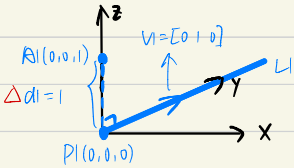
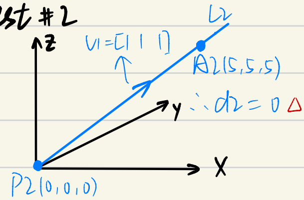
Q1:

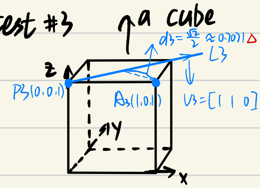
(1) test #1



(2) test #2

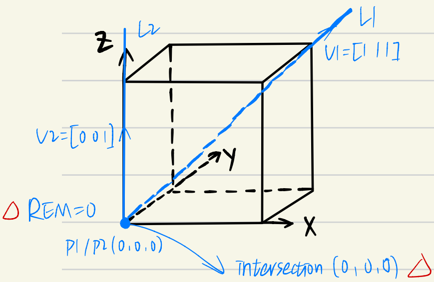


(3) test #3

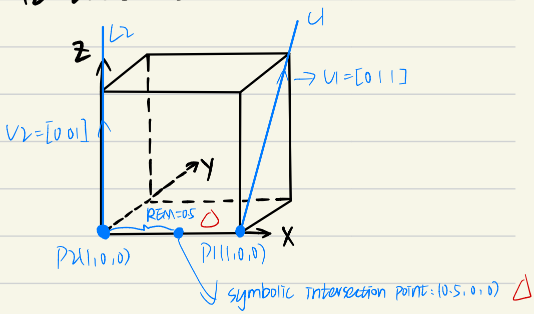


Q2:

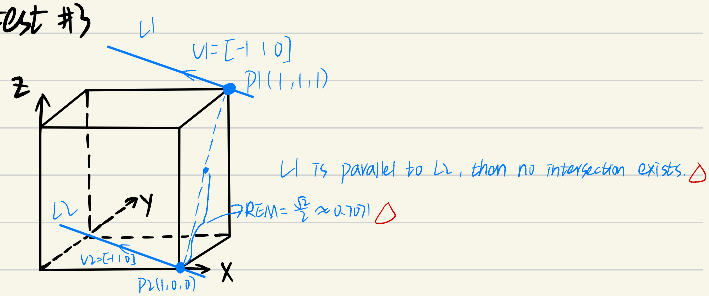
(1) test #1



(2) test #2

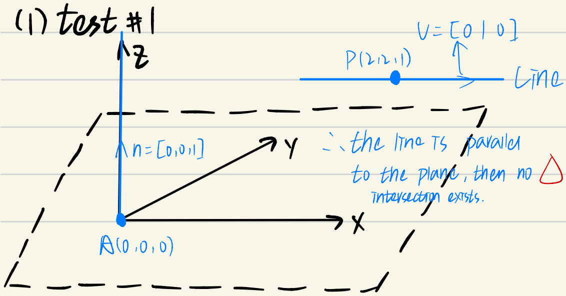


(3) test #3

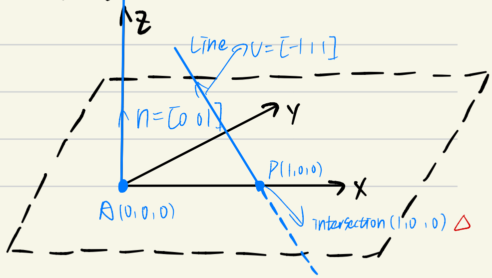


Q3:

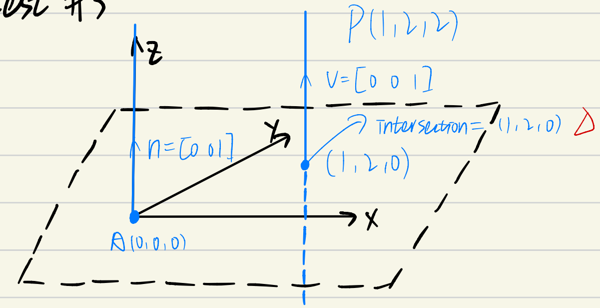
(1) test #1



(2) test #2



(3) test #3



Q4:

(1) Derive polynomial:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

And
$$\begin{cases} x = L_x = P_x + t * V_x \\ y = L_y = P_y + t * V_y \\ z = L_z = P_z + t * V_z \end{cases}$$

$$\therefore \frac{(P_x+t*V_x)^2}{a^2}+\frac{(P_y+t*V_y)^2}{b^2}+\frac{(P_z+t*V_z)^2}{c^2}=1$$

