Sparse Representation Based Face Recognition

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Due: 23:59, January 23, 2018.
No Late Submission Permitted

1 Problem Formulation

This assignment is about how a computer " sees" the world. For a computer, an image is an array of numbers, a kind of 2-dimensional signal (if we employ grayscale image). As human beings, we treat an image as a whole, rather than a set of numbers. Moreover, the more we see, the better our knowledge of the given object will be. Analogously, a computer will be supplied with a set of face photos for an individual as the training samples, to learn the face of the person. Other pictures of the same person will be used as the validation and test data.

For simplicity, we model the identification of a given face among the database as a **linear classification** problem. It translates to solving the following linear system:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$$

$\mathbf{A}$ is the matrix comprised of **features** from the training samples, and $\mathbf{x}$ is the coefficients vector, $\mathbf{y}$ is the test samples, and $\mathbf{e}$ is the error/discrepancy. Figure 1 gives an cogent representation of this problem.

2 Sparse representation — $\mathbf{x}$

We want to solve for a coefficient vector that is as **sparse** as possible while **minimizing the error** $\mathbf{e}$. A sparse vector means many zeros in the vector. Figure 2 should give you an intuitive picture of our objective:

Since we are aiming to minimize the error under the sparsity constraint, our objective function is:

$$f(x) = arg \min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1 \quad (1)$$

After you calculate the sparse coefficient vector $\mathbf{x}$, you may choose various ways to yield the final classification. We have discussed some techniques during lecture; You may use the provided linear solver for a quick solution, but <u>bonus may be credited for the self-implemented linear solver. You may also engineer new methods for better performance, in which case the detailed explanation is needed in the "Algorithm" section in your report.</u>

3 Principal Component Analysis (PCA)

As we illustrate in the lecture, PCA is a general dimensional reduction algorithm which can assist you to perform the dimensional reduction on certain types of input. We will use the linear space of all available face images to revise this concept briefly.

We may choose diverse representations of the human face, e.g., grayscale intensity, color histogram, SIFT descriptor, etc. Grayscale intensity is the simplest representation. Every concatenated vector represents an individual snapshot. If we have N individuals for reference, and one snapshot of $w \times h$ dimension from each person, the integrated face matrix A is of size $(w \times h) \times N$ dimension. Its column space is in $\mathbb{R}^{w \times h}$. $rank(\mathbf{A})$ is no larger than N, therefore, due to redundancy and other factors, $\mathbf{A}$ can be reduced along its column dimension. There are some dimension reduction

