

Modelling of Shallow Foundations for Offshore Structures

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Introduction and Applications

This paper concerns the numerical modelling of shallow circular foundations. A summary of recent work in this area at Oxford University is presented. For design purposes it is almost always necessary to devise a numerical model of foundation behaviour, however simple that might be, and the principal focus of this paper is on appropriate numerical models for modern design methods. The basic principles of the models, which are based on work-hardening plasticity theory, are described, and some problems and pitfalls discussed. Future areas of development are mentioned, and example calculations are given to illustrate the application of the models to offshore foundations. Such models must, however, be validated, and the main means of doing this is by carefully controlled laboratory tests, so this paper makes extensive reference physical modelling, although there is not space to describe the details.

The motivation for this work comes principally from the offshore oil and gas industry, where a number of shallow foundation types are employed that can reasonably be approximated as circular footings (Figure 1). The spudcan foundations of a jack-up are typically shallow cones, 20m or more in diameter for a large jack-up. In firm soils they rest on the surface, but in soft clays can penetrate deeply into the seabed (say by 50m in some circumstances). The multicellular foundations of large gravity bases are much bigger, say 120m across, and often with concrete skirts cutting 15m or more into the seabed. The overall plan of the cellular structure is often roughly circular. Finally suction caisson foundations, which have been used for a small number of jacket structures, are large circular structures, say 12m to 20m in diameter, embedded by perhaps half their diameter. Each of these structural types may be treated as a circular foundation, subjected to cyclic horizontal forces and overturning moments from wind and waves. Although embedded by a fraction of a diameter, they are essentially shallow foundations in which the foundation itself is of high rigidity compared to the soil.

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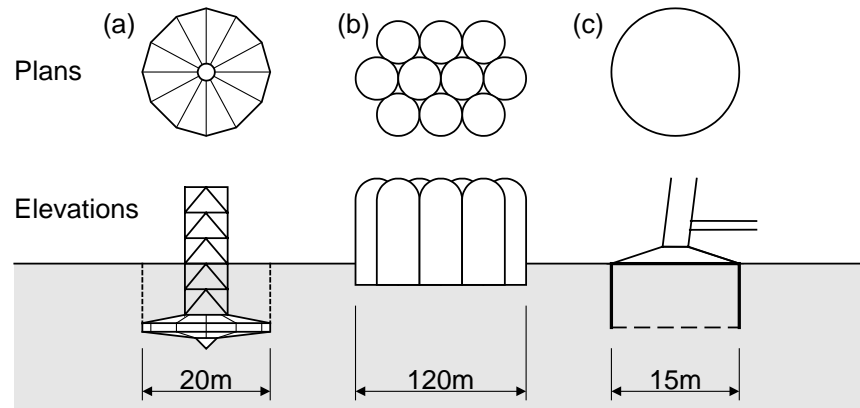


Figure 1: Shallow foundations for offshore structures (a) spudcan foundation of jack-up, (b) gravity base, (c) suction caisson

A more recent motivation comes from the emerging offshore wind power sector, in which suction caissons are being considered as an alternative to piled foundations. Two main alternatives are possible (Figure 2): a monopod foundation, which might be about 25m in diameter for a 3MW turbine, or a **quadruped foundation**, with each caisson perhaps 5m in diameter. The main novelty in the wind power case is that (compared to oil and gas installations) the vertical loads imposed by the structure are very low, but the horizontal load and overturning moment are much larger by comparison with the vertical load (Houlsby and Byrne, 2000, Byrne *et al.*, 2002).

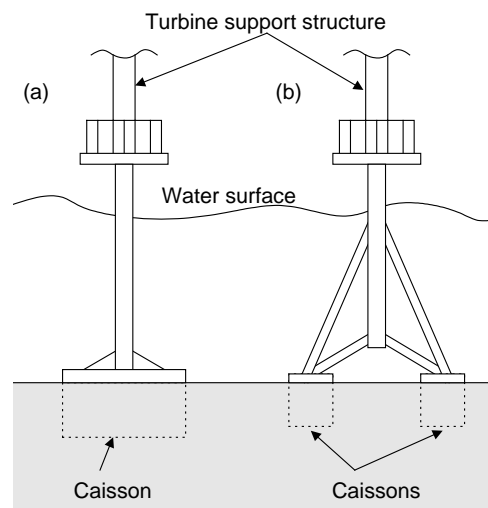


Figure2: Caisson foundations for wind turbines

Problem Definition

We shall assume that the foundation has already been designed, and the purpose here is to study the interaction between the structure and foundation. This is important certainly for jack-up structures and for the wind turbine applications

since these are dynamically sensitive structures in which the response depends critically on the stiffness of the structure/foundation system. Steel jackets and gravity bases tend to be less sensitive to dynamics since they are usually much stiffer, so they will be of less interest here.

