# The SAS System

## The VARMAX Procedure

| | |
|---|---|
| Number of Observations | 130 |
| Number of Pairwise Missing | 0 |
| Observation(s) eliminated by differencing | 1 |

**Simple Summary Statistics**

| Variable | Type | N | Mean | Standard Deviation | Min | Max | Difference |
|---|---|---|---|---|---|---|---|
| Ave_Temperature_F | Dependent | 130 | -0.03308 | 8.58099 | -17.70000 | 17.00000 | 1 |
| Ave_heating_degree_day | Dependent | 130 | 0.00154 | 6.28373 | -16.40000 | 16.70000 | 1 |
| Ave_Cooling_Degree_Day | Dependent | 130 | -0.03231 | 3.70701 | -9.50000 | 8.30000 | 1 |

**Granger-Causality Wald Test**

| Test | DF | Chi-Square | Pr > ChiSq |
|---|---|---|---|
| 1 | 4 | 11.36 | 0.0228 |

| | |
|---|---|
| Test 1: Group 1 Variables: | Ave_Temperature_F |
| Group 2 Variables: | Ave_heating_degree_day Ave_Cooling_Degree_Day |

## The SAS System

### The VARMAX Procedure

| | |
|---|---|
| **Type of Model** | VAR(2) |
| **Estimation Method** | Least Squares Estimation |

| Constant Estimates | |
|---|---|
| **Variable** | **Constant** |
| **Ave_Temperature_F** | 0.15650 |
| **Ave_heating_degree_day** | -0.08734 |
| **Ave_Cooling_Degree_Day** | 0.05867 |

| AR Coefficient Estimates | | | | |
|---|---|---|---|---|
| **Lag** | **Variable** | **Ave_Temperature_F** | **Ave_heating_degree_day** | **Ave_Cooling_Degree_Day** |
| **1** | **Ave_Temperature_F** | -0.91685 | -1.21237 | 2.00978 |
| | **Ave_heating_degree_day** | -0.00214 | 0.17738 | -0.38849 |
| | **Ave_Cooling_Degree_Day** | -0.30930 | -0.40426 | 0.92374 |
| **2** | **Ave_Temperature_F** | 1.03777 | 0.97087 | -1.14333 |
| | **Ave_heating_degree_day** | -1.05519 | -1.11970 | 0.81181 |
| | **Ave_Cooling_Degree_Day** | 0.24621 | 0.12661 | -0.60301 |

| Schematic Representation of Parameter Estimates | | | |
|---|---|---|---|
| **Variable/Lag** | **C** | **AR1** | **AR2** |
| **Ave_Temperature_F** | . | ... | ... |
| **Ave_heating_degree_day** | . | ... | ... |
| **Ave_Cooling_Degree_Day** | . | ... | ... |
| **+ is > 2*std error, - is < -2*std error, . is between, * is N/A** | | | |

