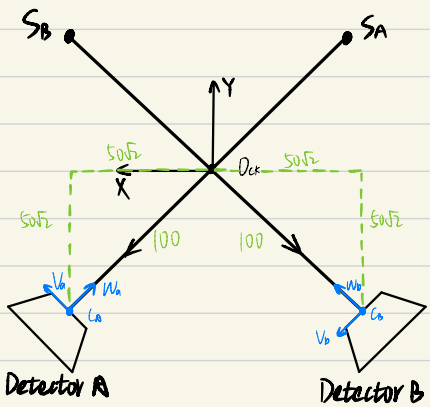
A2 Paper Work

1. Marker Reconstruction Error Sources:

- Marker size(visibility): Markers are so small that the exact location or size cannot be seen, as this leads to misalignment or even failure to locate them.
- Distortion (optical, electromagnetic): Distortion can lead to a situation where we do not get 100% of the information that the marker should convey through the computer, and where some of the information is lost, leading to errors in the reconstruction of the marker.
- Image resolution: Low image resolution can lead to the inability to observe the size and position of the markers, or even to mistake tissues that are not markers but have a high degree of similarity to the markers for them, resulting in reconstruction errors.
- Displacement (e.g. slippage of the skin): Slippage of the skin causes a shift in the marker position and the resulting reconstruction error.
- Tissue deformation (e.g. soft tissue landmarks): When markers are placed on soft tissues that are prone to deformation, the position of the marker is likely to shift, resulting in reconstruction errors.
- Human errors: Human errors that lead to biased marker reconstruction data, such as data transcription errors, improper system operation, etc.

Q2: Frame Transforms

(1) center and base vectors of A, B in the home frame



∴ center of A: $C_A(50\sqrt{2}, -50\sqrt{2}, 0)$

⇒ base vectors: $V_a(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

$W_a(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

$U_a(0, 0, 1)$

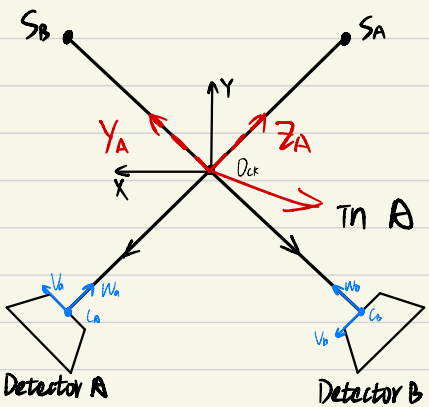
center of B: $C_B(-50\sqrt{2}, -50\sqrt{2}, 0)$

⇒ base vectors: $V_b(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0)$

$W_b(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

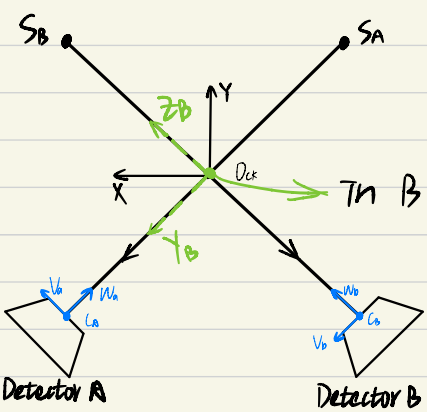
$U_b(0, 0, 1)$

(2) test #1



$P = (0, 0, 0)$ in CK frame

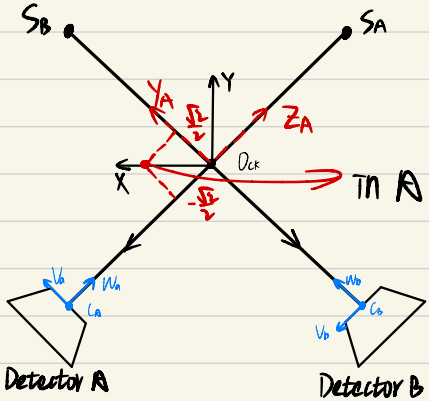
in A frame, $O_{CK} = (0, 0, 100)$ ✓



$P = (0, 0, 0)$ in CK frame

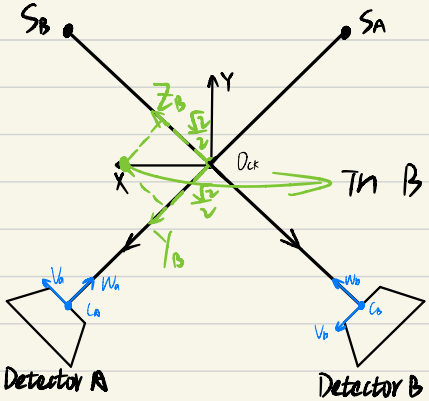
in B frame, $O_{CK} = (0, 0, 100)$ ✓

(3) test #2:



