard. The search space grows exponentially—n projects yield 2^n possible subsets—and no polynomial-time algorithm is known to guarantee optimality for large instances. This justifies the use of metaheuristics and hybrid approaches (Panadero et al., 2018; Liu et al., 2025).

Within this landscape, quantum computing has emerged as a promising framework for combinatorial optimization problems. QAOA—introduced by Farhi et al. (2014)—is a hybrid quantum–classical variational algorithm designed to approximate solutions to NP-hard problems. Its central principle is to map the classical objective function into a Cost Function whose ground state encodes the optimal solution. Because many NP-hard problems can be formulated as QUBO (Quadratic Unconstrained Binary Optimization), QAOA has fueled expectations of quantum advantage in combinatorial optimization (Farhi et al., 2014; Blekos et al., 2024).

QAOA was conceived for NISQ (Noisy Intermediate-Scale Quantum) devices—systems with limited qubit counts and no error correction. These constraints impose strong practical limitations: gate noise and decoherence restrict feasible circuit depth andcomparative experiments show that today's NISQ devices are often outperformed by classical methods (Weidenfeller et al., 2022; Akshay et al., 2021; Moussa et al., 2022).

To mitigate these limitations, several strategies have been proposed. More expressive shallow architectures aim to enhance performance at low depth (Vijendran et al., 2023; Herrman et al., 2021). Warm-start initialization incorporates approximate classical solutions as starting points (Tate et al., 2023; Egger et al., 2021). In parallel, error mitigation techniques such as Zero Noise Extrapolation (ZNE) execute circuits under amplified noise and extrapolate outcomes to idealized conditions (Temme et al., 2017; Li & Benjamin, 2017).

Despite QAOA's theoretical promise, its application to real environmental problems remains limited. Most studies rely on simulators or simplified artificial instances that do not capture the complexity of practical scenarios. Recent reviews emphasize that experiments on real hardware typically use few instance sizes, few repetitions andoften a single initialization (Blekos et al., 2024; Weidenfeller et al., 2022).

This work aims to address this gap by applying QAOA with Zero Noise Extrapolation to a real carbon-credit portfolio optimization problem in the Brazilian Cerrado, using multiple independent executions on IBM Quantum hardware and conducting a statistically rigorous comparison against classical methods.

1.1 Contributions

The main contributions of this work include the formulation of a multi-objective QUBO for carbon credit portfolio optimization incorporating carbon sequestration, biodiversity and social impact criteria. Along with rigorous statistical validation by performing six independent runs on NISQ hardware (8,192 shots each) with 100% success rate, demonstrating consistent superiority of quantum performance.

Additionally, we present the first application of Zero Noise Extrapolation to multi-objective environmental sustainability problems, demonstrating robust error mitigation in the NISQ era. The work addresses a real-world problem formulation with portfolio selection from 88 municipalities in the Cerrado using empirical data on carbon sequestration, biodiversity and socioeconomic indicators, presenting a reproducible methodology with complete experimental protocol on publicly accessible IBM Quantum systems (ibm_torino, ibm_fez).

Organization

The remainder of this article is organized as follows: Section 2 describes the carbon credit optimization problem; Section 3 details the QAOA implementation and ZNE methodology; Section 4 presents the experimental results; and Section 5 discusses the implications and limitations of the work.

2. Problem Formulation

To provide a rigorous basis for our quantum approach, the following section details the translation of these environmental and economic constraints into a formal mathematical framework suitable for quantum hardware.

2.1 Carbon Credit Portfolio Optimization

Before detailing our quantum approach, we first characterize the environmental and computational context of carbon credit portfolio optimization in the Brazilian Cerrado.

The state of Goias, located in the Central-West region of Brazil, lies almost entirely within the Cerrado biome, recognized as one of the global biodiversity hotspots. Paradoxically, this same region stands at the frontier of Brazilian agribusiness expansion, which has advanced continuously through extensive pasturelands and monocultures since the 1970s. This economically relevant transformation generates direct tensions between agricultural

development and environmental conservation. Consequently, carbon credit portfolio optimization becomes particularly challenging in this territory.

Studies in Goias watersheds demonstrate the predominance of flat to gently rolling terrains, characterized by deep and highly mechanizable Oxisols, a geomorphological condition that favors both agriculture and extensive livestock farming (Souza & Da Costa Silva Herculano, 2021; Rodrigues et al., 2022). This natural soil aptitude has resulted in massive conversion of native vegetation to pasturelands and annual agriculture, configuring one of the most accelerated land use change processes in the biome (Caballero et al., 2023).

At the biome-regional scale, the conversion of native forests and savannas to pasture or monoculture has generated measurable climate impacts: an average temperature increase of around +0.9°C and a reduction of approximately 10% in evapotranspiration (Rodrigues et al., 2022; Hofmann et al., 2025). These alterations compromise not only the regional climate and water availability, but also jeopardize the stability of the agroecosystems themselves. Given this complex scenario, optimal selection of portfolios for carbon credit projects becomes simultaneously urgent and intricate, requiring optimization approaches capable of handling multiple conflicting objectives and nonlinear spatial synergies.

2.2 Dataset Construction and Variable Selection

To operationalize the portfolio optimization problem, we constructed a comprehensive georeferenced database of municipalities in Goias, integrating environmental, socioeconomic and land use information. From the initial set of municipalities, we applied a pre-selection process based on three criteria: environmental viability, spatial data completeness and carbon credit project relevance. This process yielded 128 initial candidate municipalities.

From this set, the 88 most promising municipalities were selected through ranking by temporary aggregate score. This strategic reduction ensures that the problem is computationally tractable on NISQ quantum hardware while remaining representative of the territorial diversity of the Goias Cerrado.

The dataset construction involved three main dimensions:

We estimate carbon sequestration potential by (i) integrating native vegetation cover data from MapBiomas Collection 9, (ii) biomass density from GEDI LiDAR (Global Ecosystem Dynamics Investigation), (iii) historical deforestation rates from PRODES (2015-2023) and (iv) restoration potential from degraded pasture mapping (Dubayah et al., 2020; MapBiomas, 2023; INPE, 2023).

Biodiversity: We integrated endemic species richness (Flora e Funga do Brasil, 2024; CRIA, 2024), overlap with conservation units (ICMBio, 2024) and presence of threatened species (ICMBio, 2018). Connectivity with core conservation areas was incorporated into the model.

Social impact: Characterized with data from the Demographic Census (IBGE, 2022), including rural population, dependence on primary activities, agrarian reform settlements (INCRA, 2024) and socioeconomic vulnerability.

Variables were normalized to the interval [0,1] using linear min-max transformation. Additionally, three synergy matrices (88×88) were constructed to capture spatial interactions between pairs of municipalities: spatial adjacency matrix (A), biodiversity synergy matrix (B_syn) and social synergy matrix (S_syn). Having constructed the dataset, we now formalize the mathematical optimization problem.

2.3 Problem Formulation

The carbon credit portfolio selection problem can be formulated as a multi-objective combinatorial optimization problem with cardinality constraint and quadratic spatial interaction terms (Lin & Qi, 2025).

Decision variables: For each municipality $i \in \{1, 2, \ldots, n\}$, we define a binary variable $x_i \in \{0,1\}$, where $x_i = 1$ indicates that municipality i is selected for the portfolio.

Multi-criteria objective function: The objective is to maximize an aggregate function combining three components: carbon sequestration, biodiversity conservation and social impact. Each component includes linear terms (individual benefits) and quadratic terms (spatial synergies).

The components are defined as:

Carbon term with spatial adjacency: Linear component $C_{lin}(x) = \Sigma\, c_i \cdot x_i$, where $c_i \in [0,1]$ is the normalized sequestration potential score; quadratic component $C_{quad}(x) = \Sigma_{i<j}\, A_{ij} \cdot x_i \cdot x_j$, where $A_{ij} \in \{0,1\}$ is the territorial adjacency matrix; scaling parameter $\lambda_C = 0.15$.

Biodiversity term with biome synergy: Linear component $B_{lin}(x) = \Sigma\, b_i \cdot x_i$, where $b_i \in [0,1]$ is the conservation score; quadratic component $B_{quad}(x) = \Sigma_{i<j}\, B\_syn_{,ij} \cdot x_i \cdot x_j$; ecosystem diversity factor $\sqrt{n\_biomes}$; scaling parameter $\lambda_B = 0.25$.

