

Low-Pass Filters to Suppress Inertial and Tidal Frequencies

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ABSTRACT

A systematic way is given to design digital filters which allow clear separation of signals with periods of a few days from noise of higher frequency, particularly tidal and inertial. Several examples are given which pass little high-frequency power and none at the principal tidal frequencies. The Lanczos-cosine filter passes too much energy near diurnal frequencies; the Godin filter is better but not optimal. A longer filter is recommended, with flat low-frequency response, a sharp cut-off and very low noise. For current meter records containing inertial motions, it appears desirable to design a filter which specifically suppresses the local inertial frequency.

1. Introduction

Tides and inertial motions usually are a strong "high-frequency noise" in current-meter, temperature, or sea-level records used to study low frequency motions in the ocean. This paper discusses how to eliminate diurnal and shorter-period tides from input data (x_t) by a symmetric digital filter

$$y_t = \sum_{k=-n}^n w_k x_{t+k}, \quad w_{-k} = w_k. \quad (1)$$

The symmetry ($w_{-k} = w_k$) is imposed to preserve phase information. To correctly pass low frequencies, one constraint is

$$\sum_{k=-n}^n w_k = 1. \quad (2)$$

There are still n free variables w_1, \dots, w_n ; how should they be chosen? Groves (1955) specified that they should be chosen to minimize the total (tidal + white) noise energy which gets through the filter. The result is somewhat dependent on the amplitudes of the tidal constituents chosen, and may not give the optimal low-pass response. Nonetheless, it is surprising how little Groves' (1955) results are used—most workers seem to use a Lanczos-cosine filter.

Doodson and Warburg (1941) gave a simple filter (the " X_0 " filter) which discriminates strongly against the principal tidal constituents, but has high side bands, so does not protect well against noise. Walters and Heston (1982) tried several filters and recommend operating on the Fourier transform of the signal.

Eqs. (1) and (2) allow use of n free parameters, which is a lot of freedom; surely we ought to be able to design filters with good low-pass characteristics and protection against the local inertial frequency as well as against tidal. This paper presents a method which is convenient to use on a computer, with some examples which may be useful in themselves. The examples are compared to the few other filters in use and seem comparatively good. The Lanczos-cosine filter in common use does not protect well against diurnal tides but can be improved by adjustment. An example filter constructed here completely suppresses seven principal tidal constituents plus a local inertial frequency, and allows through only a millionth of the power of a white noise while allowing through undisturbed signals of 1.7 days period or longer.

2. Method

Assuming time is measured such that $\Delta t = 1$, and suitable properties (stationarity, etc.) of the stochastic process, we can write

$$x_t = \int_{-\pi}^{\pi} e^{i\omega t} dZ(\omega). \quad (3)$$

Let us now compute the Fourier transform of $\{y_t\}$ in terms of that (dZ) of $\{x_t\}$:

$$\begin{aligned} y_t &= \sum w_k x_{t+k} = \sum w_k \int_{-\pi}^{\pi} e^{i\omega(t+k)} dZ(\omega) \\ &= \int_{-\pi}^{\pi} e^{i\omega t} \left[\sum w_k e^{i\omega k} dZ(\omega) \right]. \end{aligned} \quad (4)$$

Therefore, the frequency decomposition of y_i is that of $\{x_i\}$ multiplied by a *filter response* factor

$$R(\omega) = \sum_{k=-n}^n w_k e^{i\omega k} = w_0 + 2 \sum_{k=1}^n w_k \cos \omega k. \quad (5)$$

We would like to make this filter response near unity at low frequencies, but small at high frequencies, particularly at inertial frequency ($\omega = \omega_0$) and at tidal frequencies ($\omega = \omega_j$). Let us therefore choose $\{w_k\}$ to minimize the error

$$E = a^2 \int_0^{\Omega_1} (R(\omega) - 1)^2 d\omega + b^2 \int_{\Omega_2}^{\pi} R^2(\omega) d\omega + \sum_j c_j^2 R^2(\omega_j), \quad (6)$$

where a , b , Ω_1 , and Ω_2 are disposable constants, and c_j are the (known) local tidal (and inertial) amplitudes. Groves (1955) used the form (6) for the case $\Omega_1 = 0$, $\Omega_2 = 0$, $b^2 = \text{noise variance}$.

