Calculus - Chapter 9 Review Selected Exercises

Concept Check

1. (a) What is a differential equation?
   (b) What is the order of a differential equation?
   (c) What is an initial condition?

2. What can you say about the solutions of the equation $y' = x^2 + y^2$ just by looking at the differential equation?

3. What is a direction field for the differential equation $y' = F(x, y)$?

4. Explain how Euler's method works.

5. What is a separable differential equation? How do you solve it?

True-False Quiz

1. All solutions of the differential equation $y' = -1 - y^4$ are decreasing functions.

2. The function $f(x) = (\ln x)/x$ is a solution of the differential equation $x^2y' + xy = 1$.

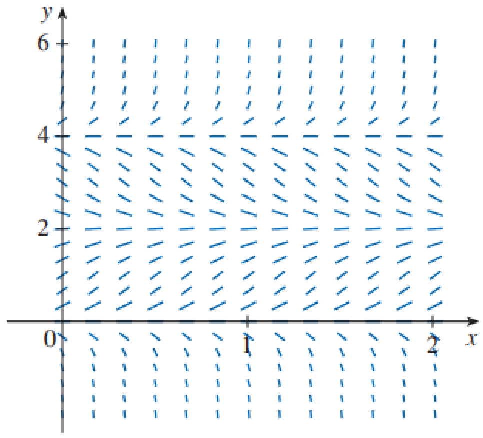
3. The function $y = 3e^{2x} - 1$ is a solution of the initial-value problem $y' - 2y = 1, y(0) = 2$.

4. The equation $y' = x + y$ is separable.

5. The equation $y' = 3y - 2x + 6xy - 1$ is separable.

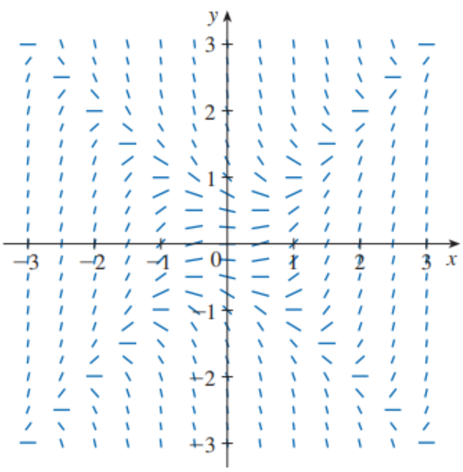
Exercises

1. (a) A direction field for the differential equation $y' = y(y-2)(y-4)$ is shown. Sketch the graphs of the solutions that satisfy the given initial conditions.
   (i) $y(0) = -0.3$ (ii) $y(0) = 1$ (iii) $y(0) = 3$ (iv) $y(0) = 4.3$
   (b) If the initial condition is $y(0) = c$, for what values of c is $\lim_{t\to\infty} y(t)$ finite? What are the equilibrium solutions?



2. (a) Sketch a direction field for the differential equation $y' = x/y$. Then use it to sketch the four solutions that satisfy the initial conditions $y(0) = 1, y(0) = -1, y(2) = 1$, and $y(-2) = 1$.
   (b) Check your work in part (a) by solving the differential equation explicitly. What type of curve is each solution curve?

3. (a) A direction field for the differential equation $y' = x^2 - y^2$ is shown. Sketch the solution of the initial-value problem $y' = x^2 - y^2, y(0) = 1$.
   (b) Use Euler's method with step size 0.1 to estimate $y(0.3)$.



   (c) On what lines are the centers of the horizontal line segments of the direction field in part (a) located? What happens when a solution curve crosses these lines?

4. (a) Use Euler's method with step size 0.2 to estimate $y(0.4)$, where $y(x)$ is the solution of the initial-value problem $y' = 2xy^2, y(0) = 1$.

(b) Repeat part (a) with step size 0.1.
(c) Find the exact solution of the differential equation and compare the value at 0.4 with the approximations in parts (a) and (b).

6. Solve the differential equation.

$$\frac{dx}{dt} = 1 - t + x - tx$$

7. Solve the differential equation.

$$2ye^{y^2}y' = 2x + 3\sqrt{x}$$

9. Solve the initial-value problem.

$$\frac{dr}{dt} + 2tr = r, \quad r(0) = 5$$

10. Solve the initial-value problem.

$$(1 + \cos x)y' = (1 + e^{-y})\sin x, \quad y(0) = 0$$

12. Solve the initial-value problem $y' = 3x^2e^y, y(0) = 1$ and graph the solution.

13. Find the orthogonal trajectories of the family of curves.

$$y = ke^x$$

14. Find the orthogonal trajectories of the family of curves.

$$y = e^{kx}$$

15. (a) Write the solution of the initial-value problem $\frac{dP}{dt} = 0.1P(1 - \frac{P}{2000}), P(0) = 100$.
(b) When does the population reach 1200?

18. A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 6 minutes?