| Model Parameter Estimates | | | | | | |
|---|---|---|---|---|---|---|
| **Equation** | **Parameter** | **Estimate** | **Standard Error** | **t Value** | **Pr > \|t\|** | **Variable** |
| **Ave_Temperature_F** | **CONST1** | 0.15650 | 0.64173 | 0.24 | 0.8077 | 1 |
| | **AR1_1_1** | -0.91685 | 1.36604 | -0.67 | 0.5034 | Ave_Temperature_F(t-1) |
| | **AR1_1_2** | -1.21237 | 1.40511 | -0.86 | 0.3899 | Ave_heating_degree_day(t-1) |
| | **AR1_1_3** | 2.00978 | 1.42498 | 1.41 | 0.1610 | Ave_Cooling_Degree_Day(t-1) |
| | **AR2_1_1** | 1.03777 | 1.36487 | 0.76 | 0.4485 | Ave_Temperature_F(t-2) |
| | **AR2_1_2** | 0.97087 | 1.40595 | 0.69 | 0.4912 | Ave_heating_degree_day(t-2) |
| | **AR2_1_3** | -1.14333 | 1.48446 | -0.77 | 0.4427 | Ave_Cooling_Degree_Day(t-2) |
| **Ave_heating_degree_day** | **CONST2** | -0.08734 | 0.51332 | -0.17 | 0.8652 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | AR1_2_1 | -0.00214 | 1.09269 | -0.00 | 0.9984 | Ave_Temperature_F(t-1) |
| | AR1_2_2 | 0.17738 | 1.12395 | 0.16 | 0.8749 | Ave_heating_degree_day(t-1) |
| | AR1_2_3 | -0.38849 | 1.13985 | -0.34 | 0.7338 | Ave_Cooling_Degree_Day(t-1) |
| | AR2_2_1 | -1.05519 | 1.09176 | -0.97 | 0.3357 | Ave_Temperature_F(t-2) |
| | AR2_2_2 | -1.11970 | 1.12462 | -1.00 | 0.3214 | Ave_heating_degree_day(t-2) |
| | AR2_2_3 | 0.81181 | 1.18742 | 0.68 | 0.4955 | Ave_Cooling_Degree_Day(t-2) |
| Ave_Cooling_Degree_Day | CONST3 | 0.05867 | 0.25572 | 0.23 | 0.8189 | 1 |
| | AR1_3_1 | -0.30930 | 0.54434 | -0.57 | 0.5709 | Ave_Temperature_F(t-1) |
| | AR1_3_2 | -0.40426 | 0.55991 | -0.72 | 0.4717 | Ave_heating_degree_day(t-1) |
| | AR1_3_3 | 0.92374 | 0.56783 | 1.63 | 0.1064 | Ave_Cooling_Degree_Day(t-1) |
| | AR2_3_1 | 0.24621 | 0.54388 | 0.45 | 0.6516 | Ave_Temperature_F(t-2) |
| | AR2_3_2 | 0.12661 | 0.56025 | 0.23 | 0.8216 | Ave_heating_degree_day(t-2) |
| | AR2_3_3 | -0.60301 | 0.59154 | -1.02 | 0.3100 | Ave_Cooling_Degree_Day(t-2) |

**Covariances of Innovations**

| Variable | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|
| Ave_Temperature_F | 52.65474 | -38.49914 | 12.45392 |
| Ave_heating_degree_day | -38.49914 | 33.69080 | -3.72720 |
| Ave_Cooling_Degree_Day | 12.45392 | -3.72720 | 8.36104 |

| Log-likelihood | -439.865 |
|---|---|

| Information Criteria | |
|---|---|
| AICC | 948.8492 |
| HQC | 965.0167 |
| AIC | 933.7292 |
| SBC | 1010.734 |
| FPEC | 66.77556 |

**Cross Covariances of Residuals**

| Lag | Variable | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| 0 | Ave_Temperature_F | 49.77518 | -36.39372 | 11.77285 |
| | Ave_heating_degree_day | -36.39372 | 31.84833 | -3.52337 |
| | Ave_Cooling_Degree_Day | 11.77285 | -3.52337 | 7.90380 |
| 1 | Ave_Temperature_F | 0.81287 | -0.96892 | -0.21246 |
| | Ave_heating_degree_day | -0.94625 | -0.48198 | -1.38841 |
| | Ave_Cooling_Degree_Day | -0.08147 | -1.42281 | -1.53241 |
| 2 | Ave_Temperature_F | 6.23424 | -2.65037 | 3.58898 |
| | Ave_heating_degree_day | -4.18486 | 2.04108 | -2.08677 |
| | | | | |

| | | | | |
|---|---|---|---|---|
| | Ave_Cooling_Degree_Day | 1.93743 | -0.62263 | 1.35098 |
| 3 | Ave_Temperature_F | 3.26698 | -0.28576 | 2.75533 |
| | Ave_heating_degree_day | -1.79215 | -2.67443 | -4.32409 |
| | Ave_Cooling_Degree_Day | 1.61328 | -3.06022 | -1.55624 |

**Cross Covariances of Residuals by Variable**

| Variable | Lag | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| Ave_Temperature_F | 0 | 49.77518 | -36.39372 | 11.77285 |
| | 1 | 0.81287 | -0.96892 | -0.21246 |
| | 2 | 6.23424 | -2.65037 | 3.58898 |
| | 3 | 3.26698 | -0.28576 | 2.75533 |
| Ave_heating_degree_day | 0 | -36.39372 | 31.84833 | -3.52337 |
| | 1 | -0.94625 | -0.48198 | -1.38841 |
| | 2 | -4.18486 | 2.04108 | -2.08677 |
| | 3 | -1.79215 | -2.67443 | -4.32409 |
| Ave_Cooling_Degree_Day | 0 | 11.77285 | -3.52337 | 7.90380 |
| | 1 | -0.08147 | -1.42281 | -1.53241 |
| | 2 | 1.93743 | -0.62263 | 1.35098 |
| | 3 | 1.61328 | -3.06022 | -1.55624 |

