Calculus - Chapter 10.2 Exercises

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1. **Exercise 1:** Find dy/dx.

$$x = 2t^3 + 3t, \quad y = 4t - 5t^2$$

2. **Exercise 3:** Find dy/dx.

$$x = te^t, \quad y = t + \sin t$$

3. **Exercise 7:** Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = t^3 + 1, \quad y = t^4 + t; \quad t = -1$$

4. **Exercise 10:** Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = e^t \sin(\pi t), \quad y = e^{2t}; \quad t = 0$$

5. **Exercise 5:** Find the slope of the tangent to the parametric curve at the indicated point.

$$x = t^2 + 2t, \quad y = 2^t - 2t; \quad (15, 2)$$

6. **Exercise 35:** Find the area enclosed by the parametric curve and the x-axis.

$$x = t^3 + 1, \quad y = 2t - t^2$$

7. **Exercise 47:** Find the exact length of the curve.

$$x = \frac{2}{3}t^3, \quad y = t^2 - 2, \quad 0 \leq t \leq 3$$

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8. **Exercise 13:** Find an equation of the tangent to the curve at the given point. Then graph the curve and the tangent.

$$x = t^2 - t, \quad y = t^2 + t + 1; \quad (0, 3)$$

9. **Exercise 29:** Show that the curve $x = \cos t, y = \sin t \cos t$ has two tangents at $(0, 0)$ and find their equations.

10. **Exercise 15:** Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

$$x = t^2 + 1, \quad y = t^2 + t$$

11. **Exercise 17:** Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

$$x = e^t, \quad y = te^{-t}$$

12. **Exercise 23:** Find the points on the curve where the tangent is horizontal or vertical.

$$x = \cos\theta, \quad y = \cos(3\theta)$$

13. **Exercise 26:** Use a graph to estimate the coordinates of the lowest point and the leftmost point on the curve $x = t^4 - 2t, y = t + t^4$. Then find the exact coordinates.

14. **Example 10.2.3:** Find the area under one arch of the cycloid $x = r(\theta - \sin\theta), y = r(1 - \cos\theta)$.

15. **Exercise 40:** Find the area of the region enclosed by the loop of the curve $x = 1 - t^2, y = t - t^3$.

16. **Exercise 48:** Find the exact length of the curve.

$$x = e^t - t, \quad y = 4e^{t/2}, \quad 0 \le t \le 2$$

17. **Exercise 50:** Find the exact length of the curve.

$$x = 3\cos t - \cos(3t), \quad y = 3\sin t - \sin(3t), \quad 0 \le t \le \pi$$

18. **Exercise 71:** Find the exact area of the surface obtained by rotating the given curve about the x-axis.

$$x = t^3, \quad y = t^2, \quad 0 \le t \le 1$$

19. **Exercise 75:** Find the surface area generated by rotating the given curve about the y-axis.

$$x = 3t^2, \quad y = 2t^3, \quad 0 \le t \le 5$$

20. **Exercise 55:** Find the distance traveled by a particle with position (x, y) as t varies in the given time interval. Compare with the length of the curve.

$$x = \sin^2 t, \quad y = \cos^2 t, \quad 0 \le t \le 3\pi$$

21. **Exercise 57:** The parametric equations give the position (in meters) of a moving particle at time t (in seconds). Find the speed of the particle at the indicated time.

$$x = 2t - 3, \quad y = 2t^2 - 3t + 6; \quad t = 5$$

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22. **Exercise 34:** Find equations of the tangents to the curve $x = 3t^2 + 1, y = 2t^3 + 1$ that pass through the point $(4, 3)$.

23. **Exercise 65:** Find the area of the region enclosed by the astroid $x = a\cos^3\theta, y = a\sin^3\theta$.

24. **Exercise 52:** Graph the curve and find its exact length.

$$x = \cos t + \ln(\tan(t/2)), \quad y = \sin t, \quad \pi/4 \le t \le 3\pi/4$$

25. **Exercise 54:** Find the length of the loop of the curve $x = 3t - t^3, y = 3t^2$.

26. **Exercise 73:** Find the exact area of the surface obtained by rotating the given curve about the x-axis.

$$x = a\cos^3\theta, \quad y = a\sin^3\theta, \quad 0 \le \theta \le \pi/2$$

27. **Exercise 79:** For a parametric curve $x = x(t), y = y(t)$, derive the formula for curvature κ:

$$\kappa(t) = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{[\dot{x}^2 + \dot{y}^2]^{3/2}}$$

where the dots indicate derivatives with respect to t.

28. **Exercise 31b:** Show that if $d < r$, then the trochoid $x = r\theta - d\sin\theta, y = r - d\cos\theta$ does not have a vertical tangent.