Lösningar till MA2043 statistik tentan 2025-03-19

1 (a) $\bar{x} = \frac{1}{7}(21+15+11+19+30+8+15) = \frac{119}{7} = \underline{\underline{17}}$

(b) x = antal vilt (kovariat), y = antal viltolyckor (respons)

$\sum x_i = 873$, $\sum x_i^2 = 127011$, $\sum y_i = 119$, $\sum y_i^2 = 2337$, $\sum x_i y_i = 15350$

$\hat{b} = \frac{7 \cdot 15350 - 873 \cdot 119}{7 \cdot 127011 - 873^2} = 0.0281$ (2)

$\hat{a} = \frac{119}{7} - 0.0281\ldots \cdot \frac{873}{7} = 13.4997$ (3)

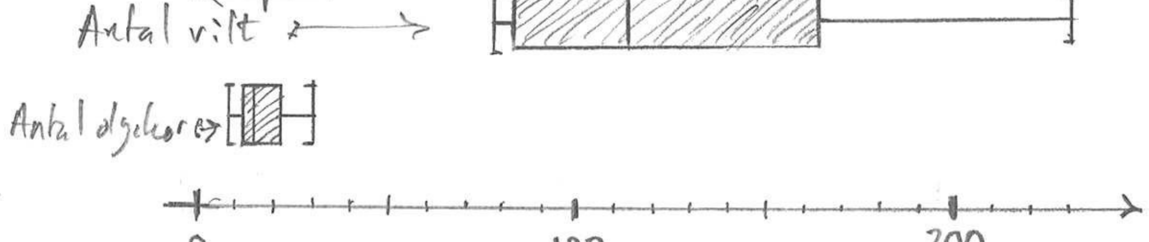
$\varepsilon_i = y_i - (\hat{a} + \hat{b}x_i)$:

Svar :
$$\begin{cases} \varepsilon_1 = 1.0450 \\ \varepsilon_2 = -1.6993 \\ \varepsilon_3 = -4.8292 \\ \varepsilon_4 = 0.8693 \\ \varepsilon_5 = 14.3111 \\ \varepsilon_6 = -7.9976 \\ \varepsilon_7 = -1.6993 \end{cases}$$

(c) x: 78, 83, 89, 114, 114, 165, 230 $\quad$ y: 8, 11, 15, 15, 19, 21, 30

$md_x = 114$	$md_y = 15$
$Q_{1,x} = 83$	$Q_{1,y} = 11$
$Q_{3,x} = 165$	$Q_{3,y} = 21$
$1.5Q = 123$	$1.5Q = 15$
$83 - 78 = 5 \not> 123$	$11 - 8 = 3 \not> 15$
$230 - 165 = 65 \not> 123$	$30 - 21 = 9 \not> 15$
inga outliers	inga outliers

Boxplot:



2. $3+4+5 = 12$ poäng dvs 9 kvar upptill 21.

$$\begin{array}{lllll}
 & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & 2\cdot\frac{3}{52}\cdot\frac{4}{51} & 2\cdot\frac{3}{52}\cdot\frac{3}{51} \\
9: & 1+8 & 2+7 & 3+6 & 4+5 \\
 & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & 2\cdot\frac{3}{52}\cdot\frac{3}{51} & 1\cdot\frac{3}{52}\cdot\frac{2}{51} \\
8: & 1+7 & 2+6 & 3+5 & 4+4 \\
 & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & 2\cdot\frac{3}{52}\cdot\frac{3}{51} & \\
7: & 1+6 & 2+5 & 3+4 & \\
 & 2\cdot\frac{4}{52}\cdot\frac{3}{51} & 2\cdot\frac{4}{52}\cdot\frac{3}{51} & 1\cdot\frac{3}{52}\cdot\frac{2}{51} & \\
6: & 1+5 & 2+4 & 3+3 & \\
 & 2\cdot\frac{4}{52}\cdot\frac{3}{51} & 2\cdot\frac{4}{52}\cdot\frac{3}{51} & & \\
5: & 1+4 & 2+3 & & \\
 & 2\cdot\frac{4}{52}\cdot\frac{3}{51} & 1\cdot\frac{4}{52}\cdot\frac{3}{51} & & \\
4: & 1+3 & 2+2 & & \\
 & 2\cdot\frac{4}{52}\cdot\frac{4}{51} & & & \\
3: & 1+2 & & & \\
 & 1\cdot\frac{4}{52}\cdot\frac{3}{51} & & & \\
2: & 1+1 & & &
\end{array}$$

