Chapter 11.1 Exercises: Sequences

James Stewart, Calculus, Metric Edition

Difficulty: Easy

1. **Exercise 3:** List the first five terms of the sequence.

$$a_n = n^3 - 1$$

2. **Exercise 4:** List the first five terms of the sequence.

$$a_n = \frac{1}{3n+1}$$

3. **Exercise 7:** List the first five terms of the sequence.

$$a_n = (-1)^{n-1}\frac{n}{2^n}$$

4. **Exercise 8:** List the first five terms of the sequence.

$$a_n = \frac{(-1)^n}{4^n}$$

5. **Exercise 13:** List the first five terms of the sequence.

$$a_1 = 1, \quad a_{n+1} = 2a_n + 1$$

6. **Exercise 14:** List the first five terms of the sequence.

$$a_1 = 6, \quad a_{n+1} = \frac{a_n}{n}$$

7. **Exercise 17:** Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots\right\}$$

8. **Exercise 18:** Find a formula for the general term a_n of the sequence.

$$\left\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \ldots\right\}$$

9. **Exercise 19:** Find a formula for the general term a_n of the sequence.

$$\left\{-\frac{2}{3}, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\right\}$$

10. **Exercise 20:** Find a formula for the general term a_n of the sequence.

$$\{5, 8, 11, 14, 17, \dots\}$$

Difficulty: Medium

11. **Exercise 27:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{5}{n+2}$$

12. **Exercise 29:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{4n^2 - 3n}{2n^2 + 1}$$

13. **Exercise 33:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3^n}{7^{2n}}$$

14. **Exercise 34:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3\sqrt{n}}{\sqrt{n} + 2}$$

15. **Exercise 37:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \sqrt{\frac{1 + 4n^2}{1 + n^2}}$$

16. **Exercise 38:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \cos\left(\frac{n\pi}{n+1}\right)$$

17. **Exercise 39:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{n^2}{\sqrt{n^3 + 4n}}$$

18. **Exercise 40:** Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = e^{2n/(n+2)}$$

19. **Exercise 41:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \frac{(-1)^n}{2\sqrt{n}}$$

20. **Exercise 42:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \frac{(-1)^{n+1}n}{n + \sqrt{n}}$$

21. **Exercise 43:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$\left\{ \frac{(2n-1)!}{(2n+1)!} \right\}$$

22. **Exercise 44:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$\left\{ \frac{\ln n}{\ln(2n)} \right\}$$

23. **Exercise 47:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$\{n^2 e^{-n}\}$$

24. **Exercise 48:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \ln(n+1) - \ln n$$

25. **Exercise 49:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \frac{\cos^2 n}{2^n}$$

26. **Exercise 51:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = n \sin(1/n)$$

27. **Exercise 53:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \left(1 + \frac{2}{n}\right)^n$$

28. **Exercise 54:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \sqrt[n]{n}$$

29. **Exercise 61:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \frac{n!}{2^n}$$

30. **Exercise 62:** Determine whether the sequence converges or diverges. If it converges, find the limit.
$$a_n = \frac{(-3)^n}{n!}$$

Difficulty: Hard

31. **Exercise 70:** (a) Determine whether the sequence defined as follows is convergent or divergent:
$$a_1 = 1 \quad a_{n+1} = 4 - a_n \text{ for } n \geq 1$$
(b) What happens if the first term is $a_1 = 2$?

32. **Exercise 75:** For what values of r is the sequence $\{nr^n\}$ convergent?

33. **Exercise 76:** (a) If $\{a_n\}$ is convergent, show that $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$. (b) A sequence $\{a_n\}$ is defined by $a_1 = 1$ and $a_{n+1} = 1/(1 + a_n)$ for $n \geq 1$. Assuming that $\{a_n\}$ is convergent, find its limit.

34. **Exercise 78:** Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?
$$a_n = \cos n$$

35. **Exercise 79:** Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?
$$a_n = \frac{1}{2n+3}$$

36. **Exercise 85:** Find the limit of the sequence
$$\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$$

37. **Exercise 86:** A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$. (a) By induction or otherwise, show that $\{a_n\}$ is increasing and bounded above by 3. Apply the Monotonic Sequence Theorem to show that $\lim_{n\to\infty} a_n$ exists. (b) Find $\lim_{n\to\infty} a_n$.

38. **Exercise 87:** Show that the sequence defined by
$$a_1 = 1 \quad a_{n+1} = 3 - \frac{1}{a_n}$$
is increasing and $a_n < 3$ for all n. Deduce that $\{a_n\}$ is convergent and find its limit.

39. **Exercise 89:** (a) Fibonacci posed the following problem: Suppose that rabbits live forever and that every month each pair produces a new pair which becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the nth month? Show that the answer is f_n, where $\{f_n\}$ is the Fibonacci sequence. (b) Let $a_n = f_{n+1}/f_n$ and show that $a_{n-1} = 1 + 1/a_{n-2}$. Assuming that $\{a_n\}$ is convergent, find its limit.

40. **Exercise 96:** Let $a_n = (1 + 1/n)^n$. (a) Show that if $0 \leq a < b$, then $\frac{b^{n+1}-a^{n+1}}{b-a} < (n+1)b^n$. (b) Deduce that $b^n[(n+1)a - nb] < a^{n+1}$. (c) Use $a = 1 + 1/(n+1)$ and $b = 1 + 1/n$ in part (b) to show that $\{a_n\}$ is increasing. (d) Use $a = 1$ and $b = 1 + 1/(2n)$ in part (b) to show that $a_{2n} < 4$. (e) Use parts (c) and (d) to show that $a_n < 4$ for all n. (f) Use the Monotonic Sequence Theorem to show that $\lim_{n\to\infty}(1 + 1/n)^n$ exists.