The minimum of (6) is attained where the gradient $(\partial E / \partial w)$ is zero, which gives a matrix equation

$$Gw = s, \quad (7)$$

where G is a $(1+n) \times (1+n)$ matrix with elements

$$L(\omega) = \begin{cases} 1, & \text{for } \omega < \Omega_1 \\ \frac{1}{2} \left(1 + \cos \frac{\pi(\omega - \Omega_1)}{\Omega_2 - \Omega_1} \right), & \text{for } \Omega_1 < \omega < \Omega_2 \\ 0, & \text{for } \Omega_2 < \omega \end{cases}. \quad (11)$$

Then choose $\{w_k\}$ to minimize the error

$$E_2 = \frac{1}{2\pi} \int_0^{\pi} [R(\omega) - L(\omega)]^2 d\omega, \quad (12)$$

with the constraint (10) and the constraints that no power should be passed at the frequencies ω_j , for $j = 1, \dots, m$,

$$R(\omega_j) = w_0 + 2 \sum_{k=1}^n w_k \cos \omega_j k = 0. \quad (13)$$

The constraints (10) and (13) are linear, so may be written as a matrix

$$Aw = b. \quad (14)$$

From (12), the partial derivatives of E_2 are

$$\frac{\partial E_2}{\partial w_0} = w_0 - s_0, \quad (15)$$

$$\frac{\partial E_2}{\partial w_k} = 2w_k - s_k, \quad (16)$$

$$\left. \begin{aligned} g_{00} &= 2C(0) + s_1(0) + s_2(0) \\ g_{0k} &= 4C(k) + 2s_1(k) - 2s_2(k) \\ g_{k0} &= 2C(k) + s_1(k) - s_2(k) \\ g_{lk} &= 2C(k+l) + 2C(k-l) + s_1(k+l) \\ &\quad + s_1(k-l) - s_2(k+l) - s_2(k-l) \end{aligned} \right\}, \quad (8)$$

and

$$\left. \begin{aligned} s_1(0) &= 2a^2 \Omega_1 \\ s_1(k) &= 2a^2 (\sin \Omega_1 k) / k \\ s_2(0) &= 2b^2 (\Omega_2 - \pi) \\ s_2(k) &= 2b^2 (\sin \Omega_2 k) / k \\ C(k) &= \sum_j c_j^2 \cos \omega_j k \end{aligned} \right\}. \quad (9)$$

The solution of (7) is then re-normalized so that

$$R(0) = w_0 + 2 \sum w_k = 1. \quad (10)$$

Eq. (10) could be applied as a constraint to minimizing (6) with a Lagrange multiplier, but this gives no noticeable advantages.

An alternative approach which does find advantage in a Lagrange multiplier, through avoiding the large matrix G in Eq. (7), may also be of use. We would like $R(\omega)$ in (5) to be near unity for low frequencies, but small at high frequencies, especially tidal frequencies. Let $L(\omega)$ be an even function thought to have a desirable shape for a filter, for example

where the vector s is the Fourier transform of $L(\omega)$:

$$s_0 = \pi^{-1} \int_0^{\pi} L(\omega) d\omega, \quad (17)$$

$$s_k = \frac{2}{\pi} \int_0^{\pi} L(\omega) \cos \omega k d\omega. \quad (18)$$

Now the minimization of (12) subject to (14) can be solved through the use of a Lagrange multiplier vector λ :

$$\frac{\partial}{\partial w} [E_2 - (Aw - b)^* \lambda] = Dw - s - A^* \lambda = 0, \quad (19)$$

where D is a diagonal matrix with $d_{00} = 1$, $d_{kk} = 2$, and $(\)^*$ denotes transpose. Eq. (19) gives w in terms of λ ; substituting in (14) gives λ in terms of b , and hence

$$w = D^{-1}s + D^{-1}A^*(AD^{-1}A^*)^{-1}(b - AD^{-1}s). \quad (20)$$

Eq. (20) gives the desired weights. The matrix $AD^{-1}A^*$ will normally be much smaller and easier to invert than G .

