1. Solve the following set of equations

$$A_i x_i = b_i, i = 1, 2, 3 \tag{1}$$

For A_1 and b_1(lineq1.dat), the solutuons for each method are

1-1. Using Gauss-Jordan Elimination(`gaussj()`)

- [nan, nan, nan, nan]

1-2. Using LU Decomposition(`ludcmp()`)

- [1.000000, -3.000000, 2.000000, 0.000000]

1-3. Using Singular Value Decomposition(`svdcmp()`)

- [1.733333, -1.533334, -0.200000, -0.733334]

For A_2 and b_2(lineq2.dat), the solutuons for each method are

1-1. Using Gauss-Jordan Elimination(`gaussj()`)

- [-2.873567, -0.612357, 0.976277, 0.635819, -0.553441]

1-2. Using LU Decomposition(`ludcmp()`)

- [-2.873566, -0.612357, 0.976277, 0.635819, -0.553441]

1-3. Using Singular Value Decomposition(`svdcmp()`)

- [-2.873566, -0.612357, 0.976278, 0.635819, -0.553441]

For A_3 and b_3(lineq3.dat), the solutuons for each method are

1-1. Using Gauss-Jordan Elimination(`gaussj()`)

- [-0.326608, 1.532293, -1.044826, -1.587447, 2.928480, -2.218931]

1-2. Using LU Decomposition(`ludcmp()`)

- [-0.326608, 1.532292, -1.044826, -1.587447, 2.928480, -2.218930]

1-3. Using Singular Value Decomposition(`svdcmp()`)

- [-0.326609, 1.532292, -1.044825, -1.587447, 2.928479, -2.218929]

Discuss empirically the advantage/disadvantage of each method

Method	Advantage	Disadvantage
Gauss-Jordan Elimination	Good for obtaining an inverse	Computations are heavy O(n^3). Cannot be used for singular one
LU Decomposition	Good for getting a solution and a determinant, Able to know whether a matrix is ill-conditioned	Substitution is needed. Cannot be used for singular one
Singular Value Decomposition	Can also be used for singular one. Can be applied to over-/under-determined.	When the dimensions are large, calculation costs are heavy

2. Apply the method of iterative improvement(`mprove()`) to the above problem and discuss the results

For A_1 and b_1(lineq1.dat), the solutuon is

- [1.000000, -3.000000, 2.000000, 0.000000]
- Because the A_1 is singular, the inverse does not exist.
- The result of LU decomposition in `mprove()` cannot be used for calculating the correct inverse, therefore.

For A_2 and b_2(lineq2.dat), the solutuon is

- [-2.873566, -0.612357, 0.976277, 0.635819, -0.553441]
- The result is same as 'Section 1'

For A_3 and b_3(lineq3.dat), the solutuon is

- [-0.326608, 1.532292, -1.044826, -1.587447, 2.928480, -2.218930]
- The result is same as 'Section 1'

3. Find the inverse and the determinant of the matrix A_i in the above problem

For A_1 and b_1(lineq1.dat), the inverse of A_1 and determinant of A_1 are

- Inverse:

$$\begin{pmatrix} -1000000002004087734272.000000 & -3750000007751532900352.000000 & 3750000007751532900352.000000 & 125000013500225945 \\ -2000000056784733601792.000000 & -75000019095251845120.000000 & 75000010299158822912.000000 & 250000048990684446 \\ 3000000041196635291648.000000 & 112500015448738234368.000000 & -112500006652645212160.000000 & -375000051495794114 \\ 1000000002004087734272.000000 & 3750000007751532900352.000000 & -37499996353486389248.000000 & -125000013500225945 \end{pmatrix}$$

- Determinant: -0.000000

For A_2(lineq2.dat), the inverse of A_2 and determinant of A_2 are

- Inverse:

$$\begin{pmatrix} 0.354536 & 0.766945 & 0.207769 & -0.595412 & 0.253128 \\ 0.035454 & 0.126695 & 0.195777 & -0.159541 & 0.050313 \\ -0.138686 & -0.098540 & -0.096715 & 0.124088 & 0.016423 \\ -0.052138 & -0.303962 & -0.023201 & 0.234619 & -0.044578 \\ 0.149114 & 0.459333 & 0.051356 & -0.171011 & 0.042492 \end{pmatrix} \tag{3}$$

- Determinant: 3835.999512

For A_3(lineq3.dat), the inverse of A_3 and determinant of A_3 are

- Inverse:

$$\begin{pmatrix} -0.162205 & 0.122801 & 0.024068 & -0.016431 & -0.022840 & 0.046132 \\ 0.169407 & -0.041117 & 0.228313 & -0.087624 & 0.180306 & -0.395655 \\ -0.011636 & 0.122745 & -0.117407 & -0.180981 & 0.015910 & 0.186766 \\ 0.105669 & -0.051726 & -0.108916 & 0.299774 & 0.000859 & -0.190541 \\ -0.053026 & -0.042362 & 0.160508 & -0.224034 & 0.161811 & 0.015024 \\ -0.062341 & -0.064694 & -0.234216 & 0.351126 & -0.364828 & 0.434633 \end{pmatrix} \tag{4}$$

- Determinant: 16178.401367

How to compile and run

1. `./build.sh`
2. `./main lineq1.dat` to run with 'lineq1.dat'
3. `./main lineq2.dat` to run with 'lineq2.dat'
4. `./main lineq3.dat` to run with 'lineq3.dat'