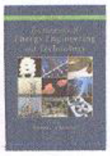

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Solar Water Heaters: Cylindrical Shell-and-Tube Type

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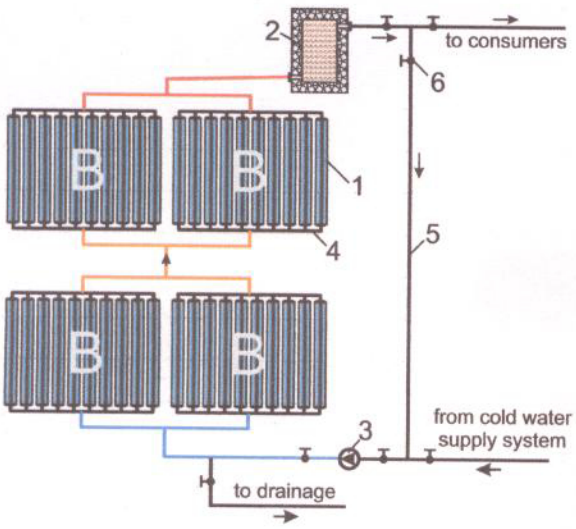

Fig. 9 The scheme and operation of a solar water heating system.

can heat M_w =80 kg/day water up to a final temperature $t_{w.fin.}$ = 21°C and in summertime up to a final temperature $t_{w.fin.}$ = 66°C.

To further increase the daily quantity of water M_w, it is necessary to add more groups of batteries with parallel tubes, and to increase final temperatures $t_{w.fin.}$, it is necessary to add more groups of batteries in series as shown in Fig. 9.

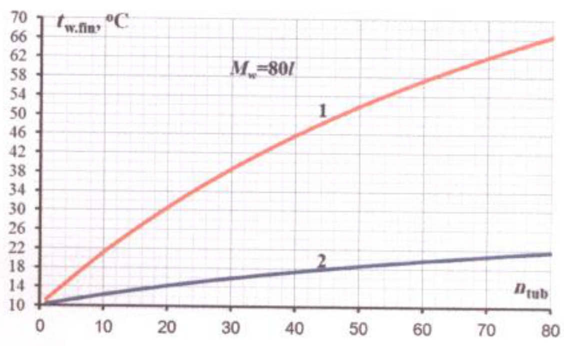

Fig. 10 Diagrams for identifying final temperatures of water $t_{w.fin}$ in batteries with different numbers of parallel tubes $n_{tub.}$ of solar heaters for heating, for instance, M_w = 80 kg/day water in summertime (1) and in wintertime (2).

CONCLUSIONS

1. The suggested method for determining the coefficients of optical losses and penetration of solar radiation can be used for calculation and design of other types of cylindrical solar water heaters (for instance, vacuum type).
2. The obtained formulas allow determining the correct values of direct and diffuse solar radiations penetrating into cylindrical shell-and-tube type solar water heater and reaching the surface of the water tube.
3. Because of the absence of free thermal convection of air in the very narrow air gap located between the water metallic tube and the internal surface of the glass tube, the heat lost from the cylindrical solar water heater is extremely low.
4. The developed method for calculation and design of the simple and cheap cylindrical shell-and-tube type solar water heater can be successfully applied for more accurate understanding of details of heat exchange phenomena that will help in designing efficient and inexpensive warm/hot water supply systems.
5. Shell-and-tube type cylindrical solar water heaters having 1–2 m of length cannot find application in cold climatic conditions with negative temperatures of outside air as they heat the water insignificantly. For heating the water up to higher temperatures, they should be much longer.
6. Instead of very long tubes, it is economically more efficient to heat the water in batteries, consisting of a large number of short tubes (1–2 m long) mounted on frames in parallel and in series. In this case, through each tube, a little flow rate of water is passed, which provide higher final temperatures of the water.
7. To prevent water from freezing at nighttime or on sunless days in cold climates, it is economically more expedient to drain the system instead of using ethylene glycol.
8. In some cases, the use of gas-fired or electric booster heaters can be inevitable for keeping the normal regime of operation of the warm/hot supply system.

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Solar Water Heaters: Cylindrical Shell-and-Tube Type

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Abstract

For making solar water heaters more affordable, a very simple and efficient structure of a cylindrical shell-and-tube type solar water heater, which can be easily constructed and located on rooftops without special technological equipment, should be used. In this entry, such a type of solar water heater is examined. A method for determining the main design characteristics of the suggested cylindrical shell-and-tube type solar water heater is developed. The method permits to evaluate the optical losses of solar radiation caused by reflection from the boundary transparent surface of cylindrical shell-and-tube type solar water heater and, as well as, provides with opportunity to determine the right values of direct and diffuse radiations penetration rates into the inside space of the shell-shaped solar water heater. Formulas for determining the correct value of the water metallic tube surface to which the penetrated solar radiation reaches are obtained. The method allows finding the final temperatures of water at the outlet of the examined solar water heater depending on its length, the initial temperature of cold water subject to heating, and the required daily mass of warm water. Economic and thermal main characteristics of solar heater battery elements are examined.

INTRODUCTION

At present, solar water heaters of a variety of structures are in use. Because solar radiation is a renewable source of energy, the economic efficiency of solar water heaters depends mainly on their initial cost. This fact stimulates the researchers to develop possibly energy-efficient and cheap structures of solar water heaters of different structures, and methods for their correct calculation and design.