The challenge is to provide a realistic modelling of the foundation, under all probable applied loads, so that it can be incorporated in a structural analysis. In practice this means creating a numerical model of the foundation for use in finite element analysis, since this is the way the structure is modelled under dynamic loads. In the past this has been achieved by making some gross simplifying assumptions: for instance the foundation may be assumed to be completely rigid (usually an unsafe assumption), or the spudcans of a jack-up may be considered as pinned to the seabed (an over-conservative assumption).

The first improvement is to treat the footing as a rigid circular foundation bearing on an elastic soil, and employ standard solutions to define stiffness factors relating the forces on the footing (Figure 3) to the corresponding displacements. Such an approach represents a considerable advance, and can be employed in finite element analyses in either the time or frequency domain. However, soil is not a linear elastic material, and modelling it as such is unrealistic, particularly under

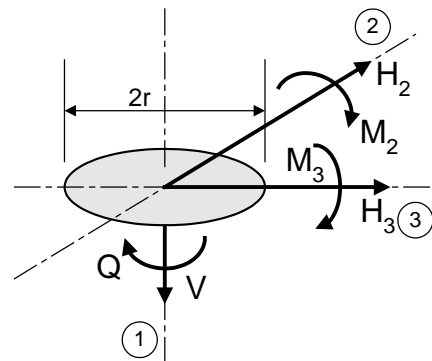


Figure 3: Forces on a circular foundation

extreme load conditions. This paper is therefore concerned with the next level of sophistication, in which the foundation itself is still treated as rigid, but the interaction with the soil is expressed in terms of the non-linear relationships between the force resultants applied to the foundation (Figure 3), and the corresponding displacements. Such “force resultant” models, which capture the non-linearities of the soil behaviour, play an important role in modelling soil structure interaction. They are usually expressed within the framework of plasticity theory, which is consistent with the way the structure is modelled. Because they involve non-linear behaviour, the analyses can no longer be carried out in the frequency domain, and full time domain analyses become necessary.

In the following we consider the key components of such models, their justification, limitations and some of the pitfalls that can be encountered. Finally we examine some future developments and give example applications of force resultant models. We do not consider here a further level of sophistication, which will be necessary in some cases, in which the soil and foundation are each discretised in a finite element analysis and a “complete” modelling of the

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problem is attempted. Although now possible, such analyses are too time consuming for routine work, and would only be employed in rather exceptional circumstances.

Force Resultant Models

The force resultant models are based on four components: a yield surface, hardening law, flow rule and elastic behaviour inside the yield surface. In developing expressions for each component, use is made of both theoretical solutions and of purely empirical data obtained in model tests. Some factors (especially relating to stiffness) are calibrated with reference to field data.

Yield surface

All geotechnical engineers are familiar with bearing capacity analysis, in which a rigid foundation “fails” at a certain vertical load. A slightly more sophisticated understanding of the problem is that, below the bearing capacity value, the foundation undergoes purely elastic (recoverable) deformation, whilst if the capacity is reached then plastic (permanent) displacement will occur. The bearing capacity value is therefore a “yield point” in plasticity theory.

If vertical load, horizontal loads and moments are applied to the foundation in some proportion, there will again be some value of load (a yield point) below which the behaviour is essentially elastic, and at which there will be the onset of plastic displacements. Traditionally this has been approached in geotechnical engineering by applying a series of reduction factors to the vertical bearing capacity, depending on the inclination and eccentricity of the load (accounting for horizontal load and moment respectively). This approach is epitomised by the work of Hansen (1970) and Vesic (1975). A more fruitful approach, which has its origins in the work of Roscoe and Schofield (1957) and later Butterfield and Tico (1979), is to treat the yield points as defining a yield surface in load space. The surface encloses the combinations of loads which would cause only elastic displacements. The typical form of such a surface for planar loading only (*i.e.* $H_2 = Q = M_3 = 0$ in Figure 3) is shown in Figure 4. The apex of the surface on the V -axis represents the bearing capacity under pure vertical loading. It is empirically found that sections of the surface including the V -axis are approximately parabolic, and there is some theoretical justification for this. Sections normal to the V -axis are approximately elliptical. The equation of the surface can therefore be written (for in-plane loading):

$$f = \left(\frac{H_3}{h_0 V_0} \right)^2 + \left(\frac{M_2}{2Rm_0 V_0} \right)^2 - 2a \left(\frac{H_3}{h_0 V_0} \right) \left(\frac{M_2}{2Rm_0 V_0} \right) - 4 \left(\frac{V}{V_0} \right)^2 \left(1 - \frac{V}{V_0} \right)^2 = 0$$