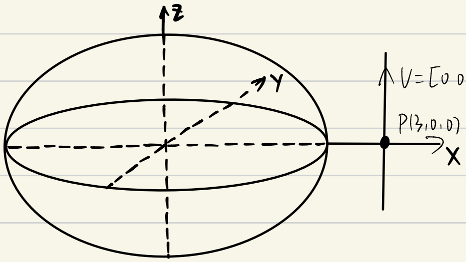
$$\Leftrightarrow b^2c^2(P_x+t*V_x)^2+a^2c^2(P_y+t*V_y)^2+a^2b^2(P_z+t*V_z)^2=a^2b^2c^2$$

$$\Leftrightarrow (bcV_x)^2t^2+(2P_xV_xb^2c^2)t+(bcP_x)^2 \quad +$$
$$(acV_y)^2t^2+(2P_yV_ya^2c^2)t+(acP_y)^2 \quad +$$
$$(abV_z)^2t^2+(2P_zV_za^2b^2)t+(abP_z)^2 - a^2b^2c^2 = 0$$

Suppose: $At^2+Bt+C=0$, then:

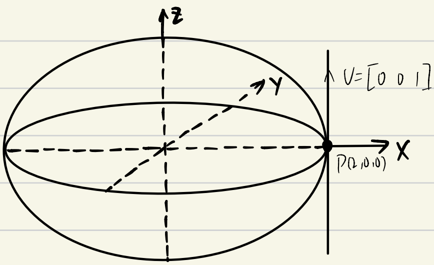
$$A=(bcV_x)^2+(acV_y)^2+(abV_z)^2$$
$$B=2(P_xV_xb^2c^2+P_yV_ya^2c^2+P_zV_za^2b^2)$$
$$C=(bcP_x)^2+(acP_y)^2+(abP_z)^2-a^2b^2c^2$$

- test #1 - no intersection



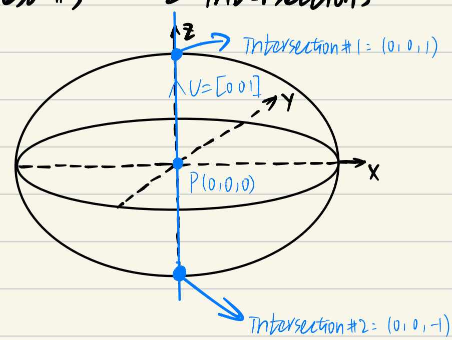
∴ no intersection △

- test #2 - 1 intersection



∴ only 1 intersection △

- test #3 - 2 intersections

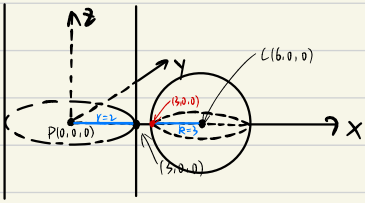


∴ 2 intersection △

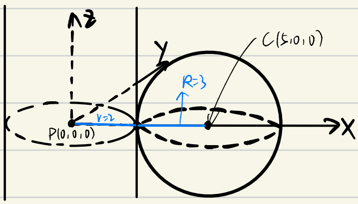
Q5:

(1) test #1: 0 intersection

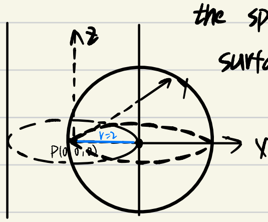
The method is explained in the program.



(2) test #2: 1 intersection



(3) test #3: infinite intersection, the cylinder and the sphere intersect, forming a surface, which means infinite point.



Q6: In this case, a paper test doesn't make sence since it's too complex to draw lots of points on a sphere on paper. Meanwhile, the set of points as input are generated randomly while testing. It's impossible to draw them out accurately on paper.

As for the test in Matlab, according to the figure generated, the sphere fits very well.

Q7: A paper test in this question is impossible since the unit vector is generated randomly, which cannot be predicted and drawn on paper. As for the test in Matlab, all the resulting points are located on the canonical unit circle (2D) or a canonical unit sphere (3D).

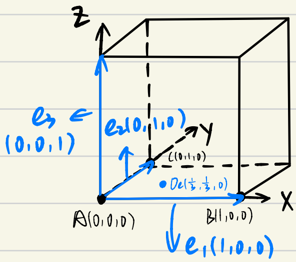
And the method is explained in Matlab.

Q8:

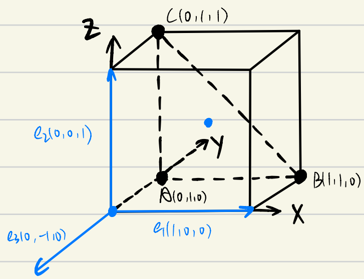
According to my test, I found when Err becomes to about 3.5, it is prone to consecutive failures. Then I should stop the simulation.

Q9:

(1) test #1:



(2) test #2:



(3) Since A(0,0,0) B(1,1,1) C(3,3,3) are collinear, so an orthonormal frame cannot be generated in this case.