SIMPLE CONSTRUCTION AND OPERATION OF THE SOLAR WATER HEATER

An example of a simple and cheap structure of a solar water heater is the cylindrical shell-and-tube type solar water heater, analyzed in this entry. Fig. 1 shows the principal scheme of a cylindrical shell-and-tube type solar water heater.

The cylindrical shell-and-tube type solar water heater consists of a radiolucent shell-shaped glass tube in which a water heating metallic tube is mounted coaxially, which is painted black. Rubber corks hermetically close the air gap between the glass and the metallic tube and, at the same time, support the coaxial metallic tube. The cylindrical solar water heater operates in the following way: the cylindrical surface of the shell-shaped glass tube reflects a part of the incident beams of solar radiation. The rest of the incident beams penetrate the inside area of the glass tube through the body shell. The body shell of the glass tube absorbs a part of the penetrated solar radiation, and another part reaches the surface of the water metallic tube. One of important features of the cylindrical shell-and-tube type solar water heater is the ability to provide insolation of same sizes for surfaces of the water metallic tube by penetrated direct radiation, regardless of the sun's position at different times of the day. This feature can be seen in the Fig. 2.

Fig. 2 also shows that the penetrated direct radiation E_{dir} (W/m^2) lights only a part (f, m^2) of the total surface of the water metallic tube. Apart from direct radiation, diffuse radiation $E_{diff.}$ (W/m^2) also penetrates into the glass tube, which lights the total surface $F_{met.tub.}$ (m^2) of the water metallic tube. For evaluating the real value of radiation, penetrating into the glass tube and reaching the surface of the water metallic tube, the values of optical losses by reflection and absorption of short waves of radiation (about 3 μm) in the body of cylindrical glass cover should be determined. The cross section of the solar water heater, shown in Fig. 3, displays the course of beams (direct radiation) in the transparent body shell of the solar water heater.

OPTICAL LOSSES OF SOLAR RADIATION

The total optical loss of incident radiation E_{lost} (W/m^2) in the solar water heater is determined by the following equation:

$$E_{lost} = E_{refl.ext.} + E_{refl.inf.} + E_{Q} \tag{1}$$

The loss of radiation $E_{refl.ext.}$ (W/m^2) by reflection from the external boundary surface of the transparent cover is evaluated by the following expression:

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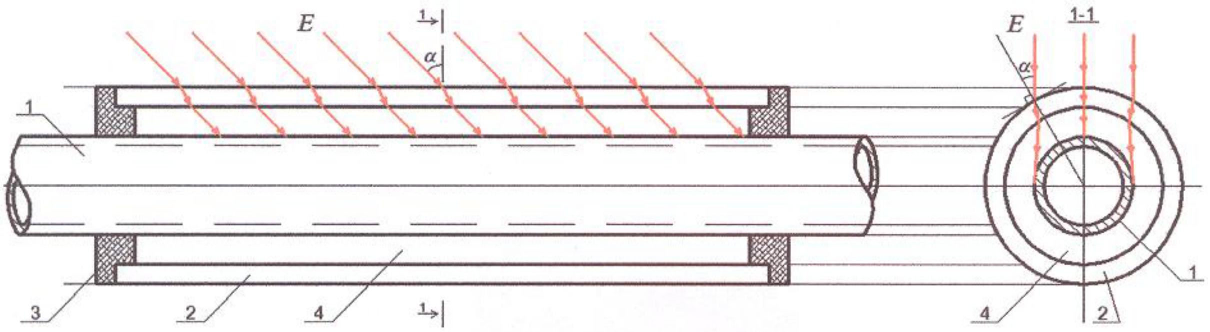

Fig. 1 Scheme of a cylindrical solar water heater. 1—water metallic tube; 2—cylindrical shell-shaped glass tube; 3—rubber sealant corks; 4—air gap; E—intensity of incident solar radiation; α—angle of incident beams.

$$E_{\text{refl.ext.}} = \rho_{\text{ext.}} E_{\text{sol.}} \tag{2}$$

The loss of radiation by reflection from the inferior surface of the transparent cover $E_{\text{refl.inf.}}$ (W/m^2) is evaluated by the following expression:

$$E_{\text{refl.inf.}} = \rho_{\text{inf.}} (E_{\text{sol.}} - E_{\text{refl.ext.}}) \tag{3}$$

Apart from the mentioned losses, there is also some loss E_Q of incidence radiation because of absorption by the transparent shell-shaped glass cover. In further analysis, the absorbed radiation lost E_Q is neglected in view of its miserable energetic value. Therefore, in the sum (1) the value of E_Q is assumed zero. Substituting the value of $E_{\text{refl.ext.}}$ from formula (2) into formula (3), the following formula is obtained for determining the value of reflected radiation $E_{\text{refl.inf.}}$ (W/m^2) from the inferior surface of the cylindrical glass tube:

$$E_{\text{refl.inf.}} = \rho_{\text{inf.}} E_{\text{sol}} (1 - \rho_{\text{ext.}}) \tag{4}$$

Substituting formulas (2) and (4) into formula (1), we obtain the following equation for evaluating the total optical losses E_{loss} of solar radiation in the solar water heater:

$$E_{\text{lost}} = E_{\text{sol.}} (\rho_{\text{ext.}} + \rho_{\text{inf.}} (1 - \rho_{\text{ext.}})) \tag{5}$$