There is no change in the form of the solution if other linear combinations of the coefficients $\{w_k\}$ are added to (14). For example, one could require $R(\omega_m) = 1$ for some low frequency ω_m , or

$$R''(0) = \sum k^2 w_k = 0. \quad (21)$$

3. Results

Since tidal amplitudes vary widely, the inertial frequency depends on latitude, and there are so many possible choices for parameters, only some examples will be chosen. The tidal amplitudes c_j and constituents ω_j used here will be those chosen by Groves (1955), excluding the fortnightly and monthly constituents, which seem part of the low frequencies to be passed. (One may wish to deal with them separately.) Groves' values were apparently chosen as typical for a low-noise tide gauge in shallow water in England. When an inertial frequency is used, it will be that of Sydney, i.e., $\omega_0 = f = 16.7^\circ \text{ h}^{-1}$. The filter lengths will be those in current use, $n = 24, 35, 60$, and 120. Since the tidal frequencies start at Q_1 with $13.4^\circ \text{ h}^{-1}$ ($=6.5 \times 10^{-5} \text{ rad s}^{-1}$), it is reasonable to take $\Omega_2 \leq 13.4^\circ \text{ h}^{-1}$. For $N = 24$, Ω_1 will have to be

rather small, since $R(\omega)$ cannot bend too fast without Gibbs' phenomenon.

a. Two-day filters

The first example will be from (7) using values of c_j , ω_j , and b ($=0.0289$) from Groves (1955)—that is, without any account of inertial noise, and with a very low error noise coming only from digitization. The filter will be called "24G113", since it is for $n = 24$, Groves' values, and $\Omega_1 = 1^\circ \text{ h}^{-1}$, $\Omega_2 = 13^\circ \text{ h}^{-1}$. Since the emphasis here will be on suppressing noise, $a = 0.005$ was taken rather smaller than b . The weights $\{w_k\}$ for "24G113" are listed in Table 1.

The second example ("24m214") now includes an inertial frequency ($\omega_0 = 16.7^\circ \text{ h}^{-1}$, $c_0 = 1$) with Groves' (1955) values, uses $\Omega_1 = 2^\circ \text{ h}^{-1}$, $\Omega_2 = 14^\circ \text{ h}^{-1}$, and $b = 0.2$ corresponding to the rather higher noise levels expected from a current meter. (If we interpret Groves units as 10 cm s^{-1} instead of 0.1 foot, this is a noise level of 2 cm s^{-1} , and inertial amplitude 10 cm s^{-1} , quite reasonable for the Sydney shelf.) Again $a = 0.0289$ is rather smaller than b . An example with $a = 0.2$, however, turned out quite similar.

The other filters illustrated are the "24-hour Gaussian filter" apparently used by Hogg (1981) and other Woods Hole people, the "D49" filter of Groves

TABLE 1. Weights $\{w_k\}$ for the filters "24G113", "24m214", and "51G113". The filters are symmetric, with $w_{-k} = w_k$, so the total filter lengths are 49, 49 and 103 points (48, 48 and 102 h).

24m214		24G113		51G113			
k	$w(k)$	k	$w(k)$	k	$w(k)$	k	$w(k)$
0	0.037242099	0	0.036509277	0	0.026345725	27	0.005346935
1	0.037100830	1	0.036470062	1	0.026292451	28	0.004687738
2	0.036674104	2	0.036137174	2	0.026132808	29	0.004082390
3	0.035955979	3	0.035400978	3	0.025868366	30	0.003530538
4	0.034944829	4	0.034353150	4	0.025502503	31	0.003031134
5	0.033652232	5	0.032836058	5	0.025039259	32	0.002582394
6	0.032109625	6	0.030960144	6	0.024483387	33	0.002182090
7	0.030369500	7	0.029098489	7	0.023840722	34	0.001827879
8	0.028499651	8	0.027200260	8	0.023117922	35	0.001516670
9	0.026571414	9	0.025256197	9	0.022322686	36	0.001245148
10	0.024645131	10	0.023537502	10	0.021463396	37	0.001010738
11	0.022757428	11	0.021992157	11	0.020548516	38	0.000810529
12	0.020914804	12	0.020563951	12	0.019587360	39	0.000640828
13	0.019096433	13	0.019378530	13	0.018589858	40	0.000498479
14	0.017266285	14	0.018168216	14	0.017565049	41	0.000380791
15	0.015391635	15	0.016667614	15	0.016521585	42	0.000284882
16	0.013462606	16	0.014958699	16	0.015468605	43	0.000208059
17	0.011506394	17	0.012951033	17	0.014414844	44	0.000147761
18	0.009590841	18	0.010711246	18	0.013368376	45	0.000101526
19	0.007814717	19	0.008702299	19	0.012337032	46	0.000067288
20	0.006285915	20	0.007030225	20	0.011328058	47	0.000042627
21	0.005092625	21	0.005646540	21	0.010348177	48	0.000024851
22	0.004275254	22	0.004805478	22	0.009403406	49	0.000012313
23	0.003807495	23	0.004477182	23	0.008498320	50	0.000003870
24	0.003593222	24	0.004442175	24	0.007636864	51	-0.000001958
				25	0.006822939		
				26	0.006059150		

