Chapter 11.8 Exercises: Power Series

James Stewart, Calculus, Metric Edition

Difficulty: Easy (5 Problems)

1. **Exercise 3:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

2. **Exercise 5:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \sqrt{n} x^n$$

3. **Exercise 7:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{n}{5^n} x^n$$

4. **Exercise 13:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

5. **Exercise 15:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$$

Difficulty: Medium (11 Problems)

6. **Exercise 9:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

7. **Exercise 11:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$$

8. **Exercise 17:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{\sqrt{n}} x^n$$

9. **Exercise 20:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$$

10. **Exercise 21:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

11. **Exercise 23:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln n}$$

12. **Exercise 26:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$$

13. **Exercise 37:** If $\sum_{n=0}^{\infty} c_n 4^n$ is convergent, can we conclude that each of the following series is convergent?
$$(a) \sum_{n=0}^{\infty} c_n (-2)^n \qquad (b) \sum_{n=0}^{\infty} c_n (-4)^n$$

14. **Exercise 38:** Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?
$$(a) \sum_{n=0}^{\infty} c_n \qquad (b) \sum_{n=0}^{\infty} c_n 8^n \qquad (c) \sum_{n=0}^{\infty} c_n (-3)^n \qquad (d) \sum_{n=0}^{\infty} (-1)^n c_n 9^n$$

15. **Exercise 44:** Suppose that the power series $\sum c_n (x-a)^n$ satisfies $c_n \neq 0$ for all n. Show that if $\lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| = R$, then R is the radius of convergence.

16. **Exercise 45:** Suppose the series $\sum c_n x^n$ has radius of convergence 2 and the series $\sum d_n x^n$ has radius of convergence 3. What is the radius of convergence of the series $\sum (c_n + d_n) x^n$?

Difficulty: Hard (5 Problems)

17. **Exercise 29:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n, \quad b>0$$

18. **Exercise 31:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} n!(2x-1)^n$$

19. **Exercise 32:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$$

20. **Exercise 34:** Find the radius of convergence and interval of convergence of the power series.
$$\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$$

21. **Exercise 43:** Show that if $\lim_{n\to\infty} \sqrt[n]{|c_n|} = c$, where $c \neq 0$, then the radius of convergence of the power series $\sum c_n x^n$ is $R = 1/c$.