The third term in this expression (involving the coefficient a) is important in that it allows the elliptical section to be rotated with respect to the H_3 and M_2 axes (Butterfield and Gottardi, 1994, Martin, 1994). For general loading this expression may be extended to:

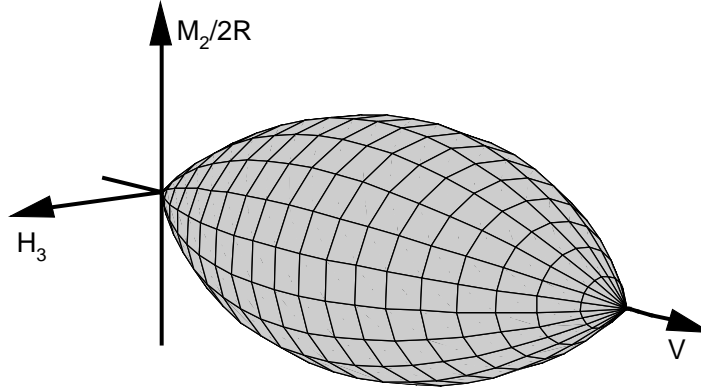


Figure 4: Yield surface for in-plane loading

$$f = h_2^2 + h_3^2 + q^2 + m_2^2 + m_3^2 + 2a(h_2m_3 - h_3m_2) - \beta_{12}^2 S (v^2)^{\beta_1} ((1-v)^2)^{\beta_2} = 0$$
 where $S = \text{sgn}[v_1(1-v_2)]$ is introduced to ensure consistency of the sign of the yield function, $\beta_{12} = (\beta_1 + \beta_2)^{(\beta_1 + \beta_2)} \beta_1^{-\beta_1} \beta_2^{-\beta_2}$, $v = V/V_0$, $h_2 = H_2/h_0V_0$, $q = Q/2Rq_0V_0$, $m_2 = M_2/2Rm_0V_0$ etc. The factors β_1 and β_2 (each close to, but just less than 1.0) are introduced to “round off” the pointed ends of the yield surface on the V -axis.

The appropriateness of the form of the above yield surface for planar loading has been confirmed by many tests carried out in a sophisticated computer-controlled loading rig at Oxford University, for a number of different footing geometries (including spudcans and caissons) and several soil types (soft clay, dense and loose sand). The main tests on clay are reported by Martin (1994) and Martin and Houlsby (2000). Those on sand are reported by Gottardi *et al.* (1999) and Byrne and Houlsby (2001). Tests involving combined vertical load and torsion have also been completed. This work is currently being extended to the full 6 degree-of-freedom

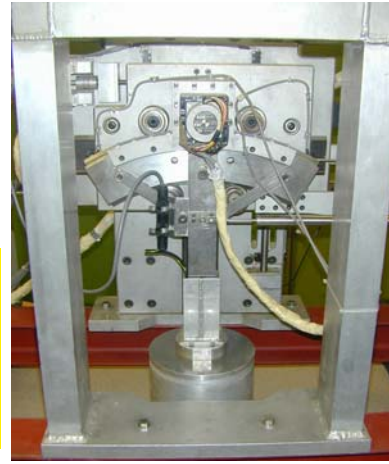


Figure 5: Computer-controlled loading rig

loading with a new, more versatile, apparatus. The principal means of verifying the yield surface shape is by “swipe tests” in which the footing position is locked vertically and the foundation rotated or translated sideways. Under these

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conditions the load point traverses across the yield surface, and relatively few tests are required to define its shape.

Confirmation of the overall shape of the yield surface has also been obtained by theoretical analysis (*e.g.* Houslsby and Puzrin, 1999) and by finite element analysis (*e.g.* Ngo Tran, 1996, Taiebat and Carter, 2000).

Hardening Law

In bearing capacity theory, the capacity of the foundation is regarded as fixed, regardless of the deformation of the foundation (although see the discussion below). In reality, once plastic deformation occurs and the foundation is pushed further into the ground, the capacity of the foundation increases. Thus the size of the yield surface is not fixed, but increases as plastic deformation occurs. It is reasonable to link this increase only to further embedment, and not for instance to horizontal movement, since there is no reason to suppose that a horizontal movement increases the capacity. In simple force resultant models the size of the yield surface is therefore simply a function of the vertical plastic strain.