(1955), and "F49" (sometimes called "Munk's tide-killer"), derived by convolving Groves' (1955) "D35" filter with a 15-point symmetric low-pass filter (Groves, personal communication, 1982). The Gaussian filter "24gaus" has weights $w_k = \exp(-\frac{1}{2}(0.0833k)^2)$, normalized by (10). Groves' filters were designed to kill tides, as here.

The power transmitted [$R^2(\omega)$] by the filters is plotted versus period in Fig. 1. This presentation, which follows Groves (1955), is useful for emphasizing the performance of the filters for the frequencies that we want to keep, and showing the recoloring necessary.

The performance of the filters is summarized in Table 2. The first column shows the ratio of the total output power at Groves' 8 diurnal constituents to that input; the second, the ratio for the 8 semi-diurnal constituents; the third, the ratio for the inertial ($16.7^\circ \text{ h}^{-1}$). The smaller the power transmitted, the better.

The fourth column of Table 2 is the high-frequency ($>1 \text{ cpd}$) noise ratio

$$\pi^{-1} \int_{\pi/12}^{\pi} R^2(\omega) d\omega = (180)^{-1} \int_{15}^{180} R^2(f) df, \quad (22)$$

computed directly from (5), with $\Delta f = 1^\circ \text{ h}^{-1}$.

The fifth column is the total noise (tidal + high frequency) passed by each filter for Groves' (1955) case of white noise of standard deviation $\sigma = 0.0289$, as might be typical of a good-quality water level recorder.

The sixth column is the total noise energy (tidal + inertial + high frequency) passed in a case with moderate noise ($\sigma = 0.2$) and inertial signal (amplitude 1, slightly larger than the M_2 tidal amplitude). The filters best at suppressing the tides are not necessarily good at suppressing a particular inertial frequency.

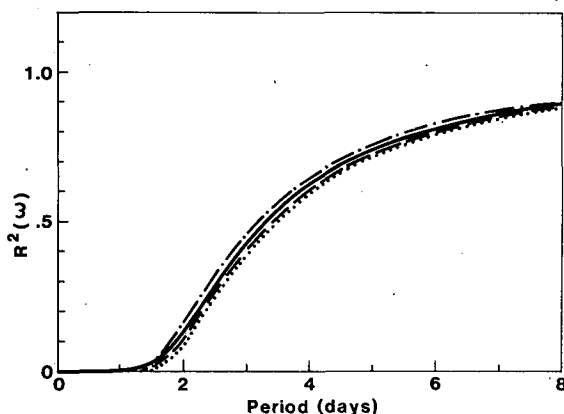


FIG. 1. Relative power transmitted, $R^2(\omega)$, versus period in days, where $R(\omega) = w_0 + 2 \sum_{k=1}^{24} w_k \cos k\omega$ is the filter response factor for the filters "24G113" (solid line), "D49" (dotted line), "24gaus" (dashed line), "24m214" (dot-dashed line).

b. Three-day filters

Godin (1972, p. 94) defined a low-pass, tide-killer filter, consisting of applying running means three times in succession. The running means are of length 24 for the first and second passes, and length 25 for the third pass. To compare Godin's filter with the others will require putting it in the form (1). The convolution of the running means of length 24 is a triangular filter of length 47; the convolution of this triangular filter with a running mean of length 25 is a symmetric filter of length 71 ($n = 35$) with

$$w_k = \begin{cases} \frac{1}{28800} [1200 - (12 - k)(13 - k) \\ - (12 + k)(13 + k)], & 0 \leq k \leq 11 \\ \frac{1}{28800} (36 - k)(37 - k), & 12 \leq k \leq 35. \end{cases} \quad (23)$$

This was checked by computing $R(\omega)$ from (5) and comparing it with

$$R(\omega) = \frac{\sin^2(12\omega) \sin(12.5\omega)}{14400 \sin^3(\frac{1}{2}\omega)}, \quad (24)$$

given by B. V. Hamon (personal communication, 1982). The performance of the Godin filter is summarized in Table 2 and Fig. 2.

