Chapter 11.4 Exercises: The Comparison Tests

James Stewart, Calculus, Metric Edition

Difficulty: Easy

1. **Exercise 3:** Use the Comparison Test to determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$$

2. **Exercise 5:** Use the Comparison Test to determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{5}{2+3^n}$$

3. **Exercise 7:** Use the Comparison Test to determine whether the series converges or diverges.
$$\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$$

4. **Exercise 9:** Use the Comparison Test to determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{n}{n^3+1}$$

5. **Exercise 11:** Use the Limit Comparison Test to determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$$

6. **Exercise 13:** Use the Limit Comparison Test to determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n}}$$

7. **Exercise 15:** Determine whether the series converges or diverges.
$$\sum_{n=1}^{\infty} \frac{n+2}{(n+1)^3}$$

8. **Exercise 17:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{3+\cos n}{3^n}$$

Difficulty: Medium

9. **Exercise 19:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{2n^2+7n}{3^n(n^2+5n-1)}$$

10. **Exercise 20:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$$

11. **Exercise 21:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

12. **Exercise 23:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$$

13. **Exercise 25:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^4+n^2}$$

14. **Exercise 26:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^3+n^2}$$

15. **Exercise 27:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$$

16. **Exercise 29:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n+1}{n2^n}$$

17. **Exercise 31:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

18. **Exercise 32:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$$

Difficulty: Hard

19. **Exercise 35:** Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

20. **Exercise 37:** Use the sum of the first 10 terms to approximate the sum of the series. Estimate the error.

$$\sum_{n=1}^{\infty} \frac{1}{1+2^n}$$

21. **Exercise 39:** For which values of p does the series $\sum_{n=2}^{\infty} \dfrac{1}{n^p \ln n}$ converge?

22. **Exercise 40:** If $a_n > 0$ and $\sum a_n$ converges, show that $\sum \ln(1+a_n)$ also converges.

23. **Exercise 41:** If $a_n > 0$ and $\sum a_n$ converges, show that $\sum (a_n/n)$ also converges.

24. **Exercise 43:** (a) Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is convergent. Prove that if $\lim_{n\to\infty} \dfrac{a_n}{b_n} = 0$, then $\sum a_n$ is also convergent.

25. **Exercise 44:** (a) Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is divergent. Prove that if $\lim_{n\to\infty} \dfrac{a_n}{b_n} = \infty$, then $\sum a_n$ is also divergent.