**Cross Correlations of Residuals**

| Lag | Variable | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| 0 | Ave_Temperature_F | 1.00000 | -0.91406 | 0.59355 |
| | Ave_heating_degree_day | -0.91406 | 1.00000 | -0.22207 |
| | Ave_Cooling_Degree_Day | 0.59355 | -0.22207 | 1.00000 |
| 1 | Ave_Temperature_F | 0.01633 | -0.02434 | -0.01071 |
| | Ave_heating_degree_day | -0.02377 | -0.01513 | -0.08751 |
| | Ave_Cooling_Degree_Day | -0.00411 | -0.08968 | -0.19388 |
| 2 | Ave_Temperature_F | 0.12525 | -0.06657 | 0.18095 |
| | Ave_heating_degree_day | -0.10511 | 0.06409 | -0.13153 |
| | Ave_Cooling_Degree_Day | 0.09768 | -0.03924 | 0.17093 |
| 3 | Ave_Temperature_F | 0.06563 | -0.00718 | 0.13891 |
| | Ave_heating_degree_day | -0.04501 | -0.08397 | -0.27254 |
| | Ave_Cooling_Degree_Day | 0.08134 | -0.19288 | -0.19690 |

**Cross Correlations of Residuals by Variable**

| Variable | Lag | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| Ave_Temperature_F | 0 | 1.00000 | -0.91406 | 0.59355 |
| | 1 | 0.01633 | -0.02434 | -0.01071 |

| | | | | |
|---|---|---|---|---|
| | 2 | 0.12525 | -0.06657 | 0.18095 |
| | 3 | 0.06563 | -0.00718 | 0.13891 |
| Ave_heating_degree_day | 0 | -0.91406 | 1.00000 | -0.22207 |
| | 1 | -0.02377 | -0.01513 | -0.08751 |
| | 2 | -0.10511 | 0.06409 | -0.13153 |
| | 3 | -0.04501 | -0.08397 | -0.27254 |
| Ave_Cooling_Degree_Day | 0 | 0.59355 | -0.22207 | 1.00000 |
| | 1 | -0.00411 | -0.08968 | -0.19388 |
| | 2 | 0.09768 | -0.03924 | 0.17093 |
| | 3 | 0.08134 | -0.19288 | -0.19690 |

**Schematic Representation of Cross Correlations of Residuals**

| Variable/Lag | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Ave_Temperature_F | +−+ | . . . | . . + | . . . |
| Ave_heating_degree_day | −+− | . . . | . . . | . . − |
| Ave_Cooling_Degree_Day | +−+ | . . − | . . . | . −− |

+ is > 2*std error, - is < -2*std error, . is between

**Portmanteau Test for Cross Correlations of Residuals**

| Up To Lag | DF | Chi-Square | Pr > ChiSq |
|---|---|---|---|
| 3 | 9 | 55.34 | <.0001 |

**Univariate Model ANOVA Diagnostics**

| Variable | R-Square | Standard Deviation | F Value | Pr > F |
|---|---|---|---|---|
| Ave_Temperature_F | 0.3147 | 7.25636 | 9.26 | <.0001 |
| Ave_heating_degree_day | 0.1950 | 5.80438 | 4.89 | 0.0002 |
| Ave_Cooling_Degree_Day | 0.3968 | 2.89155 | 13.27 | <.0001 |

**Univariate Model White Noise Diagnostics**

| Variable | Durbin Watson | Normality | | ARCH | |
|---|---|---|---|---|---|
| | | Chi-Square | Pr > ChiSq | F Value | Pr > F |
| Ave_Temperature_F | 1.95818 | 3.74 | 0.1537 | 0.85 | 0.3596 |
| Ave_heating_degree_day | 2.02908 | 14.05 | 0.0009 | 0.07 | 0.7944 |
| Ave_Cooling_Degree_Day | 2.36548 | 14.62 | 0.0007 | 1.17 | 0.2813 |