Experimental evidence is that this is satisfactory for most purposes, although in some cases there is evidence of a dependence on other plastic displacements (Cassidy, Byrne and Houslsby, 2002). The yield surface can also change in shape with vertical displacement (Byrne, 2000), although it is at present usual to neglect this effect.

The N_q term in the bearing capacity equation can be interpreted as giving an indication of the increase in bearing capacity as a foundation is pushed into the ground, since (for a cohesionless soil) one could adapt the equation $q = \frac{1}{2}\gamma BN_\gamma + \gamma DN_q$ to imply a linear increase of vertical bearing capacity with depth. In the terminology used above, with w_p as the vertical plastic penetration, this can be written $V_0 = \pi R^2(\gamma RN_\gamma + \gamma N_q w_p)$. It should be noted though that this goes beyond the original intention of the bearing capacity formula, which was purely for the ultimate capacity of a foundation installed at a particular depth.

In practice models can be implemented either by constructing (w_p, V_0) curves from bearing capacity theory (Martin 1994), or by using empirical fits to observed data, for instance the curve which was used by Cassidy (1999) to describe the behaviour of a foundation on sand:

$$\frac{V_0}{V_{0m}} = \frac{(1-f_p)(kw_p/V_{0m}) + f_p(w_p/w_{pm})^2}{(1-f_p)(1 - (2 - kw_p/V_{0m})(w_p/w_{pm})) + (w_p/w_{pm})^2}$$

where k , w_{pm} , V_{0m} and f_p are constants. This provides an excellent fit to empirical data, see Figure 6.

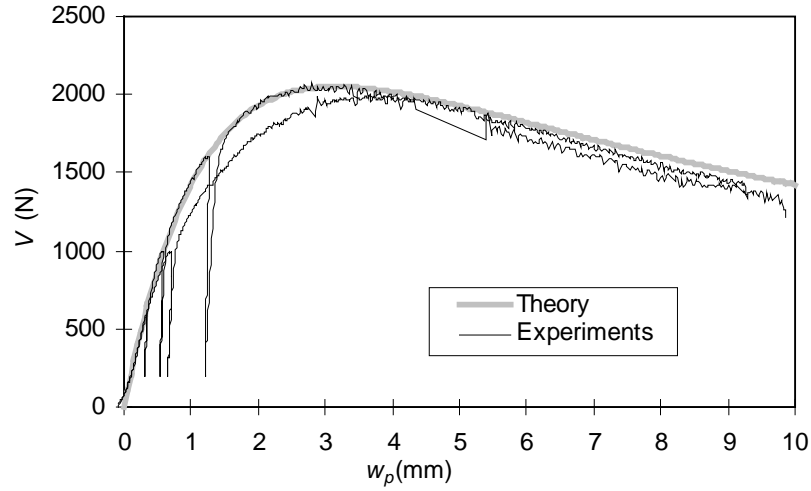


Figure 6: Comparison between empirical fit to load-displacement curve and experimental data

Flow Rule

When yield occurs, the hardening rule determines the stiffness of the response, but under multiaxial loading it is the flow rule that determines the ratios between the (plastic) displacement components. The simplest form of flow is “associated flow” in which the yield surface acts also as a “plastic potential” and (in the planar loading case):

$$dw_p = \lambda \frac{\partial f}{\partial V}, \quad du_{3p} = \lambda \frac{\partial f}{\partial H_3}, \quad d\theta_{2p} = \lambda \frac{\partial f}{\partial M_2}$$

where λ is a factor that is determined by the hardening rule.

Although associated flow offers an attractive simplicity, in that no additional relationships need to be specified, unfortunately it is insufficiently realistic to be of practical use. Associated flow does give a reasonable prediction of the ratios between horizontal movement and rotation, but it gives a very poor prediction of vertical movements.

Martin (1994) introduced a simple modification which is satisfactory for foundations on clay. The vertical displacement is adjusted by a factor $0 < \zeta \leq 1$, so that we now have $dw_p = \zeta \lambda \partial f / \partial V$, while the other equations remain unaltered. A value $\zeta \approx 0.6$ gives realistic modelling.