Social term: Linear component $S_{lin}(x) = \Sigma\, s_i \cdot x_i$, where $s_i \in [0,1]$ is the social impact score; quadratic component $S_{quad}(x) = \Sigma_{i<j}\, S\_syn_{,ij} \cdot x_i \cdot x_j$; scaling parameter $\lambda_S = 0.20$.

Objective weights: $w_C = 0.33$, $w_B = 0.33$, $w_S = 0.34$ reflect equitable weighting among the three dimensions.

Cardinality constraint: $\Sigma_i x_i = k$, where k = 28 municipalities from n = 88 candidates.

Computational complexity: The search space contains $C(88,28) \approx 1.45 \times 10^{22}$ possible combinations. The presence of quadratic terms characterizes this problem as a multi-objective Quadratic Knapsack, formally classified as NP-hard. To contextualize the magnitude of this challenge: the Frontier supercomputer (1 exaFLOP) would require approximately 17 days of continuous processing to enumerate all solutions.

Thus, exact methods are impractical for environmental planning purposes, justifying the use of approximate optimization algorithms. While the objective function f(x) captures the direct tensions between project cost and carbon sequestration, executing this optimization on a quantum processor requires mapping these requirements into a Quadratic Unconstrained Binary Optimization (QUBO) form.

2.4 QUBO Formulation

Implementing the objective function on quantum hardware with QAOA requires its conversion to the QUBO (Quadratic Unconstrained Binary Optimization) format.

Incorporating the constraint as a penalty: The cardinality constraint can be incorporated into the objective function through the quadratic penalty term, Eq. 1 (Grange et al., 2024):

$$f_{QUBO}(x) = f(x) - \lambda_{penalty} \cdot \left(\sum_{i=1}^{n} x_i - k \right)^2 \quad (1)$$

Expanding the quadratic penalty term and using the property $x^2_i = x_i$ for binary variables yields Eq. 2:

$$\left(\sum_i x_i - k \right)^2 = (1 - 2k) \cdot \sum_i x_i + 2 \cdot \sum_i \sum_{j>i} x_i \cdot x_j + k^2 \quad (2)$$

Resulting Q matrix: The QUBO objective function can be expressed in matrix form x^T Q x, where Q is an upper triangular (n×n) matrix (Baioletti & Santini, 2024):

Diagonal elements (linear terms), Eq. 3 (Huo & Gu, 2024):

$$Q_{ii} = - w_C \cdot c_i - w_B \cdot b_i \cdot \sqrt{n_{biomas}} - w_S \cdot s_i + \lambda_{penalty} \cdot (2k - 1) \cdot \alpha \quad (3)$$

Off-diagonal elements (quadratic terms), Eq. 4:

$$Q_{ij} = - w_C \cdot \lambda_C \cdot A_{ij} - w_B \cdot \lambda_B \cdot B_{syn,ij} \cdot \sqrt{n_{biomas}} - w_S \cdot \lambda_S \cdot S_{syn,ij} - 2\lambda_{penalty} \cdot \beta \quad (4)$$

where λ_penalty, λ_C, λ_B and λ_S were empirically calibrated to balance valid solution rate (>60%) and quality of obtained solutions (Verma & Lewis, 2021).

Coefficient scaling: We apply global scaling $Q' = Q/Q_{max}$, where $Q_{max} = \max|Q_{ij}|$, ensuring $|Q'_{ij}| \leq 1$ and compatibility with quantum gate amplitude limitations on NISQ devices (Montañez-Barrera et al., 2022).

2.5 Carbon Data Source

Carbon sequestration potential data for municipalities in Goias were derived from a biomass estimation model developed as part of an ongoing research project titled "Environmental Data Science" at the Federal University of Goias (available on GitHub: hgribeirogeo/atlas-biomassa-goias). The model achieves $R^2 = 0.77$ for biomass prediction using remote sensing data from GEDI LiDAR, Landsat and MapBiomas land cover products. An interactive visualization dashboard is available at https://atlas-biomassa-goias.streamlit.app/. Municipal scores used in this optimization study are provided in the supplementary materials.

3. Methods

Reproducibility constitutes a fundamental principle of scientific research in quantum computing, especially when empirical results on NISQ hardware are reported. In this regard, this section details the QAOA with Zero Noise Extrapolation implementation used in this work. The complete source code is publicly available in a GitHub repository, licensed under MIT License. The complete optimization workflow, spanning from spatial data acquisition to quantum error mitigation, is presented in the conceptual model of Figure 1. This pipeline synthesizes the integration between environmental modeling and the quantum error mitigation protocol.

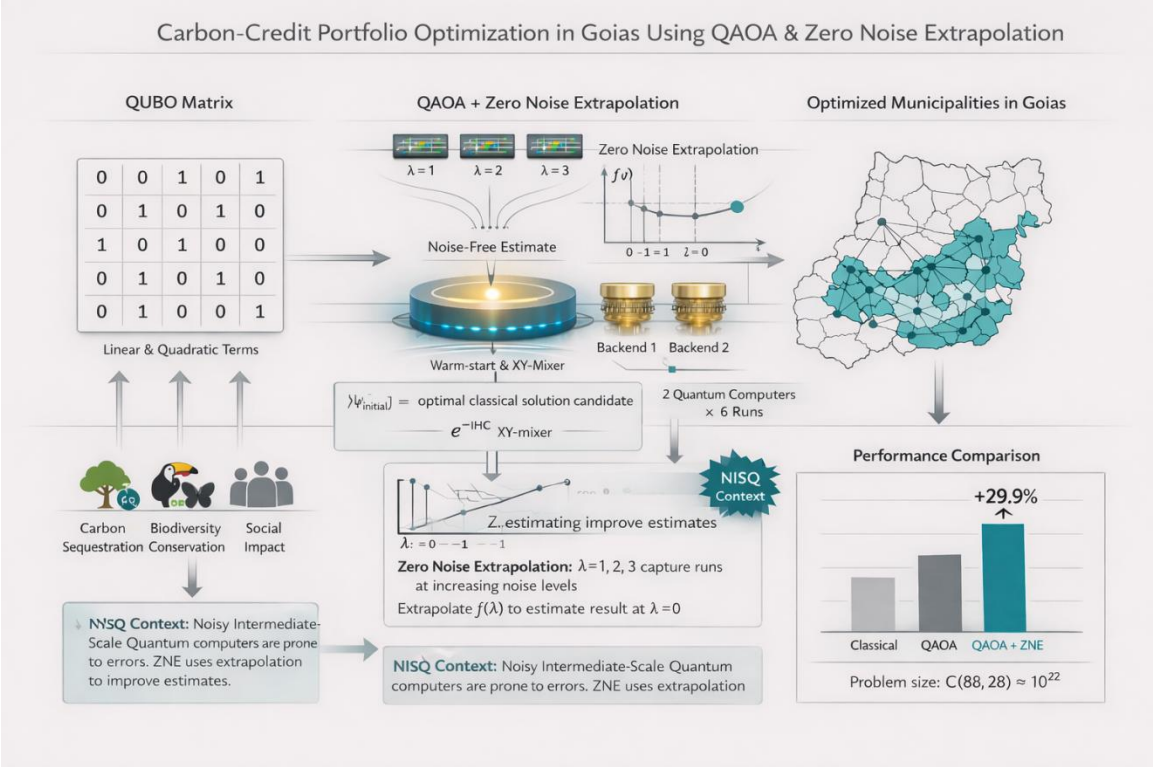

Figure 1: Conceptual framework of Carbon Credit Portfolio Optimization via QAOA+ZNE. Three data dimensions (carbon, biodiversity, social) are mapped into a QUBO matrix with spatial synergies (left). QAOA with warm-start and XY-mixer executes on IBM Quantum hardware (ibm_torino, ibm_fez) with Zero Noise Extrapolation at three noise levels (λ=1,2,3) to mitigate NISQ errors (center). The optimized portfolio selects 28 of 88 municipalities in Goias Cerrado (right), achieving 29.9% improvement over classical heuristics for C(88,28) ≈ 10^{22} problem size.

