

Algorithmic Mechanism Modeling of WeChat Red Envelope Game

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Abstract

This presentation focuses on the analysis and simulation of stochastic algorithmic mechanism behind WeChat Red Envelopes. The whole process from raw data analysis, model hypotheses testing to algorithm simulation will be included. A good approximation of WeChat Red Envelope algorithm will finally get put forward.

Outline

- 1 Introduction
- 2 Data Collecting and Analysis
- 3 Model Hypotheses
- 4 Simulations and Testing
- 5 Conclusions

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Problem Description

WeChat Red Envelope is a popular social game based on WeChat Platform. It gains popularity mainly from its randomness.

Game Process:

- 1 The sender plugs in a certain amount of money m into a virtual red envelope, sets the restriction of envelope size n - the number of people that can open this envelope, and sends it into a chat group.
- 2 Members in the group can open this envelope and grab a random amount of money, as long as the limitation of number of people is not exceeded.

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Data Analysis

- Dataset is generated within two chat groups of 36 members.
- Red envelope size is fixed as

[Amount $m = 10$ Yuan / Number of people $n = 10$]

during the analysis stage, in the purpose of regulation and normalization of data.

- Overall 202 pieces of valid data are collected.

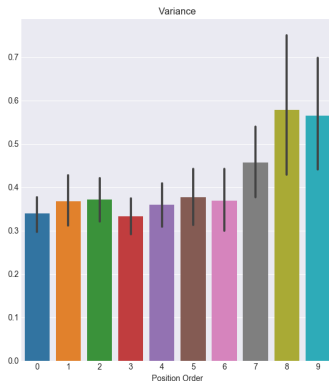
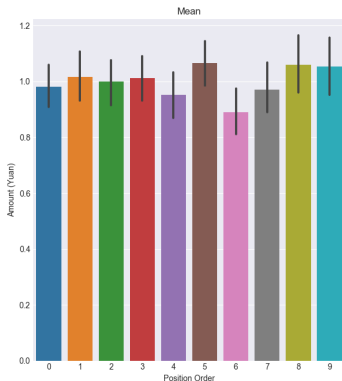
Definition

Denote the order of snatching as position $0, 1, \dots, 9$. Let A_i be the amount of money that is grabbed by position i , $i \in \{0, 1, \dots, 9\}$

Therefore $\sum_{i=0}^9 A_i = 10$ for each envelope.

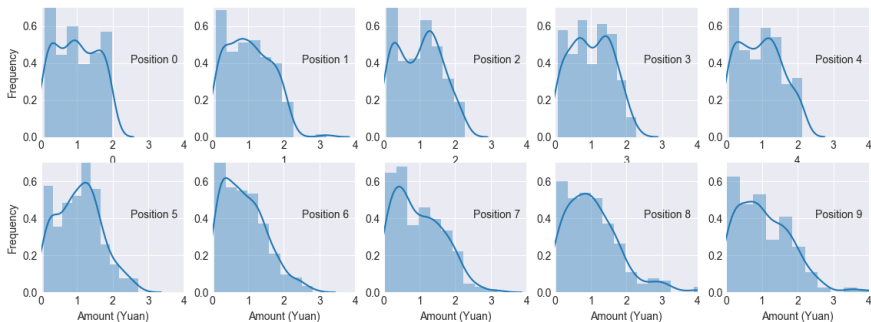
Mean and Variance at Positions

- The *mean* of A_i shows great uniformity among different positions.
- *Variance* at latter positions are generally larger, and reveals a jump at position 8.



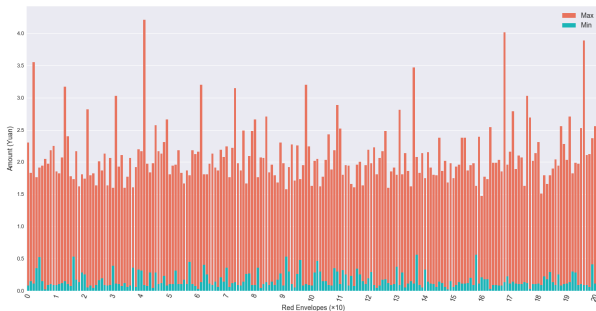
Distribution at Positions

A_i approximately follows a Uniform distribution for smaller order positions, and a truncated Normal distribution for latter positions.