The rest of the incident solar radiation $E_{\text{pen.}}$ (W/m^2) penetrates into the glass tube and finally reaches the surface of the water metallic tube. The following difference represents the value of the penetrated radiation $E_{\text{pen.}}$ (W/m^2):

$$E_{\text{pen.}} = E_{\text{sol.}} - E_{\text{lost}} \tag{6}$$

Substituting the value of total radiation lost from formula (5) into formula (6) and making some mathematical manipulations, the following formula is obtained for determining the radiation intensity, which penetrates into the glass tube and is absorbed by the water metallic tube:

$$E_{\text{pen.}} = E_{\text{sol.}} (1 - \rho_{\text{ext.}})(1 - \rho_{\text{inf.}}) \tag{7}$$

where $\rho_{\text{ext.}}$ is the coefficient of reflection of rays on the boundary surface of the shell-shaped glass cover, $\rho_{\text{inf.}}$ is the coefficient of reflection of rays from the inferior surface of the shell-shaped glass cover.

The coefficient of reflection from the boundary surface of the glass tube $\rho_{\text{ext.}}$ is determined using angles α and β, by applying Fresnel's law (see Fig. 3):[1,2]

$$\rho_{\text{ext.}} = \frac{E_{\text{refl.ext.}}}{E_{\text{sol.}}} = \frac{1}{2}\left[\frac{\sin^2(\beta - \alpha)}{\sin^2(\beta + \alpha)} + \frac{tg^2(\beta - \alpha)}{tg^2(\beta + \alpha)}\right] \tag{8}$$

The angles of incidence (α) and refraction (β) in formula (8) can be found by applying Snell's law, expressed by the following equation:

$$\frac{n_1}{n_2} = \frac{\sin\beta}{\sin\alpha} \tag{9}$$

where n_1 and n_2 are indexes of refractions of the surrounding air and glass shell cover of the solar water heater.

As can be seen from Fig. 4, the beams, refracted on the boundary surface of the glass tube, reach the inferior surface of the glass tube at angle α_1, where they are refracted again at angle β_1 and finally hit the surface of the metallic tube and absorbed by it. To determine the angles α_1 and β_1, the courses of beams in the cylindrical glass tube are analyzed using Fig. 4.

As can be seen from Fig. 4, because of convexity of the external surface of the cylindrical glass tube, the incident angles of sunrays are different. As the refraction angle α_1 is external to the triangle LOM, its value is determined by the following expression:

$$\alpha_1 = \beta + \gamma = \arcsin\left(\frac{R_{\text{ext.gl.tub.}}}{r_{\text{inf.gl.tub.}}}\sin\beta\right) \tag{10}$$

Knowing the values of angles α_1 and β_1, we can determine the value of the coefficient of reflection $\rho_{\text{inf.}}$ from the inferior surface of the transparent cylindrical tube by applying Fresnel's law as follows:

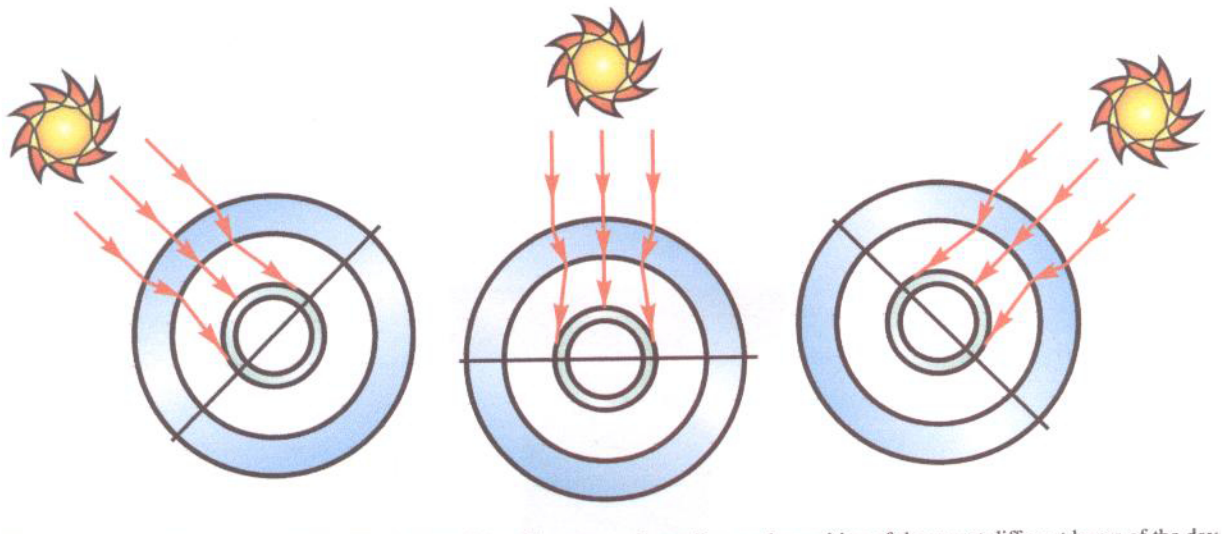

Fig. 2 Sizes of surface of the water metallic tube, lighted by sunrays depending on the position of the sun at different hours of the day.

