COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

Andrew McGregor

Lecture 9

LOCALITY SENSITIVE HASHING

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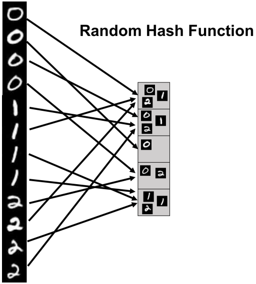

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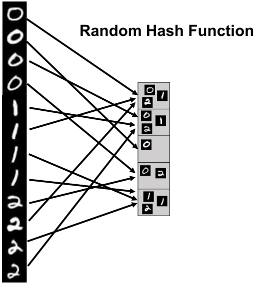

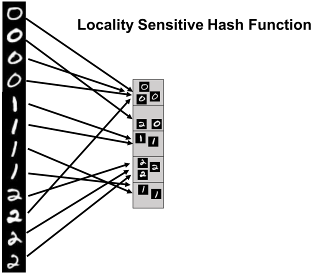

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Potential for a lot of false positives! Slows down search time.

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We want to balance a small probability of false negatives (a high hit rate) with a small probability of false positives (a small query time.)

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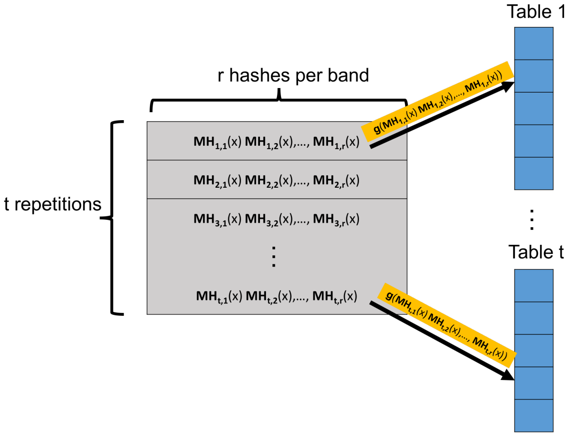

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

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$$\text{Hit Probability: } 1 - (1 - s^r)^t.$$

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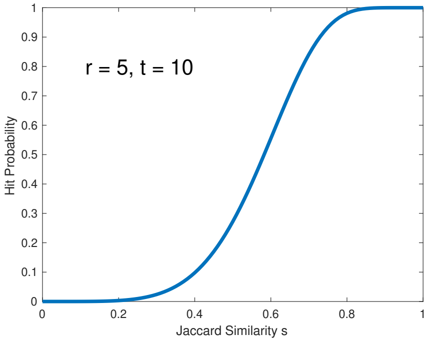

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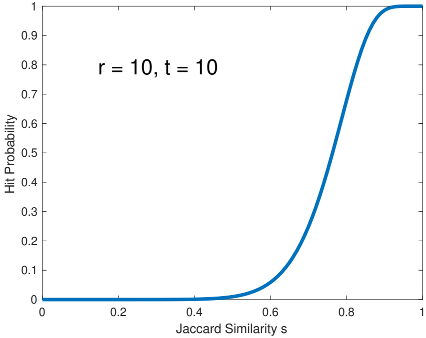

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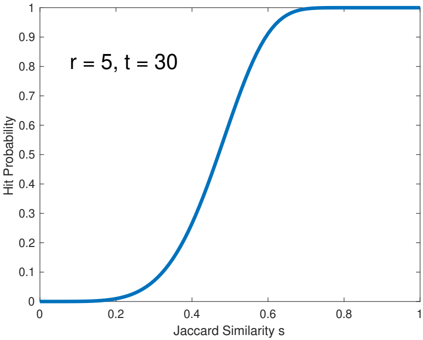

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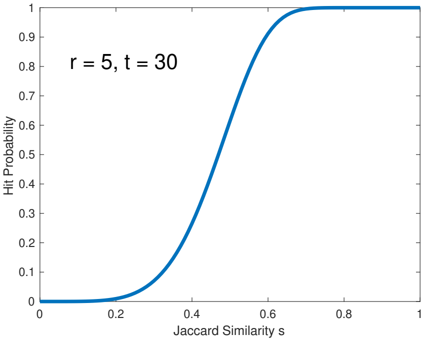

r and t are tuned depending on application. 'Threshold' when hit probability is $1/2$ is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

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GENERALIZING LOCALITY SENSITIVE HASHING

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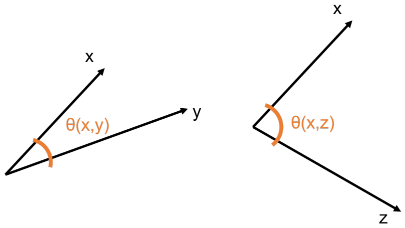

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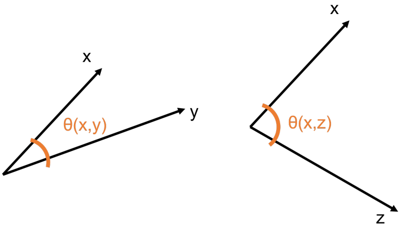

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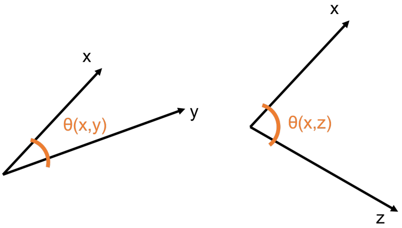