Max and Min Amount in Envelopes

- The Maximum amount of money in each red envelope is generally larger than 1.5 Yuan, and smaller than 2.0 Yuan, except for some outstanding peaks which reach the largest value of 4.21 Yuan.
- The Minimum amount is generally smaller than 0.5 Yuan, and touches the smallest possible value of 0.01 Yuan in some envelopes.



Data Features

- ① Each position has a **generally stable mean** around $m/n = 1$.
- ② **Variance is slightly increasing, and shows a large jump at the penultimate position.**
- ③ Smaller order positions, **especially the first position, reveal an Uniform distribution**, meanwhile **the latter positions show a truncated Normal distribution.**
- ④ The Min amount of money basically ranges from 0.01 Yuan to 0.5 Yuan, and has the possibility of touching the very bottom.
- ⑤ The Max amount of money basically ranges from 1.5 Yuan to 2.0 Yuan, and has the possibility of reaching a respectively large value of over 4.0 Yuan.

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Model Hypotheses

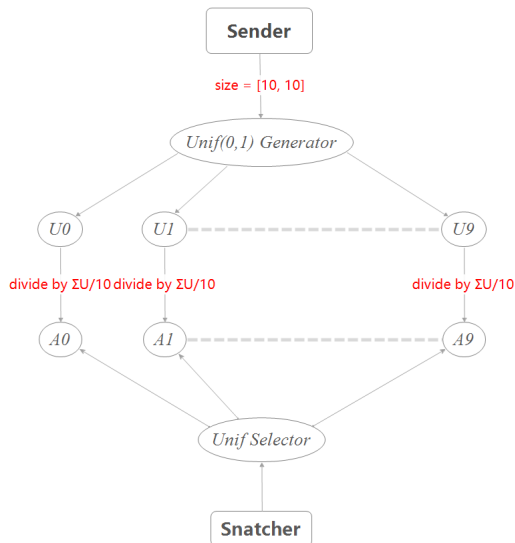
Based on the five observations, the following two hypotheses of algorithm models are proposed and will be examined mathematically.

- *Static-Uniform Model*
- *Dynamic-Updating Model*

Static-Uniform Model

Suppose the allocation of money inside every red envelope is completely computed at the exact sending time. The envelope seen by chat group members is simply static, offline package of numbers. Every time the envelope is snatched, it returns a uniformly randomly chosen number inside the package.

Static-Updating Model



Static-Uniform Model

Mathematical Verifications:

- Expectation of A_i :

$$E(A_i) = \frac{E(\sum_{j=0}^9 A_j)}{10} = 1$$

for all $i \in \{0, 1, \dots, 9\}$.

- Max and Min value in each envelope: Since $U_0, U_1, \dots, U_9 \sim \text{Unif}(0, 1)$, the j th order statistics $U_{(j)} \sim \text{Beta}(j, 10 - j + 1)$. Therefore,

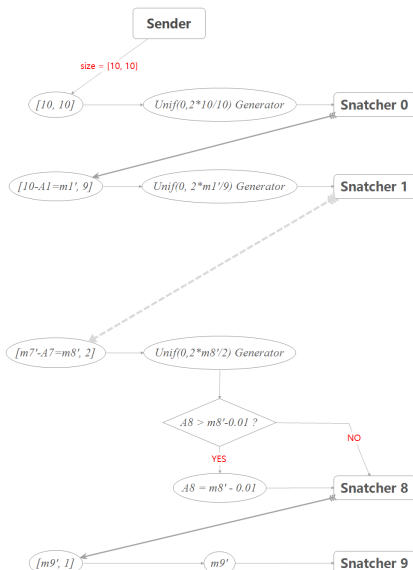
$$\begin{aligned}P(U_{\min} \leq 0.25) &= P(U_{(1)} \leq 0.25) \approx 0.947 \\P(U_{\max} \geq 0.75) &= P(U_{(10)} \geq 0.75) \approx 0.947\end{aligned}$$

which agrees with insight 4) and 5).