$P=(1,0,0)$ in CK frame

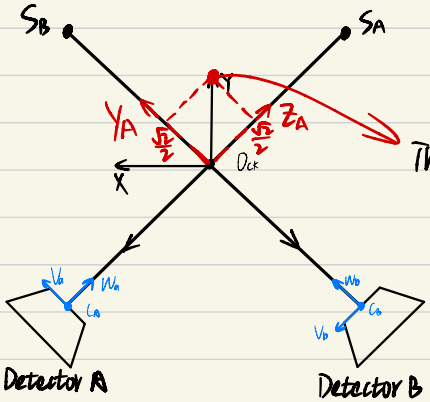
in A frame, $(0, \frac{\sqrt{2}}{2}, 100-\frac{\sqrt{2}}{2})$ ✓



$P=(1,0,0)$ in CK frame

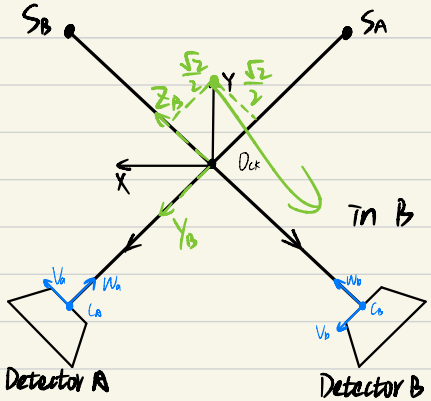
in B frame, $(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}+100)$ ✓

(4) test #3



$P=(0,1,0)$ in CK frame

in A frame, $(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}+100)$ ✓

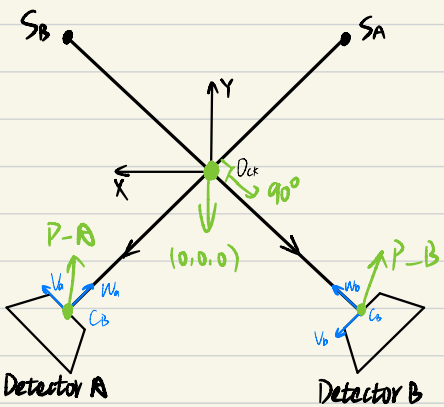


$P = (0, 1, 0)$ in CK frame

in B frame: $(0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} + 100)$ ✓

Q3: Marker Projection

(1) test #1: project (0,0,0) onto A and B



① Onto A: projected point on A is obviously C_A.

$$C_A\text{-in-CK}(50\sqrt{2}, -50\sqrt{2}, 0)$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 50\sqrt{2} \\ -50\sqrt{2} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

C_A-in-A ✓

② Onto B: projected point onto B is obviously C_B.

$$C_B\text{-in-CK}(-50\sqrt{2}, -50\sqrt{2}, 0)$$

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 100 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -50\sqrt{2} \\ -50\sqrt{2} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

C_A-in-B ✓

(2) Computation for test #2 and test #3 are too complicated (on paper). I left some codes in marker_projection.m to test its validity by calculating the sum of the squares of the two right-angled sides and the square of the hypotenuse, respectively. I didn't compare the values of those two numbers, since there are some small errors in the calculation (such as floating point calculation), which eventually lead to the failure of the comparison.

projection line

the distance of the detector and the source

the distance of the projected point and the center of the detector

Q4: Marker Reconstruction

According to the results of tests, REM are all (near) zero. Thus tests succeed.

Q5: Marker Correspondence

My program generates expected correspondence matrix.

Q6: Target Registration

My program generates expected results - the points in CT frame have the same coordinates as the points in CK frame after performing the transformation.

Q7: TRE Simulation - Discussion

The outcome is fragile and susceptible to marker segmentation errors. When I repeated the TRE simulation, the maximum allowable marker segmentation error is around 1. However, that value fluctuates widely, roughly ranging from 0.6 to 1.6.

I think it's related to the size of the screw. When there is some error, we humans may take it as an extremly small error, but because of the extremly small size of the screws, this could be one relatively large error relative to them.

At the same time, some technical limitations will also cause a large error, such as the magnification of one object in the system, the resolution of the detector, which will lead to some cases we can not clearly and accurately determine the location and size of the object, then ultimately results in the error.