A3 Paper Work

Navigation: Given tip_inTool and axis_inTool

1. Frame transform:

   To calculate the position of the tip_inCT and axis_inCT, we are supposed to calculate the transformation from tool-frame to CT-frame - $F_{CT<-tool}$.

   First, we know:

   $$F_{CT<-tool} = F_{CT<-m} * F_{m<-tool} \qquad (1)$$

   And we also know:

   $$F_{m<-tool} = F_{m<-tracker} * F_{tracker<-tool} \qquad (2)$$

   Combine (1) and (2) together, then the transformation $F_{CT<-tool}$ can be computed as below:

   $$F_{CT<-tool} = F_{CT<-m} * F_{m<-tracker} * F_{tracker <-tool}$$

2. Math for tool tip:

   $$tip\_inCT = F_{CT<-tool} * tip\_inTool$$

3. Math for tool axis:

   $$axis\_inCT = F_{CT<-tool} * axis\_inTool$$

Discussion of error simulation approaches:

We can simulate the distribution of the random error vector's magnitude in several ways:

1. Pure Gaussian distribution, fading marker halo → results in fading TRE halo, curtailed at R_0.
2. Uniform distribution, no fading of error → nonuniform distribution of TRE in a sphere of R_0
3. Constant magnitude of E_0 → highly nonuniform distribution of TRE in a sphere of R_0

I believe option#1 is the most reasonable choice. Since in real surgeries, the distribution of the random error vector's magnitude would be the most likely purely Gaussian distributed. For option #2, I think no such situation will exist because error can never be uniformly distributed. Usually, most of errors will be very small. As the errors progressively get larger, the probability of this will get lower. Here it is - markers with smaller error is closer to the original one, which eventually forms a "glow sphere" as in the lecture notes. Option #3 is not a good choice because of the same reason. In real surgeries, error vectors' magnitudes can never be constant and same.

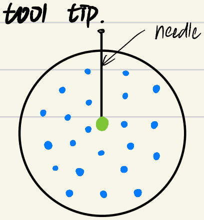
Explain my approach and design:

First of all, I choose $E0 = 26$ and I left my reason for that in comment.
For marker simulation, I also have explained that in comment.
To find the sphere of tool tip error,

1. I add some 3D random error vectors in random direction to each of the original marker.
2. Then I do registration on the true tool tip from tool-frame to m-frame by apply the following transformation.

$$(1)\ tool\_tip\_inM = F_{m \leftarrow tracker} * F_{tracker \leftarrow tool} * true\_tool\_tip\_inTool$$

After repeat that for "many" times, all of the tips will form a point cloud whose center is the true tool tip.

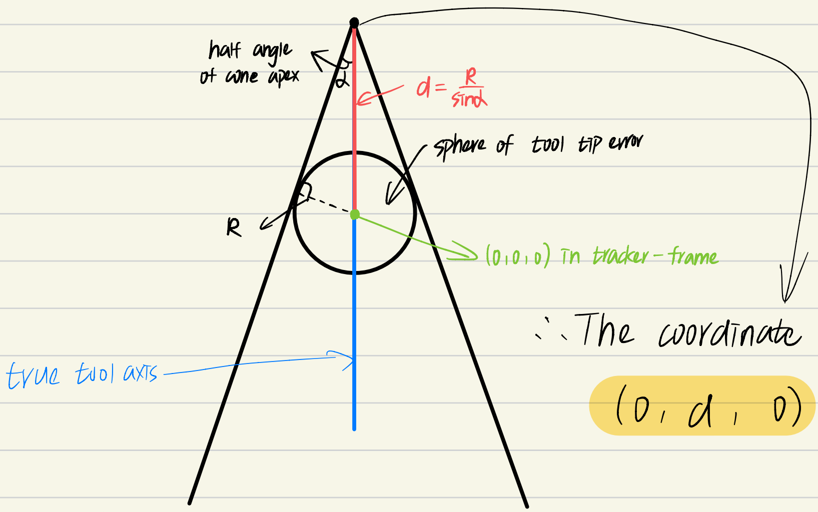


Blue points are all of the transformed tool tips.

3. Then I decide to compute, update and record the maximum distance between tool tip transformed to m-frame and the true one in m-frame, which also means I compute the supersphere to deal with this problem.

Then I'll compute the uncertainty cone of tool axis.

1. Repeat what I've done in step1, step2 of "sphere of tool tip error", now I have transformed tool axis in m-frame. Then I can compute the angle between each transformed tool axis and the true tool axis in m-frame. Meanwhile I'll update and record the maximum angle between them both. Now I have the half angle of the cone apex and the radius of the sphere of tool tip error which locates exactly inside the cone. I can figure out how to compute the coordinate of the apex.