However, Cassidy (1999) found that the picture for sands was rather more complex. He introduced instead a plastic potential which was in the same mathematical form as the yield surface, but with different constants. Even this was unsatisfactory in fitting all the data, and he found it necessary to introduce

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further relationships to alter the shape of the plastic potential, depending on the past history of movement. Such a development leads to rather complex models, and it is hoped that it can in due course be superseded by the use of multiple yield surfaces, which is discussed below.

Elastic Behaviour

Within the yield surface it is assumed (at least for the time being) that the behaviour is elastic. For the 6 degree-of-freedom problem the elastic relationship between the loads and the corresponding displacements can be expressed in the following form:

$$\begin{bmatrix} V/4GR^2 \\ H_2/4GR^2 \\ H_3/4GR^2 \\ Q/8GR^3 \\ M_2/8GR^3 \\ M_3/8GR^3 \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_3 & 0 & 0 & 0 & -k_4 \\ 0 & 0 & k_3 & 0 & k_4 & 0 \\ 0 & 0 & 0 & k_5 & 0 & 0 \\ 0 & 0 & k_4 & 0 & k_2 & 0 \\ 0 & -k_4 & 0 & 0 & 0 & k_2 \end{bmatrix} \begin{bmatrix} w/2R \\ u_2/2R \\ u_3/2R \\ \omega \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

where k_1 to k_5 are dimensionless factors which depend on the geometry of the foundation and on Poisson's ratio. Theoretical values for these factors may be derived for certain simple cases, but for more general cases they are best determined by finite element analysis (Bell, 1991, Ngo Tran 1996, Doherty and Deeks 2002). It is very important to note the role of the constant k_4 , as this introduces a coupling between the horizontal and moment terms. In other words a pure horizontal load causes some rotation of the foundation as well as horizontal movement, and a pure moment causes horizontal movement as well as rotation. These phenomena can significantly affect the behaviour of the foundation, and should not be ignored; and yet (for instance) the API recommendations for offshore foundation design make no mention of them.

Complete Models

Complete models based on the above principles include "Model B" (for footings on clay), Martin (1994), Martin and Housby (2001) and "Model C" for sands, Cassidy (1999), Housby and Cassidy (2002), Cassidy, Byrne and Housby (2002).

Problems and Pitfalls

Load Reference Point

When the model for the foundation is connected to the rest of the structure in a finite element analysis, great care is required in the definition of the "Load Reference Point" (LRP), the point at which the loads are considered to act on the foundation. For instance if the LRP is moved upwards a distance d from

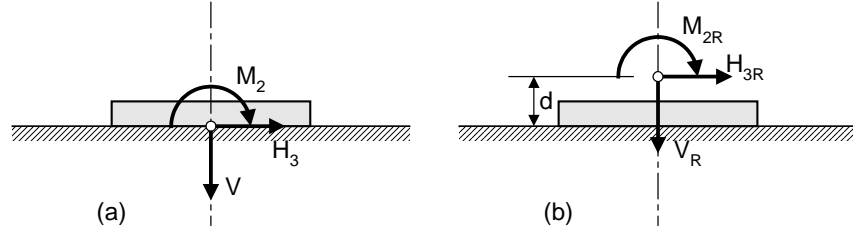


Figure 7: Load reference point

mudline level, as shown in Figure 7(b) as compared to 7(a), then for the two sets of loads to be statically equivalent we have $V_R = V$, $H_{3R} = H_3$, but $M_{2R} = M_2 - dH_{3R}$. Conversely the equivalent displacements are $w_R = w$, $u_{3R} = u_3 + d\theta_2$ and $\theta_{3R} = \theta_3$. Thus movement of the LRP changes the moment values and the horizontal displacements!

Consequences of these changes are:

- The values of k_2 , k_3 and k_4 in the elasticity expressions change. There is a particular value of d for which $k_4 = 0$ and the horizontal movements and rotations are decoupled. This position of the LRP is referred to as the elastic metacentre.
- The values of h_0 , m_0 and a in the yield surface expression change. Again there is a particular value of d for which $a = 0$, and the elliptical sections of the yield surface are symmetric about the H and M -axes. This again represents a type of decoupling of horizontal movement and rotation. This LRP is referred to as the plastic metacentre. There is no reason why the elastic and plastic metacentres should coincide.

Finally note that in some analyses (e.g. of jack-ups in soft soil) there may be large movements of the foundation with respect to mudline level, and in these cases particular care is needed in the definition of the LRP.