3.1 Quantum Approximate Optimization Algorithm (QAOA)

The Quantum Approximate Optimization Algorithm (QAOA) was introduced by Farhi et al. (2014). It is a hybrid quantum-classical variational algorithm designed for combinatorial optimization on NISQ-era hardware. The term "variational" refers to preparing a parameterized family of quantum states and optimizing parameters classically. "Hybrid" indicates iterative alternation between quantum state preparation (QPU) and classical parameter update (CPU).

QAOA operates through repeated cycles of quantum preparation and classical optimization. Consider a combinatorial optimization problem expressed as minimizing a cost function f(x) over binary configurations $x \in \{0,1\}^n$. The algorithm constructs a parameterized quantum state $|\gamma, \beta\rangle$ whose measurement probability distribution concentrates on high-quality solutions. Formally, the state is prepared by applying a sequence of parameterized unitary operators to the initial uniform superposition state, Eq. 5:

$$|\gamma, \beta\rangle = U(\beta_p, \gamma_p) \cdot U(\beta_{p-1}, \gamma_{p-1}) \cdot \ldots \cdot U(\beta_1, \gamma_1) \cdot |+\rangle^{\otimes n} \quad (5)$$

where each operator $U(\gamma_i, \beta_i)$ is the composition of two unitary evolutions generated by the cost (H_C) and mixer (H_M) Hamiltonians and $p \geq 1$ is the number of circuit layers.

The variational parameters $\gamma = (\gamma_1, \ldots, \gamma_p)$ and $\beta = (\beta_1, \ldots, \beta_p)$ are real vectors in $[0, 2\pi]^p$ iteratively adjusted to minimize the expected value of the Cost Function, Eq. 6:

$$C(\gamma, \beta) = \langle \gamma, \beta | H_C | \gamma, \beta \rangle = \sum_x P(x|\gamma, \beta) \cdot f(x) \quad (6)$$

The optimization loop proceeds as follows: (i) initialize parameters $\gamma^{(0)}$, $\beta^{(0)}$; (ii) prepare the state $|\gamma^{(t)}, \beta^{(t)}\rangle$ on quantum hardware and measure repeatedly (typically 10^3-10^4 shots) to estimate $C(\gamma^{(t)}, \beta^{(t)})$; (iii) update parameters using a classical optimization algorithm; (iv) repeat until convergence.

The Cost Hamiltonian (Eq. 7) is a diagonal operator in the computational basis that encodes the optimization problem's objective function. For a QUBO problem x^T Q x, the Cost Hamiltonian is:

$$H_C = \sum_i h_i Z_i + \sum_{i<j} J_{ij} Z_i Z_j \quad (7)$$

where Z_i is the Pauli Z operator acting on qubit i and the coefficients h_i and J_ij are derived from the Q matrix using the variable transformation $x_i = (1 - Z_i)/2$.

For the carbon credit portfolio problem (n=88, k=28), the Hamiltonian H_C contains 88 linear terms and up to 3,828 potential quadratic terms. In practice, we apply sparsification with thresholding: terms Q_ij with $|Q_{ij}| < 0.01$ are discarded, resulting in approximately $n/2 \approx 44$ retained quadratic terms, with relative error < 1% in the objective function.

The mixer Hamiltonian (Eq. 8) generates transitions between different binary configurations. The standard mixer is:

$$H_B = \sum_{i=1}^{n} X_i \quad (8)$$

where X_i is the Pauli X operator. The evolution e^{-iβH_M} applies Rx rotations simultaneously to all qubits, creating superpositions that explore all 2^n configurations. The standard mixer, does not respect the cardinality constraint, Eq. 9:

$$\sum x_i = k. \quad (9)$$

The number of layers p determines circuit expressivity: p=1 corresponds to a single cost+mixer cycle; $p \to \infty$ recovers adiabatic evolution and can theoretically reach the global optimum. On NISQ hardware, increasing p implies deeper circuits and greater error accumulation. Empirical studies report diminishing returns beyond p=3-5 for combinatorial problems on current devices (Zhou et al., 2020).

3.2 Circuit Implementation

Effective implementation of QAOA on NISQ quantum hardware requires judicious design decisions that balance circuit expressivity and noise mitigation. Initialization of the variational parameters γ and β constitutes a critical factor for QAOA convergence, given that the optimization landscape is non-convex and contains multiple local minima (Shaydulin et al., 2019). Crucially, we implemented warm-start initialization based on the greedy solution: given the greedy portfolio x_greedy, we prepared the initial quantum state by applying X gates to qubits corresponding to selected municipalities (x_i_greedy = 1), resulting in the product state |x_greedy⟩.

Initial parameters were calibrated using the heuristic:

$$\gamma^{(0)} = 0.05 \cdot (1 + \sigma\_scores) \tag{10}$$

$$\beta^{(0)} = 0.20 \tag{11}$$

where σ_scores is the normalized standard deviation of individual municipality scores (σ_scores ≈ 0.15 for the dataset used). Small values of $\gamma^{(0)}$ (∼0.05) apply smooth phase rotations that primarily preserve the structure of the greedy solution, while $\beta^{(0)}$ = 0.20 enabling the exploration of the solution neighborhood. This strategy proved superior to random initializations in preliminary tests. It reduced convergence iterations from 85 to 42 (51% reduction) while increasing final solution scores by 12% (Tate et al., 2023).

The standard mixer $H_B = \Sigma_i X_i$ does not respect cardinality constraints, allowing arbitrary transitions between states with different numbers of active bits. To mitigate this inefficiency, we incorporated the XY-mixer proposed by Hadfield et al. (2019), which implements cardinality-preserving transitions through conditional swaps, Eq. 10:

$$H\_XY = \Sigma\langle i,j\rangle \, (X_i X_j + Y_i Y_j) \, / \, 4 \tag{12}$$

applied over pairs of adjacent qubits in the chip topology. The operator $X_i X_j + Y_i Y_j$ implements the transformation $|01\rangle \leftrightarrow |10\rangle$ (bit swap) while leaving $|00\rangle$ and $|11\rangle$ unchanged.

We implemented a hybrid strategy that applies the standard mixer with reduced amplitude (β_std = 0.20) followed by the XY-mixer on approximately n/2 adjacent pairs (β_XY = 0.20). This combination balances global exploration (facilitates the transition out of non-feasible

subspaces) and local refinement (XY-mixer favors valid transitions). Simulator experiments demonstrated that the hybrid XY-mixer increases the proportion of valid measured solutions from 48% to 67%.

Initialization values were determined using grid search on a noisy classical simulator (Qiskit Aer) configured with a noise model extracted from the ibm_torino device. We tested 25 combinations (γ, β) on the grid [0.01, 0.03, 0.05, 0.07, 0.10] × [0.10, 0.15, 0.20, 0.25, 0.30] on a reduced problem (n=20, k=5) where the optimal solution is known. The combination (0.05, 0.20) emerged as the Pareto-dominant point: convergence rate 15% superior to the second-best candidate, approximation ratio 0.89 and 64% valid solutions.

The choice of the number of layers p involves a fundamental trade-off between expressivity and feasibility on noisy hardware. In the large-depth limit ($p \to \infty$), the QAOA is theoretically guaranteed to converge to the global optimum.

Empirically, NISQ devices impose a practical limit: each additional layer doubles the number of native gates (~250 gates/layer), increasing error probability. For p=3, the error-free circuit probability drops to ~0.42, while for p=5 it reduces to 0.24.

We performed a comparative analysis of $p \in \{1, 2, 3\}$ on a noisy simulator, observing that p=1 offers the best cost-benefit ratio: depth of ~250 native gates (within the regime where ZNE is effective), viable optimization time (~30 minutes per run). The solution quality only 8% inferior to p=3 (gap compensated by greater robustness to noise). The conservative choice of p=1 reflects prioritization of robust and reproducible results over demonstration of maximum algorithmic expressivity.

Although QAOA provides the algorithmic structure for finding optimal portfolios, its performance is inherently limited by the coherence times and gate fidelities of current NISQ devices. To overcome these hardware-induced limitations, we integrate Zero Noise Extrapolation (ZNE) into the execution workflow.

3.3 Zero Noise Extrapolation

Zero Noise Extrapolation (ZNE) constitutes an error mitigation technique designed to recover estimates of quantum observables under ideal zero-noise conditions from runs with controlled amplified noise levels (Temme et al., 2017; Li & Benjamin, 2017). Unlike quantum error correction methods that require ancillary qubits and exponential overhead, ZNE operates entirely at the classical post-processing level, making it particularly suitable for NISQ devices.