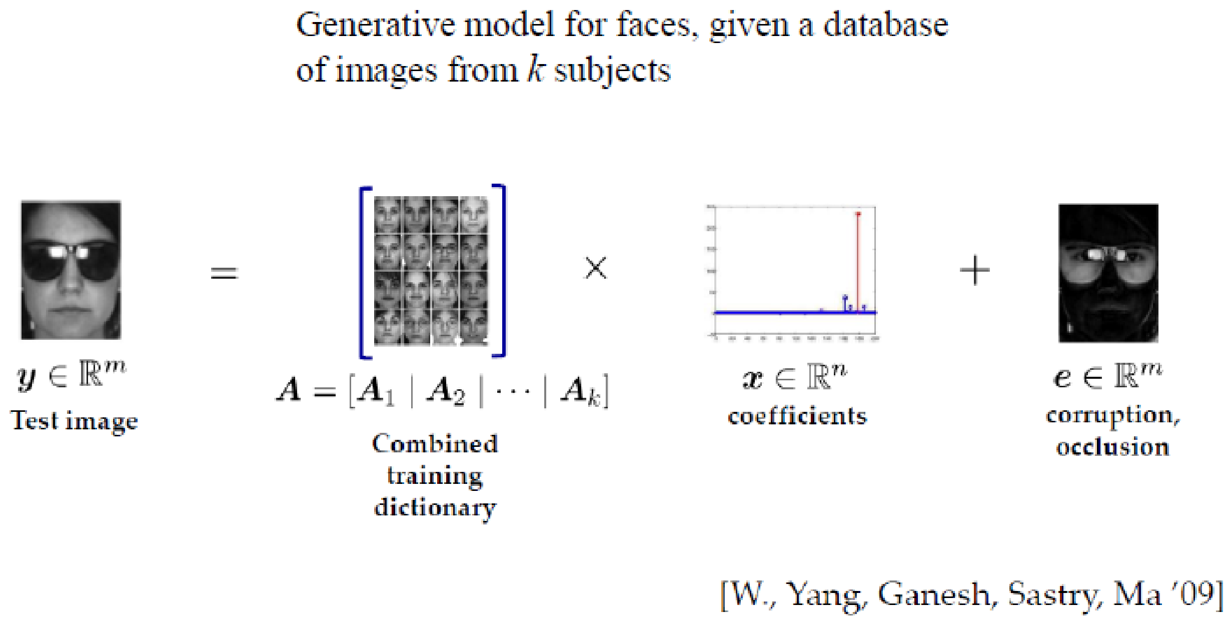

Figure 1: The problem formulation

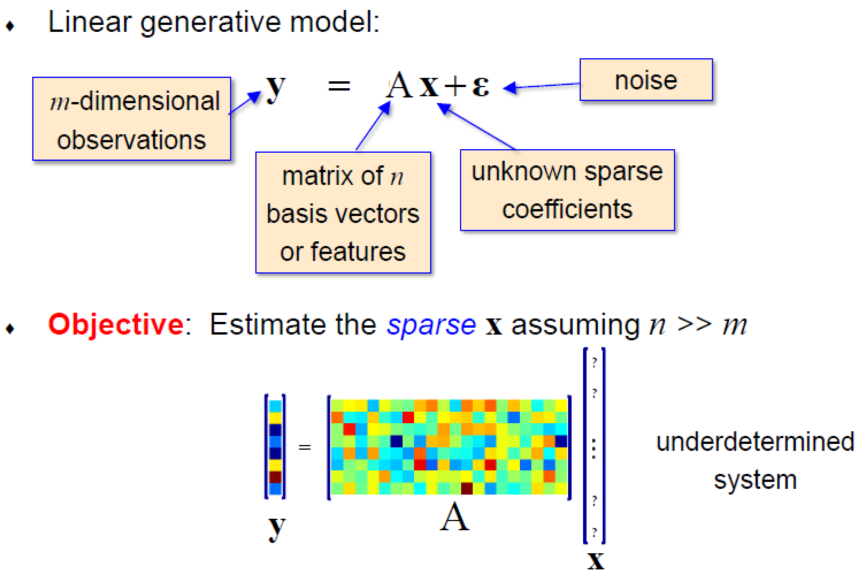

Figure 2: Interpretation of linear classification

Figure 3: A space of all available faces

algorithms, PCA is one of them. We want to effectively model the valid subspace of gray-scale representation — “Eigenface”. An eigenface representation constructs a low-dimensional linear subspace that best delineates the **variation** in the set of face images.

Why to capture (principal) variance? Let’s recall the purpose of PCA — we want a more laconic representation of faces. A base change characterizes the change between different representation. For instance, (0, 0, 1) and (1, 2, 3) may represent the same absolute position in space under different coordinate systems, but apparently, the former one is more concise. Similarly, we use the main variance to be the basis to simplify the representation of images.

How to capture the variation of pixel information? — How to find a basis of a given subspace? We can characterize a set of data by its mean and variance. After centralizing the data to the origin, the only descriptive information left is variance. We compute the covariance matrix of A which captures **all** the variance and correlation between pixel information. Its eigenvectors are the principal components of the variance space. We learn about different methods for basis construction. You may choose whichever way you want, for example, SVD or eigendecomposition. You should specify your choice and the related mathematical theories in your “Algorithm” section. Typically, we want the principal components to retain 95% fidelity.

Representation under new basis Once you have the principal variance by performing PCA on $\mathbf{A}$, you should represent the original $\mathbf{A}$ on the new basis as Eigenface $\mathbf{E}$. Also, remember to re-base the test samples. Only if they are under same linear space, the function is meaningful.

4 Assignment Requirements

4.1 Code

Three to four **MATLAB** functions, depending on whether you have self-written linear solver. Function templates are provided. **DO NOT** change provided the interface. Make sure your functions can run in the test environment: You **MAY** read the official document for insight, but **MAY NOT**

Table 1: Test Environment Specs

System	**CPU**	**Memory**	**MATLAB**
Windows 10 x64	Intel Core i7-4710MQ	16G	R2017a

copy official implementation, **NOR** call the official PCA function within your implementation. You **MAY** discuss the concept and language specs with classmates or on Piazza, but **MAY NOT** possess or reference any code from the Internet or previous hand-ins. Once found, zero score. Suspects will be asked for the oral defense.

4.2 Interface

Implement your program in the provided template, for PCA, please read the MATLAB document for the detailed meaning of input and output parameters. For SRBFR, your program should be able to read from a given directory, with a number specifying how many training samples. The given path will be the path to "CroppedYale" folder, containing all the sub-folders. For feature_sign, it should be of the same interface as the provided function.

4.3 Dataset and Exemplary Works

Download link to the dataset is provided. Some good works from the previous course are provided. You may refer to their style.

4.4 Report

Your report should adhere to the ACM SIGGRAPH style. Must be LaTeXwritten report, others not accepted. In the body part, it should contain the following sections:

- Problem Formulation: describing the problem and its mathematical formulation.
- Algorithm: your algorithm and different choices in implementations.
- Dataset Description: how does the data characterize as given, and your corresponding selection criteria for training, validation and test set.
- Performance: your performance under different parameter tuning. Better to be a diagram with analysis.
- Observation: how is the current algorithm? Any possible improvement? Any derived experiments should be specified in this section, e.g., a new representation other than grayscale intensity, or different objective function, or regularization term.
- Acknowledgement: give credit to the dataset maker and anyone you discuss with.
- Reference: use the given BibTeX file. Add any paper or material other than the provided several.

4.5 Scoring and Submission

- The full score of this assignment will add extra credit up to 5 to your overall scoring.
- Both report and code need to be submitted as **ONE** compressed package to `http://10.19.124.26:8000/u/d/b859222c839349738d24/`.
- Naming: SRBFR_studentID_email. for example: SRBFR_12345678_dingpeng
- **DO NOT** upload your dataset.

5 Handy tools

- Online LaTeX: Overleaf: `https://www.overleaf.com/signup?ref=58e82dab592a`
  *__DO NOT__ share your document links to others, once opened they will have access to your report.
- Math Symbol List: `http://web.ift.uib.no/Teori/KURS/WRK/TeX/symALL.html`
- Grammarly: `https://www.grammarly.com`
- Table Generator: `http://www.tablesgenerator.com/`
- Figures and Table Reference: `http://web.mit.edu/rsi/www/pdfs/figtab-handouts.pdf`