방과후 퀴즈 11.1.-11.10.

7문제, 30분

2025.12.8.월

1. Show that the sequence defined by

$$a_1 = 1 \quad a_{n+1} = 3 - \frac{1}{a_n}$$

is increasing and $a_n < 3$ for all n. Deduce that $\{a_n\}$ is convergent and find its limit.

2. Determine whether the series is convergent or divergent.

$$\sum_{k=1}^{\infty} ke^{-k}$$

3. Determine whether the series converges or diverges.

$$\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$$

4. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin n}{2^n}$$

5. Find the radius of convergence and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

6. Find a power series representation for the function and determine the radius of convergence.

$$f(x) = \frac{x}{(1+4x)^2}$$

7. Find the sum of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}3^n}{n5^n}$$