Additional Notes on Shrinkage Means

The "shrinkage" or BLUP mean for the response variable for a given level of a random factor is a weighted combination of the overall mean, based on the fixed effects, and the ordinary mean for the level of the random factor. The BLUP mean depends on the variance between levels of the random factor, and the variance of the observed response values for each level of the random factor (Littel et al, 1996). In the case of a variance components model (see Chapter 4), we can write the BLUP mean as follows (refer to Verbeke and Molenberghs, 2000, page 80 for additional formulae):

$$u_i = a_i \times \mu + (1 - a_i) \times \mu_i$$

We have the following notation in the equation above:

u_i is the shrinkage mean for level i of the random factor
a_i is a weighting factor for level i, calculated by σ^2_i / ($\sigma^2_{\textit{random factor}} + \sigma^2_i$)
σ^2_i is the variance for level i of the random factor, calculated by σ^2_{error} / n_i
$\sigma^2_{\textit{random factor}}$ is the variance of the random effects associated with the random factor
μ is the overall mean of the response values
μ_i is the mean of the response values for level i of the random factor

When we calculate the shrinkage means, we will use the sample estimates for the quantities in the formula above. The above formula shows how the amount of shrinkage in the BLUPs is a function of the variance of the observed response values within a level of the random factor, and the variance of the random effects. In the extreme case of no variance between levels of the random factor ($\sigma^2_{\textit{random factor}} = 0$), there is complete shrinkage, and the shrinkage estimate of the mean for each level of the random factor is equal to the overall mean. That is, there is no difference in the predicted mean for each level of the random factor.

As the equation above shows, the amount of shrinkage will be a function of the variance of the random effects and the total variance of the observed data for a given level of the random factor. If there is a lot of variability between the means of the response variable for different levels of the random factor, there will be less shrinkage than if there is less variability. In the extreme case when $\sigma^2_{\textit{random factor}} = 0$, the estimated effect of each level of the random effect will also be zero. In the case where $\sigma^2_{\textit{random factor}} > 0$, the extent to which the BLUP for a particular random effect is "shrunken" depends on the amount of information available for predicting that random effect. For example, if we are predicting BLUPs for randomly sampled clusters in our analysis, the prediction of the random cluster effect will be shrunken less for clusters with large sample sizes than for clusters with smaller sample sizes, because we have more information about the larger clusters.