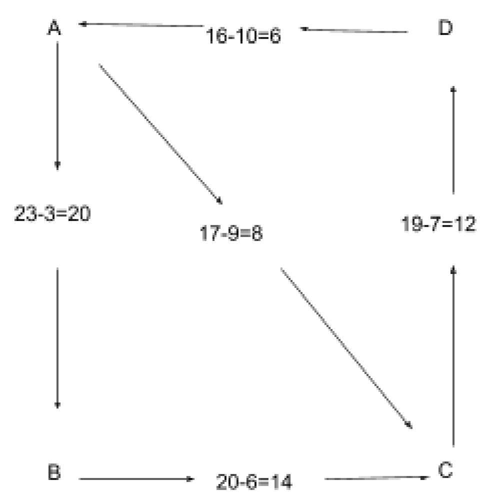
1. Problem 1
    a. Plurality - only the first place vote is counted
       a: $10 * 1 + 7 * 0 + 6 * 0 + 3 * 0 = 10$
       b: $10 * 0 + 7 * 0 + 6 * 0 + 3 * 1 = 3$
       c: $10 * 0 + 7 * 0 + 6 * 1 + 3 * 0 = 6$
       d: $10 * 0 + 7 * 1 + 6 * 0 + 3 * 0 = 7$
       a has the most so a wins.
    b. Borda - points awarded based on rank in each ordered list
       a: $10 * 4 + 7 * 3 + 6 * 2 + 3 * 1 = 76$
       b: $10 * 3 + 7 * 2 + 6 * 1 + 3 * 4 = 62$
       c: $10 * 2 + 7 * 1 + 6 * 4 + 3 * 3 = 60$
       d: $10 * 1 + 7 * 4 + 6 * 3 + 3 * 2 = 62$
       a has the most so a wins.
    c. Veto - the least times a candidate came in last
       a: $10 * 0 + 7 * 0 + 6 * 0 + 3 * 1 = 3$
       b: $10 * 0 + 7 * 0 + 6 * 1 + 3 * 0 = 6$
       c: $10 * 0 + 7 * 1 + 6 * 0 + 3 * 0 = 7$
       d: $10 * 1 + 7 * 0 + 6 * 0 + 3 * 0 = 10$
       a has the least number of vetos so a wins.
    d. Plurality with runoff - new election for two top winners based on who is ahead
       a: $10 * 1 + 7 * 0 + 6 * 0 + 3 * 0 = 10$
       d: $10 * 0 + 7 * 1 + 6 * 1 + 3 * 1 = 16$
       d wins the second election, so d wins.
    e. Draw weighted majority graph.



    f. Copeland
       A won the most pairwise elections against other candidates.

2. Problem 2 - voting rule is STV
    a. Before switch:

        first round: a: 27 b:24 c:42

        second round: a: 27 <u>c: 68</u>

       After switch:

        first round: a: 23 b: 24 c: 46

        second round: <u>b: 47</u> c: 46

       The paradoxical outcome occurs because after this switch, a gets knocked out instead of b in the first round. In the second round, this propagates to b winning over c (51 to 46) which would not have happened if the switch did not occur, where c would have won over a (68 to 27).
    b. Before switch:

        first round: a: 27 b:24 c:42

        second round: a: 27 <u>c: 68</u>

       After switch:

        first round: a: 23 b: 24 c: 42

        second round: <u>b: 47</u> c: 42

       B would win again because the same occurrence happens as in (a) where b knocks out a in the first round, then wins over c in the second round.
3. Prove that all edge weights in a WMG(P) have the same parity.

   Let there be N votes in some $WMG(P)$.

   a weighted majority graph compares any pair of candidates based on which is ahead of the other, and therefore for two candidates x and y whose number of votes over each other are $N_{x>y}$ and $N_{y>x}$ respectively, their number of votes add to the total $N_{x>y} + N_{y>x} = N$. Also let $WMG(P)_{x>y}$ be the edge weight of the weighted majority graph. Using this we know that

   $N - N_{y>x} = WMG(P)_{x>y} + N_{y>x}$, so therefore $N - WMG(P)_{x>y} = 2N_{y>x}$

   and vice versa that

   $N - (N_{x>y} - WMG(P)_{x>y}) = N_{x>y}$, so therefore $N + WMG(P)_{x>y} = 2N_{x>y}$

   Since $2N_{y>x}$ is always even, then $N - WMG(P)_{x>y}$ is also even which means that $WMG(P)_{x>y}$ shares the parity of N, proving that is is either all even or all odd for any pair of candidates.
4. Compute an NE by iteratively removing dominated state.

   C is eliminated because it is dominated by U for player 1.

   R is eliminated because it is dominated by M and L for player 2

   D is eliminated because U performs just as well or better for player 1.

   L is eliminated because it is dominated by M for player 2.

   Therefore we end with U,M = 1,3
5. Problem 5
    a. Show that the GSP is not truthful in this case.

Bidder 1: $100(10-9)=\$100$

Bidder 2: $60(9-7)=\$120$

Bidder 3: $40(7-1)=\$240$

Bidder 4: $0(1-0)=\$0$

Bidders 1 and 2 have an incentive to lie because if they report truthfully their utility is less than if they under-bid their true value, for example bidder 1 can bid 8 and their utility becomes $60(10-7)=\$180$. Bidder 1 may also bid 6, their utility will become $40(10-1)=\$360$. If bidder 2 bids 6 the utility would increase to $40(9-1)=\$320$.

b. VGG for agent j is $p_j = max_a \sum_{i \neq j} v_i(a) - \sum_{i \neq j} v_i(a^*)$ where $a^* = argmax_a SCW(a)$ and

$SCW(a) = \sum_j v_j^*(a)$

The outcome after bidding is:

bidder 1: 100 clicks

bidder 2: 60 clicks

bidder 3: 40 clicks

bidder 4: 0 clicks

bidder 1: : (100(9) + 60(7) + 40(1)) − (60(9) + 40(7)) = 540 = 5.4 pay per click

bidder 2: (100(10) + 60(7) + 40(1)) − (100(10) + 40(7)) = 180 = 1.8 pay per click

bidder 3: (100(10) + 60(9) + 40(1)) − (100(10) + 60(9)) = 40 = 0.4 pay per click

bidder 4: (100(10) + 60(9) + 40(7)) − (100(10) + 60(9) + 7(40)) = 0 pay per click