

1. Problem 1

a.

| Q1(A,1) | Q1(A,2) | Q1(A,3) | Q1(B,1) | Q1(B,2) | Q1(B,3) |
|---------|---------|---------|---------|---------|---------|
| 0 | 2 | 0 | 6 | 0 | 3 |
| Q2(A,1) | Q2(A,2) | Q2(A,3) | Q2(B,1) | Q2(B,2) | Q2(B,3) |
| 6 | 4 | 4 | 10 | 2 | 7 |

b. Let $\pi^*(s) = \operatorname{argmax}_a Q^*(s, a)$

The optimal policy at the i th iteration is the highest q-state utility starting at an action a

$\pi_1(A) = 2$, $Q_1(A, 2)$ has the max of 2

$\pi_2(B) = 1$, $Q_1(B, 1)$ has the max of 6

$\pi_1(A) = 1$, $Q_1(A, 1)$ has the max of 6

$\pi_2(B) = 1$, $Q_1(B, 1)$ has the max of 10

2. Problem 2

$$Q(s, a) = (1 - \alpha) * Q(s, a) + \alpha * [R + \gamma \max_{a'} Q(s', a')]$$

$$Q(s_1, up) = .2 * 4 + .8 * [0 + .3 * 8] = 2.72$$

$$Q(s_4, right) = .2 * 8 + .8 * [0 + .3 * 16] = 5.44$$

$$Q(s_5, down) = .2 * 4 + .8 * [0 + .3 * 10] = 3.2$$

$$Q(s_2, right) = .2 * 10 + .8 * [0 + .3 * 20] = 6.8$$

$$Q(s_3, up) = .2 * 20 + .8 * [20 + .3 * 0] = 20.0$$

3. Problem 3

a. $P(A) = P(X_1 = A) = P(X_1 = A|X_0 = A) * P(X_0 = A) + P(X_1 = A|X_0 = B) * P(X_0 = B) = 0.7$

$P(B) = P(X_1 = B) = P(X_1 = B|X_0 = A) * P(X_0 = A) + P(X_1 = B|X_0 = B) * P(X_0 = B) = 0.3$

b. $P(A) = P(X = A) = P(X = A|X_{-1} = A) * P(X = A) + P(X = A|X_{-1} = B) * P(X = B)$

$P(B) = P(X = B) = P(X = B|X_{-1} = A) * P(X = A) + P(X = B|X_{-1} = B) * P(X = B)$

$$P(X = A|X_{-1} = A) + P(X = A|X_{-1} = B) \frac{P(X=B)}{P(X=A)} = 1$$

$$P(X = B|X_{-1} = A) * \frac{P(X=A)}{P(X=B)} + P(X = B|X_{-1} = B) = 1$$

$$.8 + .6 * \frac{P(X=B)}{P(X=A)} = .4 + .2 \frac{P(X=A)}{P(X=B)}$$

$$P(A) = 3P(B), P(A) + P(B) = 1$$

$$P(A) = .75, P(B) = .25$$

4. Problem 4

a. $P(Y|X) = \frac{P(Y)*P(X|Y)}{P(X)}$

$$P((x_1, x_2)|Y) P(x_1|Y) P(x_2|Y)$$

$$P((x_1, x_2)) = P(x_1) * P(x_2)$$

$$P(Y|(x_1, x_2)) = \frac{P(x_1|Y)*P(x_2|Y)*P(Y)}{P(x_1)*P(x_2)}$$

$$P(Y = 1|x_1 = 1, x_2 = 1) = 2 * p_1 * p_2$$

b. $P(X|Y) = \frac{\text{count}(X|Y)}{\text{count}(Y)}$
 $\text{count}(x_1 = 1|Y = 1) = 2$
 $\text{count}(Y = 1) = 3$
max likelihood of p1 is $\frac{1}{3}$
max likelihood of p2 is $\frac{1}{3}$
max likelihood of q is $\frac{3}{10}$

5. Problem 5

$$W_0 = (0, 0) = 0 * 1 + 0 * 0 = 0 \rightarrow 1 \text{ correct classification}$$

$$W_1 = (0, 0) = 0 * 4 + 0 * 2 = 0 \rightarrow 1 \text{ correct classification}$$

$$W_2 = (0, 0) = 0 * 0 + 0 * -1 = 0 \rightarrow 1 \text{ incorrect, so update}$$

$$W_3 = (-4, -2) = (-4 * -1) - (2 * -1) = 6 \rightarrow 1 \text{ incorrect, so update}$$

$$W_4 = (-3, -1) = (-3 * -2) + (-1 * 1) = 5 \rightarrow 1 \text{ incorrect, so update}$$

$$W_5 = (-1, -2)$$