**Univariate Model AR Diagnostics**

| Variable | AR1 | | AR2 | | AR3 | | AR4 | |
|---|---|---|---|---|---|---|---|---|
| | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ave_Temperature_F | 0.03 | 0.8547 | 1.03 | 0.3596 | 0.85 | 0.4692 | 3.06 | 0.0193 |
| Ave_heating_degree_day | 0.03 | 0.8658 | 0.28 | 0.7598 | 0.47 | 0.7069 | 2.08 | 0.0870 |
| Ave_Cooling_Degree_Day | 5.00 | 0.0271 | 3.70 | 0.0275 | 3.40 | 0.0201 | 4.00 | 0.0044 |

**Simple Impulse Response**

| Lag | Variable Response\Impulse | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| 1 | Ave_Temperature_F | -0.91685 | -1.21237 | 2.00978 |
| | STD | 1.36604 | 1.40511 | 1.42498 |
| | Ave_heating_degree_day | -0.00214 | 0.17738 | -0.38849 |
| | STD | 1.09269 | 1.12395 | 1.13985 |
| | Ave_Cooling_Degree_Day | -0.30930 | -0.40426 | 0.92374 |
| | STD | 0.54434 | 0.55991 | 0.56783 |
| 2 | Ave_Temperature_F | 1.25934 | 1.05490 | -0.65847 |
| | STD | 1.32347 | 1.36306 | 1.39974 |
| | Ave_heating_degree_day | -0.93344 | -0.92859 | 0.37973 |
| | STD | 0.98314 | 1.01300 | 1.04333 |
| | Ave_Cooling_Degree_Day | 0.24495 | 0.05645 | -0.21429 |
| | STD | 0.56133 | 0.57794 | 0.59223 |
| 3 | Ave_Temperature_F | -0.13057 | -0.35167 | 0.36503 |
| | STD | 0.42789 | 0.43391 | 0.51056 |
| | Ave_heating_degree_day | 0.45531 | 0.56358 | -0.78377 |
| | STD | 0.53167 | 0.54545 | 0.58374 |
| | Ave_Cooling_Degree_Day | 0.17461 | 0.06899 | -0.25917 |
| | STD | 0.24370 | 0.25235 | 0.27495 |

**Simple Impulse Response by Variable**

| Variable Response\Impulse | Lag | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| Ave_Temperature_F | 1 | -0.91685 | -1.21237 | 2.00978 |
| | STD | 1.36604 | 1.40511 | 1.42498 |
| | 2 | 1.25934 | 1.05490 | -0.65847 |
| | STD | 1.32347 | 1.36306 | 1.39974 |
| | 3 | -0.13057 | -0.35167 | 0.36503 |
| | STD | 0.42789 | 0.43391 | 0.51056 |
| Ave_heating_degree_day | 1 | -0.00214 | 0.17738 | -0.38849 |
| | STD | 1.09269 | 1.12395 | 1.13985 |
| | 2 | -0.93344 | -0.92859 | 0.37973 |
| | STD | 0.98314 | 1.01300 | 1.04333 |
| | | | | |

| | | | | |
|---|---|---|---|---|
| | 3 | 0.45531 | 0.56358 | -0.78377 |
| | STD | 0.53167 | 0.54545 | 0.58374 |
| Ave_Cooling_Degree_Day | 1 | -0.30930 | -0.40426 | 0.92374 |
| | STD | 0.54434 | 0.55991 | 0.56783 |
| | 2 | 0.24495 | 0.05645 | -0.21429 |
| | STD | 0.56133 | 0.57794 | 0.59223 |
| | 3 | 0.17461 | 0.06899 | -0.25917 |
| | STD | 0.24370 | 0.25235 | 0.27495 |