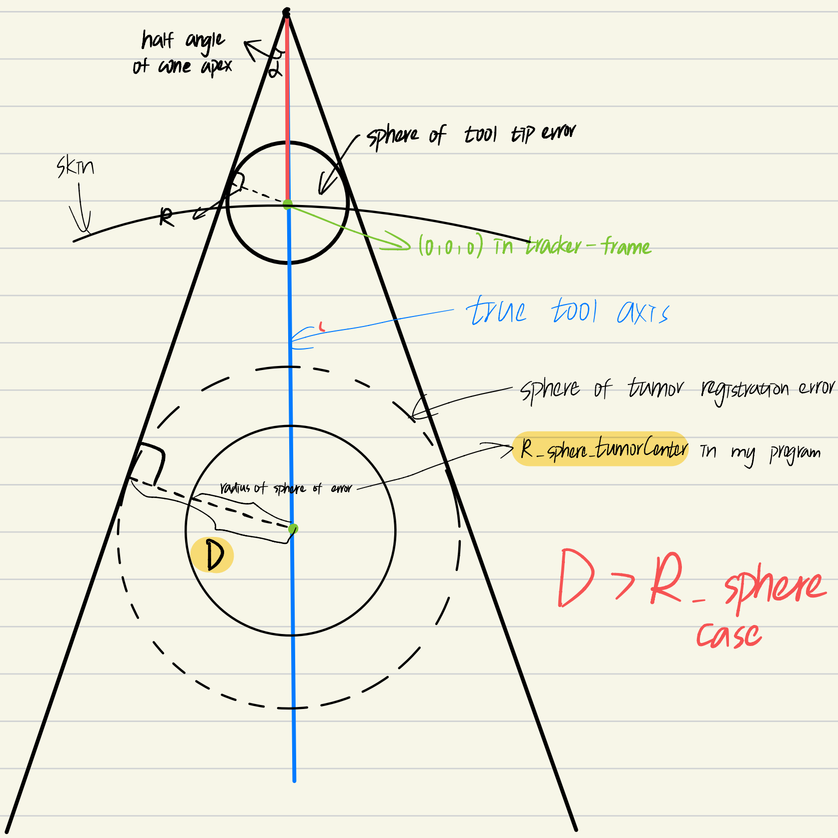


∵ The coordinate of the apex:

$(0, d, 0)$

When I have the angle of apex and the coordinate of apex, I can make sure I succeed computing the uncertainty cone.

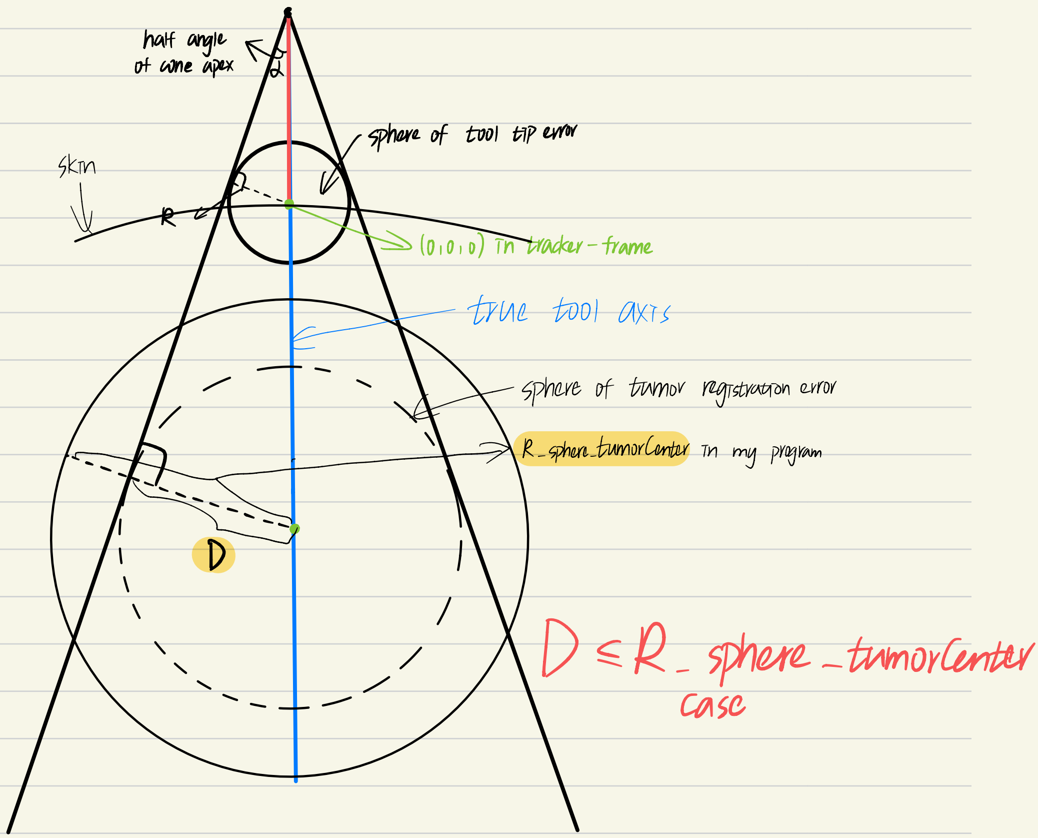
After all of those. I'll compute the sphere of tumor registration error.
Repeat what I've done in "sphere of tool tip error". I can compute the radius of that sphere by calculating the supersphere.