$$\rho_{\text{inf.}} = \frac{E_{\rho'}}{E_\tau} = \frac{1}{2}\left[\frac{\sin^2(\beta_1 - \alpha_1)}{\sin^2(\beta_1 + \alpha)} + \frac{tg^2(\beta_1 - \alpha_1)}{tg^2(\beta_1 + \alpha_1)}\right] \quad (11)$$

As $R_{\text{ext.gl.tub.}}/r_{\text{inf.gl.tub.}} > 1$, $\beta > \alpha_1$ too (see Fig. 4). Therefore, the rays, refracted from the inferior surface of the glass tube, pass from an optically denser medium into an optically less dense medium (air). In this case, the total reflection of rays from the inferior surface of the glass tube can take place. According to Snell's law, this can happen with those rays the incidence angles of which on the exterior surface of the glass tube correspond to an incidence angle $\alpha_{\text{total.refl.}}$ in case of which total reflection takes place. That anlgle is determined by the following equation:

$$\alpha_{\text{total.refl.}} = \arcsin\left(\frac{r_{\text{inf.gl.tub.}}}{R_{\text{ext.gl.tub.}}}\right) \quad (12)$$

For instance, in the case of glass tube with diameters $d_{\text{ext.gl.tub.}} = 0.044$ m and $d_{\text{inf.gl.tub.}} = 0.04$ m, the total reflection of the rays from the inferior surface of the glass tube occurs at the incidence angle on the exterior surface $\alpha_{\text{total.refl.}} = 65°23'$. A part of the rays, penetrated into the glass tube, do not touch the water metallic tube. As can be seen from Fig. 4, only the rays having marginal incidence angles ($\alpha_{\text{marg.}}$) on the exterior surface reach the surface of the water metallic tube, limited by the arc chord AB.

The value of the marginal incidence angles is determined by the following expression:

$$\alpha_{\text{marg.}} = \arcsin\left(\frac{R_{\text{ext.met.tub.}}}{R_{\text{ext.gl.tub.}}}\right) \quad (12')$$

where: $R_{\text{ext.met.tub.}}$ and $R_{\text{ext.gl.tub.}}$ are external radius of the water metallic tube and glass tube.

Therefore, only the rays that spread between the rays with incidence marginal angles $\alpha_{\text{marg.}}$ penetrate through the surface of segment A-B of glass tube and reach the surface of the water metallic tube at the points M-M (see Fig. 4). Hence, the solar direct radiation penetrates

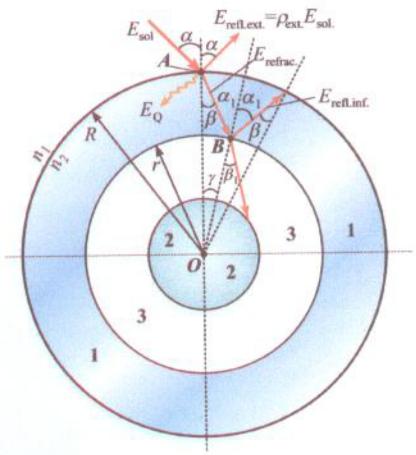

Fig. 3 Regularities of reflection, refraction, and absorption of sun beams in shell-shaped transparent cylindrical cover. 1—transparent body shell with exterior radius R and inferior radius r; 2—water metallic tube; 3—air gap between the water metallic tube and the inferior surface of the transparent body shell; $E_{\text{sol.}}$—total intensity (W/m^2) of the incident rays under course angle α; $E_{\text{refl.ext.}}$—intensity (W/m^2) of radiation, reflected from the external (boundary) surface of the shell-shaped glass tube; $E_{\text{refrac.}}$—intensity of radiation (W/m^2) refracted in the body shell at course angle β; $E_{\text{refl.inf.}}$—intensity of radiation (W/m^2) reflected from the shell-shaped glass tube inferior surface; E_Q—intensity of radiation absorbed by the body of the glass tube.

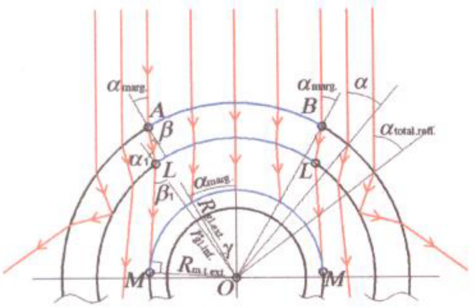

Fig. 4 Course of beams in the cylindrical glass tube.

into the glass tube through the surface of a rectangle with the width equal to the arc chord AB (see Fig. 4).

$$f_{\text{met.tub.}} = d_{\text{ext.met.tub.}} l_{\text{met.tub.}} \tag{13a}$$

Apart from direct radiation there is also diffuse radiation $E_{\text{diff.}}$(W/m^2) on the whole external surface of the glass tube which penetrates into glass tube and insolates the surface $F_{\text{diff.}}$ of the metal tube. The value of $F_{\text{diff.}}$, m^2 can be determined by the following formula:

$$F_{\text{diff.}} = \pi l_{\text{met.tub.}} d_{\text{ext.met.tub.}} \tag{13b}$$

HEAT GAINS BY SOLAR RADIATION

The infinite series of reflection and refraction of rays between the exterior and inferior surfaces of the shell-shaped cylindrical glass cover and corresponding values of coefficients of reflection and refraction are shown in Fig. 5.[2,3]