Dynamic-Updating Model

Suppose the amount of money snatched from the red envelope is generated dynamically at the time when a member opens it. Under this scenario, the amount of money A_0, A_1, \dots, A_9 grabbed by different positions are dependant with each other. Therefore, every generating of A_i is influenced by its position i and the rest money m'_i . A significant restriction that might get neglected is that every person should at least grab 0.01 Yuan, therefore when the algorithm reaches the stage A_8 , it needs some adjustments to ensure $A_9 \geq 0.01$.

Dynamic-Updating Model



Dynamic-Updating Model

Mathematical Verifications:

- Expectation of A_i : For the first position, $A_0 \sim \text{Unif}(0, 2\frac{m}{n})$,

$$E(A_0) = \frac{0 + 2 \times \frac{m}{n}}{2} = \frac{0 + 2 \times \frac{10}{10}}{2} = 1$$

Conditioning on the first position, $A_1|A_0 = a \sim \text{Unif}(0, 2\frac{m-a}{n-1})$.
So according to Adam's Law, for the position 1, we have

$$\begin{aligned} E(A_1) &= E(E(A_1|A_0)) \\ &= E\left(\frac{10 - A_0}{9}\right) \\ &= \frac{10 - E(A_0)}{9} \\ &= 1 \end{aligned}$$

and similar for the rest positions.

Dynamic-Updating Model

Mathematical Verifications:

- The variance of A_0 is

$$\text{Var}(A_0) = \frac{(2 - 0)^2}{12} = \frac{1}{3}$$

and according to Eve's Law, variance of A_1 at position 1 will be

$$\begin{aligned}\text{Var}(A_1) &= E(\text{Var}(A_1|A_0)) + \text{Var}(E(A_1|A_0)) \\ &= E\left(\frac{(2\frac{10-A_0}{9} - 0)^2}{12}\right) + \text{Var}\left(\frac{10 - A_0}{9}\right) \\ &= \frac{247}{9^3} \approx 0.339 > \frac{1}{3} = \text{Var}(A_0)\end{aligned}$$

which agrees with the increasing variance in insight 2).

Dynamic-Updating Model

- Also, as the result of extra adjusting at position 8, A_8 and A_9 suffers more uncertainty than previous positions, which perfectly answers the jump of variance at position 8.
- The truncated Normal distribution at latter positions is also a natural consequence under this model. Detailed proof will be omitted.

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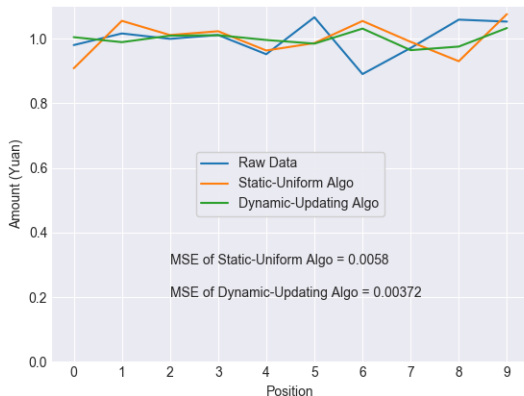
Simulation and Testing

Simulation programs of both Static-Uniform model and Dynamic-Updating model are accomplished using *Python* language. In order to conduct hypotheses testing, both programs are run with the same size of red envelope $[10, 10]$ and the same number of samples 202. After acquiring the simulation results, the following three targets are evaluated and compared to the original data:

- **Uniformity of Mean**
- **Regression Curve of Variance**
- **Probability Distribution at Positions**

Uniformity of Mean

The mean values of amount of money A_0, A_1, \dots, A_9 from original raw data and two simulation models are as follows:



Uniformity of Mean

Definition

Denote \tilde{A}_i to be the mean values of raw data, and \hat{A}_i to be the mean values of simulated data, for $i \in \{0, 1, \dots, 9\}$. Mean square error (MSE) of simulation results compared to raw data is defined as

$$MSE = \frac{1}{10} \sum_{j=0}^9 (\hat{A}_j - \tilde{A}_j)^2$$

which represents the deviation of mean values from raw data.

