Calculus - Chapter 9.3 Representative Exercises (Separable Equations)

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1. **Exercise 1:** Solve the differential equation.

$$\frac{dy}{dx} = 3x^2y^2$$

2. **Exercise 3:** Solve the differential equation.

$$\frac{dy}{dx} = x\sqrt{y}$$

3. **Exercise 8:** Solve the differential equation.

$$\frac{dy}{dx} = 2x(y^2 + 1)$$

4. **Exercise 13:** Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = xe^y, \quad y(0) = 0$$

5. **Exercise 14:** Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dP}{dt} = \sqrt{Pt}, \quad P(1) = 2$$

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6. **Exercise 9:** Solve the differential equation.

$$\frac{dp}{dt} = t^2p - p + t^2 - 1$$

7. **Exercise 10:** Solve the differential equation.

$$\frac{dz}{dt} + e^{t+z} = 0$$

8. **Exercise 18:** Find the solution of the differential equation that satisfies the given initial condition.

$$x + 3y^2\sqrt{x^2+1}\frac{dy}{dx} = 0, \quad y(0) = 1$$

9. **Exercise 20:** Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = \frac{x \sin x}{y}, \quad y(0) = -1$$

10. **Exercise 21:** Find an equation of the curve that passes through the point (0, 2) and whose slope at (x, y) is x/y.

11. **Exercise 33:** Find the orthogonal trajectories of the family of curves.

$$y = \frac{k}{x}$$

12. **Exercise 39:** Solve the initial-value problem in Exercise 9.2.27 to find an expression for the charge at time t. Find the limiting value of the charge.

13. **Exercise 40:** Solve the differential equation that models the temperature of a 95°C cup of coffee in a 20°C room.

14. **Exercise 41:** Solve the differential equation for the learning model $\frac{dP}{dt} = k(M - P)$.

15. **Exercise 47:** A tank contains 1000 L of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank (a) after t minutes and (b) after 20 minutes?

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16. **Exercise 23:** Solve the differential equation $y' = x + y$ by making the change of variable $u = x + y$.

17. **Exercise 31:** Find the orthogonal trajectories of the family of curves.

$$x^2 + 2y^2 = k^2$$

18. **Exercise 32:** Find the orthogonal trajectories of the family of curves.

$$y^2 = kx^3$$

19. **Exercise 42:** In an elementary chemical reaction, $A + B \to C$. The law of mass action states that $\frac{dx}{dt} = k(a - x)(b - x)$.

(a) Assuming that $a \neq b$, find x as a function of t. Use the fact that the initial concentration of C is 0.

(b) Find $x(t)$ assuming that $a = b$.

20. **Exercise 52(a):** An object of mass m is moving horizontally through a medium which resists the motion with a force $m\frac{dv}{dt} = f(v)$.

(a) Suppose that the resisting force is proportional to the velocity, $f(v) = -kv$. Let $v(0) = v_0$ and $s(0) = s_0$. Determine v and s at any time t. What is the total distance that the object travels from time $t = 0$?