퀴즈 2차 (현경서t)

미적분학1

반: __________ **번호:** __________ **이름:** ____________________

1. (10 points) Prove the next theorem.

> Suppose f is a continuous, positive, decreasing function on $[1, \infty]$ and let $a_n = f(n)$.
>
> (i) If $\int_1^\infty f(x)\,dx$ is **convergent**, then $\sum_{n=1}^\infty a_n$ is **convergent**.
>
> (ii) If $\int_1^\infty f(x)\,dx$ is **divergent**, then $\sum_{n=1}^\infty a_n$ is **divergent**.

2. (10 points) Prove the next theorem.

> Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.
>
> (i) If $\sum b_n$ is **convergent** and $a_n \leq b_n$ for all n, then $\sum a_n$ is also **convergent**.
>
> (ii) If $\sum b_n$ is **divergent** and $a_n \geq b_n$ for all n, then $\sum a_n$ is also **divergent**.

3. (10 points) Prove the next theorem.

If the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n (b_n > 0)$ satisfies

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim_{n\to\infty} b_n = 0$

then the series is convergent.

4. (10 points) Prove the next theorem.

Let $\sum a_n$ be a series and suppose that the following limit exists:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$$

(i) If $L < 1$, then the series $\sum a_n$ is absolutely convergent.

(ii) If $L > 1$ or $L = \infty$, then the series $\sum a_n$ is divergent.

5. (10 points) Find the Macalurin seires for next function.

(a) (2 points) $\dfrac{1}{1-x} =$

(b) (2 points) $e^x =$

(c) (2 points) $\sin x =$

(d) (2 points) $\cos x =$

(e) (2 points) $(1+x)^k =$

— 수고하셨습니다 —