Calculus - Chapter 9.4 Representative Exercises (Models for Population Growth)

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1. **Exercise 1:** A population grows according to the given logistic equation, where t is measured in weeks.

$$\frac{dP}{dt} = 0.04P\left(1 - \frac{P}{1200}\right), \quad P(0) = 60$$

   (a) What is the carrying capacity? What is the value of k?

   (b) Write the solution of the equation.

   (c) What is the population after 10 weeks?

2. **Exercise 2:** A population grows according to the given logistic equation, where t is measured in weeks.

$$\frac{dP}{dt} = 0.02P - 0.0004P^2, \quad P(0) = 40$$

3. **Exercise 6:** Suppose a population $P(t)$ satisfies $\frac{dP}{dt} = 0.4P - 0.001P^2, P(0) = 50$, where t is measured in years.

   (a) What is the carrying capacity?

   (c) When will the population reach 50% of the carrying capacity?

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4. **Exercise 3:** Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$, where t is measured in weeks.

   (a) What is the carrying capacity? What is the value of k?

   (c) Use the direction field to sketch solutions for initial populations of 20, 40, 60, 80, 120, and 140. Which solutions have inflection points?

   (d) What are the equilibrium solutions?

5. **Exercise 5:** The Pacific halibut fishery has been modeled by the differential equation $\frac{dy}{dt} = ky(1 - \frac{y}{M})$ where $y(t)$ is the biomass, $M = 8 \times 10^7$ kg, and $k = 0.71$ per year.
   (a) If $y(0) = 2 \times 10^7$ kg, find the biomass a year later.
   (b) How long will it take for the biomass to reach 4×10^7 kg?
6. **Exercise 7:** Suppose a population grows according to a logistic model with initial population 1000 and carrying capacity 10,000. If the population grows to 2500 after one year, what will the population be after another three years?
7. **Exercise 9:** The population of the world was about 6.1 billion in 2000. Assume the carrying capacity is 20 billion.
   (a) Write the logistic differential equation for these data.
   (b) Use the logistic model to estimate the world population in the year 2010.
8. **Exercise 11:** One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not.
   (a) Write a differential equation that is satisfied by y.
   (b) Solve the differential equation.
9. **Exercise 17:** A population is modeled by $\frac{dP}{dt} = kP - m$ where $k = \alpha - \beta$.
   (a) Find the solution that satisfies $P(0) = P_0$.
   (b) What condition on m will lead to an exponential expansion?

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10. **Exercise 13:**
    (a) Show that if P satisfies the logistic equation, then $\frac{d^2P}{dt^2} = k^2P\left(1 - \frac{P}{M}\right)\left(1 - \frac{2P}{M}\right)$.
    (b) Deduce that a population grows fastest when it reaches half its carrying capacity.
11. **Exercise 19:** Consider the modified logistic equation: $\frac{dP}{dt} = 0.08P\left(1 - \frac{P}{1000}\right) - 15$.
    (a) Explain the meaning of the term -15 if $P(t)$ is a fish population.
    (c) What are the equilibrium solutions?
    (d) Describe what happens to the fish population for various initial populations.
12. **Exercise 21:** Consider the modified logistic model $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right)$.
    (a) Show that any solution is increasing if $m < P < M$ and decreasing if $0 < P < m$.
    (b) For $k = 0.08$, $M = 1000$, and $m = 200$, draw a direction field and sketch several solution curves. What are the equilibrium solutions?