Homework 3

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I. Introduction

II. Part 1

III. Part 2

$$\begin{aligned} F_1 &: x, y \mapsto -x + f(\omega_2 y + I) \\ F_2 &: x, y \mapsto -y + f(\omega_1 x + I) \end{aligned} \tag{1}$$

$$J(x, y) = \begin{pmatrix} \frac{\partial F_1}{\partial x}(x, y) & \frac{\partial F_1}{\partial y}(x, y) \\ \frac{\partial F_2}{\partial x}(x, y) & \frac{\partial F_2}{\partial y}(x, y) \end{pmatrix} \tag{2}$$

$$\begin{aligned} \frac{\partial F_1}{\partial x}(x, y) &= \frac{\partial F_2}{\partial y}(x, y) = -1 \\ \frac{\partial F_1}{\partial y}(x, y) &= 50\omega_2 \sigma(\omega_2 y + I)(1 - \sigma(\omega_2 y + I)) \\ \frac{\partial F_2}{\partial x}(x, y) &= 50\omega_1 \sigma(\omega_1 x + I)(1 - \sigma(\omega_1 x + I)) \end{aligned} \tag{3}$$

Under $\omega_1 = \omega_2 = \omega$ and $x = y$:

$$\frac{\partial F_1}{\partial y}(x, x) = \frac{\partial F_2}{\partial x}(x, x) = 50\omega\sigma(\omega x + I)(1 - \sigma(\omega x + I)) \tag{4}$$

$$\forall x, y, \operatorname{tr}(J(x, y)) = -2 < 0 \tag{5}$$

$$\forall x, \det(J(x, x)) = 1 - 2500\omega^2 \sigma(\omega x + I)^2 (1 - \sigma(\omega x + I))^2 \tag{6}$$