

NOTES AND CORRESPONDENCE

Removing Tidal-Period Variations from Time-Series Data Using Low-Pass Digital Filters

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ABSTRACT

Several low-pass, digital filters are examined for their ability to remove tidal period variations from a time-series of water surface elevation for San Francisco Bay. The most efficient filter is the one which is applied to the Fourier coefficients of the transformed data, and the filtered data recovered through an inverse transform. The ability of the filters to remove the tidal components increased in the following order: 1) cosine-Lanczos filter; 2) cosine-Lanczos squared filter; 3) Godin filter; and 4) a transform filter. The Godin filter is not sufficiently sharp to prevent severe attenuation of 2–3 day variations in surface elevation resulting from weather events.

1. Introduction

Associated with the increasing interest in long-term (residual) circulation in estuaries is a need to separate these patterns from the generally strong tidal component. For our own purposes we wish to low-pass filter several hydrodynamic parameters for forcing a residual circulation model (Walters and Cheng, 1980). These parameters are derived from the time-averaged (or filtered) terms in the governing equations and are of the form $\langle u_i u_j \rangle$ and $\langle y^2 \rangle$, where u_i is the tidal velocity component in the i th direction, y is the surface elevation, and $\langle \rangle$ denotes the filtering operation. The time increment associated with the data is typically 2 min to 1 h; it needs to be expressed in terms of a semidaily-to-daily time increment. A particularly stringent requirement, which requires a relatively steep filter slope, is to discriminate variations with periods of 2–3 days or longer. The key problem is the elimination of tidal components in the diurnal band without severe attenuation at low frequency.

Our purpose here is to examine specific realizations of several filters that are commonly used by researchers. Because two of these filters meet our requirements, we have not found it necessary to make any subsequent modifications of the filters. Three of the filters are applied to the discrete time series by forming the convolution

$$\bar{x}[t_{k+(m/2)}] = \sum_{i=0}^m w_i x(t_{k+i}), \quad (1)$$

where $\bar{x}(t_k)$ is the filtered series at time $t_k = k\Delta t$, w_i

are the filter weights, x is the input time series, Δt is the time increment for this series, and the number of weights is $m + 1$. Equivalently, one can apply the filter in frequency space as

$$\bar{X}(f) = W(f)X(f), \quad (2)$$

where \bar{X} and X are the Fourier transform of \bar{x} and x , and $W(f)$ is the frequency response of the filter. The filtered time series is then recovered through an inverse Fourier transform. In most cases, we have found that it is computationally more efficient to apply the filter in frequency space (2), than to use weights such as in (1). For more information concerning filters, see Godin (1972) and the references therein.

There are several ways to approach the filtering problem. The simplest way is to use block averages in time; however, as pointed out by Godin (1972) among others, there are serious aliasing problems in this case, particularly with hourly data spacing and 24 h averages. Godin suggests, rather, the use of the tidal eliminator denoted as $A_{25}^2 A_{24} / (25^2 \cdot 24)$ which here will be called the Godin filter. The factor A_n represents the arithmetical summation of n observations so that this filter processes the data three times: first using 24-point averages (A_{24}), then twice using 25-point averages (A_{25}). Although this filter completely eliminates the diurnal and higher frequencies (Fig. 1), it has the undesirable effect of attenuating the motions with considerably longer periods. (The amplitude is 0.7 for 100 h periods.) As a result, three other filters with sharper cutoff are examined in hopes of recovering more information

in the 2–4 day period range, which is of importance in wind forcing.

The first of these is the cosine-Lanczos filter as applied by Mooers *et al.* (1968). This filter is the Fourier transform of the product of a cosine and Lanczos filter, and uses 121 weighting points, for which there are 61 unique weights because of symmetry about the center weight. For 1 h data spacing, the half-power point is at 40 h (Fig. 1). Although this filter is much sharper than the Godin filter, note that the high frequency tail creeps across the diurnal tidal band, the effect of which will be seen shortly. In a desire to remove this tail, the filter was squared; i.e., the data were filtered twice. As may be seen in Fig. 1, this removes the tail, increases the filter slope and raises the half power point to about 45 h. (Note, however, that the length of the filter is now approximately doubled).