**Decomposition of Prediction Error Covariances**

| Lead | Variable | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| 1 | Ave_Temperature_F | 52.65474 | 0.00000 | 0.00000 |
| | Ave_heating_degree_day | 28.14911 | 5.54169 | 0.00000 |
| | Ave_Cooling_Degree_Day | 2.94561 | 5.22037 | 0.19506 |
| 2 | Ave_Temperature_F | 63.07917 | 3.02050 | 0.78789 |
| | Ave_heating_degree_day | 30.78453 | 5.76265 | 0.02944 |
| | Ave_Cooling_Degree_Day | 5.15329 | 6.56346 | 0.36150 |
| 3 | Ave_Temperature_F | 68.89324 | 3.97859 | 0.87246 |
| | Ave_heating_degree_day | 32.21250 | 7.50071 | 0.05757 |
| | Ave_Cooling_Degree_Day | 6.38565 | 6.69070 | 0.37046 |
| 4 | Ave_Temperature_F | 71.27972 | 3.97863 | 0.89845 |
| | Ave_heating_degree_day | 33.27615 | 7.71607 | 0.17739 |
| | Ave_Cooling_Degree_Day | 6.59376 | 6.87538 | 0.38356 |
| 5 | Ave_Temperature_F | 71.95825 | 4.00522 | 0.89858 |
| | Ave_heating_degree_day | 33.72312 | 7.77289 | 0.17802 |
| | Ave_Cooling_Degree_Day | 6.61158 | 6.88145 | 0.38364 |
| 6 | Ave_Temperature_F | 72.12137 | 4.00935 | 0.90785 |
| | Ave_heating_degree_day | 33.79994 | 7.80407 | 0.19556 |
| | Ave_Cooling_Degree_Day | 6.62446 | 6.93341 | 0.38550 |

**Decomposition of Prediction Error Covariances by Variable**

| Variable | Lead | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| Ave_Temperature_F | 1 | 52.65474 | 0.00000 | 0.00000 |
| | 2 | 63.07917 | 3.02050 | 0.78789 |
| | 3 | 68.89324 | 3.97859 | 0.87246 |
| | 4 | 71.27972 | 3.97863 | 0.89845 |
| | 5 | 71.95825 | 4.00522 | 0.89858 |
| | 6 | 72.12137 | 4.00935 | 0.90785 |
| Ave_heating_degree_day | 1 | 28.14911 | 5.54169 | 0.00000 |

| | | | | |
|---|---|---|---|---|
| | 2 | 30.78453 | 5.76265 | 0.02944 |
| | 3 | 32.21250 | 7.50071 | 0.05757 |
| | 4 | 33.27615 | 7.71607 | 0.17739 |
| | 5 | 33.72312 | 7.77289 | 0.17802 |
| | 6 | 33.79994 | 7.80407 | 0.19556 |
| Ave_Cooling_Degree_Day | 1 | 2.94561 | 5.22037 | 0.19506 |
| | 2 | 5.15329 | 6.56346 | 0.36150 |
| | 3 | 6.38565 | 6.69070 | 0.37046 |
| | 4 | 6.59376 | 6.87538 | 0.38356 |
| | 5 | 6.61158 | 6.88145 | 0.38364 |
| | 6 | 6.62446 | 6.93341 | 0.38550 |

**Proportions of Prediction Error Covariances**

| Lead | Variable | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| 1 | Ave_Temperature_F | 1.00000 | 0.00000 | 0.00000 |
| | Ave_heating_degree_day | 0.83551 | 0.16449 | 0.00000 |
| | Ave_Cooling_Degree_Day | 0.35230 | 0.62437 | 0.02333 |
| 2 | Ave_Temperature_F | 0.94306 | 0.04516 | 0.01178 |
| | Ave_heating_degree_day | 0.84164 | 0.15755 | 0.00080 |
| | Ave_Cooling_Degree_Day | 0.42666 | 0.54341 | 0.02993 |
| 3 | Ave_Temperature_F | 0.93422 | 0.05395 | 0.01183 |
| | Ave_heating_degree_day | 0.80995 | 0.18860 | 0.00145 |
| | Ave_Cooling_Degree_Day | 0.47488 | 0.49757 | 0.02755 |
| 4 | Ave_Temperature_F | 0.93596 | 0.05224 | 0.01180 |
| | Ave_heating_degree_day | 0.80827 | 0.18742 | 0.00431 |
| | Ave_Cooling_Degree_Day | 0.47599 | 0.49632 | 0.02769 |
| 5 | Ave_Temperature_F | 0.93620 | 0.05211 | 0.01169 |
| | Ave_heating_degree_day | 0.80921 | 0.18652 | 0.00427 |
| | Ave_Cooling_Degree_Day | 0.47645 | 0.49590 | 0.02765 |
| 6 | Ave_Temperature_F | 0.93617 | 0.05204 | 0.01178 |
| | Ave_heating_degree_day | 0.80862 | 0.18670 | 0.00468 |
| | Ave_Cooling_Degree_Day | 0.47510 | 0.49726 | 0.02765 |

