

## Calculus - Chapter 10.1 Exercises

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### 난이도 하

1. **Exercise 3:** Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as  $t$  increases.

$$x = 1 - t^2, \quad y = 2t - t^2, \quad -1 \leq t \leq 2$$

2. **Exercise 7:** (a) Sketch the curve by using the parametric equations to plot points. (b) Eliminate the parameter to find a Cartesian equation of the curve.

$$x = 2t - 1, \quad y = \frac{1}{2}t + 1$$

3. **Exercise 9:** (a) Sketch the curve. (b) Eliminate the parameter to find a Cartesian equation.

$$x = t^2 - 3, \quad y = t + 2, \quad -3 \leq t \leq 3$$

4. **Exercise 11:** (a) Sketch the curve. (b) Eliminate the parameter to find a Cartesian equation.

$$x = \sqrt{t}, \quad y = 1 - t$$

5. **Exercise 13:** (a) Eliminate the parameter to find a Cartesian equation of the curve. (b) Sketch the curve and indicate the direction.

$$x = 3 \cos t, \quad y = 3 \sin t, \quad 0 \leq t \leq \pi$$

6. **Exercise 21:** (a) Eliminate the parameter. (b) Sketch the curve.

$$x = \sin^2 t, \quad y = \cos^2 t$$

7. **Exercise 37(b):** Find parametric equations to represent the line segment from  $(-2, 7)$  to  $(3, -1)$ .
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8. **Exercise 15:** (a) Eliminate the parameter. (b) Sketch the curve.

$$x = \cos \theta, \quad y = \sec^2 \theta, \quad 0 \leq \theta < \pi/2$$

9. **Exercise 19:** (a) Eliminate the parameter. (b) Sketch the curve.

$$x = \ln t, \quad y = \sqrt{t}, \quad t \geq 1$$

10. **Exercise 25:** Describe the motion of a particle with position  $(x, y)$  as  $t$  varies in the given interval.

$$x = 5 + 2 \cos(\pi t), \quad y = 3 + 2 \sin(\pi t), \quad 1 \leq t \leq 2$$

11. **Exercise 27:** Describe the motion of a particle with position  $(x, y)$  as  $t$  varies in the given interval.

$$x = 5 \sin t, \quad y = 2 \cos t, \quad -\pi \leq t \leq 5\pi$$

12. **Exercise 34(a, b):** Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices.

(a)  $x = t^4 - t + 1, \quad y = t^2$

(b)  $x = t^2 - 2t, \quad y = \sqrt{t}$

13. **Exercise 34(c, d):** Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices.

(c)  $x = t^3 - 2t, \quad y = t^2 - t$

(d)  $x = \cos(5t), \quad y = \sin(2t)$

14. **Exercise 41(a, c):** Find parametric equations for the path of a particle that moves along the circle  $x^2 + (y - 1)^2 = 4$  in the manner described.

(a) Once around clockwise, starting at  $(2, 1)$ .

(c) Halfway around counterclockwise, starting at  $(0, 3)$ .

15. **Exercise 42(a):** Find parametric equations for the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

16. **Exercise 45(b):** Sketch the graph of each curve and explain how the curves differ from one another. (i)  $x = t^2, y = t$  (ii)  $x = t, y = \sqrt{t}$

17. **Exercise 49:** A curve traced out by a point P at a distance  $d$  from the center of a circle of radius  $r$  as the circle rolls along a straight line is called a trochoid. Show that its parametric equations are:

$$x = r\theta - d \sin \theta, \quad y = r - d \cos \theta$$

18. **Exercise 58(a):** If a gun is fired with  $\alpha = 30^\circ$  and  $v_0 = 500$  m/s, when will the bullet hit the ground? How far from the gun will it hit the ground?

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19. **Exercise 28:** Describe the motion of a particle with position  $(x, y)$  as  $t$  varies in the given interval.

$$x = \sin t, \quad y = \cos^2 t, \quad -2\pi \leq t \leq 2\pi$$

20. **Exercise 34(e, f):** Match the parametric equations with the graphs labeled I-VI.

(e)  $x = t + \sin(4t), \quad y = t^2 + \cos(3t)$

(f)  $x = t + \sin(2t), \quad y = t + \sin(3t)$

21. **Exercise 51:** Find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle  $\theta$  as the parameter.
22. **Exercise 53:** A curve, called a witch of Maria Agnesi, consists of all possible positions of the point P in the figure. Show that parametric equations for this curve can be written as:

$$x = 2a \cot \theta, \quad y = 2a \sin^2 \theta$$

23. **Exercise 54:** Find parametric equations for the set of all points P as shown in the figure such that  $|OP| = |AB|$ . (This curve is called the cissoid of Diocles).
24. **Exercise 56:** The position of one particle is  $x = 3 \sin t, y = 2 \cos t$  and a second particle is  $x = -3 + \cos t, y = 1 + \sin t$  for  $0 \leq t \leq 2\pi$ .
- (a) Graph the paths. At how many points do they intersect?
- (b) Do the particles collide? If so, find the collision points.
25. **Exercise 59:** Investigate the family of curves defined by the parametric equations  $x = t^2, y = t^3 - ct$ . How does the shape change as  $c$  increases?