Assignment 6B

- **Problem 1 (50 points): Laplace's Equation in a Box**

  An empty square box has conducting walls, 1 m each. All of the walls are grounded at 0 statvolts, except for the wall at the top, which is at voltage $V = 1$ statvolt.

  Note: The small gaps between the top wall and the others in the figure indicate that they are insulated from one another, but you can assume that these gaps have negligible width in your calculation.

  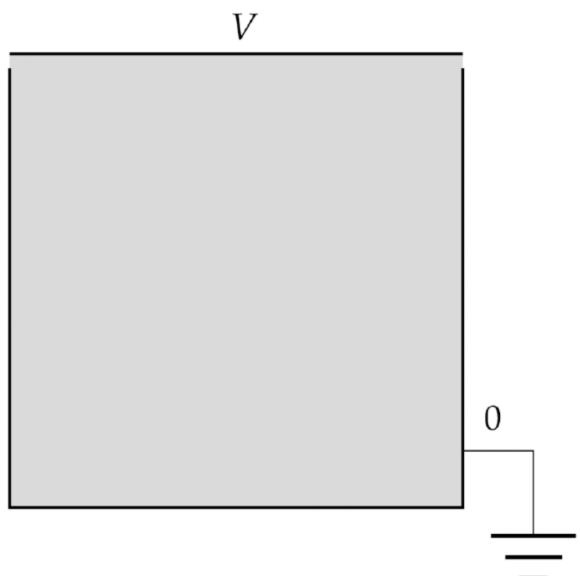

  Figure 1: Box with top wall at voltage V and other walls at 0 statvolts.

  For the two-dimensional case, solve Laplace's equation for the electrostatic potential ϕ, subject to boundary conditions $\phi = V$ on the top wall and $\phi = 0$ on the other walls.

  (a) Write a program to compute the solution to this problem ***using the combined overrelaxation/Gauss–Seidel method***.

  Use the following parameters: The grid spacing $a = 1$ cm, so there are 100 grid points on each side (101 if we count the points at both the beginning and the end).

  Continue iterating until the value of the electric potential changes by no more than $\delta = 10^{-6}$ statvolt at any grid point.

  (b) Experiment with different values of the overrelaxation parameter ω to find which value gives the fastest solution. Report the optimal value of ω among those you tried.

  Note: Larger values generally speed up convergence, but the calculation may become unstable if ω is too large.

  (c) Make a density plot of the final solution.

- **Problem 2 (50 points): Electrostatic Potential of a Capacitor**

  Consider a model of an electronic capacitor consisting of two flat metal plates enclosed in a square metal box. For simplicity, model the system in two dimensions.

  The walls of the box are at 0 statvolts, while the two capacitor plates (with negligible thickness) are at fixed potentials of +1 statvolt and −1 statvolt, respectively.

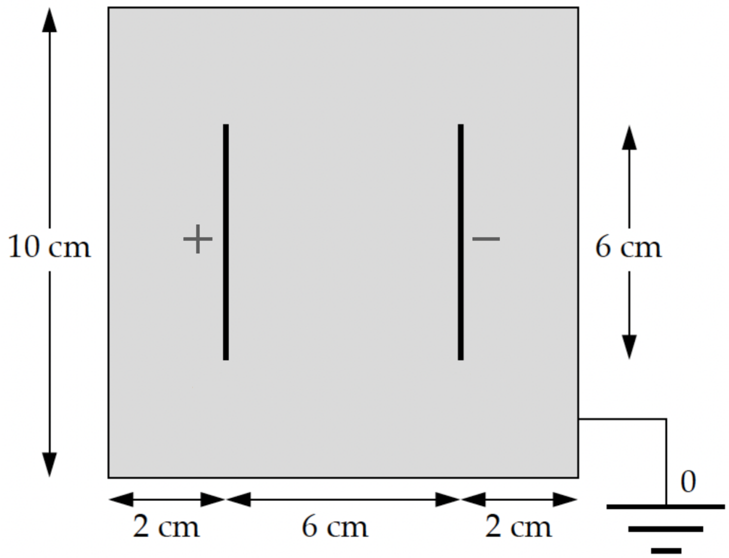

Figure 2: Capacitor plates inside a square box with 0 statvolt walls and fixed plate potentials of +1 and −1 statvolt.

For the two-dimensional case, solve Laplace's equation for the electrostatic potential ϕ with boundary conditions of $\phi = +1$ statvolt on the positive plate, $\phi = -1$ statvolt on the negative plate, and $\phi = 0$ on the walls.

(a) Write a program, or modify the previous one, to solve Laplace's equation for this problem on a grid of 100×100 points. Calculate the potential at each grid point to a precision of 10^{-6} statvolt.

(b) Make a density plot of the resulting potential.

Hint: Notice that the capacitor plates are at fixed potential, not fixed charge, so they are part of the boundary condition. Treat the plates in the same way as the walls of the box, with their potentials fixed at the specified values.