Chapter 11.5 Exercises: Alternating Series and Absolute Convergence

James Stewart, Calculus, Metric Edition

Difficulty: Easy (6 Problems)

1. **Exercise 2:** Test the series for convergence or divergence.

$$\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \frac{2}{11} - \dots$$

2. **Exercise 3:** Test the series for convergence or divergence.

$$-\frac{2}{5} + \frac{4}{6} - \frac{6}{7} + \frac{8}{8} - \frac{10}{9} + \dots$$

3. **Exercise 5:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n}$$

4. **Exercise 6:** Test the series for convergence or divergence.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}+1}$$

5. **Exercise 22:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^4}$$

6. **Exercise 23:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{2/3}}$$

Difficulty: Medium (11 Problems)

7. **Exercise 7:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$$

8. **Exercise 9:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} (-1)^n e^{-n}$$

9. **Exercise 10:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{2n+3}$$

10. **Exercise 11:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^2}{n^3+4}$$

11. **Exercise 14:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty}(-1)^{n-1} \arctan n$$

12. **Exercise 24:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1} n^2}{n^2+1}$$

13. **Exercise 27:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$$

14. **Exercise 30:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2+4}$$

15. **Exercise 32:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+2}}$$

16. **Exercise 37:** Show that the series is convergent. How many terms of the series do we need to add in order to find the sum to the indicated accuracy?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6} \quad (|\text{error}| < 0.00005)$$

17. **Exercise 41:** Approximate the sum of the series correct to four decimal places.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$$

Difficulty: Hard (5 Problems)

18. **Exercise 17:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty}(-1)^n \sin\left(\frac{\pi}{n}\right)$$

19. **Exercise 18:** Test the series for convergence or divergence.

$$\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$$

20. **Exercise 29:** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{1 + 2\sin n}{n^3}$$

21. **Exercise 46:** For what values of p is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$$

22. **Exercise 53:** Suppose the series $\sum a_n$ is conditionally convergent.

    (a) Prove that the series $\sum n^2 a_n$ is divergent.

    (b) Conditional convergence of $\sum a_n$ is not enough to determine whether $\sum n a_n$ is convergent. Show this by giving an example of a conditionally convergent series such that $\sum n a_n$ converges and an example where $\sum n a_n$ diverges.