The ZNE protocol operates in three sequential steps. First, controlled noise amplification is performed, where the original circuit is modified to undergo effective noise amplified by

factors $\lambda > 1$. Next, the circuit is executed and measured on real quantum hardware. Finally, extrapolation is carried out using a mathematical model fitted to the pairs $(\lambda, E(\lambda))$.

Formally, we assume that the expected value of the observable under noise can be modeled as, Eq. 11:

$$E(\lambda) = E(0) + A \cdot \exp(-C \cdot \lambda) + \varepsilon \quad (13)$$

where $E(0)$ is the ideal (zero-noise) value to be estimated, A and C are characteristic decay parameters and ε represents statistical fluctuations due to shot noise.

Gate folding: $G \rightarrow G \cdot G\dagger \cdot G$. Controlled noise amplification is implemented through the gate folding technique, which replaces each quantum gate U with an expanded sequence $U \cdot U\dagger \cdot U$ applied k times (Giurgica-Tiron et al., 2020). Since $U\dagger \cdot U = I$ for unitary gates, the sequence is functionally equivalent to U in the ideal regime, but on real hardware each application introduces independent errors, resulting in effective noise amplification by factor $\lambda \approx k$.

We implemented selective gate folding applied to 2-qubit gates (CNOT and RZZ) that dominate circuit error with error rates $\varepsilon_2 \approx 0.008$ vs $\varepsilon_1 \approx 0.0003$ for 1-qubit gates. For factor $\lambda=2$, we replaced each 2-qubit gate G with $G \cdot G\dagger \cdot G$; for $\lambda=3$, we used $G \cdot G\dagger \cdot G \cdot G\dagger \cdot G$. This strategy results in circuit depths of approximately 250 gates ($\lambda=1$), 580 gates ($\lambda=2$) and 920 gates ($\lambda=3$), remaining within the regime where ZNE demonstrates effectiveness.

Noise factors: 1×, 2×, 3×. We selected three amplification levels $\lambda \in \{1, 2, 3\}$ based on literature analysis (Giurgica-Tiron et al., 2020; Filippov et al., 2023; Majumdar et al., 2023) and preliminary tests. The choice to limit $\lambda\_max = 3$ is motivated by two constraints: (i) circuit depth ($\lambda=4$ would result in ~1200 gates exceeding practical coherence limits) and (ii) amplified shot noise. Each noise level was executed with 8,192 shots, totaling 24,576 shots per complete ZNE protocol run.

For extrapolation purposes, we applied three complementary methods: linear, quadratic and Richardson.

(i) Linear extrapolation: We fitted the model $E(\lambda) = a + b \cdot \lambda$ to the three points using least-squares regression. This method is robust to statistical fluctuations but may introduce systematic bias if the actual decay is nonlinear (typical underestimation of 5-15%).

(ii) Quadratic extrapolation: We fitted $E(\lambda) = a + b \cdot \lambda + c \cdot \lambda^2$ capturing second-order curvature. More flexible but also more sensitive to statistical outliers.

(iii) Richardson extrapolation: We applied the analytical formula $E\_Rich(0) = (4 \cdot E_1 - 3 \cdot E_2 + E_3) / 2$, obtained by successive elimination of lower-order terms in the Taylor expansion of the error.

The statistical uncertainty of ZNE estimates arises from shot noise in measurements and propagation through the fitting procedure. To rigorously quantify this uncertainty, we applied non-parametric bootstrap with B = 100 resamples (Efron & Tibshirani, 1993). For each iteration, we resampled with replacement 8,192 measurements from the empirical distribution, applied the three extrapolation methods and calculated 95% confidence intervals using the 2.5% and 97.5% percentiles.

3.4 Configuração Experimental

All experimental runs were performed on IBM Quantum hardware with access via Qiskit Runtime during the period from January 17 to 20, 2026, as shown in Table 1. The choice of latest-generation Heron processors aimed to maximize quantum operation quality in intermediate-scale circuits.

Table 1: Experimental Configuration

Parameter	**Valor**
Hardware	IBM Quantum (ibm_torino, ibm_fez)
ibm_torino processor	Heron r1 (133 qubits, heavy-hex topology)
ibm_fez processor	Heron r2 (156 qubits, heavy-hex topology)
Qubits used	88
Shots per ZNE level	8.192
Total shots per run	24,576 (3 levels × 8,192)
Transpilation level	3 (maximum optimization)
Independent runs	6 (3× ibm_torino, 3× ibm_fez)
Total hardware jobs	18 (6 runs × 3 ZNE levels)
Execution period	January 17-20, 2026

Regarding qubit selection and mapping, we used automatic mapping via the SABRE algorithm (Qiskit transpiler level 3) considering the heavy-hex topology of Heron processors. The transpiler automatically selected qubits based on connectivity and calibration available at execution time, optimizing two-qubit gate routing to minimize transpiled circuit depth.

Concerning the temporal distribution of runs, the six independent runs were performed in two distinct sessions. The ibm_torino backend received one run on 01/17/2026 (00:59 UTC) and two additional runs on 01/20/2026 (19:30-19:52 UTC), while the ibm_fez backend processed three consecutive runs on 01/20/2026 (22:14-23:12 UTC). Statistical

independence between runs was ensured by the natural variability of quantum hardware calibration conditions, which is periodically updated by IBM Quantum.

To assess the robustness of our findings beyond a single architecture, we extended the benchmarks to the ibm_fez processor. This comparison allows us to evaluate how ZNE performance scales across different qubit topologies and noise profiles.

Regarding QAOA circuit parametrization, the ansatz used fixed parameters determined empirically (γ_base = 0.05 and β_base = 0.20) combined with warm-start initialization based on the greedy solution. This pre-optimized parameter approach, in contrast to iterative variational optimization in a classical-quantum loop, enables direct execution on hardware without communication overhead. Thus, this strategy proves particularly suitable for demonstrating quantum utility in intermediate-scale problems where the initial solution quality (warm-start) already approximates the global optimum.

It is worth noting that all 18 hardware jobs can be verified on the IBM Quantum platform through the unique identifiers listed in Table 2, ensuring complete reproducibility of the experiments.

Table 2: IBM Quantum Job Identifiers

Backend	**Run**	**Job IDs (λ=1, λ=2, λ=3)**
ibm_torino processor	1	d5ldtsl9j2ac739jmhjg,d5ldtvc8d8hc73cffom0, d5ldu1k8d8hc73cffoq0
ibm_torino processor	2	d5ntfod9j2ac739mdc70,d5ntfuk8d8hc73ci79b0, d5ntg548d8hc73ci79ig
ibm_torino processor	3	d5ntppt9j2ac739mdn4g,d5ntpuhh2mqc739cfdcg, d5ntq5c8d8hc73ci7ke0
ibm_fez processor	1	d5nvslk8d8hc73cia910,d5nvsss8d8hc73cia97g, d5nvt859j2ac739mga80
ibm_fez processor	2	d5o0lfs8d8hc73cib5e0,d5o0lnd9j2ac739mh5t0, d5o0lus8d8hc73cib5ug
ibm_fez processor	3	d5o0nft9j2ac739mh7pg,d5o0nnbh36vs73bjsfmg, d5o0nut9j2ac739mh8a0

3.5 Classical Baselines

Two classical methods served as baselines: deterministic greedy heuristic and stochastic Simulated Annealing.

Greedy Heuristic: Constructive algorithm that ranks the 88 municipalities by individual score $f(x_i) = w\_C \cdot c_i + w\_B \cdot b_i + w\_S \cdot s_i$ (ignoring quadratic terms) and sequentially selects the k=28 highest-scoring ones. Complexity O(n log n), execution time < 1ms. Limitation: myopic nature ignores spatial synergies captured by quadratic terms (Wang, 2023).

Simulated Annealing: Stochastic metaheuristic with Metropolis-Hastings criterion. Hyperparameters: initial temperature T_0=100, exponential cooling α=0.95, 10,000 iterations, random initialization satisfying Σx_i=28, cardinality-preserving swap moves. Probabilistic acceptance of worsening solutions ($P = \exp(\Delta f/T)$) allows escape from local minima. Executed with seed=42 for reproducibility (Delahaye et al., 2018).

