Calculus - Chapter 10.1 Representative Exercises

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1. **Exercise 3:** Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

$$x = 1 - t^2, \quad y = 2t - t^2, \quad -1 \leq t \leq 2$$

2. **Exercise 7:** (a) Sketch the curve by using the parametric equations to plot points. (b) Eliminate the parameter to find a Cartesian equation of the curve.

$$x = 2t - 1, \quad y = \frac{1}{2}t + 1$$

3. **Exercise 9:** (a) Sketch the curve. (b) Eliminate the parameter to find a Cartesian equation.

$$x = t^2 - 3, \quad y = t + 2, \quad -3 \leq t \leq 3$$

4. **Exercise 11:** (a) Sketch the curve. (b) Eliminate the parameter to find a Cartesian equation.

$$x = \sqrt{t}, \quad y = 1 - t$$

5. **Exercise 13:** (a) Eliminate the parameter to find a Cartesian equation of the curve. (b) Sketch the curve and indicate the direction.

$$x = 3\cos t, \quad y = 3\sin t, \quad 0 \leq t \leq \pi$$

6. **Exercise 21:** (a) Eliminate the parameter. (b) Sketch the curve.

$$x = \sin^2 t, \quad y = \cos^2 t$$

7. **Exercise 37(b):** Find parametric equations to represent the line segment from $(-2, 7)$ to $(3, -1)$.

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8. **Exercise 15:** (a) Eliminate the parameter. (b) Sketch the curve.

$$x = \cos\theta, \quad y = \sec^2\theta, \quad 0 \le \theta < \pi/2$$

9. **Exercise 19:** (a) Eliminate the parameter. (b) Sketch the curve.

$$x = \ln t, \quad y = \sqrt{t}, \quad t \ge 1$$

10. **Exercise 25:** Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = 5 + 2\cos(\pi t), \quad y = 3 + 2\sin(\pi t), \quad 1 \le t \le 2$$

11. **Exercise 27:** Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = 5\sin t, \quad y = 2\cos t, \quad -\pi \le t \le 5\pi$$

12. **Exercise 34(a, b):** Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices.

    (a) $x = t^4 - t + 1, \quad y = t^2$

    (b) $x = t^2 - 2t, \quad y = \sqrt{t}$

13. **Exercise 34(c, d):** Match the parametric equations with the graphs labeled I-VI. Give reasons for your choices.

    (c) $x = t^3 - 2t, \quad y = t^2 - t$

    (d) $x = \cos(5t), \quad y = \sin(2t)$

14. **Exercise 41(a, c):** Find parametric equations for the path of a particle that moves along the circle $x^2 + (y-1)^2 = 4$ in the manner described.

    (a) Once around clockwise, starting at $(2, 1)$.

    (c) Halfway around counterclockwise, starting at $(0, 3)$.

15. **Exercise 42(a):** Find parametric equations for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

16. **Exercise 45(b):** Sketch the graph of each curve and explain how the curves differ from one another. (i) $x = t^2, y = t$ (ii) $x = t, y = \sqrt{t}$

17. **Exercise 49:** A curve traced out by a point P at a distance d from the center of a circle of radius r as the circle rolls along a straight line is called a trochoid. Show that its parametric equations are:

$$x = r\theta - d\sin\theta, \quad y = r - d\cos\theta$$

18. **Exercise 58(a):** If a gun is fired with $\alpha = 30^\circ$ and $v_0 = 500$ m/s, when will the bullet hit the ground? How far from the gun will it hit the ground?

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19. **Exercise 28:** Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = \sin t, \quad y = \cos^2 t, \quad -2\pi \leq t \leq 2\pi$$

20. **Exercise 34(e, f):** Match the parametric equations with the graphs labeled I-VI.

    (e) $x = t + \sin(4t), \quad y = t^2 + \cos(3t)$

    (f) $x = t + \sin(2t), \quad y = t + \sin(3t)$

21. **Exercise 51:** Find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle θ as the parameter.

22. **Exercise 53:** A curve, called a witch of Maria Agnesi, consists of all possible positions of the point P in the figure. Show that parametric equations for this curve can be written as:

$$x = 2a \cot \theta, \quad y = 2a \sin^2 \theta$$

23. **Exercise 54:** Find parametric equations for the set of all points P as shown in the figure such that $|OP| = |AB|$. (This curve is called the cissoid of Diocles).

24. **Exercise 56:** The position of one particle is $x = 3 \sin t, y = 2 \cos t$ and a second particle is $x = -3 + \cos t, y = 1 + \sin t$ for $0 \leq t \leq 2\pi$.

    (a) Graph the paths. At how many points do they intersect?

    (b) Do the particles collide? If so, find the collision points.

25. **Exercise 59:** Investigate the family of curves defined by the parametric equations $x = t^2, y = t^3 - ct$. How does the shape change as c increases?