A filter "35G113" was generated from (7) with $n = 35$, $\Omega_1 = 1^\circ \text{ h}^{-1}$, $\Omega_2 = 13^\circ \text{ h}^{-1}$. Its performance is also summarized in Table 2. In Fig. 2, its power curve would lie too close to the "Godin" curve, but slightly below it.

Since (23) could also be regarded as having $n = 36$ (or 37) with end weights of zero, "36G113", "37G113", and "38G113" were also generated and tested. The power based in each of the diurnal, semi-diurnal, and high-frequency bands decreased by a factor of close to 2 for each unit increment in n .

c. Four-day filters

The largest G in (7) that would fit in memory was for $n = 51$. The filter "51G113" was generated with $n = 51$ and other parameters as before, and its performance is summarized in Table 2 and Fig. 2. It is a remarkable filter, passing only 5×10^{-12} of the diurnal tidal power, 6×10^{-14} of the semi-diurnal tidal power, and 4×10^{-10} of the high-frequency noise. The weights $\{w_k\}$ for "51G113" are listed in Table 1.

Another filter for $n = 51$, "51g514" was generated, with more emphasis on passing signals of a few-days period. This had Ω_1 increased to 5° h^{-1} , Ω_2 to 14° h^{-1} , and σ to 0.05, with the other parameters as before. Its response is also summarized in Table 2 and Fig. 2.

TABLE 2. Transmitted power in critical bands.

Filter	Diurnal power ratio $\times 10^6$	Semi-diurnal power ratio $\times 10^6$	Inertial power ratio $\times 10^6$	High frequency power ratio $\times 10^6$	$\sigma = 0.0289$ total noise power passed $\times 10^6$	$\sigma = 0.2$ and inertial; total noise power passed $\times 10^6$
D49	0.10	0.102	20.9	487	0.56	41
F49	1.67	0.037	138.3	69	0.87	142
24gaus	58.83	55.588	88.3	40	85.78	176
24G113	0.27	0.003	163.8	57	0.17	166
24m214	13.03	0.564	0.2	17	6.66	7
Godin	0.023	0.000	1.76	2.85	0.013	1.89
35G113	0.002	0.000	0.87	0.57	0.002	0.90
51G113	0.000	0.000	0.00	0.00	0.000	0.00
51g514	0.044	0.001	40.34	6.80	0.028	40.63
Lancz7	1113.115	0.025	36.39	1.38	516.977	553.42
Lancz6	27.852	0.009	0.00	0.71	12.945	12.98
60gau6	0.043	0.011	0.03	0.00	0.054	0.08
60i312	0.001	0.000	0	1.53	0.002	0.06
(Lancz7) ²	4.014	0.000	0.001	1.38	1.866	1.92
120i913	0.002	0.000	0	1.06	0.003	0.04

d. Five-day filters

For $n = 60$, the Lanczos-cosine filter "Lancz7" is defined by

$$w_k = \left[1 + \cos\left(\frac{k\pi}{60}\right) \right] \sin\left(\frac{pk\pi}{12}\right), \quad (25)$$

normalized by (10), with $p = 0.7$. It seems to be widely used in oceanography, e.g. by Bryden (1979). Fig. 3 and Table 2 show that it is a poor filter for suppressing diurnal tides, especially the O_1 and Q_1 constituents. The coefficient $p = 0.7$ seemed rather arbitrary; some trial produced "Lancz6", which comes from (25), with $p = 0.6$ (see Fig. 4).

There does not seem to be a five-day Gaussian filter in common use, but "60gau6" with $a = 0.06$ in

$$w_k = \exp[-1/2(ak)^2], \quad (26)$$

normalized by (10), seems a likely choice.