**Proportions of Prediction Error Covariances by Variable**

| Variable | Lead | Ave_Temperature_F | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
|---|---|---|---|---|
| Ave_Temperature_F | 1 | 1.00000 | 0.00000 | 0.00000 |
| | 2 | 0.94306 | 0.04516 | 0.01178 |
| | 3 | 0.93422 | 0.05395 | 0.01183 |
| | 4 | 0.93596 | 0.05224 | 0.01180 |

| | | | | |
|---|---|---|---|---|
| | 5 | 0.93620 | 0.05211 | 0.01169 |
| | 6 | 0.93617 | 0.05204 | 0.01178 |
| Ave_heating_degree_day | 1 | 0.83551 | 0.16449 | 0.00000 |
| | 2 | 0.84164 | 0.15755 | 0.00080 |
| | 3 | 0.80995 | 0.18860 | 0.00145 |
| | 4 | 0.80827 | 0.18742 | 0.00431 |
| | 5 | 0.80921 | 0.18652 | 0.00427 |
| | 6 | 0.80862 | 0.18670 | 0.00468 |
| Ave_Cooling_Degree_Day | 1 | 0.35230 | 0.62437 | 0.02333 |
| | 2 | 0.42666 | 0.54341 | 0.02993 |
| | 3 | 0.47488 | 0.49757 | 0.02755 |
| | 4 | 0.47599 | 0.49632 | 0.02769 |
| | 5 | 0.47645 | 0.49590 | 0.02765 |
| | 6 | 0.47510 | 0.49726 | 0.02765 |

**Forecasts**

| Variable | Obs | Time | Forecast | Standard Error | 95% Confidence Limits | |
|---|---|---|---|---|---|---|
| Ave_Temperature_F | 132 | JUL2020 | 84.37588 | 7.25636 | 70.15368 | 98.59808 |
| | 133 | AUG2020 | 90.20011 | 12.89959 | 64.91739 | 115.48284 |
| | 134 | SEP2020 | 92.57325 | 18.45130 | 56.40936 | 128.73714 |
| | 135 | OCT2020 | 93.96056 | 23.60054 | 47.70435 | 140.21677 |
| | 136 | NOV2020 | 95.18787 | 28.23376 | 39.85072 | 150.52503 |
| Ave_heating_degree_day | 132 | JUL2020 | -4.12863 | 5.80438 | -15.50500 | 7.24774 |
| | 133 | AUG2020 | -8.67729 | 9.23474 | -26.77704 | 9.42247 |
| | 134 | SEP2020 | -10.79176 | 12.31255 | -34.92391 | 13.34039 |
| | 135 | OCT2020 | -11.56526 | 15.34767 | -41.64614 | 18.51561 |
| | 136 | NOV2020 | -12.01405 | 18.22855 | -47.74135 | 23.71325 |
| Ave_Cooling_Degree_Day | 132 | JUL2020 | 13.61054 | 2.89155 | 7.94321 | 19.27787 |
| | 133 | AUG2020 | 14.61727 | 5.58531 | 3.67027 | 25.56427 |
| | 134 | SEP2020 | 14.79942 | 7.73890 | -0.36855 | 29.96739 |
| | 135 | OCT2020 | 15.39817 | 9.49208 | -3.20597 | 34.00232 |
| | 136 | NOV2020 | 16.10029 | 11.04574 | -5.54895 | 37.74954 |

## The SAS System

### The VARMAX Procedure

| | |
|---|---|
| Number of Observations | 130 |
| Number of Pairwise Missing | 0 |
| Observation(s) eliminated by differencing | 1 |