Elasticity and Cyclic Loading

The treatment of the behaviour within the yield surface as elastic can at best be seen as an interim measure until better approaches are available. Soil cannot be regarded as linear elastic except at the very smallest of strains, and this is in turn reflected in foundation behaviour. Tests on foundations which have been unloaded to well within the yield surface show significant amounts of nonlinearity, especially under horizontal and moment loading. At best the yield surface can only be treated as a boundary between load states for which the plastic deformations are relatively small, and those for which the plastic deformation dominates.

In many offshore problems, the dominant loading is cyclic in nature, and (except for a few extreme events) will involve loading within the yield surface. If realistic modelling of the system stiffness and damping is to be achieved, then

the nonlinearity within the yield surface must be modelled. Several approaches have been suggested in the past, but the two main alternatives are (a) boundary surface models and (b) multiple yield surfaces. Although boundary surface models are attractive in some respects, they are incapable of capturing some of the effects of past history on the stiffness of the foundation, and so the preferred approach is use of multiple yield surfaces. Experimental evidence (Byrne, 2000) suggests that models based on this concept could capture most of the features of cyclic loading.

Although preliminary multi-surface models have been developed, and an example is given below, the most important developments in the near future are likely to be more rigorously calibrated multiple surface plasticity models to represent the nonlinearity at low loads. This will allow more accurate modelling of stiffness and damping at serviceability loads. This is important, both in estimating the displacements of dynamically sensitive structures, and in fatigue analyses. Work is in progress applying the “continuous hyperplasticity” approach (Puzrin and Housby, 2001a,b), which allows a compact mathematical representation of models with an infinite number of yield surfaces: thus modelling smooth changes of stiffness as loading amplitude increases.

Rate Effects

One might reasonably expect that most practical loading cases on clay soils would be **close to undrained conditions**, but on sands one would expect partial drainage to be significant. Estimates show that typical loading periods for foundations on sand (mainly related to the period of large waves) may be broadly comparable with t_{50} drainage times for excess pore water pressures.

The experimental evidence (Mangal, 1999, Mangal and Housby, 1999, Byrne, 2000) is, however, that across a remarkably **large range of loading rates there is little difference in the performance of the foundation under combined loads**. There are some minor rate effects, which tend to increase the capacity by a few percent as the loading period changes from much longer than t_{50} to much shorter, but there is no dramatic change of response. **A full explanation of the unexpected lack of sensitivity to loading rate is not yet possible, but it is probably related (at least for dense sands) to relatively small excess pore pressures being caused, even under undrained conditions.**

Special cases

Although the force resultant models are able to capture many of the features of the behaviour of foundations under combined loads, it is important to realise their fundamental limitations. Because no attention is paid to modelling the detail of soil response, but this is lumped together into a “smeared” model, there are inevitably features of foundation behaviour that cannot be modelled. For instance, it is clearly not possible to capture effects such as scour. However,

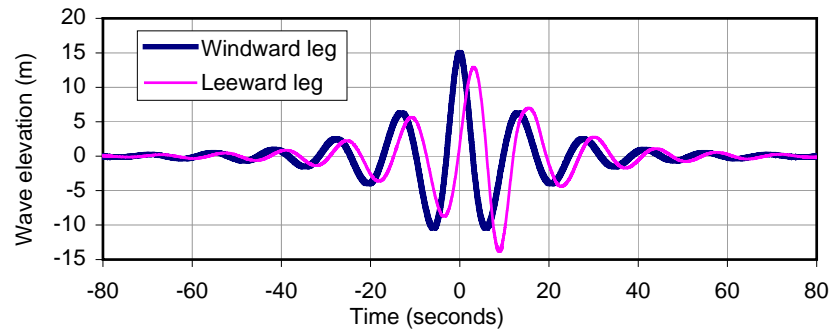


Figure 8: Time history of loading on legs of jack-up

some special aspects can be included in the models, and work is in progress on extending the models in certain areas.

Of particular interest at present are the special features of caisson foundations at low vertical loads (Byrne *et al.* 2003) and in particular the complexities of the behaviour at the tension/compression transition (see Byrne, 2000, Byrne and Houlsby, 2002). Both these issues are important in the application of caissons as foundations for offshore wind turbines.

Example Analyses

Models of the type described above have been used to describe the response of the foundations of jack-up units in complete static analyses by Martin (1994), and under dynamic conditions by Thompson (1996), Williams *et al.* (1998, 1999), Cassidy (1999). Back analyses of case records of jack-ups on both clay and sand have been carried out to calibrate the most important parameter in the model, which turns out to be the chosen value of the shear modulus, G , Cassidy, Houlsby, Hoyle and Marcom (2002). By including modelling of the foundation in a unified way with the analysis of the complete unit, studies have been possible of the relative importance of assumptions made about the structure, foundation and wave loading, leading to an assessment of those parameters which most affect the reliability of jack-up units, Cassidy *et al.* (2001a,b), Cassidy, Taylor *et al.* (2002).

