Introductgion to Artificial Intelligence - Homework 2

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1 If there are multiple winners you should list all of them. Please briefly describe your calculations(or proofs). You will get 0 point without calculations. Consider the following profile P:

$$P = 10@[a \succ b \succ c \succ d] + 7@[d \succ a \succ b \succ c] + 6@[c \succ d \succ a \succ b] + 3@[b \succ c \succ d \succ a]$$

a Calculate the winner for plurality

$$a_{points} = 10(1) + 7(0) + 6(0) + 3(0) = 10$$
$$b_{points} = 10(0) + 7(0) + 6(0) + 3(0) = 0$$
$$c_{points} = 10(0) + 7(0) + 6(1) + 3(0) = 6$$
$$d_{points} = 10(0) + 7(1) + 6(0) + 3(0) = 7$$

Since a (10) has the most 1^{st} place votes, it is the plurality winner

b Calculate the winner for borda

$$a_{points} = 10(4) + 7(3) + 6(2) + 3(1) = 76$$
$$b_{points} = 10(3) + 7(2) + 6(1) + 3(2) = 56$$
$$c_{points} = 10(2) + 7(1) + 6(4) + 3(3) = 60$$
$$d_{points} = 10(1) + 7(4) + 6(3) + 3(2) = 62$$

A is the winner, because it has the most points by the borda system.

c Calculate the winner for veto

$$a_{veto} = 10(0) + 7(0) + 6(0) + 3(1) = 3$$

$$b_{veto} = 10(0) + 7(0) + 6(1) + 3(0) = 6$$

$$c_{veto} = 10(0) + 7(1) + 6(0) + 3(0) = 7$$

$$d_{veto} = 10(1) + 7(0) + 6(0) + 3(0) = 10$$

A is the winner, because it was vetoed the least

d Calculate the winner for plurality with runoff

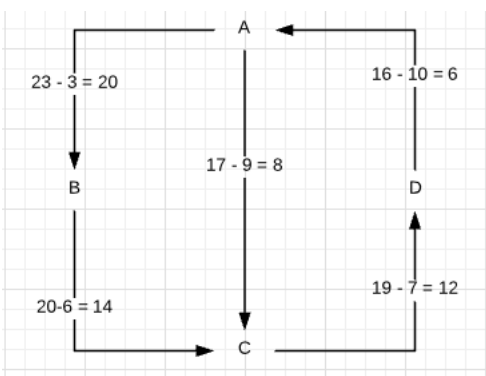
Two top winners: A and D New election calculation

$$a_{points} = 10(4) + 7(0) + 6(0) + 3(0) = 10$$

$$d_{points} = 10(0) + 7(1) + 6(1) + 3(1) = 16$$

D is the new winner of plurality with runoff.

e Draw the weighted majority graph. You only need to show positive edges and weights



f Calculate the winner(s) for Copeland

A won the most pairwise elections.

2 Let the voting rule be STV

a Consider the following profile

$$27@[a \succ b \succ c] + 42@[c \succ a \succ b] + 24@[b \succ c \succ a]$$

What paradoxical outcome occurs when four votes switch from $a \succ b \succ c$ to $c \succ a \succ b$? After this change, after the first round, a gets knocked out instead of b. After the second round, b is the winner, because it has 51 first place votes to c's 42. Originally, c would have won with 42 + 24 votes to a's 27.

b For the same profile in (a), what paradoxical outcome occurs when four voters with $a \succ b \succ c$ dont vote?

In this case, the first round of voting boots a, instead of b. After the second round, b would win again, because it would have 24 + 23 votes to c's 42.

3 Prove that for any profile P, let $WMG(P)$ denote the weighted majority graph. Prove that one of the following two cases must hold: (1) weights on all edges of in $WMG(P)$ are even numbers; or (2) weights on all edges of in $WMG(P)$ are odd numbers.

Let there be N votes in $WMG(P)$.
For any pair of candidates x, y, let W_{xy} denote the votes that prefer x to y. We know that the weighted edge WMG_{xy} between x and y equals $W_{xy} - W_{yx}$. Additionally, $N = W_{xy} + W_{yx}$.
From this, $N - W_{yx} = W_{xy}$ and $N - W_{xy} = W_{yx}$ We can expand this to

$$N - W_{yx} = WMG_{xy} + W_{yx} => N - WMG_{xy} = 2W_{yx}$$

and

$$N - (W_{xy} - WMG_{xy}) = W_{xy} => N + WMG_{xy} = 2W_{xy}$$

Since $2W_{yx}$ is always even, $N - WMGxy$ must also be even. If N is odd, then all WMG_{xy} must be odd. If N is even, all WMG_{xy} must also be even.

4 Consider the following game. Compute an NE by iteratively removing dominated state

First we eliminate C, because it is strongly dominated by U for player 1. Then we eliminate R, because it is dominated by M and L for player 2. We eliminate D, because U performs just as well or better for player 1. We then eliminate L, because it is dominated by M for player two. We are thus left with U,M = 1,3

5 Problem

a Suppose n = 4 and m = 3; s1 = 100; s2 = 60; s3 = 40; v1 = 10; v2 = 9; v3 = 7; v4 = 1. Show that GSP is not truthful in this case. That is, when all bidders report truthfully, at least one of the bidders have incentive to lie. Identify all bidders who have incentive to lie.

Bidder 1: $100(10-9) = \$100$
Bidder 2: $60(9-7) = \$120$
Bidder 3: $40(7-1) = \$240$
Bidder 4: $0(1-NA) = \$0$

Bidders 1 and 2 have incentive to lie, because their utility, if they report truthfully, is less than if they under-bid their true value. I.e. bidder 1 can bid 8 and his/her utility becomes $60(10-7) = \$180$. Bidder 1 may also bid 6, and his/her utility would become $40(10-1) = \$360$. Finally, Bidder 2 may bid 6 and increase his/her utility to $40(9-1) = \$320$.

b What is the VCG outcome and payments (note: pay-per-click payments, not the total payments) given that all bidders report their true values?

VCG for agent j

$$p_j = max_a \sum_{i \neq j} v_i(a) - \sum_{i \neq j} v_i(a^*)$$

Where $a^* = argmax_a SCW(a)$
and $SCW(a) = \sum_j v_j^*(a)$

The outcome after bidding is
bidder 1: 100 clicks
bidder 2: 60 clicks
bidder 3: 40 clicks
bidder 4: 0 clicks
Bidder 1: $(100(9) + 60(7) + 40(1)) - (60(9) + 40(7)) = 540 => \frac{540}{100}$ pay per click = 5.4 pay per click
Bidder 2: $(100(10) + 60(7) + 40(1)) - (100(10) + 40(7)) = 180 => \frac{180}{100}$ pay per click = 1.8 pay per click
Bidder 3: $(100(10) + 60(9) + 40(1)) - (100(10) + 60(9)) = 40 => \frac{40}{100}$ pay per click = 0.4

pay per click
Bidder 4: $(100(10) + 60(9) + 40(7)) - (100(10) + 60(9) + 7(40)) = 0 => \frac{0}{100}$ pay per click $= 0.0$ pay per click