**Simple Summary Statistics**

| Variable | Type | N | Mean | Standard Deviation | Min | Max | Difference |
|---|---|---|---|---|---|---|---|
| Ave_heating_degree_day | Dependent | 130 | 0.00154 | 6.28373 | -16.40000 | 16.70000 | 1 |
| Ave_Cooling_Degree_Day | Dependent | 130 | -0.03231 | 3.70701 | -9.50000 | 8.30000 | 1 |
| Ave_Temperature_F | Independent | 130 | -0.03308 | 8.58099 | -17.70000 | 17.00000 | 1 |

## The SAS System

### The VARMAX Procedure

| | |
|---|---|
| **Type of Model** | VARX(2,1) |
| **Estimation Method** | Least Squares Estimation |

| Constant Estimates | |
|---|---|
| **Variable** | **Constant** |
| **Ave_heating_degree_day** | 0.02694 |
| **Ave_Cooling_Degree_Day** | 0.02166 |

| Coefficient Estimates of Independent Variables | | |
|---|---|---|
| **Lag** | **Variable** | **Ave_Temperature_F** |
| **0** | **Ave_heating_degree_day** | -0.73252 |
| | **Ave_Cooling_Degree_Day** | 0.23652 |
| **1** | **Ave_heating_degree_day** | -0.53334 |
| | **Ave_Cooling_Degree_Day** | -0.09281 |

| AR Coefficient Estimates | | | |
|---|---|---|---|
| **Lag** | **Variable** | **Ave_heating_degree_day** | **Ave_Cooling_Degree_Day** |
| **1** | **Ave_heating_degree_day** | -0.56687 | 0.94418 |
| | **Ave_Cooling_Degree_Day** | -0.11788 | 0.44874 |
| **2** | **Ave_heating_degree_day** | -0.10558 | -0.34312 |
| | **Ave_Cooling_Degree_Day** | -0.10380 | -0.33177 |

| Schematic Representation of Parameter Estimates | | | | | |
|---|---|---|---|---|---|
| **Variable/Lag** | **C** | **XL0** | **XL1** | **AR1** | **AR2** |
| **Ave_heating_degree_day** | . | - | . | .+ | -- |
| **Ave_Cooling_Degree_Day** | . | + | . | .. | -- |
| **+ is > 2*std error, - is < -2*std error, . is between, * is N/A** | | | | | |

| Model Parameter Estimates | | | | | | |
|---|---|---|---|---|---|---|
| **Equation** | **Parameter** | **Estimate** | **Standard Error** | **t Value** | **Pr > \|t\|** | **Variable** |
| **Ave_heating_degree_day** | **CONST1** | 0.02694 | 0.20862 | 0.13 | 0.8975 | 1 |
| | **XL0_1_1** | -0.73252 | 0.02948 | -24.85 | 0.0001 | Ave_Temperature_F(t) |
| | **XL1_1_1** | -0.53334 | 0.39277 | -1.36 | 0.1770 | Ave_Temperature_F(t-1) |
| | **AR1_1_1** | -0.56687 | 0.40539 | -1.40 | 0.1646 | Ave_heating_degree_day(t-1) |
| | **AR1_1_2** | 0.94418 | 0.41942 | 2.25 | 0.0262 | Ave_Cooling_Degree_Day(t-1) |

| | | | | | | |
|---|---|---|---|---|---|---|
| | AR2_1_1 | -0.10558 | 0.03609 | -2.93 | 0.0041 | Ave_heating_degree_day(t-2) |
| | AR2_1_2 | -0.34312 | 0.07038 | -4.88 | 0.0001 | Ave_Cooling_Degree_Day(t-2) |
| Ave_Cooling_Degree_Day | CONST2 | 0.02166 | 0.20585 | 0.11 | 0.9164 | 1 |
| | XL0_2_1 | 0.23652 | 0.02909 | 8.13 | 0.0001 | Ave_Temperature_F(t) |
| | XL1_2_1 | -0.09281 | 0.38756 | -0.24 | 0.8111 | Ave_Temperature_F(t-1) |
| | AR1_2_1 | -0.11788 | 0.40001 | -0.29 | 0.7687 | Ave_heating_degree_day(t-1) |
| | AR1_2_2 | 0.44874 | 0.41385 | 1.08 | 0.2804 | Ave_Cooling_Degree_Day(t-1) |
| | AR2_2_1 | -0.10380 | 0.03561 | -2.91 | 0.0042 | Ave_heating_degree_day(t-2) |
| | AR2_2_2 | -0.33177 | 0.06945 | -4.78 | 0.0001 | Ave_Cooling_Degree_Day(t-2) |