The final filter is denoted as the transform filter. In this case the data are Fourier transformed, the filter is applied to the Fourier coefficients in frequency space, and the filtered data are found through an inverse transform. For an ideal filter, all the coefficients above the cutoff frequency are set to zero. Unfortunately, this causes ringing through the entire data set. For this reason, the Fourier coefficients are reduced to zero linearly over a range of frequencies (between periods of 40 and 30 h). In a sense, the smoothness in the filter transition leads to a reduction in ringing in the filtered data. Whereas the linear reduction is used here, good results also are obtained with other tapers—notably a cosine taper.

As a final note, Roberts and Roberts (1978) discuss the relative merits of a specific application of the Butterworth squared filter as compared to the

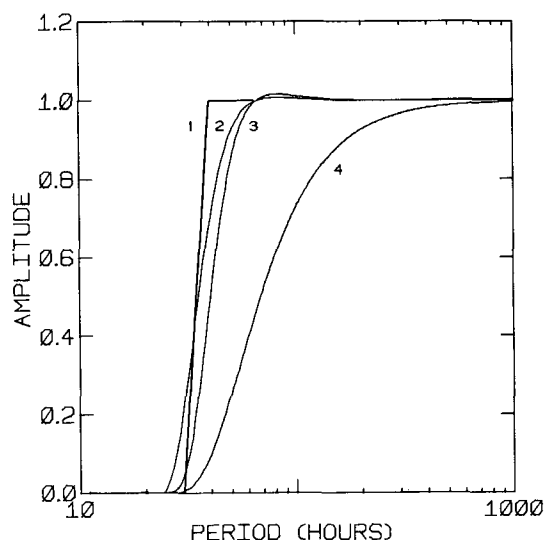


FIG. 1. Amplitude response for three filters: (1) transform (linear taper), (2) cosine-Lanczos, (3) cosine-Lanczos squared, and (4) Godin.

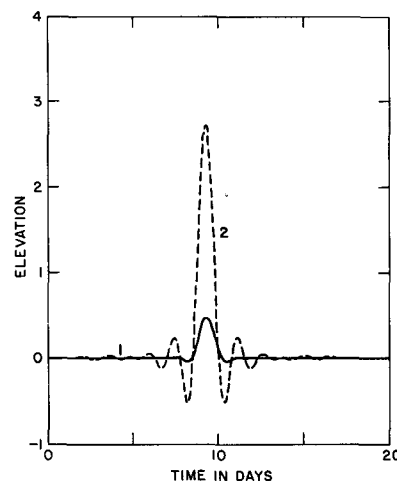


FIG. 2. Impulse response for (1) the cosine-Lanczos squared, and (2) the transform filter (linear taper). The input is a 512-point record of hourly values containing all zero's except point 200, which has a value of 50 [see also Roberts and Roberts (1978), Fig. 3].

cosine-Lanczos, Gaussian and ideal filters. Because their form of the filter does not attenuate the diurnal tidal components as well as the cosine-Lanczos filter, it has not been included in this comparison. For problems which do not require as much attenuation, its use may be desirable for its greater efficiency. Nonetheless, the impulse response for the cosine-Lanczos squared and transform filters are computed here (Fig. 2) and may be compared with the impulse response for the other filters as computed by Roberts and Roberts (1978, their Fig. 3). The response of the transform filter is similar to that of the cosine-Lanczos and the Butterworth squared; the response of the cosine-Lanczos squared is highly damped owing to the two-pass nature of the filter.

2. Results

As a basis for comparison, a portion of the record for tidal elevation at the entrance to San Francisco Bay is used. The different low-pass values for surface elevation and their corresponding power spectra are compared to that of the unfiltered tidal elevation. The specific period chosen was the 190-day period beginning on 17 May 1975. This record contains discrete hourly values and was filtered using the various tapers described above. Note that there is a varying amount of data lost at the start and end of the record, the quantity depending upon the number of filter points. The most data lost was with the cosine-Lanczos squared where 5 days were lost at each end of the record. For the power spectra, the low-pass filtered values were decimated to 4 h increments and 1024 points were transformed using a FFT. The unfiltered data were not decimated and 4096 points