Q10:

$$\begin{cases} \sin 90^\circ = 1 \\ \cos 90^\circ = 0 \end{cases}$$

(1) test #1: axis: x, angle: 90°

∴ Rotation matrix:

$$RM_x(90^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90^\circ & -\sin 90^\circ \\ 0 & \sin 90^\circ & \cos 90^\circ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

And homogenous rotation matrix:

$$HRM_x(90^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90^\circ & -\sin 90^\circ & 0 \\ 0 & \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) test #2: axis: y, angle: 90°

∴ Rotation matrix:

$$RM_y(90^\circ) = \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ \\ 0 & 1 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

And homogenous rotation matrix:

$$HRM_y(90^\circ) = \begin{bmatrix} \cos 90^\circ & 0 & \sin 90^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) test #3: axis: z, angle: 90°

∴ Rotation matrix:

$$RM_z(90^\circ) = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And homogenous rotation matrix:

$$HRM_z(90^\circ) = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$O_h(0,0,0)$ $h_1(1,0,0)$ $h_2(0,1,0)$ $h_3(0,0,1)$

Q11: Sorry, I temporarily found some problems with my method, the test on paper and test in Matlab quite match.

(1) test #1: pure translation, the translation vector: $d(-1,-1,-2)$

$[O_v(1,1,2), V_1(2,1,2), V_2(1,2,2), V_3(1,1,3)] \rightarrow (O_h, h_1, h_2, h_3)$

Here, the homogeneous translation matrix is:

$$hm = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow d$$

to test if it's right, I'll show the translation - $O_v \rightarrow O_h$

$$hm * \underbrace{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}}_{O_v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow O_h$, hence checked!

(2) test #2: pure translation, the translation vector: $d(2,3,4)$

$[O_v(-2,-3,-4), V_1(-1,-3,-4), V_2(-2,-2,-4), V_3(-2,-3,-3)] \rightarrow (O_h, h_1, h_2, h_3)$

Here, the homogeneous translation matrix is:

$$hm = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow d$$

to test if it's right, I'll show the translation - $O_v \rightarrow O_h$

$$hm * \underbrace{\begin{bmatrix} -1 \\ -3 \\ -4 \\ 1 \end{bmatrix}}_{V_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow h_1$, hence checked!

(3) test #4: pure rotation. 45° rotation about z-axis

$[O_v(0,0,0), V_1(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0), V_2(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0), V_3(0,0,1)] \rightarrow (O_h, h_1, h_2, h_3)$

Here, the homogeneous rotation matrix is:

$$hm = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

to test if it's right, I'll show the rotation $V_1 \rightarrow h_1$

$$hm * \underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \\ 1 \end{bmatrix}}_{V_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

h_1, hence checked!

(4) test #4: pure rotation. -45° rotation about z-axis

$[O_v(0,0,0), V_1(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0), V_2(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0), V_3(0,0,1)] \rightarrow (O_h, h_1, h_2, h_3)$

Here, the homogeneous rotation matrix is:

$$hm = \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) & 0 & 0 \\ \sin(-45^\circ) & \cos(-45^\circ) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

to test if it's right, I'll show the rotation $V_2 \rightarrow h_2$

$$hm * \underbrace{\begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \\ 1 \end{bmatrix}}_{V_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow h_2$, hence checked

(5) test #5: combine translation and rotation.

$[O_v(3,2,3), V_1(\frac{\sqrt{2}}{2}+3, \frac{\sqrt{2}}{2}+2, 3), V_2(-\frac{\sqrt{2}}{2}+3, \frac{\sqrt{2}}{2}+2, 3), V_3(3,2,4)] \rightarrow [O_h, h_1, h_2, h_3]$

Ov translated from (3,2,3) to Oh first, then -45° around 'z' so that the perspective is taken from 'V' to home (h).

Then the homogeneous matrix should be:

$$hm = hrm * htm = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(-45°) & -\sin(-45°) & 0 & 0 \\ \sin(-45°) & \cos(-45°) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(-45°) & -\sin(-45°) & 0 & -3 \\ \sin(-45°) & \cos(-45°) & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(6) test #6: combine translation and rotation.

$[O_v(-1,3,-2), V_1(\frac{\sqrt{2}}{2}-1, \frac{\sqrt{2}}{2}+3, -2), V_2(-\frac{\sqrt{2}}{2}-1, \frac{\sqrt{2}}{2}+3, -2), V_3(-1,3,-1)] \rightarrow [O_h, h_1, h_2, h_3]$

Ov translated from (-1,3,-2) to Oh first, then 45° around 'z' so that the perspective is taken from 'V' to home (h).

Then the homogeneous matrix should be:

$$hm = hrm * htm = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos45° & -\sin45° & 0 & 0 \\ \sin45° & \cos45° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos45° & -\sin45° & 0 & 1 \\ \sin45° & \cos45° & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$