Calculus - Chapter 10.3 Exercises

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1. **Exercise 1:** Plot the point whose polar coordinates are given. Then find two other pairs of polar coordinates of this point, one with $r > 0$ and one with $r < 0$.

$$(a)(1, \pi/4) \quad (b)(-2, 3\pi/2) \quad (c)(3, -\pi/3)$$

2. **Exercise 3:** Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

$$(a)(2, 3\pi/2) \quad (b)(\sqrt{2}, \pi/4) \quad (c)(-1, -\pi/6)$$

3. **Exercise 5:** The Cartesian coordinates of a point are given. Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.

$$(a)(-4, 4) \quad (b)(3, 3\sqrt{3})$$

4. **Exercise 7:** Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.

$$1 < r \leq 3$$

5. **Exercise 17:** Identify the curve by finding a Cartesian equation for the curve.

$$r = 5\cos\theta$$

6. **Exercise 21:** Find a polar equation for the curve represented by the given Cartesian equation.

$$x^2 + y^2 = 7$$

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7. **Exercise 33:** Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in Cartesian coordinates.

$$r = -2\sin\theta$$

8. **Exercise 35:** Sketch the curve with the given polar equation.

$$r = 2(1 + \cos\theta)$$

9. **Exercise 37:** Sketch the curve with the given polar equation.

$$r = \theta, \quad \theta \geq 0$$

10. **Exercise 39:** Sketch the curve with the given polar equation.

$$r = 3\cos(3\theta)$$

11. **Exercise 41:** Sketch the curve with the given polar equation.

$$r = 2\cos(4\theta)$$

12. **Exercise 45:** Sketch the curve with the given polar equation.

$$r^2 = 9\sin(2\theta)$$

13. **Exercise 49:** Sketch the curve with the given polar equation.

$$r = \sin(\theta/2)$$

14. **Exercise 23:** Find a polar equation for the curve represented by the given Cartesian equation.

$$y = \sqrt{3}x$$

15. **Exercise 25:** Find a polar equation for the curve represented by the given Cartesian equation.

$$x^2 + y^2 = 4y$$

16. **Exercise 29:** The figure shows a graph of r as a function of θ in Cartesian coordinates. Use it to sketch the corresponding polar curve.

17. **Exercise 31:** The figure shows a graph of r as a function of θ in Cartesian coordinates. Use it to sketch the corresponding polar curve.

18. **Exercise 56 (a, c):** Match the polar equations with the graphs labeled I-IX. Give reasons for your choices.

$$(a)\ r = \cos(3\theta) \quad (c)\ r = \cos(\theta/2)$$

19. **Exercise 58:** Show that the curves $r = a\sin\theta$ and $r = a\cos\theta$ intersect at right angles.

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20. **Exercise 51:** Show that the polar curve $r = 4 + 2\sec\theta$ (a conchoid) has the line $x = 2$ as a vertical asymptote by showing that $\lim_{r\to\pm\infty} x = 2$. Use this fact to help sketch the conchoid.

21. **Exercise 53:** Show that the curve $r = \sin\theta\tan\theta$ (a cissoid of Diocles) has the line $x = 1$ as a vertical asymptote. Use this fact to help sketch the cissoid.

22. **Exercise 55:** (a) In Example 10 the graphs suggest that the limaçon $r = 1 + c\sin\theta$ has an inner loop when $|c| > 1$. Prove that this is true, and find the values of θ that correspond to the inner loop. (b) From Figure 18 it appears that the limaçon loses its dimple when $c = 1/2$. Prove this.

23. **Exercise 57:** Show that the polar equation $r = a\sin\theta + b\cos\theta$, where $ab \neq 0$, represents a circle. Find its center and radius.

24. **Exercise 66:** Use a graph to estimate the y-coordinate of the highest points on the curve $r = \sin(2\theta)$. Then use calculus to find the exact value.

25. **Exercise 67:** Investigate the family of curves with polar equations $r = 1 + c\cos\theta$ where c is a real number. How does the shape change as c changes?