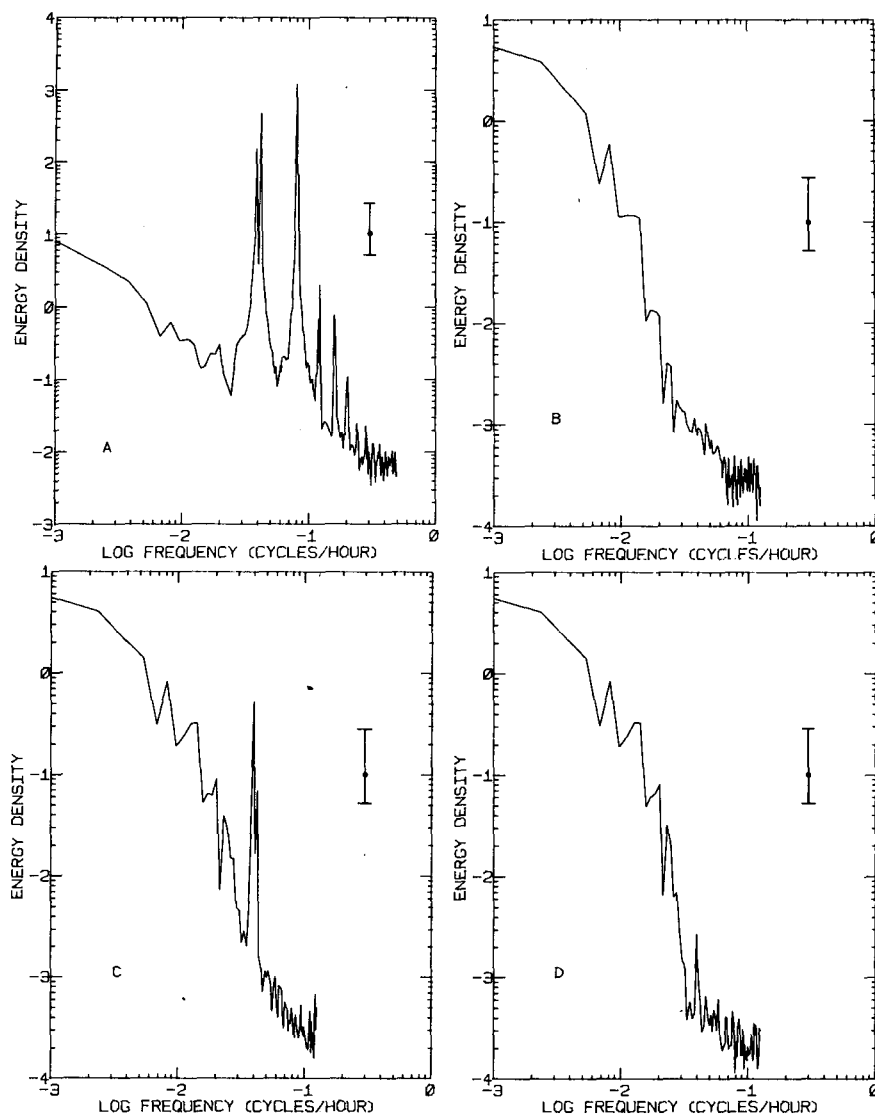


FIG. 3. (a) Power spectrum for the unfiltered data set, (b) Power spectrum for the data filtered with the Godin filter, (c) Power spectrum for the data filtered with the cosine-Lanczos filter, (d) Power spectrum for the data filtered with the cosine-Lanczos squared filter. The 95% confidence interval is shown by the vertical line.

were transformed. Both the filtered and unfiltered data sets were realigned on day 5 of the record (Julian day 142) to compensate for data lost during filtering. In addition, the spectra were smoothed using a non-overlapping boxcar average with six points in each average.

The power spectra for the various records are compared in Fig. 3. Fig. 3a contains the spectrum for the unfiltered record and indicates the large amount of attenuation needed to remove the tidal constituents. Also, note the presence of many tidal harmonics. In the unfiltered spectrum, each of the harmonics can be traced to the primary diurnal and semidiurnal species; O_1 , K_1 , M_2 , S_2 , and N_2 . On the other hand,

there are variations with a period of 2–3 days that are related to meteorological forcing; these motions are to be retained after filtering. The Godin filter (Fig. 3b) completely attenuates all the diurnal and shorter period tidal species. However, there is appreciable roll-off out to at least 100 h periods. For a record with little or no information in the 2–4 day periods, this filter is undoubtedly an excellent choice. For the cosine-Lanczos filter (Fig. 3c) the combination of the filter tail and the large diurnal energy peaks leaves appreciable signal at diurnal frequencies. Similar to the 24 h boxcar average, daily values of this filtered data lead to serious aliasing problems. Finally, the square of this filter (Fig. 3d) provides