The matrix G of (7) would not fit in the memory of the HP-85, so "60i312" was generated from (20), with 7 zeros (at Q_1 , O_1 , K_1 , N_2 , M_2 , S_2 , and local inertial), and $\Omega_1 = 3.5^\circ \text{ h}^{-1}$, $\Omega_2 = 12.3^\circ \text{ h}^{-1}$. This was actually better with the zero imposed at $16.7^\circ \text{ h}^{-1}$ than without.

e. Ten-day filters

In an effort to suppress the excessive diurnal power passed by the "Lancz7" filter of (25), Walters and Heston (1982) convolved it with itself to make a ten-day long ($n = 120$) filter "(Lancz7)²". The response

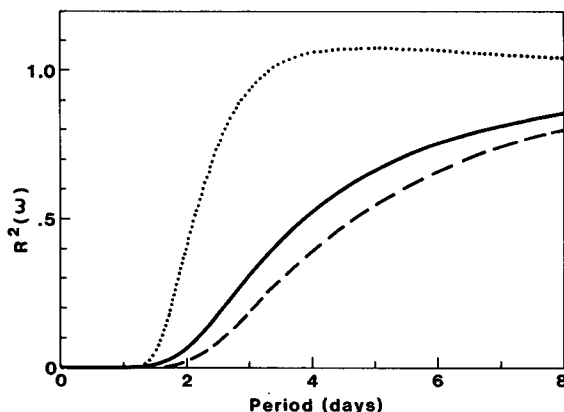


FIG. 2. As in Fig. 1, but where $R(\omega) = w_0 + 2 \sum w_k \cos k\omega$ is the filter response factor for the Godin filter (solid), "51g514" (dotted), and "51G113" (dashed).

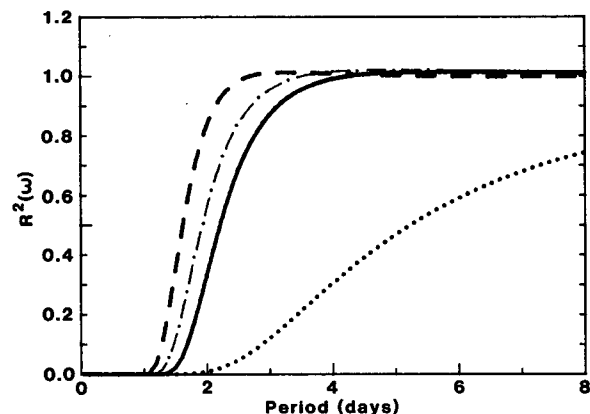


FIG. 3. As in Fig. 1, but where $R(\omega) = w_0 + 2 \sum_{k=1}^{60} w_k \cos k\omega$ is the filter response factor, for the filters "60i312" (solid), "60gau6" (dotted), "Lancz7" (dashed), and "Lancz6" (dot-dashed).

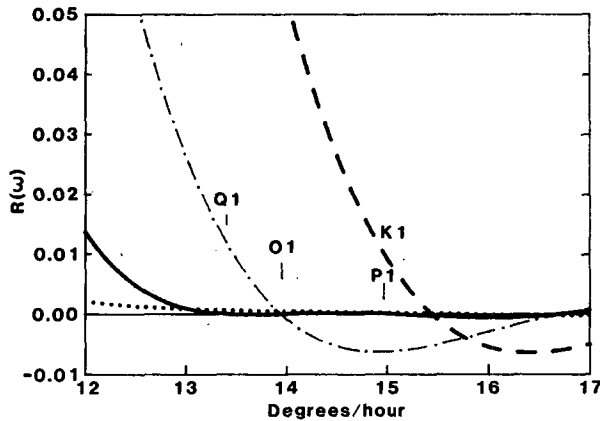


FIG. 4. Filter response factor $R(\omega) = w_0 + 2 \sum_{k=1}^{60} w_k \cos k\omega$ versus ω in the diurnal band ($\omega = 12^\circ \text{ h}^{-1}$ to 17° h^{-1}), for the filters "60i312" (solid), "60gau6" (dotted), "Lancz7" (dashed), and "Lancz6" (dot-dashed).

$R(\omega)$ is the square of "Lancz7"; its square is plotted in Fig. 5, and the power passed in Table 2.

