Chapter 11.6 Exercises: The Ratio and Root Tests

James Stewart, Calculus, Metric Edition

Difficulty: Easy (5 Problems)

1. **Exercise 3:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n}{5^n}$$

2. **Exercise 4:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$$

3. **Exercise 7:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{k=1}^{\infty} \frac{1}{k!}$$

4. **Exercise 8:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{k=1}^{\infty} ke^{-k}$$

5. **Exercise 21:** Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$$

Difficulty: Medium (10 Problems)

6. **Exercise 9:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$$

7. **Exercise 10:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n!}{100^n}$$

8. **Exercise 12:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$$

9. **Exercise 13:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$$

10. **Exercise 22:** Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$$

11. **Exercise 24:** Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$$

12. **Exercise 25:** Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$$

13. **Exercise 27:** Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$$

14. **Exercise 31:** Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} (\frac{n}{\ln n})^n$$

15. **Exercise 33:** Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^2}$$

Difficulty: Hard (5 Problems)

16. **Exercise 14:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

17. **Exercise 16:** Use the Ratio Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$$

18. **Exercise 35:** The terms of a series are defined recursively by the equations

$$a_1 = 2 \qquad a_{n+1} = \frac{5n+1}{4n+3} a_n$$

Determine whether $\sum a_n$ converges or diverges.

19. **Exercise 40:** For which positive integers k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

20. **Exercise 42:** Let $\sum a_n$ be a series with positive terms and let $r_n = a_{n+1}/a_n$. Suppose that $\lim_{n\to\infty} r_n = L < 1$, so $\sum a_n$ converges by the Ratio Test. As usual, we let R_n be the remainder after n terms, that is,

$$R_n = a_{n+1} + a_{n+2} + a_{n+3} + \dots$$

   (a) If $\{r_n\}$ is a decreasing sequence and $r_{n+1} < 1$, show, by summing a geometric series, that

$$R_n \le \frac{a_{n+1}}{1 - r_{n+1}}$$

   (b) If $\{r_n\}$ is an increasing sequence, show that

$$R_n \le \frac{a_{n+1}}{1 - L}$$