Chapter 11.2 Exercises: Series

James Stewart, Calculus, Metric Edition

Difficulty: Easy

1. **Exercise 3:** Calculate the sum of the series $\sum_{n=1}^{\infty} a_n$ whose partial sums are given.

$$s_n = 2 - 3(0.8)^n$$

2. **Exercise 5:** Calculate the first eight terms of the sequence of partial sums correct to four decimal places. Does it appear that the series is convergent or divergent?

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

3. **Exercise 11:** Find at least 10 partial sums of the series. Graph both the sequence of terms and the sequence of partial sums on the same screen. Does it appear that the series is convergent or divergent? If it is convergent, find the sum.

$$\sum_{n=1}^{\infty} \frac{6}{(-3)^n}$$

4. **Exercise 19:** Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

5. **Exercise 23:** Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$$

6. **Exercise 24:** Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$$

7. **Exercise 27:** Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} 12(0.73)^{n-1}$$

8. **Exercise 33:** Determine whether the series is convergent or divergent.

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \cdots$$

9. **Exercise 53:** Express the number as a ratio of integers.

$$0.\bar{8} = 0.8888\ldots$$

Difficulty: Medium

10. **Exercise 15:** Let $a_n = \frac{2n}{3n+1}$. (a) Determine whether $\{a_n\}$ is convergent. (b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

11. **Exercise 21:** Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$$

12. **Exercise 29:** Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$$

13. **Exercise 31:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{e^{2n}}{6^{n-1}}$$

14. **Exercise 35:** Determine whether the series is convergent or divergent.

$$-\frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \frac{16}{625} - \frac{32}{3125} + \cdots$$

15. **Exercise 37:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$$

16. **Exercise 38:** Determine whether the series is convergent or divergent.

$$\sum_{k=1}^{\infty} \frac{k^2}{k^2 - 2k + 5}$$

17. **Exercise 41:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{4 + e^{-n}}$$

18. **Exercise 45:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$$

19. **Exercise 47:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \arctan(n)$$

20. **Exercise 48:** Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right)$$

21. **Exercise 49:** Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right)$$

22. **Exercise 55:** Express the number as a ratio of integers.

$$2.5\overline{16} = 2.516516516\ldots$$

23. **Exercise 59:** Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=1}^{\infty} (-5)^n x^n$$

24. **Exercise 61:** Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$$

25. **Exercise 69:** If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n-1}{n+1}$, find a_n and $\sum_{n=1}^{\infty} a_n$.

26. **Exercise 71:** A doctor prescribes a 100-mg antibiotic tablet to be taken every eight hours. It is known that the body eliminates 75% of the drug in eight hours. (a) How much of the drug is in the body just after the second tablet is taken? After the third tablet? (b) If Q_n is the quantity of the antibiotic in the body just after the nth tablet is taken, find an equation that expresses Q_{n+1} in terms of Q_n. (c) What quantity of the antibiotic remains in the body in the long run?

Difficulty: Hard

27. **Exercise 22:** Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{1}{n^3 - n}$$

28. **Exercise 52:** A sequence of terms is defined by $a_1 = 1, a_n = (5-n)a_{n-1}$. Calculate $\sum_{n=1}^{\infty} a_n$.

29. **Exercise 67:** Use the partial fraction command on a CAS to find a convenient expression for the partial sum, and then use this expression to find the sum of the series.
$$\sum_{n=1}^{\infty} \frac{3n^2 + 3n + 1}{(n^2 + n)^3}$$

30. **Exercise 77:** Find the value of c if $\sum_{n=2}^{\infty}(1 + c)^{-n} = 2$.

31. **Exercise 82:** Graph the curves $y = x^n, 0 \leq x \leq 1$, for $n = 0, 1, 2, 3, 4, \ldots$ on a common screen. By finding the areas between successive curves, give a geometric demonstration of the fact, shown in Example 2, that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$.

32. **Exercise 83:** The figure shows two circles C and D of radius 1 that touch at P. The line T is a common tangent line... Find an expression for the diameter of C_n and thus provide another geometric demonstration of Example 2.

33. **Exercise 84:** A right triangle ABC is given with $\angle A = \theta$ and $|AC| = b$. CD is drawn perpendicular to AB, DE is drawn perpendicular to BC, EF $\perp$ AB, and this process is continued indefinitely. Find the total length of all the perpendiculars $|CD| + |DE| + |EF| + |FG| + \cdots$ in terms of b and θ.

34. **Exercise 86:** Suppose that $\sum_{n=1}^{\infty} a_n$ $(a_n \neq 0)$ is known to be a convergent series. Prove that $\sum_{n=1}^{\infty} 1/a_n$ is a divergent series.

35. **Exercise 92:** The Fibonacci sequence was defined by $f_1 = 1, f_2 = 1, f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. Show that each of the following statements is true. (a) $\frac{1}{f_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_n} - \frac{1}{f_nf_{n+1}}$ (b) $\sum_{n=2}^{\infty} \frac{1}{f_{n-1}f_{n+1}} = 1$ (c) $\sum_{n=2}^{\infty} \frac{f_n}{f_{n-1}f_{n+1}} = 2$