Eq. (20) was used to generate a comparable filter, with emphasis on passing signals with periods of a few days, rather than particularly on suppressing tides. The filter "120i913" has $R(\omega_i) = 0$ for the seven tidal constituents (O_1 , K_1 , Q_1 , P_1 , M_2 , S_2 , N_2) and local inertial frequency ($16.7^\circ \text{ h}^{-1}$), has $R'(0) = 0$ [Eq. (21)], and uses (11) with $\Omega_1 = 8.7^\circ \text{ h}^{-1}$ and $\Omega_2 = 12.9^\circ \text{ h}^{-1}$. [Numerically, it is necessary to avoid $k\Omega$ or $k(\Omega_2 - \Omega_1)$ being a multiple of 180° .] The energy passed is shown in Fig. 5 and Table 2. The weights $\{w_k\}$ are listed in Table 3.

4. Discussion

The Lanczos-cosine "Lancz7" filter has apparently been in common use because it passes periods longer

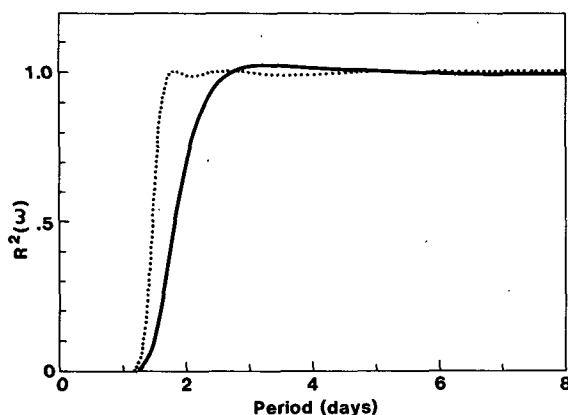


FIG. 5. Relative power transmitted, $R^2(\omega)$, versus period in days, where $R(\omega) = w_0 + 2 \sum_{k=1}^{120} w_k \cos k\omega$ is the filter response factor for the filters ("Lancz7")² (solid) and "120i913" (dotted).

than two days with little attenuation (Fig. 3). This is important when visually comparing filtered records for sudden changes. Unfortunately, "Lancz7" passes an unacceptable amount of diurnal tidal power, as can be seen in Table 2, and as noted previously by Walters and Heston (1982). Fig. 4 shows why: the first zero of the transfer function $R(\omega)$ occurs at $15.4^\circ \text{ h}^{-1}$, completely past the diurnal band. The form (24) can be improved considerably (Table 2) by changing the value of p so as to shift the first zero to near 14° h^{-1} (near O_1) to get the filter "Lancz6". This filter passes a factor of 40 less power in the diurnal and local inertial bands, a factor of 3 less power in the semi-diurnal band, and a factor of 2 less high-frequency noise power, and even a flatter response for periods of a few days—though at some cost in passing a two-day period signal. For some further cost, "60i312" passes a great deal less noise, so would be better for processing leading to spectral estimates of low-frequency power, or if good suppression of noise is required to look at low frequencies.

If one prefers a short filter, Table 2, column 5 shows that "24G113" passes the least overall noise for $n = 24$ as well as least semi-diurnal noise (column 2), so it might be recommended for water-level recorders for which the inertial signal is negligible. For use with current meter records with inertial energy, collected at the latitude of Sydney, the recommendation would be "24m214". If one looks at the positive features of the filters in Fig. 1, one sees that the power passed at long periods is much the same for all of the records. The response for "F49" plots on top of that for "24m214", to within the width of the inked line, though slightly below. The order of the power passed for periods longer than a day is $24m214 > F49 > 24G113 > 24\text{gaus} > D49$ uniformly, so it appears that the filters recommended here for $n = 24$ are better than Groves' (1955).

For longer filters, the noise passed by "51G113" is so low (10^{-12}) that one might find it the best filter for finding very weak signals or for spectral estimation with re-coloring.

Suppressing tidal energy may be more important than suppressing the high-frequency energy, because the aliases of the filtered high-frequency power will be swallowed in the (normally) red signal spectrum, whereas the tides will come through as spikes. For instance, with subsampling at 24-hour intervals, O_1 will alias to 14.2 day period, Q_1 to 9.4 days and M_2 to 14.8 days.

