Assignment 8B

- **Problem 1 (50 points):** Mean Value Method for Integration

  Consider the integral:

$$I = \int_0^2 \sin^2\left[\frac{1}{x(2-x)}\right] dx\,. \tag{1}$$

  (a) (40 points) Estimate the value of this integral using the mean value method with $N = 10,000$ points.

  (b) (10 points) Evaluate the error in your estimation. Using the formula from the lecture notes, the standard error of the Monte Carlo estimate is given by:

$$\sigma_I \approx \frac{b-a}{\sqrt{N}}\sqrt{\langle f^2\rangle - \langle f\rangle^2}\,. \tag{2}$$

  Hint: The expectation values appearing at the numerator of the formula above are not known a priori, but you estimate them from the sample itself, using:

$$\langle f\rangle \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i)\,, \tag{3}$$

$$\langle f^2\rangle \approx \frac{1}{N}\sum_{i=1}^{N} f^2(x_i)\,. \tag{4}$$