Assignment 3B

- **Problem 1 (50 points): The Lotka–Volterra equations**

  The Lotka–Volterra equations are a mathematical model of predator–prey interactions between biological species. Let two variables x and y be proportional to the size of the populations of two species, traditionally called "rabbits" (the prey) and "foxes" (the predators). You could think of x and y as being the population in thousands, say, so that $x = 2$ means there are 2000 rabbits. Strictly the only allowed values of x and y would then be multiples of 0.001, since you can only have whole numbers of rabbits or foxes. But 0.001 is a pretty close spacing of values, so it's a decent approximation to treat x and y as continuous real numbers so long as neither gets very close to zero.

  In the Lotka–Volterra model, the rabbits reproduce at a rate proportional to their population, but are eaten by the foxes at a rate proportional to both their own population and the population of foxes:

  $$\frac{dx}{dt} = \alpha x - \beta xy \tag{1}$$

  where α and β are constants. At the same time, the foxes reproduce at a rate proportional to the rate at which they eat rabbits—because they need food to grow and reproduce—but also die of old age at a rate proportional to their own population:

  $$\frac{dy}{dt} = \gamma xy - \delta y \tag{2}$$

  where γ and δ are also constants.

  (a) Write a program to solve these equations using the 4th-Order Runge–Kutta method for the case $\alpha = 1$, $\beta = \gamma = 0.5$, and $\delta = 2$, starting from the initial condition $x = y = 2$. Have the program make a graph showing both x and y as a function of time on the same axes from $t = 0$ to $t = 30$.

  (b) Describe in words what is going on in the system, in terms of rabbits and foxes.

- **Problem 2 (50 points): The driven pendulum**

  A pendulum (like the one in lecture notes) can be driven by, for example, exerting a small oscillating force horizontally on the mass. Then the equation of motion for the pendulum becomes

  $$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta + C\cos\theta\sin\Omega t \tag{3}$$

  where C and Ω are constants.

  (a) Write a program to solve this equation for θ as a function of time with $l = 10\,\mathrm{cm}$, $C = 2\,\mathrm{s}^{-2}$, and $\Omega = 5\,\mathrm{s}^{-1}$ and make a plot of θ as a function of time from $t = 0$ to $t = 100\,\mathrm{s}$. Start the pendulum at rest with $\theta = 0$ and $d\theta/dt = 0$. Using the 4th-Order Runge–Kutta is a better choice.

  (b) Now change the value of Ω, while keeping C the same, to find a value for which the pendulum resonates with the driving force and swings widely from side to side. Make a plot for this case also.