Fig. 5 displays the total value of radiation penetrating through the convex shell-shaped cylindrical glass tube into the inside space, where it falls on the surface of the water metallic tube. The total value of penetrated radiation is the sum of infinite number of rays refracted into the inside space of the cylindrical glass tube:

$$\begin{aligned} E_{\text{pen.}} = E_{\text{sol.}}\big((1-\rho)(1-\rho') &+ (1-\rho)(1-\rho')\rho\rho' \\ &+ (1-\rho)(1-\rho')\rho^2\rho'^2 + \cdots \\ &+ (1-\rho)(1-\rho')(\rho\rho')^{n-1}\big) \end{aligned} \tag{14}$$

Denoting the value in the brackets of Eq. (14) by τ_p we obtain the following formula for determining the intensity of finally penetrated radiation:

$$E_{\text{pen.}} = \tau_p E_{\text{sol.}} \tag{15}$$

where

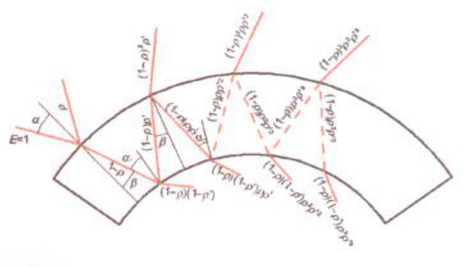

Fig. 5 The course of rays in the body shell of the glass tube, showing their reflection, refraction, and penetration into the cylindrical glass tube.

$$\begin{aligned} \tau_p = (1-\rho)(1-\rho') &+ (1-\rho)(1-\rho')\rho\rho' \\ &+ (1-\rho)(1-\rho')\rho^2\rho'^2 + \cdots \\ &+ (1-\rho)(1-\rho')(\rho\rho')^{n-1} \end{aligned} \tag{15'}$$

is the coefficient of penetration of solar radiation into the internal space of the solar heater, which is determined by the following expression:

$$\tau_p = (1-\rho)(1-\rho')\sum_{n=1}^{n=\infty}(\rho\cdot\rho')^{n-1} \tag{16}$$

The last expression is the sum of a descending geometric progression, the first term of which is $b_1 = (1-\rho)(1-\rho')$ and common ratio is $q = \rho\cdot\rho'$. The sum of the descending geometric progression is determined by the following equation:

$$\tau_p = \frac{b_1}{1-q} \quad \text{or} \quad \tau_p = \frac{(1-\rho)(1-\rho')}{1-\rho\cdot\rho'} \tag{17}$$

Consequently, the total value $E_{\text{tot.pen.dir.}}$ (W) of direct radiation, which penetrates into the internal space of the cylindrical solar water heater and insolates the surface of the water metallic tube, is given by

$$E_{\text{tot.pen.dir.}} = \tau_p E_{\text{sol.dir.}} \tag{18}$$

where τ_p is the coefficient of penetration of solar radiation into the glass tube (transparency).

The values of coefficients of reflection from the exterior (ρ) and inferior (ρ') surfaces within the scope of the shell-shaped cylindrical transparent cover, were evaluated by the help of obtained equations, assuming the external diameter of glass tube $d_{\text{ext.gl.tub.}} = 0.044$ m and inferior diameter $d_{\text{inf.gl.tub.}} = 0.04$ m. The results of calculations for different incidence angles α of solar radiation are represented in Fig. 6.

Fig. 6 shows that the average value of the coefficient of penetration of solar radiation into the glass tube is about

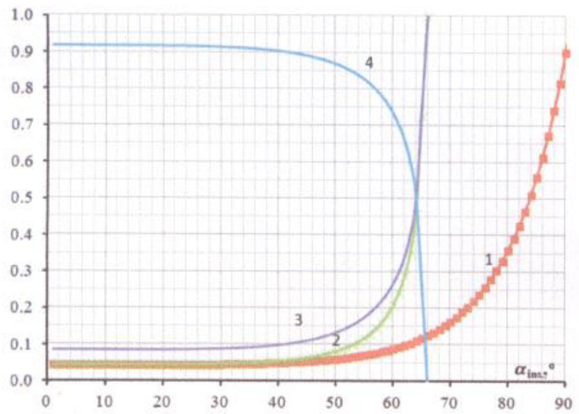

Fig. 6 Graphs of rates of reflection and penetration of solar radiation within the scope of shell-shaped cylindrical transparent cover of the solar water heater, depending on the incident angle α of solar radiation. 1—rates of reflection ρ (boundary reflection) from the exterior surface of the cylindrical glass tube; 2—rates of reflection ρ' from the inferior surface of the cylindrical glass tube; 3—rates of total boundary reflection from the shell-shaped glass cover; 4—coefficient of penetration τ_p of solar radiation into the glass tube.