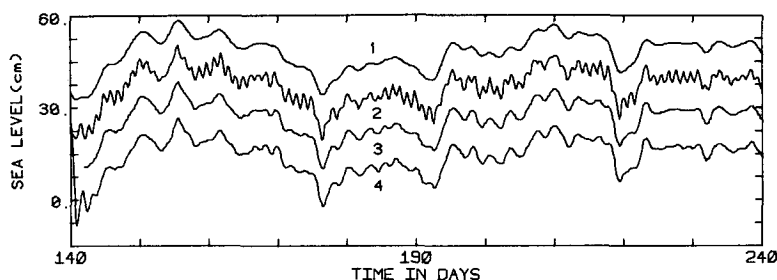


FIG. 4. Time series of the filtered data using: (1) the Godin filter, (2) the cosine-Lanczos filter, (3) the cosine-Lanczos squared filter and (4) the transform filter.

the desired attenuation characteristics: 1) sharp roll-off in the 1–2 day range, and 2) nearly complete attenuation of the tidal constituents.

For the transform filter, the Fourier coefficients are reduced to zero between the periods of 30–40 h (Fig. 1). As a consequence, the power spectrum looks virtually like that in Fig. 3a except that the amplitude is zero above 0.025 cph. As it turns out, this filter is more efficient in computer time for this 4096 point data set because of the speed of the Fast Fourier Transform (FFT) subroutine.

As a final comparison, the mean surface elevation defined by the four filters is shown in Fig. 4. (Note that the data are offset for clarity.) In this figure one can see clearly what the results for the power spectra indicate. First, the cosine-Lanczos filter did not completely remove the tidal signal as is evidenced in trace two. On the other hand, the Godin filter attenuates components with periods up to at least 100 h (compare the first and the third trace). While the energy in that range is small in many cases, there are significant variations here with periods of the order of 2–3 days at ~day 180 and 200.

The cosine-Lanczos squared filter was successful in eliminating the diurnal tidal signal while not having much attenuation in the 2–3 day range of periods. However, this filter uses a large number of data points such that five days of data are lost at each end of the record and the computations take somewhat longer than the other filters.

As may be expected, the transform filter did the best job of separating the tidal from the low frequency components. Note the oscillations with periods of ~1.5 days at day 170, which are attenuated in the other records. While there is theoretically no data loss with this filter, in practice one must deal

with the Gibbs phenomena at both ends of the record. The Fourier transform considers this a finite length record which is zero everywhere else. At the ends of the record, the signal then goes to zero, leaving a step in time. When the high-frequency Fourier coefficients are filtered out, it is no longer possible to represent this step accurately and oscillations occur (see Fig. 4, trace 4, at day 140). As an estimate, about three days of data are lost at each end of the record for the case presented here.

Ultimately, the type of filter used depends on the characteristics of the data and the time scales of the phenomena sought. Our requirement for complete attenuation of the tidal constituents, while having little effect on lower frequencies, has necessitated the use of a filter with fairly sharp cutoff. Both the square of the common cosine-Lanczos filter and the transform filter meet these needs, with the latter being more computationally efficient.

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REFERENCES

- Godin, G., 1972: *The Analysis of Tides*. University of Toronto Press, 264 pp.
- Mooers, C. N. K., L. M. Bogert, R. L. Smith and J. G. Pattullo, 1968: A compilation of observations from moored current meters and thermographs. Data Rep. 30, Dept. Oceanogr., Oregon State University, Corvallis, 91–92.
- Roberts, J., and T. D. Roberts, 1978: Use of the Butterworth low-pass filter for oceanographic data. *J. Geophys. Res.*, **83**, 5510–5514.
- Walters, R. A., and R. T. Cheng, 1980: Calculations of estuarine residual currents using the finite element method. *Proc. Third Int. Conf. on Finite Elements in Flow Problems*, Calgary, D. H. Norrie, Ed., University of Calgary, 60–69.