The Fourier transform method of Walters and Heston (1982) seems like a good idea if the whole record will fit in memory, and one uses a carefully rounded $R(\omega)$ to avoid "ringing". For instance, one notes that the "transform filter" trial in their Fig. 4 shows initial ringing for about two days. The same "ringing" would occur if a sharp front occurred in the middle of a record. The length of data is not likely

TABLE 3. Weights $\{w_k\}$ for the filter "120i913". The filter is symmetric, with $w_{-k} = w_k$, so the total filter length is 241 points (240 hours).

k	$w(k)$	k	$w(k)$	k	$w(k)$	k	$w(k)$
0	0.059983763	31	-0.003767607	61	-0.002797825	91	-0.000927041
1	0.059619747	32	-0.002083090	62	-0.002405415	92	-0.000884426
2	0.058536421	33	-0.000455716	63	-0.001949182	93	-0.000828245
3	0.056759542	34	0.001062503	64	-0.001447799	94	-0.000766179
4	0.054330728	35	0.002427191	65	-0.000921726	95	-0.000702370
5	0.051305694	36	0.003602933	66	-0.000392583	96	-0.000637083
6	0.047752118	37	0.004564057	67	0.000117891	97	-0.000567392
7	0.043747394	38	0.005294783	68	0.000589202	98	-0.000488692
8	0.039376438	39	0.005788778	69	0.001003568	99	-0.000396663
9	0.034729681	40	0.006048247	70	0.001347253	100	-0.000289156
10	0.029901225	41	0.006082735	71	0.001611461	101	-0.000167514
11	0.024987093	42	0.005907861	72	0.001792637	102	-0.000036947
12	0.020083436	43	0.005544148	73	0.001892088	103	0.000094135
13	0.015284565	44	0.005016057	74	0.001915023	104	0.000215700
14	0.010680746	45	0.004351217	75	0.001869252	105	0.000318158
15	0.006355800	46	0.003579781	76	0.001763818	106	0.000394386
16	0.002384577	47	0.002733785	77	0.001607903	107	0.000441163
17	-0.001169390	48	0.001846365	78	0.001410196	108	0.000459624
18	-0.004255895	49	0.000950771	79	0.001178827	109	0.000454605
19	-0.006839458	50	0.000079158	80	0.000921769	110	0.000433032
20	-0.008899966	51	-0.000738753	81	0.000647487	111	0.000401795
21	-0.010432516	52	-0.001476983	82	0.000365519	112	0.000365685
22	-0.011446533	53	-0.002114382	83	0.000086704	113	0.000325996
23	-0.011964332	54	-0.002635339	84	-0.000177148	114	0.000280202
24	-0.012019343	55	-0.003030001	85	-0.000414169	115	0.000222875
25	-0.011654239	56	-0.003294021	86	-0.000613794	116	0.000147642
26	-0.010919123	57	-0.003427916	87	-0.000768246	117	0.000049698
27	-0.009869873	58	-0.003436243	88	-0.000873771	118	-0.000071792
28	-0.008566628	59	-0.003326761	89	-0.000931275	119	-0.000212081
29	-0.007072349	60	-0.003109790	90	-0.000946117	120	-0.000360537
30	-0.005451338						

to be a multiple of all tidal periods, so Walter and Heston's Fourier transform will not have each of the tidal constituents precisely on a Fourier component, and it may not be possible to set zero response for each constituent.

Walters and Heston (1982) tried using a ten-day filter ($n = 120$), ("Lancz7")². In Table 2, this is seen to still pass too much diurnal power, but the suggestion to use a longer filter seems good. In fact, a convolution filter of form (1) with $n = 120$ only costs 5 times as much with $n = 24$. Short filters used to be necessary to avoid excess computational effort and because long records were comparatively rare. A year of hourly data presently costs about A\$4 to filter with $n = 24$ (Murray Greig, private communication, 1982), so $n = 120$ will cost about \$16 more to do a better job of processing a record which took a year and several thousand dollars to collect. According to Table 2 and Fig. 5, if one uses "120i913" instead of, say, "F49", one gets much better suppression of tidal frequencies and noise, and complete passage of periods down to 1.7 days—with no need to recolor!

A reasonable recommendation on filtering out tides, then, is to use "120i913"—and then extend into the 5 days at each end of the record with "60i312" and "24G113". These will act conservatively for in-

termediate frequencies, but will keep the tidal and high-frequency noise from coming through, while allowing recovery of 8 of the 10 days otherwise lost from the ends of the record.

In conclusion, it appears that filters can be designed which are considerably better than traditional tide-killer low-pass filters. In particular, "120i913" seems worth general use.

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