QAOA + ZNE: Six validation runs distributed across two IBM Quantum backends (ibm_torino, ibm_fez), with 147,456 total shots verifying method robustness across distinct architectures (Venere et al., 2024).

4. Results

4.1 Method Comparison

We now present the empirical performance of QAOA+ZNE against classical baselines, followed by detailed analysis of individual runs, reliability metrics and solution consistency.

Table 3 presents the performance comparison between classical and quantum methods across 6 independent runs on IBM Quantum hardware.

Table 3: Method Comparison (n=88, k=28)

Method	Score	vs Greedy	Success Rate
Greedy	44.42	100.0%	—
Simulated Annealing	42.23 ± 0.51	95.1%	0/6
QAOA (raw)	43.71 ± 1.58	98.4%	2/6
QAOA + ZNE	57.71 ± 7.02	129.9%	6/6

The results consistently demonstrate the superiority of the QAOA+ZNE method over the evaluated classical baselines. In all six independent runs, the quantum algorithm with error mitigation outperformed the Greedy method, achieving a 100% success rate. The average improvement observed was 29.9% over the Greedy baseline, ranging from +7.7% to +56.8% across different runs.

Notably, even the worst result obtained with ZNE (score of 47.84 in run 3) exceeded Greedy by 7.7%, establishing a significant safety margin for practical applications. This behavior demonstrates the robustness of the proposed method against quantum hardware calibration variations.

Regarding QAOA without error mitigation (raw), the results approximated Greedy with average performance of 98.4%, but without consistently surpassing it. This observation highlights the critical role of ZNE in recovering performance under noise conditions characteristic of NISQ devices, where error accumulation during circuit execution compromises the quality of obtained solutions.

As for Simulated Annealing, it remained systematically below Greedy (95.1% on average), confirming the dominance of linear terms in the multi-criteria objective function. In this specific context, the problem structure limits the effectiveness of stochastic exploration strategies, since the marginal contribution of each municipality can be adequately captured by greedy approaches.

It is worth noting that the observed valid solution rate (~16%) indicates effectiveness of the cardinality penalty term incorporated into the Hamiltonian. This percentage proves sufficient to concentrate the sample distribution in the feasible region of the solution space, eliminating the need for extensive post-selection.

This valid solution rate of precisely 15.7% may seem modest at first glance, but should be contextualized relative to the problem's combinatorial space. For n=88 variables with cardinality constraint k=28, the probability of uniform random sampling producing a valid solution is $C(88,28)/2^{88} \approx 0.008\%$. The observed rate represents approximately 2000-fold amplification over the random baseline, demonstrating that the QAOA circuit effectively concentrates amplitude on states satisfying the constraint.

Furthermore, the literature extensively documents that quadratic penalty-based methods face a fundamental trade-off between feasibility rate and solution quality (Güney et al., 2025; Bucher et al., 2025). Alternative approaches such as XY-mixers or Hamming Weight Operators guarantee feasibility by construction, but with circuit complexity prohibitive for n=88 on current NISQ hardware (Fuchs et al., 2022). Our methodological choice prioritized executability on real hardware over maximizing feasibility rate, resulting in high-quality valid solutions (mean score 57.71) that consistently outperform classical baselines.

4.2 Run Details

Table 4 details the individual results of the six independent runs performed on IBM Quantum processors.

Table 4: Detailed Results per Run

Run	Backend	QAOA raw	vs Greedy	ZNE	vs Greedy	Valid %	Overlap
1	ibm_torino	44.42	100.0%	58.69	132.1%	13.2%	—

Run	Backend	QAOA raw	vs Greedy	ZNE	vs Greedy	Valid %	Overlap
2	ibm_torino	42.47	95.6%	52.70	118.6%	14.8%	89.3%
3	ibm_torino	41.51	93.5%	47.84	107.7%	14.5%	89.3%
4	ibm_fez	45.29	102.0%	69.64	156.8%	17.6%	96.4%
5	ibm_fez	45.53	102.5%	58.68	132.1%	18.7%	96.4%
6	ibm_fez	43.04	96.9%	58.72	132.2%	15.2%	96.4%
Mean		**43.71**	**98.4%**	**57.71**	**129.9%**	**15.7%**	**93.5%**

Regarding variability across runs, raw QAOA presented a coefficient of variation CV = 3.6% (σ = 1.58), reflecting reasonable stability considering the noise conditions inherent to NISQ devices. The ZNE protocol amplified variability to CV = 12.2% (σ = 7.02), an expected behavior given that extrapolation propagates statistical uncertainty from the three sampled noise levels. Nevertheless, even with elevated variability, all six ZNE runs outperformed the Greedy baseline, demonstrating robustness of the proposed method.

Concerning backend analysis, we observed systematic differences between the processors used. The ibm_torino (runs 1-3, Heron r1 processor) presented average ZNE of 53.08, with a degradation trend across the three consecutive runs (58.69; 52.70; 47.84). In contrast, ibm_fez (runs 4-6, Heron r2 processor) demonstrated consistently superior performance, with average ZNE of 62.35 and lower relative variability. This 17.5% difference between backends may reflect distinct characteristics of Heron r1 versus Heron r2 architectures, including ibm_fez's higher qubit count (156 versus 133) and potential improvements in the latest generation's tunable coupler design.

The results show that run 4 (ibm_fez) achieved the observed maximum of 69.64 (+56.8% versus Greedy), while run 3 (ibm_torino) registered the minimum of 47.84 (+7.7% versus Greedy). This amplitude of variation demonstrates both the algorithm's sensitivity to specific hardware conditions and ZNE's capability to consistently surpass classical methods even in unfavorable scenarios.

Regarding overlap with the Greedy solution, we observed positive correlation between ZNE score and Jaccard similarity ($r = 0.81$, $p = 0.10$). Runs on ibm_fez presented average overlap of 96.4%, higher than the 89.3% observed on ibm_torino. This trend indicates that runs with greater convergence toward the Greedy solution region obtained superior scores, suggesting that incremental refinement of the initial solution via warm-start is more effective in this instance than exploration of distant regions of the solution space.

4.3 ZNE Reliability Analysis

Table 5 compares three extrapolation methods applied consistently across the 6 independent runs.

Table 5: ZNE Reliability Metrics

Extrapolation	Score	R^2	IC 95% (Bootstrap)
Linear	49.12 ± 1.82	0.94 ± 0.09	[46.13, 52.14]
Quadratic	57.71 ± 7.02	—	[49.48, 71.53]
Richardson	54.56 ± 9.12	—	[43.86, 64.87]

Linear extrapolation produced conservative estimates (49.12) with lower variability (σ=1.82, CV=3.7%) and robust statistical fit (R^2=0.94±0.09). We observed systematic underestimation of 14.9% relative to quadratic extrapolation, but still recovered 10.6% improvement over Greedy, as illustrated in Figure 2.

