Chapter 11.9 Exercises: Representations of Functions as Power Series

James Stewart, Calculus, Metric Edition

Difficulty: Easy (6 Problems)

1. **Exercise 3:** Find a power series representation for the function and determine the interval of convergence.
$$f(x) = \frac{1}{1+x}$$

2. **Exercise 4:** Find a power series representation for the function and determine the interval of convergence.
$$f(x) = \frac{x}{1+x}$$

3. **Exercise 7:** Find a power series representation for the function and determine the interval of convergence.
$$f(x) = \frac{2}{3-x}$$

4. **Exercise 16(a):** Use Equation 1 to find a power series representation for $f(x) = \ln(1-x)$. What is the radius of convergence?

5. **Exercise 27:** Evaluate the indefinite integral as a power series. What is the radius of convergence?
$$\int \frac{t}{1-t^8} dt$$

6. **Exercise 28:** Evaluate the indefinite integral as a power series. What is the radius of convergence?
$$\int \frac{t}{1+t^3} dt$$

Difficulty: Medium (12 Problems)

7. **Exercise 9:** Find a power series representation for the function and determine the interval of convergence.
$$f(x) = \frac{x^2}{x^4+16}$$

8. **Exercise 13:** Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.
$$f(x) = \frac{2x-4}{x^2-4x+3}$$

9. **Exercise 15(a):** Use differentiation to find a power series representation for
$$f(x) = \frac{1}{(1+x)^2}$$
What is the radius of convergence?

10. **Exercise 17:** Find a power series representation for the function and determine the radius of convergence.
$$f(x) = \frac{x}{(1+4x)^2}$$

11. **Exercise 21:** Find a power series representation for the function and determine the radius of convergence.
$$f(x) = \ln(5-x)$$

12. **Exercise 22:** Find a power series representation for the function and determine the radius of convergence.
$$f(x) = x^2 \tan^{-1}(x^3)$$

13. **Exercise 31:** Use a power series to approximate the definite integral to six decimal places.
$$\int_0^{0.3} \frac{x}{1+x^3} dx$$

14. **Exercise 36:** Use the result of Example 5 to compute $\ln 1.1$ correct to four decimal places.

15. **Exercise 40:** The Bessel function of order 1 is defined by
$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$$
   (a) Find the domain of J_1.
   (b) Show that J_1 satisfies the differential equation
   $$x^2 J_1''(x) + xJ_1'(x) + (x^2-1)J_1(x) = 0$$
   (c) Show that $J_0'(x) = -J_1(x)$.

16. **Exercise 49:** Use the power series for $\tan^{-1} x$ to prove the following expression for π as the sum of an infinite series:
$$\pi = 2\sqrt{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}$$

17. **Exercise 50(a):** By completing the square, show that
$$\int_0^{1/2} \frac{dx}{x^2-x+1} = \frac{\pi}{3\sqrt{3}}$$

Difficulty: Hard (6 Problems)

19. **Exercise 39(a):** Show that J_0 (the Bessel function of order 0 given in Example 8) satisfies the differential equation

$$x^2 J_0''(x) + x J_0'(x) + x^2 J_0(x) = 0$$

20. **Exercise 42:** If $f(x) = \sum_{n=0}^{\infty} c_n x^n$, where $c_{n+4} = c_n$ for all $n \geq 0$, find the interval of convergence of the series and a formula for $f(x)$.

21. **Exercise 43:** A function f is defined by

$$f(x) = 1 + 2x + x^2 + 2x^3 + x^4 + \dots$$

that is, its coefficients are $c_{2n} = 1$ and $c_{2n+1} = 2$ for all $n \geq 0$. Find the interval of convergence of the series and find an explicit formula for $f(x)$.

22. **Exercise 45:** Let

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

Find the intervals of convergence for f, f', and f''.

23. **Exercise 46(c):** Find the sum of each of the following series.

$$(i) \sum_{n=2}^{\infty} n(n-1)x^n, \quad |x| < 1 \qquad (ii) \sum_{n=2}^{\infty} \frac{n^2 - n}{2^n} \qquad (iii) \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

24. **Exercise 50(b):** By factoring $x^3 + 1$ as a sum of cubes, rewrite the integral in part (a). Then express $1/(x^3 + 1)$ as the sum of a power series and use it to prove the following formula for π:

$$\pi = \frac{3\sqrt{3}}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{8^n} \left(\frac{2}{3n+1} + \frac{1}{3n+2} \right)$$