In the following we consider an example of a dynamic analysis of a jack-up unit subjected to a “NewWave” loading. In-plane loading only is considered, and for this simple example there is just one upwind and one downwind leg of the jack-up. The time history of the wave elevation on both the upwind and downwind legs is shown in Figure 8, where it can be seen that the wave, focussed on the upwind leg, arrives a little later and attenuated at the downwind leg. This induces a complex dynamic response within the jack-up, resulting in the load paths shown in Figure 9 on the windward and leeward legs. As moment

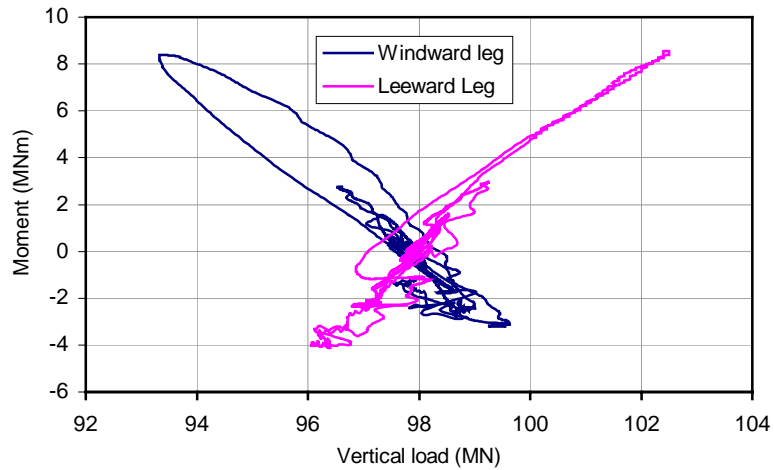


Figure 9: Load paths followed by windward and leeward footings of jack-up

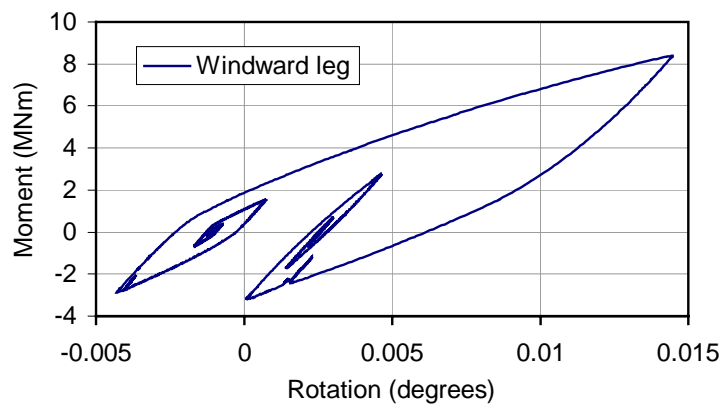


Figure 10: Moment rotation response of jack-up foundation

is applied to the foundations the vertical load on the windward leg reduces and that on the leeward leg increases.

The foundation is in this case modelled by a preliminary version of a multiple yield surface model, so it exhibits nonlinearity at small displacements. The moment-rotation response is shown in Figure 10, where it can be seen that non-linear behaviour is shown even at small moments. When the first small waves arrive at the jack-up the displacements are mainly (but not entirely) recoverable, but when the large NewWave passes through there is a permanent offset in the rotation of the spudcan.

The result in terms of horizontal movement of the deck of the jack-up is shown in Figure 11, although the permanent rotation of the foundation has

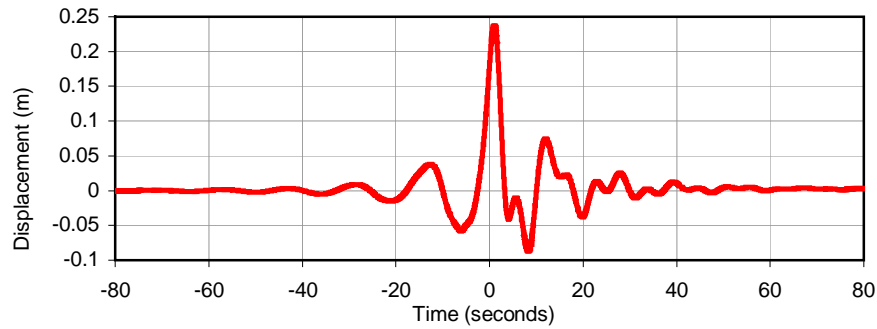


Figure 11: Horizontal displacement of jack-up deck

resulted in a small permanent offset in the deck position, this is not perceptible in the figure.

