

There Is No Largest Prime Number

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27th International Symposium of Prime Numbers

OUTLINE

1 MOTIVATION

- The Basic Problem That We Studies

WHAT ARE PRIME NUMBERS?

DEFINITION

A **prime number** is a number that has exactly two divisors.

EXAMPLE

- 2 is prime (two divisors: 1 and 2).

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- 3 is prime (two divisors: 1 and 3).

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- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (**three** divisors: 1, 2, and 4).

THERE IS NO LARGEST PRIME NUMBER

THE PROOF USES *reductio ad absurdum*.

PROOF.

- ① Suppose p were the largest prime number.
- ② But $p + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers. □

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WHAT'S STILL TO DO?

ANSWERED QUESTIONS

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Is every even number the sum of two prime?

WHATS STILL TO DO?

- Answered Questions
 - How many primes are there?
- Open Questions
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ANSWERED QUESTIONS

How many primes are there?

OPEN QUESTIONS

Is every even number the sum of two primes?

OPEN QUESTIONS

Is every even number the sum of two primes?[1]

AN ALGORITHM FOR FINDING PRIME NUMBERS.

```
int main (void)
{
    std::vector<bool> is_prime (100, true);
    for (int i = 2; i < 100; i++)
        if (is_prime[i])
        {
            std::cout << i << " ";
            for (int j = i; j < 100; is_prime [j] = false, j+=i);
        }
    return 0;
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[Goldbach, 1742] Christian Goldbach.

A problem we should try to solve before the ISPN 43 deadline,
Letter to Leonhard Euler, 1742.