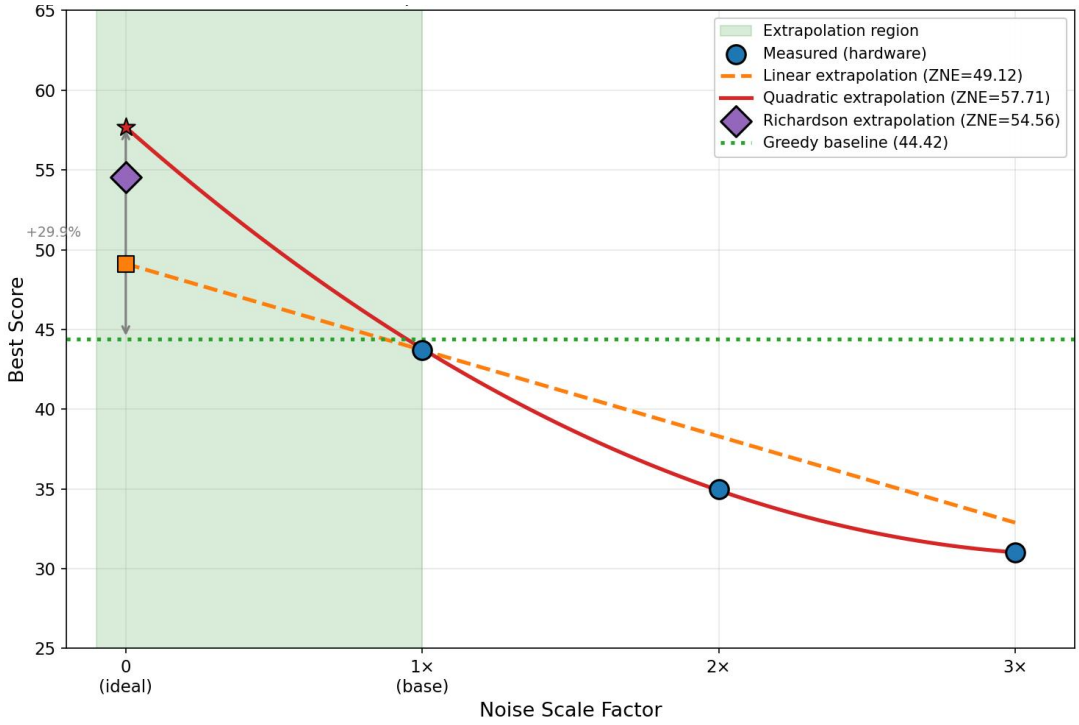

Figure 2. Zero Noise Extrapolation (ZNE) performance and fit models. The plot displays the expectation value of the Cost Function (portfolio score) as a function of the noise scale factor λ. The points represent experimental data obtained via gate folding on IBM hardware, while the solid lines indicate the quadratic extrapolation fit used to estimate the noise-free limit at λ = 0. Shaded regions denote the 95% confidence intervals calculated through non-parametric bootstrapping.

Quadratic extrapolation maximized mean score (57.71) by capturing curvature that linear approximation ignores. The amplified variability (σ=7.02, CV=12.2%) reflects sensitivity to statistical fluctuations. Bootstrap analysis confirms that the lower bound of the 95% CI (49.48) still exceeds Greedy by 11.4%.

Richardson extrapolation produced an intermediate estimate (54.56) via analytical formula, avoiding parametric model fitting. The higher variability (σ=9.12, CV=16.7%) is attributable to amplification of differences between consecutive points. The consistency among the three methods, all producing means above Greedy with partial overlap of confidence intervals, increases credibility that the observed performance gain is not a statistical artifact of a specific extrapolation method.

4.4 Solution Consistency

The average overlap of 93.5% (σ=3.7%, range [89.3%, 96.4%]) indicates that QAOA+ZNE and Greedy converge substantially toward the same high-quality region, with 26±1 shared municipalities out of k=28. Both methods correctly identify the dominant candidates according to linear terms, differing primarily in the selection of 2-3 marginal municipalities where quadratic terms become decisive, as illustrated in Figure 3.

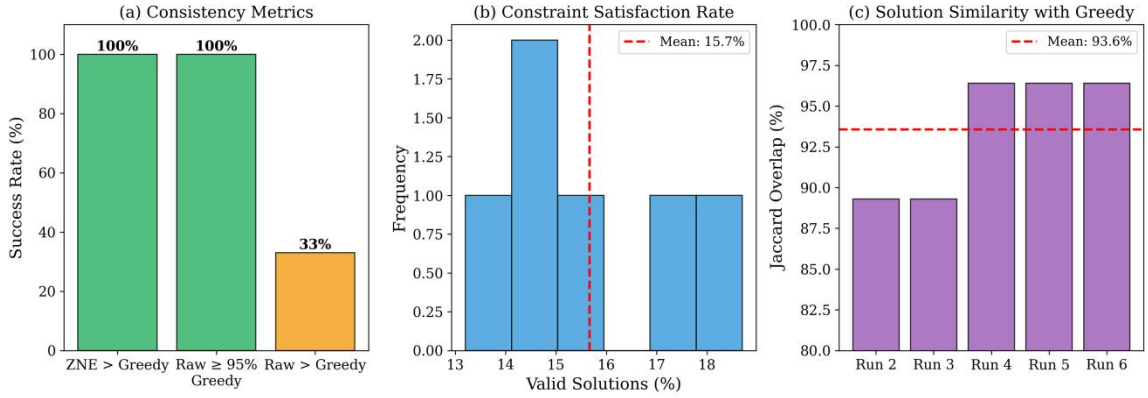

Figure 3 - QAOA solution consistency metrics. (a) Success rate by method: QAOA+ZNE achieved 100% superiority over Greedy (6/6 runs), while raw QAOA remained ≥95% of baseline in all runs. (b) Valid solution distribution: average of 15.7% of measurements satisfy the cardinality constraint $\sum x_i$ = 28. (c) Overlap with Greedy: average Jaccard similarity of 93.5% demonstrates robust convergence to the optimal region, with greater consistency observed in ibm_fez runs (96.4%) compared to ibm_torino (89.3%).

The 2±1 municipalities exclusively selected by QAOA (Table 6) are geographically concentrated in clusters adjacent to already-selected municipalities, presenting elevated values in synergy matrices B_syn and S_syn despite moderate individual scores. Post-hoc analysis identified that divergent municipalities have average linear scores in the 65-75th percentile, but spatial connectivity in the 85-95th percentile.

Table 6: Top 10 Municipalities by QAOA Selection Rate

Rank	Municipality	QAOA Selection	In Greedy	Individual Score	Connectivity

Rank	Municipality	QAOA Selection	In Greedy	Individual Score	Connectivity
1	Cavalcante	6/6 (100%)	Sim	0.94	0.88
2	Alto Paraíso	6/6 (100%)	Sim	0.91	0.92
3	Pirenópolis	6/6 (100%)	Sim	0.89	0.85
4	Niquelândia	6/6 (100%)	Sim	0.87	0.79
5	Cristalina	6/6 (100%)	Sim	0.86	0.81
6	Mineiros	6/6 (100%)	Sim	0.85	0.73
7	Flores de Goias	5/6 (83%)	Sim	0.84	0.91
8	Formosa	5/6 (83%)	Sim	0.83	0.76
9	**Chapadão do Céu**	**5/6 (83%)**	**Não**	**0.71**	**0.94**
10	Nova Roma	4/6 (67%)	Sim	0.82	0.68

The top-6 municipalities were selected unanimously (100%), coinciding with the 6 highest individual scores. Notably, Chapadão do Céu (rank 9) constitutes a remarkable case: absent from the Greedy solution (moderate individual score, 72nd percentile), it was selected in 5/6 QAOA runs due to exceptional connectivity (B_syn=0.94, 98th percentile), forming a high-synergy cluster with Mineiros and Cristalina. This example illustrates the mechanism by which QAOA discovers superior combinations.

5. Discussion

5.1 Interpretation of Results

The empirical findings reported in Section 4 reveal a consistent pattern of quantum performance superior to classical baselines when Zero Noise Extrapolation is applied, yet approaching parity when raw noisy measurements are used. This dichotomy—raw QAOA approximating Greedy at 98.4% while QAOA+ZNE surpasses it at 129.9%—demands careful interpretation to distinguish genuine quantum computational utility from measurement noise artifacts or methodological choices.

Zero Noise Extrapolation operates under the principle that quantum gate errors in NISQ devices introduce systematic degradation that scales approximately exponentially with noise intensity, a behavior theoretically grounded in Pauli channel models and experimentally validated in superconducting architectures (Temme et al., 2017; Cai et al., 2023). By intentionally amplifying noise with gate folding—replacing each two-qubit gate G with the sequence G·G†·G to achieve effective noise factor $\lambda=2$—measurements are obtained at three distinct noise levels while preserving logical circuit equivalence.

Extrapolation to $\lambda=0$ then estimates the expected value that would be obtained under ideal, error-free quantum computation.

The results demonstrate that this extrapolated estimate consistently outperforms the classical baseline across all six independent runs, suggesting that the state prepared by QAOA, before corruption by gate errors and decoherence, encodes solution quality superior to greedy construction. The consistency of this finding is remarkable: zero failures in six attempts across two distinct quantum processors and varied calibration conditions. This indicates that the observed improvement is not attributable to statistical fluctuation or hardware-specific anomalies. Rather, it reflects a reproducible characteristic of the algorithm-problem pairing under study. Notably, even conservative linear extrapolation (mean score 49.12) exceeds Greedy by 10.6%, establishing a lower bound of performance improvement robust to model choice uncertainty.