| Covariances of Innovations | | |
|---|---|---|
| Variable | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
| Ave_heating_degree_day | 5.56211 | 5.37858 |
| Ave_Cooling_Degree_Day | 5.37858 | 5.41543 |

| Log-likelihood | -132.049 |
|---|---|

| Information Criteria | |
|---|---|
| AICC | 303.6622 |
| HQC | 317.7981 |
| AIC | 298.0986 |
| SBC | 346.5831 |
| FPEC | 1.326095 |

| Cross Covariances of Residuals | | | |
|---|---|---|---|
| Lag | Variable | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
| 0 | Ave_heating_degree_day | 5.25793 | 5.08444 |
| | Ave_Cooling_Degree_Day | 5.08444 | 5.11928 |
| 1 | Ave_heating_degree_day | -1.47750 | -1.46077 |
| | Ave_Cooling_Degree_Day | -1.40682 | -1.41738 |
| 2 | Ave_heating_degree_day | 0.37069 | 0.44661 |
| | Ave_Cooling_Degree_Day | 0.38328 | 0.39253 |
| 3 | Ave_heating_degree_day | -2.46361 | -2.45554 |
| | Ave_Cooling_Degree_Day | -2.42050 | -2.40663 |

| Cross Correlations of Residuals | | | |
|---|---|---|---|
| Lag | Variable | Ave_heating_degree_day | Ave_Cooling_Degree_Day |
| 0 | Ave_heating_degree_day | 1.00000 | 0.98001 |
| | Ave_Cooling_Degree_Day | 0.98001 | 1.00000 |
| 1 | Ave_heating_degree_day | -0.28100 | -0.28156 |

| | | | |
|---|---|---|---|
| | Ave_Cooling_Degree_Day | -0.27116 | -0.27687 |
| 2 | Ave_heating_degree_day | 0.07050 | 0.08608 |
| | Ave_Cooling_Degree_Day | 0.07388 | 0.07668 |
| 3 | Ave_heating_degree_day | -0.46855 | -0.47330 |
| | Ave_Cooling_Degree_Day | -0.46655 | -0.47011 |

| Schematic Representation of Cross Correlations of Residuals | | | | |
|---|---|---|---|---|
| Variable/Lag | 0 | 1 | 2 | 3 |
| Ave_heating_degree_day | ++ | -- | . . | -- |
| Ave_Cooling_Degree_Day | ++ | -- | . . | -- |
| + is > 2*std error, - is < -2*std error, . is between | | | | |

| Portmanteau Test for Cross Correlations of Residuals | | | |
|---|---|---|---|
| Up To Lag | DF | Chi-Square | Pr > ChiSq |
| 3 | 4 | 56.56 | <.0001 |

| Univariate Model ANOVA Diagnostics | | | | |
|---|---|---|---|---|
| Variable | R-Square | Standard Deviation | F Value | Pr > F |
| Ave_heating_degree_day | 0.8671 | 2.35841 | 131.58 | <.0001 |
| Ave_Cooling_Degree_Day | 0.6093 | 2.32711 | 31.45 | <.0001 |

| Univariate Model White Noise Diagnostics | | | | | |
|---|---|---|---|---|---|
| Variable | Durbin Watson | Normality | | ARCH | |
| | | Chi-Square | Pr > ChiSq | F Value | Pr > F |
| Ave_heating_degree_day | 2.54492 | 0.63 | 0.7293 | 3.73 | 0.0557 |
| Ave_Cooling_Degree_Day | 2.54045 | 0.69 | 0.7066 | 2.73 | 0.1009 |

| Univariate Model AR Diagnostics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Variable | AR1 | | AR2 | | AR3 | | AR4 | |
| | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F | F Value | Pr > F |
| Ave_heating_degree_day | 10.92 | 0.0012 | 5.45 | 0.0054 | 18.42 | <.0001 | 24.31 | <.0001 |
| Ave_Cooling_Degree_Day | 10.53 | 0.0015 | 5.28 | 0.0063 | 17.82 | <.0001 | 25.36 | <.0001 |