$\tau_p = 0.8836$. This means that about 90% of the incident solar radiation within the surface of a rectangle with the width equal to the arc chord AB (see Fig. 4) penetrates into the glass tube. Thus, the surface of the water metallic tube absorbs the following sum $E_{tot.pen.}$ (W) of penetrated direct and diffuse solar radiations:

$$E_{tot.pen.} = f_{met.tub.dir.}E_{pen.dir.} + F_{met.tub.}E_{pen.diff.} \tag{19}$$

The surface $f_{met.tub.dir.}$ (m^2) of the metallic tube, insolated by penetrated direct radiation, is given by

$$f_{met.tub.dir.} = d_{ext.met.tub.}l_{met.tub.} \tag{20}$$

The surface $F_{met.tub.diff.}$ (m^2) of the metal tube, lighted by penetrated diffuse radiation, is the following:

$$F_{met.tub.diff.} = \pi d_{ext.met.tub.}l_{met.tub.} \tag{21}$$

Therefore, the sum of penetrated direct and diffuse radiations $E_{tot.pen.}$ (W/m^2), insolating the surface of the water metallic tube, is given by

$$E_{tot.pen.} = d_{ext.met.tub.}l_{met.tub.}\tau_p E_{sol.dir.} + \pi d_{ext.met.tub.}l_{met.tub.}\tau_p E_{sol.diff.} \tag{22}$$

Formula (22) can be written as follows:

$$E_{tot.pen.} = d_{ext.met.tub.}l_{met.tub.}\tau_p E_{sol.dir.}\left(1 + \pi\frac{E_{sol.diff.}}{E_{sol.dir.}}\right) \tag{23}$$

where $d_{ext.met.tub.}$ and $l_{met.tub.}$ are the external diameter and length of water metallic tube (m), respectively; $E_{sol.dir.}$ and $E_{sol.diff.}$ are the intensities of incident direct and diffuse solar radiations (W/m^2), respectively.

According to data of the construction norms and rules of Armenia,[4] for winter period (November to March) the average hourly ratio of $E_{sol.diff.}/E_{sol.dir.}$ is about 0.4. Substituting this ratio in formula (23), the following formula is obtained for determining the penetrated total radiation $E_{tot.pen.}$ reaching the surface of the water tube:

$$E_{tot.pen.} = d_{met.tub.}l_{met.tub.}\tau_p E_{sol.dir.}(1 + 0.4\pi) \tag{24}$$

Taking into account that the average hourly intensity of solar direct radiation is $E_{sol.dir.} = 250$ W/m^2, according to formula (24) the value of radiation penetrating into the cylindrical solar water heater is

$$E_{tot.pen.} = 470.4 d_{ext.met.tub.}l_{met.tub.} \tag{25}$$

HEAT LOST BY HEAT TRANSFER

Besides heat gain by solar radiation $E_{tot.pen.}$ (W), there is also a simultaneous heat loss Q_l (W) by heat transfer from the warmer water of the metallic tube to the outside colder air. To determine the value of heat lost Q_l from water of the metallic tube to the outside colder air, the absence of free thermal convection of air in the very narrow air gap located between the water metallic tube and the internal surface of the glass tube should be taken into account. Therefore, the heat transfer through the air gap from warmer water to outside air occurs mainly by heat conductance. The value of heat lost Q_l from the water in the metallic tube is determined by the Eq. 26 (below),[5] where: l-length of the cylindrical glass tube of the water heater (m); $t_{w.av.}$ is the average temperature of water, flowing inside the metallic tube (°C); t_{out} is the temperature of outside air (°C); α_w is the coefficient of heat convection on the internal surface of the metallic tube (W/m^2 °C); $d_{int.met.tub}$ is the internal diameter of the water metallic tube (m); $\lambda_{met.}$ is the coefficient of heat conductivity of the metal tube (W/m °C); $d_{ext.met.tub.}$ is the external diameter of the water metallic tube (m); $\lambda_{equiv.air} = \lambda_{air}$ is the coefficient of equivalent heat conductivity of air in

$$Q_l = \frac{\pi l(t_{w.av.} - t_{out})}{\left(\frac{1}{\alpha_w d_{int.met.tub.}} + \frac{1}{2\lambda_{met.}}\ln\frac{d_{ext.met.tub.}}{d_{int.met.tub.}} + \frac{1}{2\lambda_{equiv.air}}\ln\frac{d_{int.gl.tub.}}{d_{ext.met.tub.}} + \frac{1}{2\lambda_{gl}}\ln\frac{d_{ext.gl.tub.}}{d_{int.gl.tub.}} + \frac{1}{\alpha_{air.out}d_{ext.gl.tub.}}\right)} \tag{26}$$

the gap (W/m °C); $d_{int.gl.tub.}$ is the internal diameter of the cylindrical glass tube (m); λ_{gl} is the coefficient of heat conductivity of glass (W/m °C); $d_{ext.gl.tub.}$ is the external diameter of the cylindrical glass tube (m); and $\alpha_{air.out}$ is the coefficient of heat convection on the outside boundary surface of the glass tube (W/m^2 °C).

The value of equivalent coefficient $\lambda_{equiv.}$ of heat conductivity through the narrow air gap is determined by the following formula:

$$\lambda_{equiv.} = \varepsilon_{conv.}\lambda_{air} \tag{27}$$

where $\varepsilon_{conv.}$ is the dimensionless quantity, called the coefficient of convection, characterizing the impact of miserable convection of air in the gap.

Because the convection of air in the glass tube is conditioned only by the difference of densities of warm and cold air particles, the coefficient of convection $\varepsilon_{conv.}$ has to be defined using Grashof and Prandtl numbers (GrPr), $\varepsilon_{conv.} = f(\mathrm{Gr}_{air}\cdot \mathrm{Pr}_{air})$.