Uniformity of Mean

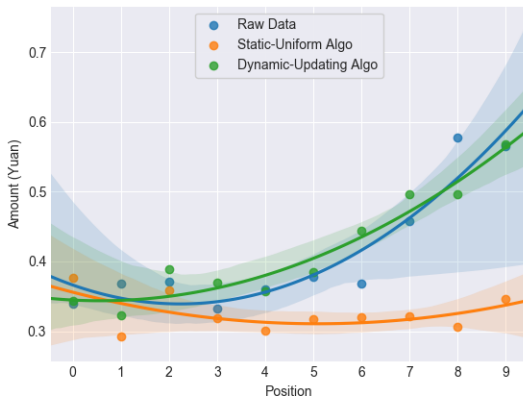
Comparison:

- MSE of Dynamic-Updating model ≈ 0.00372
- MSE of Static-Uniform model ≈ 0.00580 .

This reveals that the Dynamic-Updating model generally performs better than Static-Uniform model on approximating mean values.

Variance Regression

The variance of A_0, A_1, \dots, A_9 are sampled and estimated using an order-2 regression. The regression plot is shown below:



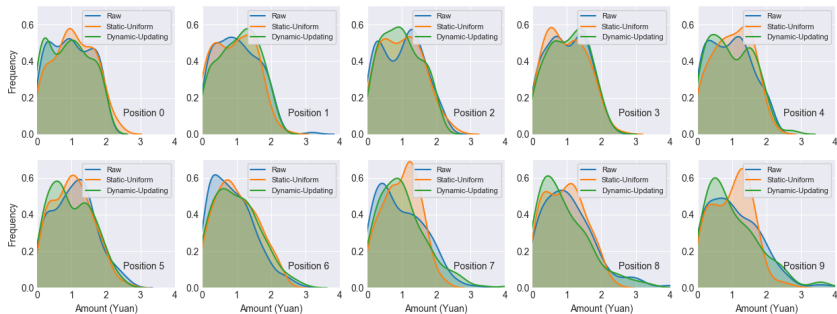
Variance Regression

Comparison:

It can be observed that the output of Dynamic-Updating model follows a perfect regression curve, meanwhile the Static-Uniform model does not reproduce the increasing and jumping variance. Therefore, Dynamic-Updating model performs much better than Static-Uniform model on approximating the fluctuation of amount of money.

Distribution at Positions

The distribution of A_0, A_1, \dots, A_9 are from original raw data and two simulation models are shown below:



Distribution at Positions

Comparison:

Both Static-Uniform and Dynamic-Updating models perform well in approximating distributions at different positions. However, Static-Uniform algorithm does not reveal the Uniform distribution at earlier positions and the truncated Normal distribution at latter positions as clearly as Dynamic-Updating algorithm. Therefore, Dynamic-Updating model still performs better than Static-Uniform model on approximating the distribution at each position.

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Conclusions

In summary, the Dynamic-Updating model is a perfect approximation of the Wechat Red Envelope algorithm mechanism.

- It is an online algorithm that generates a uniformly random amount of money ranging from 0.01 Yuan to twice the mean of rest envelope $2 \frac{m'_i}{n-i}$.
- It satisfies the uniformity of mean and the increasing variance along the positions, and especially answers the jump of variance at the penultimate position.

Acknowledgement

- ① During this project, I collaborated and discussed with classmates Xinyu Zhang, Ruoyu Wang, Zikun Xiao and Haoyi Zhu.
- ② I also adopted part of contents from Zhihu to Section III about the idea of dynamic algorithm mechanism.
- ③ Original red envelope data is collected in two chat groups on WeChat with in total 36 student members. I appreciate Ruoyu Wang and Yuyan Zhou for helping to collect the raw data.

References



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