The second example concerns the use of full 6-degree-of-freedom analysis. A horizontal load is first applied to a circular foundation in the H_2 direction. The load is applied above the foundation level so that it also causes a moment M_3 . A second horizontal is now applied, and we consider four different directions of the second load, ranging from parallel to the first load component to perpendicular to it, see cases A, B, C and D in Figure 12. The H_3 component of this second load also causes a (negative) moment M_2 . Such a type of loading could occur, for instance, when the directions of the wind and waves acting on a structure were not coincident. The loadings applied are typical of those that might be applied to a large wind turbine foundation. Figure 13 shows the moment-rotation response in terms of (θ_2, M_2) , demonstrating that the

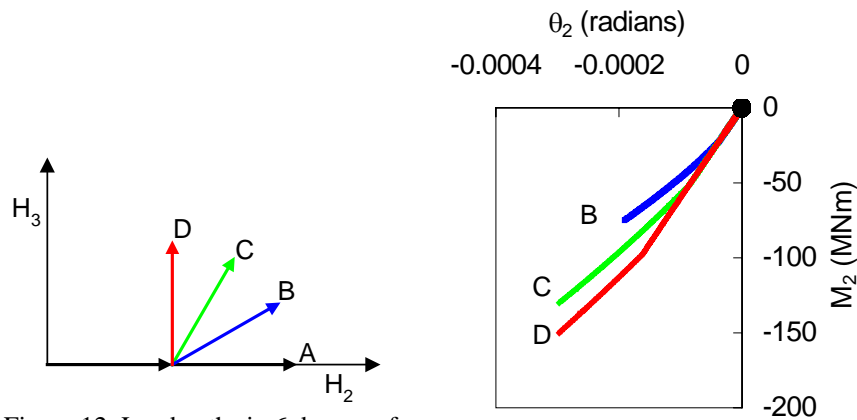


Figure 12: Load paths in 6 degree-of-freedom analysis

Figure 13: Moment rotation relationship

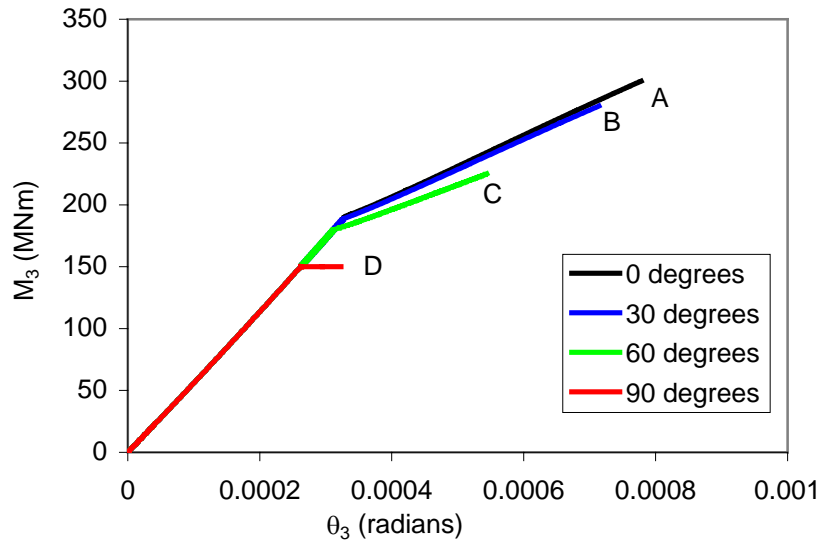


Figure 14: Moment-rotation relationship

more flexible response occurs for the cases where the new loading is approximately parallel to the original load. Figure 14 shows the equivalent moment-rotation response in terms of (θ_3, M_3) . In this plot the stiffest behaviour is for the load path approximately parallel to the original load. This sort of complexity of behaviour could only be predicted by a relatively sophisticated model of this type.

Conclusions

Much progress has been made in the numerical modelling of shallow foundations on clay for the offshore industry. Force resultant models can be combined with structural analyses to predict the behaviour of the entire structure/foundation system under dynamic loading conditions imposed by waves. Further work is required, particularly on the modelling of realistic behaviour under cyclic loading, and on special applications such as caissons for offshore wind turbine applications.

Acknowledgements

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