The value of Grashof number is the following ratio: (The Prandtl number for air is Pr = 0.7.[5])

$$\mathrm{Gr} = \frac{\beta g \delta^3_{air} t_{air.gap.}}{\nu^2_{air}} \tag{27'}$$

where $\beta = 1/(273 + t_{air})$ is the rate of volumetric thermal expansion of air (1/°C), $g = 9.8$ m/s^2 is the acceleration due to gravity, $\delta_{air} = (d_{int.gl.tub.} - d_{ext.met.tub.})/2$ is the air gap thickness (m) in the cylindrical solar water heater between the metallic and glass tubes (m), $\nu_{air} = 14.16\times 10^{-6}$ m^2/s is the kinematic viscosity of air in the range of temperatures 10–60°C, and $t_{air.gap}$ is the temperature of air in the gap (°C).

The value of the average temperature of air in the gap $t_{air.gap.av.}$ is conditioned by the average temperature of the air layer contacting the surface of the water metallic tube and the internal surface of the glass tube. Therefore, the value of the average temperature of air in the gap $t_{air.gap.av.}$ can be determined by the following equation:

$$t_{air.gap.av.} = \frac{(t_{w.in} + t_{w.fin})/2 + t_{out}}{2} \quad \text{or} \quad t_{air.gap.av.} = \frac{t_{w.in} + t_{w.fin} + 2t_{out}}{4} \tag{28}$$

where $(t_{w.in} + t_{w.fin})/2$ is the average temperature of water in the metallic tube, assumed equal to the temperature of the thin layer air gap, surrounding the metallic tube (°C) and t_{out} is the temperature of the glass tube, assumed equal to the outside air temperature (°C).

To determine the coefficient of convection $\varepsilon_{conv.}$, first the value of $\mathrm{Gr}_{air}\cdot\mathrm{Pr}_{air}$ is needed. Preliminary calculations proved that $\mathrm{Gr}_{air}\cdot\mathrm{Pr}_{air} < 1000$. In such a range of values of GrPr, the coefficient of convection in the air gap is recommended to be $\varepsilon_{conv.} = 1.0$.[5]

Substituting the value of $\varepsilon_{conv.} = 1$ and the value $t_{w.av.} = (t_{w.fin} + t_{w.in})/2$ into Eq. (26), the following Eq. (29) (below) for determining the heat lost Q_l (W) from the water of the metallic tube to the outside air by heat transfer is obtained.

METHOD FOR CALCULATION AND DESIGN

The difference between radiation penetrated by solar energy $E_{tot.pen.}$ (heat gains) and heat lost Q_l is the useful heat Q_{us} (W), which is absorbed by the water in the metallic tube:

$$Q_{us} = E_{tot.pen.} - Q_l \tag{30}$$

Substituting the values of $E_{tot.pen.}$ from formula (25) and of heat lost Q_l from formula (29) into formula (30), we obtain the following formula for determining the useful heat:

$$Q_{us} = 470.4 d_{ext.met.tub.} l_{met.tub.} - \frac{\pi l (t_{w.in} + t_{w.fin} - 2t_{out})}{2A} \tag{31}$$

where

$$A = \frac{1}{\alpha_w d_{int.met.tub.}} + \frac{1}{2\lambda_{met}}\ln\frac{d_{ext.met.tub.}}{d_{int.met..tub.}} + \frac{1}{2\lambda_{air}}\ln\frac{d_{int.gl.tub.}}{d_{ext.met.tub.}} + \frac{1}{2\lambda_{gl}}\ln\frac{d_{ext.gl.tub.}}{d_{int.gl.tub.}} + \frac{1}{\alpha_{air.out} d_{ext.gl.tub.}} \tag{31'}$$

As the absorbed useful heat finally rises the temperature of water from initial value $t_{w.in}$ to a final value $t_{w.fin}$, it can be represented by the following formula:

$$Q_{us} = g_w c_w (t_{w.fin} - t_{w.in}) \tag{32}$$

where g_w is the water mass flow rate in the metallic tube, $c_w = 4180$ J/kg °C is the specific heat of water, and $t_{w.fin.}$

$$Q_l = \frac{\pi l (t_{w.in} + t_{w.fin} - 2t_{out})}{2\left(\frac{1}{\alpha_w d_{int.met.tub.}} + \frac{1}{2\lambda_{met}}\ln\frac{d_{ext.met.tub.}}{d_{int.met.tub.}} + \frac{1}{2\lambda_{air}}\ln\frac{d_{in.gl.tub.}}{d_{ext.met.tub.}} + \frac{1}{2\lambda_{gl}}\ln\frac{d_{ext.gl.tub.}}{d_{int.gl.tub.}} + \frac{1}{\alpha_{air.out} d_{ext.gl.tub.}}\right)} \tag{29}$$

and $t_{w.in}$ are final and initial temperatures of water in the solar heater (°C), respectively.