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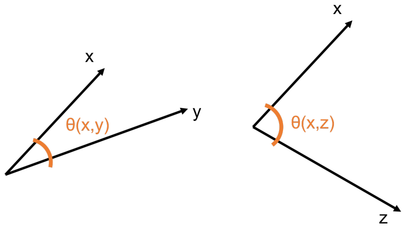

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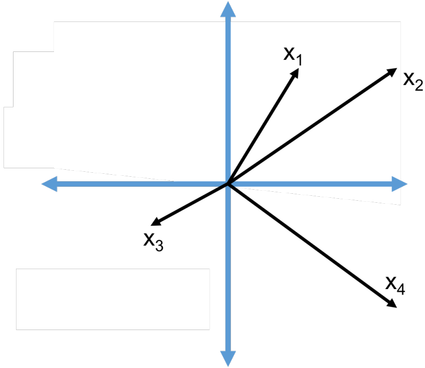

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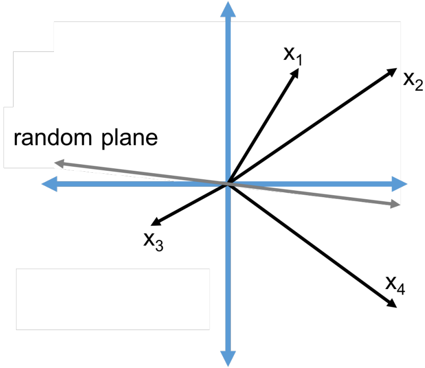

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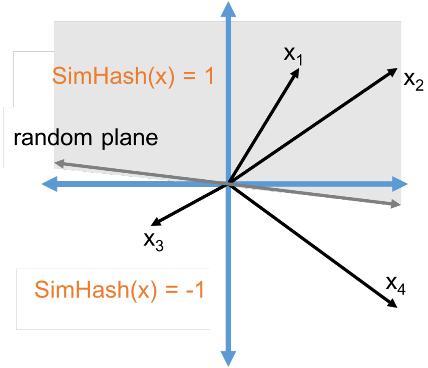

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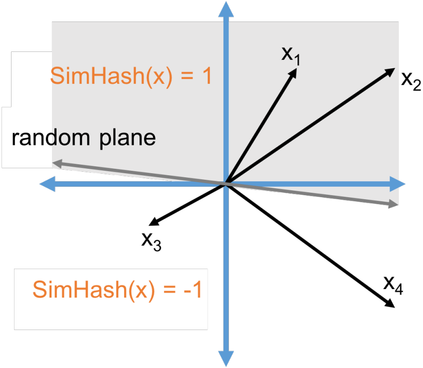

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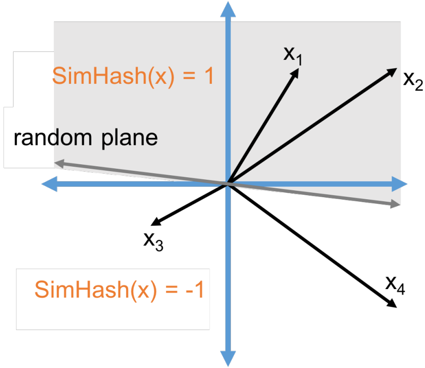

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SIMHASH FOR COSINE SIMILARITY

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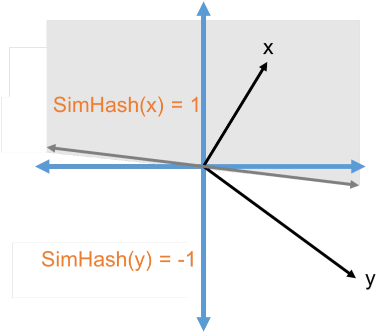

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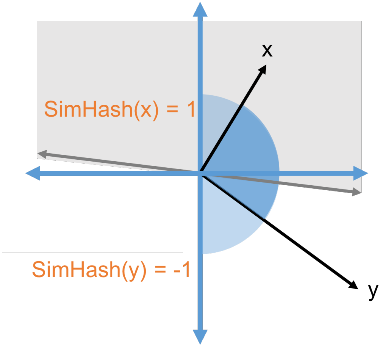

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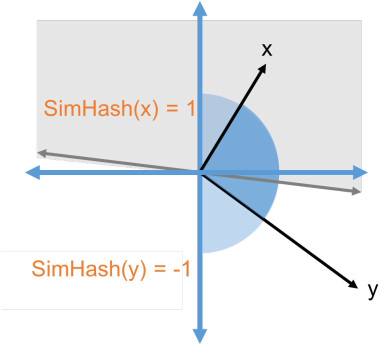

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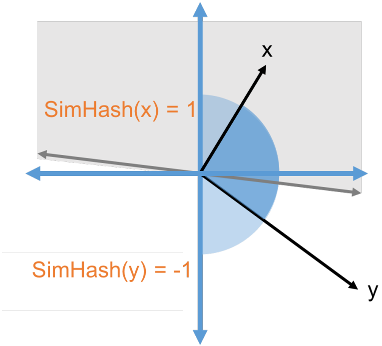

- $\Pr[SimHash(x) \neq SimHash(y)] = \frac{\theta(x,y)}{180}$
- $\Pr[SimHash(x) = SimHash(y)] = 1 - \frac{\theta(x,y)}{180}$

Questions on MinHash and Locality Sensitive Hashing?