

Assignment 6A

- **Problem 1 (50 points): Laplace's Equation in a Box**

An empty square box has conducting walls, 1 m each. All of the walls are grounded at 0 statvolts, except for the wall at the top, which is at voltage $V = 1$ statvolt (in the Gauss unit system 1 statvolt corresponds to ≈ 299.79 volts in SI units).

Note: The small gaps between the top wall and the others in the figure indicates that they are insulated from one another, but you can assume that these gaps have negligible width in your calculation.

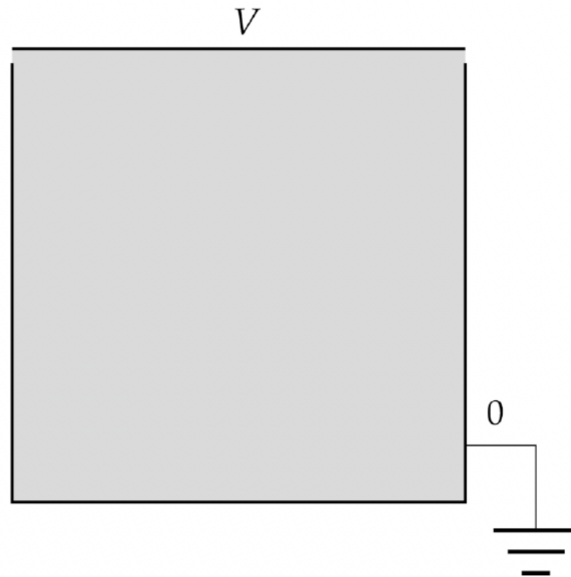


Figure 1: Box with top wall at voltage V and other walls at 0 statvolts.

For the two-dimensional case, solve Laplace's equation for the electrostatic potential ϕ , subject to boundary conditions $\phi = V$ on the top wall and $\phi = 0$ on the other walls.

- (a) Write a program to compute the solution to the two-dimensional electrostatics problem using the Jacobi method.

Use the following parameters: The grid spacing $a = 1$ cm, so there are 100 grid points on each side (101 if we count the points at both the beginning and the end).

- (b) Make a density plot of the result.

Hint: This program may take a while to run, so start with a sparse grid to ensure your program is functioning correctly. Once confirmed, use the tighter grid specified above.

• **Problem 2 (50 points): Poisson's Equation with Square Charges**

Two square charges are placed inside a square two-dimensional box. The potential is zero at the walls, and the charges are defined such that $4\pi\rho = +1$ and $4\pi\rho = -1$ (surface charge density in statcoulomb/cm², where 1 statcoulomb $\approx 3.33564 \times 10^{-10}$ coulombs in SI) at the locations of the positive and negative charges, respectively.

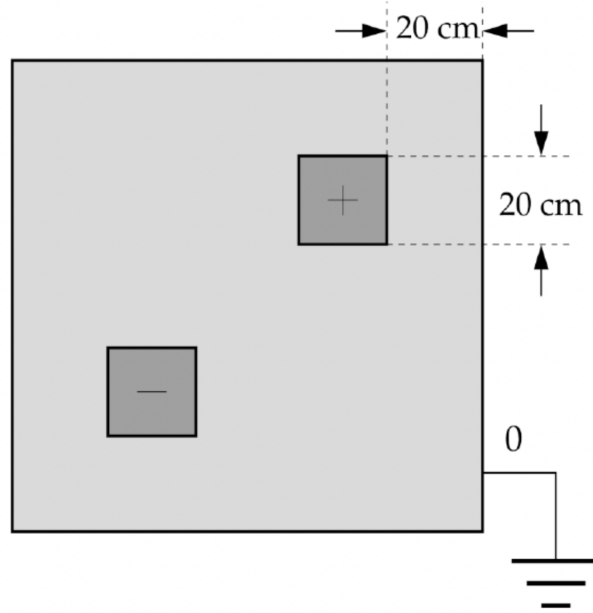


Figure 2: Two square charges inside a box with zero potential at the walls.

- (a) Write a program, or modify the previous one, to solve Poisson's equation using Jacobi's method, in the presence of the charge density specified above.

Continue iterating until the solution for the electric potential changes by less than 10^{-6} statvolt per step at every grid point.

Note: In the Gauss system of units the Poisson's equation is written as:

$$\nabla^2\phi = -4\pi\rho$$

- (b) Make a density plot of the result.