Kontroll:

$$\sum_{k=1}^{8} k = \frac{8\cdot 9}{2} = 36$$

$$2\cdot7 + 2\cdot6 + 2\cdot1 + 3\cdot2 + 2\cdot1 = 36 \text{ ok}$$

Så totalt blir

$P(2 \text{ kort med} \leq 9 \text{ poäng}) =$

$$= 7\cdot2\cdot\frac{4}{52}\cdot\frac{4}{51} + (6\cdot2+2\cdot1)\frac{4}{52}\cdot\frac{3}{51} + 3\cdot2\cdot\frac{3}{52}\cdot\frac{3}{51} + 2\cdot1\cdot\frac{3}{52}\cdot\frac{2}{51}$$

$$= \frac{14\cdot16 + 14\cdot12 + 6\cdot9 + 2\cdot6}{52\cdot51}$$

$$= \frac{458}{2652} = \frac{229}{1326} = \underline{\underline{0.1727}}$$

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3. (a) $3\cdot5 = 15$ lökar så antal tulpaner: $X \in Bin(15, 0.7)$

$$P(X \geq 10) = \binom{15}{10}0.7^{10}\cdot0.3^{5} + \binom{15}{11}0.7^{11}\cdot0.3^{4} + \binom{15}{12}0.7^{12}0.3^{3} + \binom{15}{13}0.7^{13}0.3^{2} + \binom{15}{14}0.7^{14}0.3^{1} + \binom{15}{15}0.7^{15}\cdot0.3^{0}$$

$$= \underbrace{3003\cdot(1) + 1365\cdot(2) + 455\cdot(3)}_{(1)} + 105\cdot(2) + 15\cdot(3) + 1\cdot0.00474\ldots$$

$$= \underline{\underline{0.7216}}$$

(b) $30\cdot5 = 150$ så $X \in Bin(150, 0.7)$

Emellertid är X en summa av slumpvariabler

och $E(X) = 150\cdot0.7 = 105$ och $V(X) = 150\cdot0.7\cdot0.3 = 31.5$

varmed $P(X \geq 100) \approx 1 - P(X \leq 99) =$

$$= 1 - \Phi\left(\frac{99-105}{\sqrt{31.5}}\right) = 1 - \Phi(-1.07) = \underline{\underline{0.8577}}$$

4. (a) $\begin{cases} H_0: \mu = 0 \\ H_1: \mu > 0 \end{cases}$ $\bar{\varepsilon} = \frac{8.2190}{1078} = 0.0076\ldots$ (1)

$$s_\varepsilon^2 = \frac{1}{1077}\left(84.6305 - 1078 \cdot 0.0076\ldots^2\right) = 0.0785\ldots \text{(2)}$$

$$u = \frac{\sqrt{1078}\,(0.0076\ldots - 0)}{\sqrt{0.0785\ldots}} = 0.8933\ldots$$

$t_{0.05,100} = 1.6602$ dvs $u \not> \lambda_{0.05}$ så

H_0 kan ej förkastas dvs man kan ej bevisa att avrundningsfelet är >0 på 5% sign.nivå. $\underbrace{0.6770}_{t_{0.25,100}} < 0.8933 < \underbrace{1.2901}_{t_{0.10,100}}$ så p-värdet $\in (0.1, 0.25)$

(b) "Fuffens" betyder här avvikelse från jämn fördelning. Så

$\begin{cases} H_0: \text{jämnt fördelade avrundningsfel} \\ H_1: \text{ojämnt fördelade avrundningsfel} \end{cases}$

Totalt 1078 st så förväntat antal vid jämn fördelning blir $\frac{1078}{5} = 215.6$ observationer i varje intervall. Vi får

$$u = \sum_{k=1}^{5} \frac{(O_k - E_k)^2}{E_k} = \frac{(200-215.6)^2}{215.6} + \frac{(203-215.6)^2}{215.6} + \frac{(233-215.6)^2}{215.6} + \frac{(245-215.6)^2}{215.6} + \frac{(197-215.6)^2}{215.6}$$

$$= 1.13 + 0.74 + 1.40 + 4.01 + 1.60$$

$$= 8.88 \not> 13.2767 = \chi^2_{0.01,5-1}$$

dvs man kan ej förkasta H_0 dvs kan ej bevisa fuffens med avrundningsfelen på 1% sign.nivå

p-värdet $\in (0.05, 0.10)$ eftersom $\underbrace{7.7794}_{\chi^2_{0.1,4}} < 8.88 < \underbrace{9.4877}_{\chi^2_{0.05,4}}$

(c) $\#\{\varepsilon_i > 0.1\} = 245 + 197 = 442$

$p = \frac{442}{1078} = 0.4100$ (1) $\quad \lambda_{0.05/2}\sqrt{\frac{p(1-p)}{n}} = 1.959964\sqrt{\frac{0.41 \cdot 0.59}{1078}} = 0.02936\ldots$

$(0.4100\ldots - 0.02936\ldots,\ 0.4100\ldots + 0.02936\ldots)$

$(0.3806,\ 0.4394)$