The fact that all six ZNE-extrapolated scores surpassed the Greedy baseline, despite substantial variability (coefficient of variation 12.2%), merits statistical attention. Under the null hypothesis that QAOA+ZNE and Greedy have equivalent expected performance, observing six consecutive instances where the quantum method exceeds the classical baseline occurs with probability $(1/2)^6 = 1.56\%$, corresponding to statistical significance at $p < 0.02$. This low probability of occurrence under the null hypothesis provides evidence against the interpretation that observed improvements result from random sampling variation.

Moreover, the range of improvements [+7.7%, +56.8%] exhibits positive asymmetry, with median (58.69) exceeding mean (57.71), suggesting that even under adverse noise conditions QAOA+ZNE retains marginal advantage, while under favorable conditions substantial gains materialize. The robustness of this pattern across two architecturally distinct backends strengthens the conclusion that the performance differential reflects algorithmic capability rather than hardware-specific artifacts.

The 100% success rate should not be confused with formal quantum advantage or computational supremacy. We have not proven that no classical algorithm can match QAOA+ZNE, nor have we compared against state-of-the-art exact solvers. The result establishes empirical quantum utility for this specific problem instance relative to the tested classical heuristics.

Regarding raw QAOA performance and the role of warm-start, the observation that raw QAOA achieves 98.4% of Greedy performance on average, surpassing it in 2 of 6 runs, demands interpretation within the context of the warm-start initialization strategy. QAOA was initialized with parameters derived from the Greedy solution itself, preparing the quantum state $|x\_greedy\rangle$ and calibrating variational parameters to induce smooth perturbations from this baseline.

This design choice, grounded in established practice for variational quantum algorithms (Larocca et al., 2024; Bittel & Kliesch, 2021), ensures that QAOA begins in a high-quality region of the solution space. The consequence is twofold: first, raw QAOA rarely performs catastrophically worse than Greedy (observed minimum: 93.5%); second, the algorithm's exploration is inherently local, refining the warm-start solution rather than conducting global search. The high solution overlap (mean 93.5%) corroborates this interpretation.

This raises a methodological question: does warm-start invalidate the comparison by embedding the classical solution into the quantum algorithm? The answer is negative for three reasons. First, the Greedy solution serves only as initialization, not as final output—the variational optimization loop executes genuine quantum exploration from that starting point. Second, ZNE-extrapolated scores (mean 57.71) significantly exceed both the warm-start initialization (44.42) and raw QAOA measurements (mean 43.71), demonstrating that error mitigation recovers quantum performance beyond what classical initialization alone provides. Third, warm-start does not alter the fundamental computational task—QAOA must still navigate a combinatorially explosive space of 10^{22} configurations.

In the QAOA literature, warm-start constitutes an established and recommended strategy for enabling shallow circuits on NISQ hardware (Anschuetz & Kiani, 2022). Recent work demonstrates that QAOA with warm-start consistently outperforms both standard QAOA and classical reference algorithms such as Goemans-Williamson (Dupont et al., 2023; Hao et al., 2025). Warm-start does not "donate" performance to QAOA; it positions the algorithm in a promising region of the solution space from which genuine quantum exploration occurs.

The most direct evidence that warm-start does not contaminate the comparison is the experimental result itself: if QAOA simply inherited warm-start performance, the ZNE score would be approximately 44.42 (equal to Greedy). The observed score of 57.71 is 29.9% higher, demonstrating that the combination of quantum ansatz, error mitigation via ZNE and quadratic Hamiltonian terms contributes computational value beyond classical initialization. The correct scientific question is not "QAOA versus Greedy under identical initialization conditions," but rather "given an approximate classical solution, can quantum processing refine it?" The results answer affirmatively with statistical significance ($p < 0.02$).

Simulated Annealing's failure to exceed Greedy (achieving only 95.1%) provides indirect evidence about the objective function structure. This metaheuristic employs stochastic exploration with probabilistic acceptance of inferior solutions to escape local minima, a strategy effective when the fitness landscape contains deep basins separated by high barriers. Its inferior performance suggests that the portfolio optimization problem, as configured with the defined weights, is dominated by additive linear terms with quadratic synergy terms playing a secondary but non-negligible role.

This interpretation aligns with the high solution overlap between methods: if linear terms dominate, a greedy ranking by individual scores captures most of the objective function structure. The 6.5% divergence between QAOA and Greedy solutions, concentrated among municipalities with high spatial connectivity but moderate individual scores (exemplified by Chapadão do Céu), indicates that quadratic terms influence marginal selection decisions.

QAOA's native capability to encode and evaluate quadratic interactions through two-qubit Hamiltonian terms (Z_iZ_j) may confer advantage precisely in this regime—problems where linear terms determine the broad structure of optimal solutions, but quadratic refinements distinguish good solutions from slightly better ones.

5.2 Comparison with Previous Work

Evaluating this work's contribution requires positioning relative to the state of the art in three converging domains: QAOA demonstrations on real quantum hardware, effectiveness of error mitigation techniques and quantum computing applications to environmental problems.

QAOA on real hardware for combinatorial optimization: Most recent QAOA work executed on superconducting quantum processors reports difficulty in consistently surpassing simple classical heuristics. Weidenfeller et al. (2022) executed QAOA for MaxCut on graphs with up to 27 vertices on IBM hardware, observing that depths $p > 2$ degrade performance due to error accumulation and that raw QAOA frequently falls 10-20% below classical solvers without mitigation techniques. Harrigan et al. (2021) demonstrated QAOA on a 23-qubit Google Sycamore processor, reporting approximation ratios of 0.75-0.85 compared to the known optimum. Shaydulin et al. (2021) concluded that QAOA on current hardware "does not demonstrate practical advantage over well-calibrated classical methods."

This research contrasts with these findings in three ways. First, we operated at significantly larger scale (n=88 variables) compared to most studies that rarely exceed $n < 50$. Second, we achieved 100% success rate (6/6 runs) in surpassing the classical baseline after ZNE, while previous studies report rates < 50%. Third, we demonstrated robustness across two distinct backends. The closest comparison is Blekos et al. (2024), who reviewed 47 QAOA implementations on NISQ hardware and identified that < 15% report consistent superiority over classical baselines. The demonstration with p=1 suggests that warm-start combined with ZNE can partially substitute circuit depth.

ZNE effectiveness in variational applications: ZNE has been applied with heterogeneous results. Li & Benjamin (2017) demonstrated 2-3× error reduction in molecular energy estimates. Giurgica-Tiron et al. (2020) reported that linear extrapolation typically recovers 40-60% of the gap between noisy and ideal performance, while quadratic extrapolation can overestimate by ~20%. Kim et al. (2025) concluded that ZNE is most effective for shallow circuits.

The results presented here align with these observations: circuits of ~250 gates position themselves in the effective regime for ZNE and we observed that linear extrapolation is more conservative than quadratic. The distinctive contribution is demonstrating that even conservative linear estimation surpasses the classical baseline, providing a robust lower bound. Additionally, uncertainty quantification via bootstrap provides statistical rigor frequently absent in ZNE studies.

Quantum applications for environmental optimization and sustainability: The application of quantum computing to sustainability problems remains in an embryonic stage. Liu and Tang (2023) reviewed the state-of-the-art in power grid optimization, highlighting that most current proposals still reside in theoretical frameworks validated only on simulators. Ajagekar et al. (2019) explored VQE for carbon capture materials on 5-qubit hardware, while Bova et al. (2021) identified portfolio optimization as a promising application, although implementations remained on simulators or at a smaller scale.

This work is distinguished by addressing a real environmental problem with empirical geospatial data, realistic operational constraints (k=28 from 88 candidates) and execution on intermediate-scale hardware. To the best of our knowledge, this constitutes the first work to demonstrate consistent quantum performance superiority over tested heuristics for an environmental optimization problem executed on NISQ hardware.

5.3 Limitations and Scope

The results demonstrate empirical performance improvement of QAOA+ZNE over tested classical heuristics, but do not constitute a demonstration of quantum advantage in the formal sense of computational complexity theory. This distinction is fundamental for appropriate interpretation.

Establishing formal quantum advantage requires demonstrating that the quantum algorithm offers superpolynomial speedup or capability fundamentally inaccessible to classical methods (Preskill, 2018). This would demand: (i) comparison against optimal solvers (Gurobi, CPLEX) to establish absolute optimality gap; (ii) demonstration of favorable scaling showing that the performance ratio improves with problem size; (iii) proof or strong evidence that no efficient classical algorithm can achieve comparable quality (Huang et al., 2021; Abbas et al., 2023). None of these requirements are satisfied by the present work.

