# Chapter 11.3 Exercises: The Integral Test and Estimation of Sums

James Stewart, Calculus, Metric Edition

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## Difficulty: Easy (6 Problems)

1. **Exercise 1:** Draw a picture to show that $\sum_{n=2}^{\infty} \frac{1}{n^{1.5}} < \int_1^{\infty} \frac{1}{x^{1.5}} dx$. What can you conclude about the series?

2. **Exercise 3:** Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} n^{-3}$$

3. **Exercise 5:** Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{2}{5n-1}$$

4. **Exercise 11:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$$

5. **Exercise 13:** Determine whether the series is convergent or divergent.

$$1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \dots$$

6. **Exercise 21:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4}$$

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## Difficulty: Medium (11 Problems)

7. **Exercise 7:** Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{n^2}{n^3+1}$$

8. **Exercise 8:** Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

9. **Exercise 9:** Use the Integral Test to determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$$

10. **Exercise 17:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2}$$

11. **Exercise 23:** Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

12. **Exercise 24:** Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

13. **Exercise 25:** Determine whether the series is convergent or divergent.

$$\sum_{k=1}^{\infty} ke^{-k}$$

14. **Exercise 27:** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n^3}$$

15. **Exercise 38(a):** Find the partial sum $s_{10}$ of the series $\sum_{n=1}^{\infty} 1/n^4$. Estimate the error in using $s_{10}$ as an approximation to the sum of the series.

16. **Exercise 39(a):** Use the sum of the first 10 terms to estimate the sum of the series $\sum_{n=1}^{\infty} 1/n^2$. How good is this estimate?

17. **Exercise 40:** Find the sum of the series $\sum_{n=1}^{\infty} ne^{-2n}$ correct to four decimal places.

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## Difficulty: Hard (5 Problems)

18. **Exercise 31:** Find the values of $p$ for which the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^p}$ is convergent.

19. **Exercise 32:** Find the values of $p$ for which the series $\sum_{n=3}^{\infty} \dfrac{1}{n \ln n[\ln(\ln n)]^p}$ is convergent.

20. **Exercise 33:** Find the values of $p$ for which the series $\sum_{n=1}^{\infty} n(1+n^2)^p$ is convergent.

21. **Exercise 47:** Find all positive values of $b$ for which the series $\sum_{n=1}^{\infty} b^{\ln n}$ converges.

22. **Exercise 48:** Find all values of $c$ for which the following series converges: $\sum_{n=1}^{\infty}(\dfrac{c}{n} - \dfrac{1}{n+1})$.