Chapter 11.4 Exercises: The Comparison Tests

James Stewart, Calculus, Metric Edition

Difficulty: Easy (6 Problems)

1. **Exercise 7:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + 8}$$

2. **Exercise 8:** Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$$

3. **Exercise 9:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$$

4. **Exercise 11:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{9^n}{3 + 10^n}$$

5. **Exercise 12:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$$

6. **Exercise 18:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{3n^4 + 1}}$$

Difficulty: Medium (13 Problems)

7. **Exercise 10:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3 + 1}$$

8. **Exercise 15:** Determine whether the series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3 + 4k + 3}}$$

9. **Exercise 16:** Determine whether the series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$$

10. **Exercise 19:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n - 2}$$

11. **Exercise 21:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

12. **Exercise 22:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$$

13. **Exercise 23:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n+1}{n^3+n}$$

14. **Exercise 24:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^4+n^2}$$

15. **Exercise 28:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n+3^n}{n+2^n}$$

16. **Exercise 29:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{e^n+1}{ne^n+1}$$

17. **Exercise 31:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2}$$

18. **Exercise 33:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} (1+\frac{1}{n})^2 e^{-n}$$

19. **Exercise 41:** Use the sum of the first 10 terms to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{5+n^5}$. Estimate the error.

Difficulty: Hard (6 Problems)

20. **Exercise 17:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{1+\cos n}{e^n}$$

21. **Exercise 34:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$

22. **Exercise 35:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

23. **Exercise 36:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

24. **Exercise 37:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \sin(\frac{1}{n})$$

25. **Exercise 40:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$$

26. **Exercise 48:** (a) Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is convergent. Prove that if
$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$
then $\sum a_n$ is also convergent.
(b) Use part (a) to show that the series converges.
$$\text{(i)} \sum_{n=1}^{\infty} \frac{\ln n}{n^3} \qquad \text{(ii)} \sum_{n=1}^{\infty} \left(1 - \cos\frac{1}{n^2}\right)$$

27. **Exercise 49:** (a) Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is divergent. Prove that if
$$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$$
then $\sum a_n$ is also divergent.
(b) Use part (a) to show that the series diverges.
$$\text{(i)} \sum_{n=2}^{\infty} \frac{1}{\ln n} \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{\ln n}{n}$$

28. **Exercise 50:** Give an example of a pair of series $\sum a_n$ and $\sum b_n$ with positive terms where $\lim_{n\to\infty}(a_n/b_n) = 0$ and $\sum b_n$ diverges, but $\sum a_n$ converges. (Compare with Exercise 48.)

29. **Exercise 55:** Let $\sum a_n$ and $\sum b_n$ be series with positive terms. Is each of the following statements true or false? If the statement is false, give an example that disproves the statement.
   (a) If $\sum a_n$ and $\sum b_n$ are divergent, then $\sum a_n b_n$ is divergent.
   (b) If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum a_n b_n$ diverges.
   (c) If $\sum a_n$ and $\sum b_n$ are convergent, then $\sum a_n b_n$ is convergent.