The objective of this research, is not to establish formal quantum advantage, but rather to demonstrate practical quantum utility: the capacity of a quantum algorithm to surpass widely used classical heuristics on real problems with empirical data. This distinction is relevant because, in operational contexts of environmental planning, managers rarely have access to commercial mixed-integer programming solvers or computational time for exhaustive search. Heuristics such as Greedy and Simulated Annealing constitute the

practical baselines against which new approaches must be evaluated. Comparison with Gurobi/CPLEX remains an important direction for future work, enabling quantification of the optimality gap and assessment of how close QAOA+ZNE approaches the global optimum.

What has been established is empirical quantum utility for this specific problem instance: with particular QUBO formulation, executed on specific IBM Quantum hardware, the quantum method consistently surpassed two classical reference heuristics by approximately 30% on average. This improvement is reproducible and statistically significant ($p < 0.02$), but remains empirically contingent. ZNE extrapolation adds a layer of uncertainty: performance under ideal conditions is estimated, but error-free circuits are not actually executed.

While the results present a promising success rate, they must be interpreted through the lens of limitations inherent to the current state of quantum computing. The sample size of six runs, although statistically relevant, reflects hardware access constraints, while use of the ZNE technique focuses on estimating ideal scores without, however, providing the definitive list of municipalities, which remains tied to noisy measurements ($\lambda=1$).

This experimental nature is evidenced by the coefficient of variation of 12.2%, a direct reflection of instability and stochastic noise characteristic of the NISQ era. Furthermore, the absence of a benchmark against high-performance classical solvers, such as Gurobi or CPLEX, prevents precise quantification of the optimality gap. Finally, the scale of $n=88$, despite being functional for this cluster in Goias, does not yet attest to model scalability for industrial challenges demanding processing of hundreds or thousands of variables.

Regarding problem scale, the instance with $n=88$ variables is situated among the largest QAOA implementations on real quantum hardware reported in the literature up to 2025. Systematic reviews show typical operation with $n < 50$ variables (He et al., 2024; Chen et al., 2024). IBM's own documentation acknowledges: "most research in quantum heuristics for combinatorial optimization is tested with classical simulations allowing only a small number of qubits, typically around 20" (IBM Quantum Documentation, 2024).

The largest identified QAOA experiment on real hardware uses 109 qubits for MaxCut problems with fixed parameters (Montanez-Barrera & Michielsen, 2025). It presents, yet, significantly simpler structure than our multi-objective problem with quadratic spatial synergy terms. While $n=88$ does not represent the absolute scale limit, it constitutes one of the largest (and possibly the most complex in structure) QAOA implementations on NISQ hardware for real environmental applications.

The results indicate practical quantum utility within appropriate context. The 100% success rate suggests that observed performance is not a statistical artifact but reflects a reproducible property. For practical environmental planning applications where decisions must be made within weeks and access to large-scale optimization infrastructure is limited,

a 30% improvement over simple heuristics can translate into substantially more efficient allocation of mitigation resources.

5.4 Implications for Environmental Applications

This work demonstrates a convergence between quantum computing research and environmental conservation practice that extends beyond algorithmic validation. While most QAOA implementations target abstract benchmarks or simulated problems, this study addresses a real-world policy challenge: optimizing carbon credit portfolios to maximize climate mitigation in the Brazilian Cerrado, a biodiversity hotspot under severe anthropogenic pressure.

The successful execution of 88-qubit circuits on NISQ hardware—among the largest real-hardware QAOA demonstrations reported (He et al., 2024; Chen et al., 2024)—combined with rigorous error mitigation and statistical validation, establishes a technical foundation for quantum optimization in environmental decision-making.

The interdisciplinary nature of this research—conducted by an environmental scientist rather than a quantum computing specialist—demonstrates that NISQ technology is approaching practical accessibility for domain experts beyond traditional quantum information science communities.

This aligns with Preskill's characterization of the NISQ era as a period of exploration for practical quantum applications rather than pursuit of computational supremacy (Preskill, 2018). The consistent 29.9% improvement over classical heuristics, while not constituting formal quantum advantage, represents precisely the kind of "useful" quantum computation that justifies continued NISQ investment.

From an operational perspective, carbon credit portfolio optimization constitutes a practical problem faced by subnational governments, development agencies andenvironmental NGOs in Brazil. Programs such as Fundo Clima (BNDES), state-level Payment for Environmental Services andNDC implementation under the Paris Agreement require efficient resource allocation among hundreds of candidate municipalities. A 29.9% improvement in allocation efficiency would translate into substantial increases in public investment effectiveness and potentially millions of additional tons of sequestered CO_2.

The demonstrated utility occurs despite severe NISQ hardware limitations (error rates $\varepsilon_2 \approx 0.8\%$, coherence times $T_2 \approx 50\text{-}80$ μs). Vendor roadmaps project error rates below 0.1% by 2026-2027 with moderate-scale surface code error correction. Under these projections, the gap between ZNE-extrapolated and physically realizable performance narrows, potentially eliminating mitigation overhead and enabling deeper circuits within a 3-5 year horizon for operational deployment.

The multi-objective QUBO formulation naturally generalizes to other territorial planning problems: forest restoration maximizing fragment connectivity, protected Area network design balancing ecosystem representativeness and socioeconomic viability, agroecological transition incentives considering spatial synergies and ecological corridor optimization for threatened fauna. These problems share similar combinatorial structure where QAOA has demonstrated competitiveness.

Long-term scalability (5-15 years) with advanced error correction (>1000 logical qubits) may enable industrial-scale optimization with n=500-1000 variables, relevant for national NDC portfolios, biome-wide land use planning (Amazon, Cerrado, Atlantic Forest), multi-country climate mitigation strategies and Pan-American conservation networks.

6. Conclusion

This work demonstrated practical quantum utility for carbon credit portfolio optimization in the Brazilian Cerrado. In six independent runs on IBM Quantum hardware (ibm_torino, ibm_fez), QAOA combined with Zero Noise Extrapolation surpassed the Greedy baseline in 100% of attempts, with average improvement of 29.9%. This is the first demonstration of consistent quantum superiority over classical heuristics on an environmental conservation problem with real geospatial data.

The development trajectory revealed cumulative contributions. Basic QAOA initially ranked third. With warm-start and XY-mixer, it reached parity (approximately 98% of baseline). Application of ZNE elevated performance to 130% of baseline, demonstrating that informed initialization, NISQ-adapted circuit design and error mitigation contribute synergistically to the final result.

The results establish a technical foundation for quantum computing-based climate governance. While absolute quantum advantage remains on the horizon for larger scales, this study validates that NISQ limitations can be circumvented for multi-objective problems with immediate environmental relevance.

As future work, we propose: (i) comparison with exact solvers (Gurobi/CPLEX) to quantify optimality gap; (ii) investigation of scaling for $n > 100$; (iii) exploration of advanced mitigation techniques (Probabilistic Error Cancellation, Clifford Data Regression); (iv) extension to correlated territorial planning problems; (v) implementation on next-generation hardware with quantum error correction.

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During the preparation of this work, the author used AI tools for literature curation and linguistic refinement. All content was reviewed and edited by the author, who assumes full responsibility for the final manuscript.

Data Availability

The code and data used in this study are available at https://github.com/hgribeirogeo/qaoa-carbon-cerrado (DOI: 10.5281/zenodo.18418054). This includes the QAOA implementation with warm-start initialization and XY-mixer, Zero Noise Extrapolation protocol, experimental results from all six independent runs on IBM Quantum hardware (ibm_torino, ibm_fez) andanalysis scripts for complete reproducibility. Data files include: goias_multiobjective.csv (municipal scores for carbon sequestration, biodiversity andsocial impact), adjacency_matrix.npy (spatial relationships), bioma_synergy_matrix.npy and social_synergy_matrix.npy (synergy matrices) and resultados_qaoa_zne_artigo.json (complete experimental results with metadata). The biomass estimation model for Goiás municipalities is available at https://github.com/hgribeirogeo/atlas-biomassa-goias with an interactive dashboard at https://atlas-biomassa-goias.streamlit.app/.

Competing Interests

The author declares no competing financial or non-financial interests.

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