Calculus - Chapter 10 Review Selected Exercises

Concept Check

1. **(1)** (a) What is a parametric curve? (b) How do you sketch a parametric curve?

2. **(2)** (a) How do you find the slope of a tangent to a parametric curve? (b) How do you find the area under a parametric curve?

3. **(3)** Write an expression for each of the following: (a) The length of a parametric curve (b) The area of the surface obtained by rotating a parametric curve about the x-axis (c) The speed of a particle traveling along a parametric curve

4. **(4)** (a) Use a diagram to explain the meaning of the polar coordinates (r, θ) of a point. (b) Write equations that express the Cartesian coordinates (x, y) of a point in terms of the polar coordinates. (c) What equations would you use to find the polar coordinates of a point if you knew the Cartesian coordinates?

5. **(5)** (a) How do you find the area of a region bounded by a polar curve? (b) How do you find the length of a polar curve? (c) How do you find the slope of a tangent line to a polar curve?

6. **(6)** (a) Give a geometric definition of a parabola. (b) Write an equation of a parabola with focus $(0, p)$ and directrix $y = -p$. What if the focus is $(p, 0)$ and the directrix is $x = -p$?

7. **(7)** (a) Give a definition of an ellipse in terms of foci. (b) Write an equation for the ellipse with foci $(\pm c, 0)$ and vertices $(\pm a, 0)$.

8. **(8)** (a) Give a definition of a hyperbola in terms of foci. (b) Write an equation for the hyperbola with foci $(\pm c, 0)$ and vertices $(\pm a, 0)$. (c) Write equations for the asymptotes of the hyperbola in part (b).

True-False Quiz

1. **(1)** If the parametric curve $x = f(t), y = g(t)$ satisfies $g'(1) = 0$, then it has a horizontal tangent when $t = 1$.

2. **(2)** If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $\frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{d^2x/dt^2}$.

3. **(3)** The length of the curve $x = f(t), y = g(t), a \leq t \leq b$, is $\int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}dt$.

4. **(5)** If a point is represented by (x, y) in Cartesian coordinates (where $x \neq 0$) and (r, θ) in polar coordinates, then $\theta = \tan^{-1}(y/x)$.

5. **(7)** The equations $r = 2$, $x^2 + y^2 = 4$, and $x = 2\sin(3t), y = 2\cos(3t)$ $(0 \leq t \leq 2\pi)$ all have the same graph.

6. **(9)** The graph of $y^2 = 2y + 3x$ is a parabola.

7. **(10)** A tangent line to a parabola intersects the parabola only once.

8. **(11)** A hyperbola never intersects its directrix.

Exercises

1. **(1)** Sketch the parametric curve and eliminate the parameter to find a Cartesian equation of the curve.

$$x = t^2 + 4t, \quad y = 2 - t, \quad -4 \leq t \leq 1$$

2. **(4)** Sketch the parametric curve and eliminate the parameter.

$$x = 2\cos\theta, \quad y = 1 + \sin\theta$$

3. **(11)** Sketch the polar curve $r = 1 + \sin\theta$.

4. **(13)** Sketch the polar curve $r = \cos(3\theta)$.

5. **(19)** Find a polar equation for the curve represented by the Cartesian equation $x + y = 2$.

6. **(23)** Find the slope of the tangent line to the curve $x = \ln t, y = 1 + t^2$ at $t = 1$.

7. **(25)** Find the slope of the tangent line to the polar curve $r = e^{-2\theta}$ at $\theta = \pi$.

8. **(27)** Find dy/dx and d^2y/dx^2 for $x = t + \sin t, y = t - \cos t$.

9. **(31)** At what points does the curve $x = 2a\cos t - a\cos(2t), y = 2a\sin t - a\sin(2t)$ have vertical or horizontal tangents?

10. **(33)** Find the area enclosed by the curve $r^2 = 9\cos(5\theta)$.

11. **(37)** Find the area of the region that lies inside both of the circles $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$.

12. **(39)** Find the length of the curve $x = 3t^2, y = 2t^3, 0 \leq t \leq 2$.

13. **(41)** Find the length of the curve $r = 1/\theta, \pi \leq \theta \leq 2\pi$.

14. **(45)** Find the area of the surface obtained by rotating the curve $x = 4\sqrt{t}, y = \frac{t^3}{3} + \frac{1}{2t^2}, 1 \leq t \leq 4$ about the x-axis.

15. **(49)** Find the foci and vertices and sketch the graph of $\frac{x^2}{9} + \frac{y^2}{8} = 1$.
16. **(50)** Find the foci and vertices and sketch the graph of $4x^2 - y^2 = 16$.
17. **(53)** Find an equation of the ellipse with foci $(\pm 4, 0)$ and vertices $(\pm 5, 0)$.
18. **(54)** Find an equation of the parabola with focus $(2, 1)$ and directrix $x = -4$.
19. **(55)** Find an equation of the hyperbola with foci $(0, \pm 4)$ and asymptotes $y = \pm 3x$.