

6.24

X_1, X_2, \dots, X_n oberoende alla följande
 $\mu = E(X_i) = 0.1$, $\sigma = D(X_i) = 8$ så enl. CGS är

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \stackrel{\text{appr.}}{\sim} N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N\left(0.1, \frac{8}{\sqrt{n}}\right)$$

$$P(|\bar{X}_n| > 0.08) = 1 - P(|\bar{X}_n| \leq 0.08) =$$

$$= 1 - \left(\Phi\left(\frac{0.08 - 0.1}{8/\sqrt{n}}\right) - \Phi\left(\frac{-0.08 - 0.1}{8/\sqrt{n}}\right) \right)$$

$$= 1 - \left(\Phi\left(-\frac{0.02}{8} \sqrt{n}\right) + \Phi\left(-\frac{0.18}{8} \sqrt{n}\right) \right)$$

$$= 1 - \left(1 - \Phi(0.0025\sqrt{n}) + 1 - \Phi(0.0225\sqrt{n}) \right)$$

$$= 1 + \Phi(0.0025\sqrt{n}) - \Phi(0.0225\sqrt{n})$$

$$= \begin{cases} 1 + \Phi(0.0025\sqrt{16}) - \Phi(0.0225\sqrt{16}) & n=16 \\ 1 + \Phi(0.0025\sqrt{1600}) - \Phi(0.0225\sqrt{1600}) & n=1600 \\ 1 + \Phi(0.0025\sqrt{160000}) - \Phi(0.0225\sqrt{160000}) & n=160000 \end{cases}$$

$$= \begin{cases} 1 + \Phi(0.01) - \Phi(0.09) \\ 1 + \Phi(0.1) - \Phi(0.9) \\ 1 + \Phi(1) - \Phi(9) \end{cases}$$

$$= \begin{cases} 1 + 0.5040 - 0.5359 \\ 1 + 0.5398 - 0.8159 \\ 1 + 0.8413 - 1 \end{cases}$$

$$= \begin{cases} 0.9681 \\ 0.7239 \\ 0.8413 \end{cases}$$

6.16

$$X \in N(150, 3) \perp Y \in N(100, 4)$$

$$(a) \quad X + Y \in N(150 + 100, \sqrt{3^2 + 4^2}) = N(250, 5)$$

$$X - Y \in N(150 - 100, \sqrt{3^2 + 4^2}) = N(50, 5)$$

$$\frac{X+Y}{2} \in N\left(\frac{150+100}{2}, \frac{1}{2}\sqrt{3^2+4^2}\right) = N(125, 2.5)$$

$$(b) \quad P(X+Y < 242.6) = \Phi\left(\frac{242.6 - 250}{5}\right) =$$

$$= 1 - \underbrace{\Phi(1.48)}_{0.9306} = \underline{\underline{0.0694}}$$

$$P(|X-Y| < 40) = P(-40 < X-Y < 40) =$$

$$= \Phi\left(\frac{40-50}{5}\right) - \Phi\left(\frac{-40-50}{5}\right)$$

$$= \Phi(-2) - \Phi(-18) = 1 - \Phi(2) = 0.02275$$

$$P\left(\left|\frac{X+Y}{2} - 125\right| > 5\right) = 1 - P\left(\left|\frac{X+Y}{2} - 125\right| \leq 5\right) =$$

$$= 1 - P(-5 \leq \frac{X+Y}{2} - 125 \leq 5)$$

$$= 1 - P(120 \leq \frac{X+Y}{2} \leq 130)$$

$$= 1 - \left(\Phi\left(\frac{130-125}{2.5}\right) - \Phi\left(\frac{120-125}{2.5}\right) \right)$$

$$= 1 - \left(2 \underbrace{\Phi(2)}_{0.97725} - 1 \right) = \underline{\underline{0.0455}}$$