Chapter 11.3 Exercises: The Integral Test and Estimation of Sums

James Stewart, Calculus, Metric Edition

Difficulty: Easy

1. **Exercise 3:** Use the Integral Test to determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$
2. **Exercise 5:** Use the Integral Test to determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} \frac{1}{(n+3)^{5/2}}$$
3. **Exercise 7:** Use the Integral Test to determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$
4. **Exercise 9:** Use the Integral Test to determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$
5. **Exercise 13:** Determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} \frac{1}{2n+3}$$
6. **Exercise 15:** Determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$$
7. **Exercise 16:** Determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$

Difficulty: Medium

8. **Exercise 4:** Explain why the Integral Test can't be used to determine whether the series is convergent.
$$\sum_{n=1}^{\infty} \frac{n \sin n}{1+n^2}$$

9. **Exercise 10:** Use the Integral Test to determine whether the series is convergent or divergent.
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

10. **Exercise 11:** Use the Integral Test to determine whether the series is convergent or divergent.
$$\sum_{n=1}^{\infty} ne^{-n}$$

11. **Exercise 17:** Determine whether the series is convergent or divergent.
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

12. **Exercise 21:** Find the values of p for which the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^p}$ is convergent.

13. **Exercise 23(a):** Find the partial sum s_{10} of the series $\sum_{n=1}^{\infty} 1/n^2$. Estimate the error in using s_{10} as an approximation to the sum of the series.

14. **Exercise 23(b):** Use (2) with $n = 10$ to give an improved estimate of the sum.

15. **Exercise 23(c):** Find a value of n so that s_n is within 0.0001 of the sum.

16. **Exercise 27(a):** Use the sum of the first 10 terms to estimate the sum of the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^2}$. Estimate the error.

Difficulty: Hard

17. **Exercise 31:** How many terms of the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^2}$ would you need to add to find its sum to within 0.01?

18. **Exercise 33:** If $\sum a_n$ is a convergent series with positive terms, is it true that $\sum \sin(a_n)$ is also convergent?

19. **Exercise 35(a):** Show that $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^p}$ converges if $p > 1$ and diverges if $p \leq 1$. (This is the p-series test for integrals.)

20. **Exercise 37(a):** Find the partial sum s_{10} of the series $\sum_{n=1}^{\infty} 1/n^4$. Estimate the error in using s_{10} as an approximation to the sum of the series.

21. **Exercise 37(b):** Use (2) with $n = 10$ to give an improved estimate of the sum.

22. **Exercise 40:** Find all positive values of b for which the series $\sum_{n=1}^{\infty} b^{\ln n}$ converges.