Substituting the value of Q_{us} from formula (32) into formula (31), we obtain the following equation:

$$g_w c_w \left(t_{w.fin} - t_{w.in}\right) = 470.4 d_{ext.met.tub.} l_{met.tub.} - \frac{\pi l \left(t_{w.in} + t_{w.fin} - 2t_{out}\right)}{2A} \quad (32')$$

After some mathematical manipulations, from the previous equation the following formula is obtained, which allows determining the required length $l_{tub.}$ (m) of the cylindrical shell- and tube-shaped solar water heater of given diameters of tubes, water mass flow rate g_w (kg/s), and initial temperature $t_{w.in}$, as well as the needed final temperature $t_{w.fin}$:

$$t_{w.fin} = \frac{2A\left(g_w c_w t_{w.in} + 470.4 d_{ext.met.tub.} l_{met.tub.}\right) + \pi l \left(2t_{out} - t_{w.in}\right)}{2A g_w c_w + \pi l} \quad (33)$$

The mass flow rate of water through each tube g_w (kg/s) is determined by the following formula:

$$g_w = \frac{M}{n_{tub.} Z \cdot 3600} \text{ kg/s} \quad (34)$$

where $n_{tub.}$ is the number of tubes in batteries, joined in parallel; and Z is the sunny hours of the day (hr). During winter, Z=6 hr.

For finding out the possible values of required lengths of the solar water heater $l_{tub.}$ (m), calculations were made by the obtained Eq. (33), assuming that the solar water heater is installed in climatic conditions of Armenia having wintertime design temperature $t_{out} = -19$°C, summertime temperature $t_{out} = +35$°C and average hourly intensities of direct and diffuse solar radiations, respectively, $E_{dir} = 250\text{W/m}^2$ and $E_{diff.} = 100\text{W/m}^2$ were made. The diameter of glass tube is $d_{ext.gl.tub.} = 0.044$ m and of water metallic tube is $d_{ext.met.tub.} = 0.021$ m; $G_w =$ variable, $t_{w.in} = 10$°C, $t_{w.fin} =$ var, $d_{ext.met.tub} = 0.021$ m, $\tau_p = 0.8836$, $\lambda_{air} = 0.027$ W/m °C, $\lambda_{gl} = 0.745$ W/m °C, and $\alpha_{air.out} = 23$ W/m^2 °C. The results of calculations for both winter ($t_{out} = -19$°C) and summer ($t_{out} = +35$°C) conditions are illustrated in Figs. 7 and 8, respectively.

SOLAR HEATER DESIGN IN THE FORM OF BATTERIES

Fig. 7 shows that as much water M_w (kg/day) daily is required to heat, as lower final temperatures of water $t_{w.fin}$, can be provided. Longer solar water heaters provide higher final temperatures of water. However, even in impossibly long heaters (for instance, $l_{tub.} = 800$ m), the water is heated up to a limit final temperature. As can be seen from Fig. 7, in the case of outside temperature $t_{out} = -19$°C, the limit temperatures ranged from 33°C to 37°C depending on the daily mass M (kg/day) of required water. Fig. 8 shows the same regularities in summertime as in wintertime. It proves the possibility of heating the water up to temperatures higher than 100°C if the solar heater has proper length. However, the use of too long heaters for providing high temperatures is not practical and technically feasible. Another way for preparing high-temperature water is reducing the mass flow rate g_w in the tube. From this point of view, it is becoming highly expedient to heat the water in batteries, consisting of a large number of short tubes mounted parallel on frames. In this case, through each tube, a little quantity g_w (kg/s) of water is passed and is heated up to higher temperatures $t_{w.fin.}$ (°C). Such a system of warmed water supply is illustrated in Fig. 9. The volume of the tank is enough for storing the daily required mass M_w (kg/day) of warmed water.

Fig. 10 shows that a battery with $n_{tub.} = 80$ parallel tubes with length $l_{tub.} = 1.2$ m each, in wintertime,

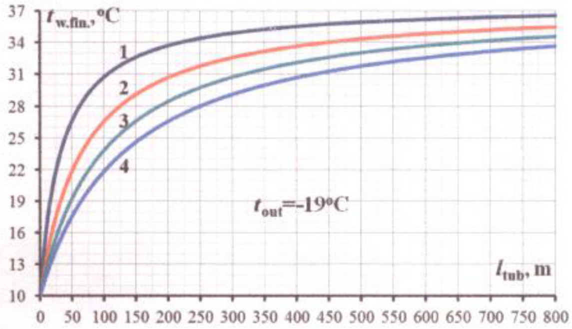

Fig. 7 Diagrams for identifying final temperatures of water $t_{w.fin}$ and lengths $l_{tub.}$ of solar water heater in wintertime. Curves of required daily mass M_w (kg/day) of warmed water: 1—M_w = 20 kg/day; 2—M_w = 40 kg/day; 3—M_w = 60 kg/day; 4—M_w = 80 kg/day.

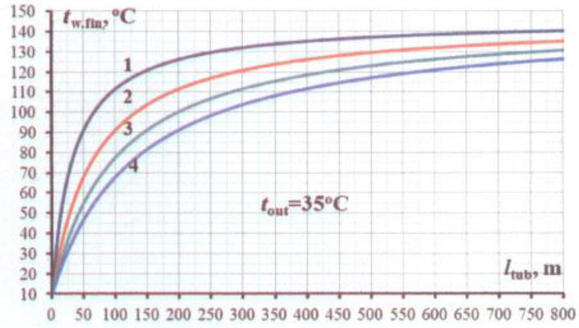

Fig. 8 Diagrams for identifying final temperatures of water $t_{w.fin}$ and lengths $l_{tub.}$ of solar water heater in summertime. Curves of mass M_w (kg/day) of hot water required for a day: 1—M_w = 20 kg/day; 2—M_w = 40 kg/day; 3—M_w = 60 kg/day; 4—M_w = 80 kg/day.