Assignment 7A

- **Problem 1 (100 points):** Thermal diffusion in the Earth's crust.

  A classic example of a diffusion problem with a time-varying boundary condition is the diffusion of heat into the crust of the Earth, as surface temperature varies with the seasons.

  In this assignment we are going to model this system by solving numerically the heat equation:

  $$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2} \,,$$

  and making the following physical assumptions:

  - Assume that the mean daily temperature at a particular point on the surface varies as:

    $$T_{\mathrm{surf}}(t) = A + B \sin\left(\frac{2\pi t}{\tau}\right) \,,$$

    where $\tau = 365$ days, $A = 10°\mathrm{C}$, and $B = 12°\mathrm{C}$.
  - Assume that at a depth of 20 m below the surface almost all annual temperature variation is ironed out, and the temperature is, to a good approximation, a constant 11°C (which is higher than the mean surface temperature of 10°C due to heating from the hot core of the planet).
  - The thermal diffusivity of the Earth's crust varies somewhat from place to place, but for our purposes, we will treat it as constant with a value $D = 0.1\ \mathrm{m}^2\ \mathrm{day}^{-1}$.

  Write a program to calculate the temperature profile of the crust as a function of depth up to 20 m and time up to 10 years by solving the heat equation using the Forward-Time Centered-Space (FTCS) method.

  The initial condition should be set such that the temperature varies linearly with depth from the average surface temperature to the deepest boundary temperature. That is, for an initial temperature profile $T(x)$ at depth x:

  $$T(x) = T_{\mathrm{surf}} + (T_{\mathrm{deep}} - T_{\mathrm{surf}}) \frac{x}{20} \,,$$

  where $T_{\mathrm{surf}} = A$ is the mean surface temperature, and $T_{\mathrm{deep}} = 11°\mathrm{C}$ is the constant temperature at a depth of 20 m.

  Then run your program for the first nine simulated years to allow it to settle down into whatever pattern it reaches. For the tenth and final year, plot four temperature profiles taken at 3-month intervals on a single graph to illustrate how the temperature changes as a function of depth and time.

  Hint: Choose appropriate values for the number of space grid points and the time-step h to ensure sufficient accuracy. In particular, take into account the von Neumann stability criteria discussed in class, but remember that it is a necessary condition for stability, not a sufficient condition for accuracy.