Calculus - Chapter 10.4 Exercises

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1. **Exercise 1:** Find the area of the region that is bounded by the given curve and lies in the specified sector.

$$r = \sqrt{2\theta}, \quad 0 \leq \theta \leq \pi/2$$

2. **Exercise 9:** Sketch the curve and find the area that it encloses.

$$r = 4\cos\theta$$

3. **Exercise 17:** Find the area of the region enclosed by one loop of the curve.

$$r = 4\cos(3\theta)$$

4. **Exercise 23:** Find the area of the region that lies inside the first curve and outside the second curve.

$$r = 4\sin\theta, \quad r = 2$$

5. **Exercise 29:** Find the area of the region that lies inside both curves.

$$r = 3\sin\theta, \quad r = 3\cos\theta$$

6. **Exercise 49:** Find the exact length of the polar curve.

$$r = 2\cos\theta, \quad 0 \leq \theta \leq \pi$$

7. **Exercise 63:** Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

$$r = 2\cos\theta, \quad \theta = \pi/3$$

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8. **Exercise 11:** Sketch the curve and find the area that it encloses.

$$r = 3 - 2\sin\theta$$

9. **Exercise 13:** Graph the curve and find the area that it encloses.

$$r = 2 + \sin(4\theta)$$

10. **Exercise 21:** Find the area of the region enclosed by the inner loop of the curve.

$$r = 1 + 2\sin\theta$$

11. **Exercise 27:** Find the area of the region that lies inside the first curve and outside the second curve.

$$r = 3\cos\theta, \quad r = 1 + \cos\theta$$

12. **Exercise 31:** Find the area of the region that lies inside both curves.

$$r = \sin(2\theta), \quad r = \cos(2\theta)$$

13. **Exercise 35:** Find the area inside the larger loop and outside the smaller loop of the limaçon $r = \frac{1}{2} + \cos\theta$.

14. **Exercise 37:** Find all points of intersection of the given curves.

$$r = \sin\theta, \quad r = 1 - \sin\theta$$

15. **Exercise 39:** Find all points of intersection of the given curves.

$$r = 2\sin(2\theta), \quad r = 1$$

16. **Exercise 51:** Find the exact length of the polar curve.

$$r = \theta^2, \quad 0 \le \theta \le 2\pi$$

17. **Exercise 52:** Find the exact length of the polar curve.

$$r = 2(1 + \cos\theta)$$

18. **Exercise 55:** Find the exact length of the curve. Use a graph to determine the parameter interval.

$$r = \cos^4(\theta/4)$$

19. **Exercise 65:** Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

$$r = 1/\theta, \quad \theta = \pi$$

20. **Exercise 67:** Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

$$r = \cos(2\theta), \quad \theta = \pi/4$$

21. **Exercise 69:** Find the points on the given curve where the tangent line is horizontal or vertical.

$$r = \sin\theta$$

22. **Exercise 71:** Find the points on the given curve where the tangent line is horizontal or vertical.

$$r = 1 + \cos\theta$$

23. **Exercise 72:** Find the points on the given curve where the tangent line is horizontal or vertical.

$$r = e^{\theta}$$

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24. **Exercise 22:** Find the area enclosed by the loop of the strophoid $r = 2\cos\theta - \sec\theta$.

25. **Exercise 36:** Find the area between a large loop and the enclosed small loop of the curve $r = 1 + 2\cos(3\theta)$.

26. **Exercise 41:** Find all points of intersection of the given curves.

$$r^2 = 2\cos(2\theta), \quad r = 1$$

27. **Exercise 47:** The points of intersection of the cardioid $r = 1 + \sin\theta$ and the spiral loop $r = 2\theta, -\pi/2 \leq \theta \leq \pi/2$, can't be found exactly. Use a graph to find the approximate values of θ at which the curves intersect. Then use these values to estimate the area that lies inside both curves.

28. **Exercise 48:** When recording live performances, sound engineers often use a microphone with a cardioid pickup pattern... Suppose the microphone is placed 4 m from the front of the stage and the boundary of the optimal pickup region is given by the cardioid $r = 8 + 8\sin\theta$... The musicians want to know the area they will have on stage within the optimal pickup range... Answer their question.

29. **Exercise 73:** Let P be any point (except the origin) on the curve $r = f(\theta)$. If ψ is the angle between the tangent line at P and the radial line OP, show that

$$\tan\psi = \frac{r}{dr/d\theta}$$

30. **Exercise 75:** (a) Use Formula 10.2.9 to show that the area of the surface generated by rotating the polar curve $r = f(\theta), a \leq \theta \leq b$ about the polar axis is

$$S = \int_a^b 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

(b) Use the formula in part (a) to find the surface area generated by rotating the lemniscate $r^2 = \cos(2\theta)$ about the polar axis.