

Calculus - Chapter 9.5 Representative Exercises (Linear Equations)

난이도 하

1. **Exercise 2:** Determine whether the differential equation is linear. If it is linear, then write it in the form of Equation 1.

$$y' - x = y \tan x$$

2. **Exercise 5:** Solve the differential equation.

$$y' + y = 1$$

3. **Exercise 6:** Solve the differential equation.

$$y' - y = e^x$$

4. **Exercise 7:** Solve the differential equation.

$$y' = x - y$$

5. **Exercise 17:** Solve the initial-value problem.

$$xy' + y = 3x^2, \quad y(1) = 4$$

난이도 중

6. **Exercise 9:** Solve the differential equation.

$$xy' + y = \sqrt{x}$$

7. **Exercise 12:** Solve the differential equation.

$$y' - 3x^2y = x^2$$

8. **Exercise 13:** Solve the differential equation.

$$t^2 \frac{dy}{dt} + 3ty = \sqrt{1+t^2}, \quad t > 0$$

9. **Exercise 19:** Solve the initial-value problem.

$$x^2 y' + 2xy = \ln x, \quad y(1) = 2$$

10. **Exercise 21:** Solve the initial-value problem.

$$t \frac{du}{dt} = t^2 + 3u, \quad t > 0, u(2) = 4$$

11. **Exercise 23:** Solve the initial-value problem.

$$xy' = y + x^2 \sin x, \quad y(\pi) = 0$$

12. **Exercise 31:** In the circuit shown in Figure 4, a battery supplies a constant voltage of 40 V, the inductance is 2 H, the resistance is 10 Ω , and $I(0) = 0$.

- (a) Find $I(t)$.
- (b) Find the current after 0.1 seconds.

13. **Exercise 33:** For the given RC circuit, find the charge and the current at time t . (Given: $R = 5\Omega$, $C = 0.05$ F, $E(t) = 60$ V, $Q(0) = 0$ C)

14. **Exercise 35:** Solve the learning curve differential equation $\frac{dP}{dt} = k[M - P(t)]$ as a linear differential equation.

15. **Exercise 39(a):** An object with mass m is dropped from rest, and air resistance is proportional to the speed, giving the DE $m \frac{dv}{dt} = mg - cv$.

- (a) Solve this as a linear equation to show that $v = \frac{mg}{c}(1 - e^{-ct/m})$.

난이도 상

16. **Exercise 28:** Solve the Bernoulli differential equation $xy' + y = -xy^2$.

17. **Exercise 29:** Solve the Bernoulli differential equation $y' + \frac{2}{x}y = \frac{y^3}{x^2}$.

18. **Exercise 32:** In the circuit in Figure 4, a generator supplies $E(t) = 40 \sin 60t$ V, the inductance is 1 H, the resistance is 20 Ω , and $I(0) = 1$ A.

- (a) Find $I(t)$.
- (b) Find the current after 0.1 seconds.

19. **Exercise 37:** A tank contains 100 L of water. A solution with concentration 0.4 kg/L is added at 5 L/min. The mixed solution is drained at 3 L/min. Show that y (amount of salt) satisfies $\frac{dy}{dt} = 2 - \frac{3y}{100+2t}$. Solve this equation.

20. **Exercise 41:**

- (a) Show that the substitution $z = 1/P$ transforms the logistic DE $P' = kP(1 - P/M)$ into the linear DE $z' + kz = \frac{k}{M}$.
- (b) Solve the linear DE in part (a) to obtain an expression for $P(t)$.