CSCI 4150: Introduction to Artificial Intelligence, 2017 Spring
Homework 2: Game Theory, Mechanism Design, and Social Choice
Total points: 100. Bonus point: 10.

We only accept electronic submission at Submitty. Please try to ask questions on Piazza. If Piazza is not helpful, please contact the TAs.

Problem 1 (30 pt): If there are multiple winners you should list all of them. Please briefly describe your calculations (or proofs). You will get 0 point without calculations. Consider the following profile P:

$$P = 10@[a \succ b \succ c \succ d] + 7@[d \succ a \succ b \succ c] + 6@[c \succ d \succ a \succ b] + 3@[b \succ c \succ d \succ a]$$

1. Calculate the winner(s) for plurality.
2. Calculate the winner(s) for Borda.
3. Calculate the winner(s) for veto.
4. Calculate the winner(s) for plurality with runoff.
5. Draw the weighted majority graphs. You only need to show positive edges and their weights.
6. Calculate the winner(s) for Copeland.

Problem 2 (10 pt) Let the voting rule be STV.

1. Consider the following profile:

   $$27@[a \succ b \succ c] \qquad 42@[c \succ a \succ b] \qquad 24@[b \succ c \succ a]$$

   What paradoxical outcome occurs when four votes switch from $a \succ b \succ c$ to $c \succ a \succ b$?
2. For the same profile in (a), what paradoxical outcome occurs when four voters with $a \succ b \succ c$ don't vote?

Problem 3 (10 pt) Prove that for any profile P, let $WMG(P)$ denote the weighted majority graph. Prove that one of the following two cases must hold: (1) weights on all edges of in $WMG(P)$ are even numbers; or (2) weights on all edges of in $WMG(P)$ are odd numbers.

Problem 4 (30 pt). Consider the following game. Compute an NE by iteratively removing dominated strategies. Every time a strategy is removed, you must show which strategy (which can be a mixed strategy) dominates it.

	L	M	R
U	5, 0	1, 3	4, 0
C	2, 4	1, 4	3, 5
D	0, 1	1, 0	5, 0

Problem 5 (20pt) In an *ad auction* there are n bidders bidding for $m < n$ slots. Each bidder is interested in getting only one slot. The slots are ranked from the top (first) to the bottom (last) on the right side of a webpage. The ith slot will get s_i clicks. We can assume that $s_1 > s_2 > \cdots > s_m$.

An outcome consists in two parts: an allocation of m slots to m different bidders, and the **pay-per-click payment** p_i for the ith slot. Each bidder j has a private value v_j^* for each **click**, thus her utility for getting the ith slot is $s_i(v_j^* - p_i)$.

The *generalized second price auctions (GSP)* is a popular mechanism for ad auctions currently used at many companies including Google. It works as follows.

- Rank the bids from high to low. Let $b'_1 > b'_2 > \cdots > b'_n$ denote these bids.
- Allocate the 1st slot to the bidder with the highest bid b'_1, the 2nd slot to the bidder with the second highest bid b'_2, . . ., allocate the mth slot to the bidder with bid b'_m.
- For each $i \leq m$, let $p_i = b_{i+1}$.

Questions.

(a) Suppose $n = 4$ and $m = 3$; $s_1 = 100, s_2 = 60, s_3 = 40$; $v_1^* = 10, v_2^* = 9, v_3^* = 7, v_4^* = 1$. Show that GSP is not truthful in this case. That is, when all bidders report truthfully, at least one of the bidders have incentive to lie. Identify all bidders who have incentive to lie.

(b) What is the VCG outcome and payments (note: pay-per-click payments, not the total payments) given that all bidders report their true values?

Problem 6 Bonus question (hard 10pt): Let $\vec{s}_B = (m-1, \ldots, 0)$ denote the scoring vector for Borda.

1. Prove that for any $p > 0, q \in \mathbb{R}$, the positional scoring rule r with the scoring vector $p \cdot \vec{s}_B + q = (p(m-1) + q, p(m-2) + q, \ldots, q)$ is equivalent to Borda. That is, for any profile P, $r(P) = \text{Borda}(P)$.
2. Prove the reverse of (a). That is, prove that a position scoring rule r with scoring vector $\vec{s} = (s_1, \ldots, s_m)$ is equivalent to Borda only if there exist $p > 0, q \in \mathbb{R}$ such that $\vec{s} = p \cdot \vec{s}_B + q$.
   Hint: prove that $s_1 - s_2 = s_2 - s_3 = \cdots = s_{m-1} - s_m$.