D > R_sphere_tumorCenter case

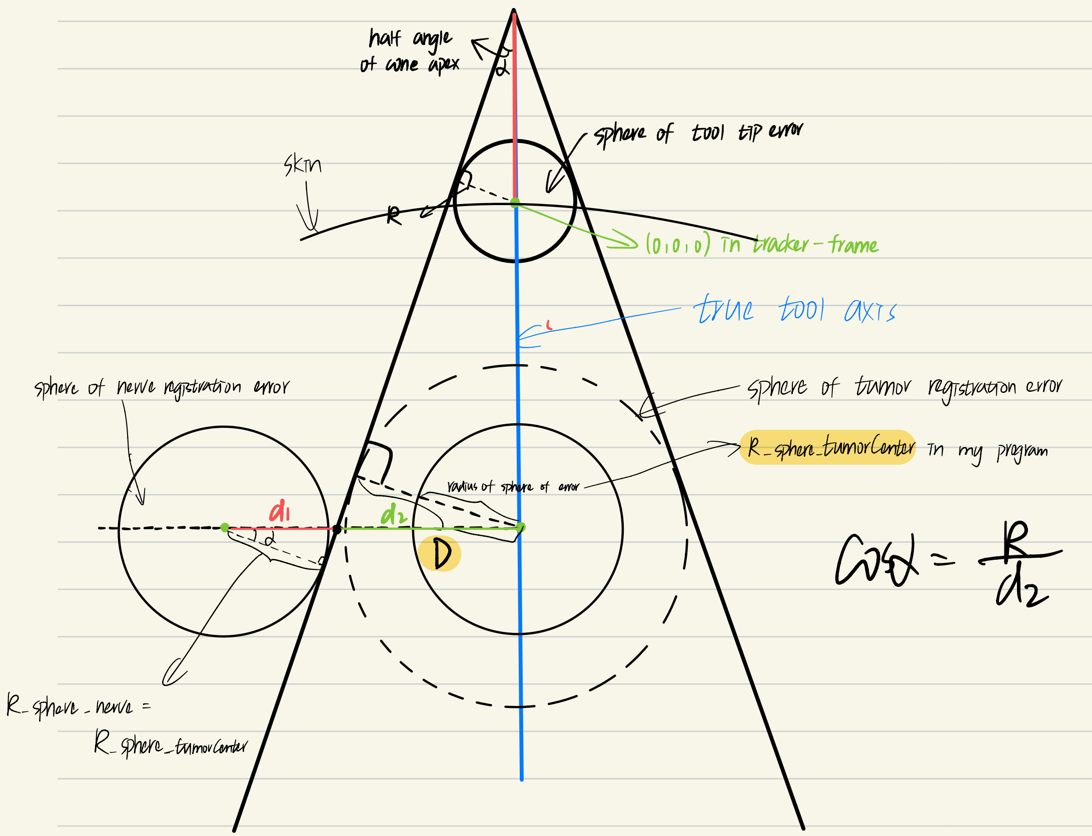
$D > R\_sphere\_tumorCenter$, the smallest distance is: $D - R\_sphere\_tumorCenter$

∴ $D \leq R\_sphere\_tumorCenter$, we can ignore the smallest radius since no matter what it is, we can hit the tumor with "near certainty".



Eventually, it's "sphere of nerve registration error".

About the radius of this sphere of error, Prof Fitchtinger says we could let it be equal to the radius of sphere of tumor registration error.



If we don't want to hit the nerve, then we should avoid the needle trajectory from passing through the sphere of nerve registration error. Since within this sphere, any point may be a nerve because of the presence of errors. Thus we cannot touch this sphere using the needle.

(1) If the needle trajectory is the line tangent to the sphere of nerve registration error, then the closest distance would be R_sphere_nerve.

(2) If the needle trajectory is the true tool axis, the closest distance would be:

$$d_1 + d_2 = \frac{R\_sphere\_nerve}{\cos\alpha} + \frac{D}{\cos\alpha}$$

It is worth mentioning here that - in my program, situation - "the sphere of tumor registration error is beyond the uncertainty cone" is more likely to occur. So if you want to check another case - "the sphere of tumor is inside the cone", please run navigation_error_simulator a few more times.