Chapter 11.10 Exercises: Taylor and Maclaurin Series

James Stewart, Calculus, Metric Edition

Difficulty: Easy (10 Problems)

1. **Exercise 3:** If $f^{(n)}(0) = (n+1)!$ for $n = 0, 1, 2, \ldots$, find the Maclaurin series for f and its radius of convergence.

2. **Exercise 5:** Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = xe^x$ centered at $a = 0$.

3. **Exercise 8:** Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = \ln x$ centered at $a = 1$.

4. **Exercise 11:** Find the Maclaurin series for $f(x) = (1 - x)^{-2}$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

5. **Exercise 13:** Find the Maclaurin series for $f(x) = \cos x$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

6. **Exercise 14:** Find the Maclaurin series for $f(x) = e^{-2x}$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

7. **Exercise 16:** Find the Maclaurin series for $f(x) = \sin 3x$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

8. **Exercise 39:** Use a Maclaurin series in Table 1 to obtain the Maclaurin series for the function $f(x) = \arctan(x^2)$.

9. **Exercise 41:** Use a Maclaurin series in Table 1 to obtain the Maclaurin series for the function $f(x) = x \cos 2x$.

10. **Exercise 83:** Find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

Difficulty: Medium (21 Problems)

11. **Exercise 6:** Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = \frac{1}{1+x}$ centered at $a = 2$.

12. **Exercise 7:** Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = \sqrt[3]{x}$ centered at $a = 8$.

13. **Exercise 9:** Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = \sin x$ centered at $a = \pi/6$.

14. **Exercise 15:** Find the Maclaurin series for $f(x) = 2x^4 - 3x^2 + 3$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

15. **Exercise 17:** Find the Maclaurin series for $f(x) = 2^x$ using the definition of a Maclaurin series. Also find the associated radius of convergence.

16. **Exercise 21:** Find the Taylor series for $f(x) = x^5 + 2x^3 + x$ centered at $a = 2$.

17. **Exercise 23:** Find the Taylor series for $f(x) = \ln x$ centered at $a = 2$.

18. **Exercise 25:** Find the Taylor series for $f(x) = e^{2x}$ centered at $a = 3$.

19. **Exercise 27:** Find the Taylor series for $f(x) = \sin x$ centered at $a = \pi$.

20. **Exercise 35:** Use the binomial series to expand the function $f(x) = \sqrt[4]{1-x}$ as a power series. State the radius of convergence.

21. **Exercise 37:** Use the binomial series to expand the function $f(x) = \dfrac{1}{(2+x)^3}$ as a power series. State the radius of convergence.

22. **Exercise 44:** Use a Maclaurin series in Table 1 to obtain the Maclaurin series for the function $f(x) = x^2 \ln(1 + x^3)$.

23. **Exercise 47:** Use a Maclaurin series in Table 1 to obtain the Maclaurin series for the function $f(x) = \sin^2 x$. [Hint: Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.]

24. **Exercise 51:** Find the Maclaurin series of $f(x) = \cos(x^2)$ and the associated radius of convergence. Graph f and its first few Taylor polynomials on the same screen.

25. **Exercise 55:** Use the Maclaurin series for $\cos x$ to compute $\cos 5^\circ$ correct to five decimal places.

26. **Exercise 59:** Evaluate the indefinite integral as an infinite series.

$$\int \sqrt{1 + x^3} dx$$

27. **Exercise 63:** Use series to approximate the definite integral to within the indicated accuracy.

$$\int_0^{\frac{1}{2}} x^3 \arctan x dx \quad \text{(four decimal places)}$$

28. **Exercise 67:** Use series to evaluate the limit.

$$\lim_{x\to 0} \frac{x - \ln(1+x)}{x^2}$$

29. **Exercise 85:** Find the sum of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 3^n}{n 5^n}$$

Difficulty: Hard (11 Problems)

32. **Exercise 29:** Find the Taylor series for $f(x) = \sin 2x$ centered at $a = \pi$.

33. **Exercise 30:** Find the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 16$.

34. **Exercise 48:** Find the Maclaurin series for $f(x) = \begin{cases} (x - \sin x)/x^3 & \text{if } x \neq 0 \\ 1/6 & \text{if } x = 0 \end{cases}$.

35. **Exercise 50:** Use the formula $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$ and the Maclaurin series for $\ln(1+x)$ to show that

$$\tanh^{-1} x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$$

36. **Exercise 57:** (a) Use the binomial series to expand $1/\sqrt{1-x^2}$. (b) Use part (a) to find the Maclaurin series for $\sin^{-1} x$.

37. **Exercise 61:** Evaluate the indefinite integral as an infinite series.

$$\int \frac{\cos x - 1}{x} dx$$

38. **Exercise 71:** Use series to evaluate the limit.

$$\lim_{x\to 0} \frac{x^3 - 3x + 3\tan^{-1} x}{x^5}$$

39. **Exercise 74:** Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for $y = \sec x$.

40. **Exercise 77:** Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for $y = (\arctan x)^2$.

41. **Exercise 96:** (a) Show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is not equal to its Maclaurin series. (b) Graph the function in part (a) and comment on its behavior near the origin.

42. **Exercise 97:** Use the following steps to prove Theorem 17 (Binomial Series).

(a) Let $g(x) = \sum_{n=0}^{\infty} \binom{k}{n} x^n$. Differentiate this series to show that

$$g'(x) = \frac{kg(x)}{1+x} \quad -1 < x < 1$$

(b) Let $h(x) = (1+x)^{-k} g(x)$ and show that $h'(x) = 0$.

(c) Deduce that $g(x) = (1+x)^k$.