COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

Andrew McGregor

Lecture 7

- Estimate $\#$ distinct elements d in stream $x_1, \ldots, x_n \in U$

DISTINCT ELEMENTS RECAP

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- **Median Trick:** If an algorithm returns a sufficiently accurate numerical answer with probability at least 3/4, run it $O(\log(1/\delta))$ times and take the median answer. This will have the required accuracy with probability at least $1-\delta$.

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The more distinct hashes we see, the higher we expect this maximum to be.

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Note: Careful averaging of estimates from multiple hash functions.

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- Set the maximum # of trailing zeros to the maximum in the two sketches.

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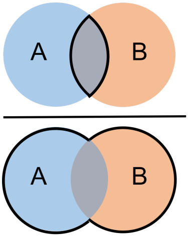
Questions on distinct elements counting?

Jaccard Index: A similarity measure between two sets.

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\#\text{ shared elements}}{\#\text{ total elements}}.$$



Natural measure for similarity between bit strings – interpret an n bit string as a set, containing the elements corresponding the positions of its ones. $J(x, y) = \frac{\#\text{ shared ones}}{\text{total ones}}$.

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- **Near Neighbor Search:** Have a database of n sets/bit strings and given a set A, want to find if it has high Jaccard similarity to anything in the database. $\Omega(n)$ time with a linear scan.
- **All-pairs Similarity Search:** Have n different sets/bit strings and want to find all pairs with high Jaccard similarity. $\Omega(n^2)$ time if we check all pairs explicitly.

Will speed up via randomized locality sensitive hashing.

Document Similarity:

- E.g., detecting plagiarism, copyright infringement, spam.
- Use Shingling + Jaccard similarity.

The quick brown fox jumped high. → {the quick brown, quick brown fox, brown fox jumped, fox jumped high}

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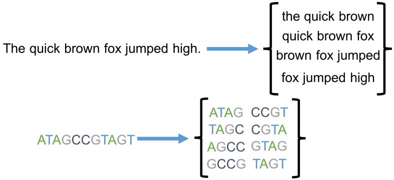
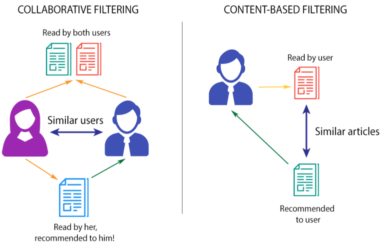
- Use Shingling + Jaccard similarity. (n-grams, k-mers)



APPLICATION: COLLABORATIVE FILTERING

Online recommendation systems are often based on **collaborative filtering**. Simplest approach: find similar users and make recommendations based on those users.



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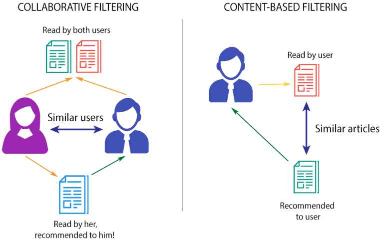

- Twitter: represent a user as the set of accounts they follow. Match similar users based on the Jaccard similarity of these sets. Recommend that you follow accounts followed by similar users. Netflix: look at sets of movies watched. Amazon: look at products purchased, etc.

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APPLICATION: ENTITY RESOLUTION

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- E.g. data on individuals from voting registrations, property records, and social media accounts. Names and addresses may not exactly match, due to typos, nicknames, moves, etc.
- Still want to match records that all refer to the same person using all pairs similarity search.

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See Section 3.8.2 of *Mining Massive Datasets* for a discussion of a real world example involving 1 million customers. Naively this would be $\binom{1000000}{2} \approx 500$ billion pairs of customers to check!

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- **Fake Reviews**: Very common on websites like Amazon. Detection often looks for (near) duplicate reviews on similar products, which have been copied. 'Near duplicate' measured with shingles + Jaccard similarity.
- **Lateral phishing**: Phishing emails sent to addresses at a business coming from a legitimate email address at the same business that has been compromised.
  - One method of detection looks at the recipient list of an email and checks if it has small Jaccard similarity with any previous recipient lists. If not, the email is flagged as possible spam.

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- Let $\mathbf{h} : U \rightarrow [0, 1]$ be a random hash function
- $\mathbf{s} := 1$
- For $x_1, \ldots, x_{|A|} \in A$
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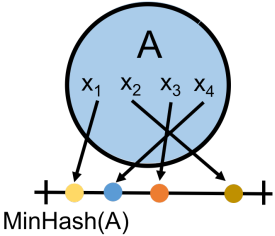

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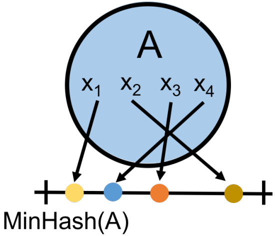

Identical to our distinct elements sketch!

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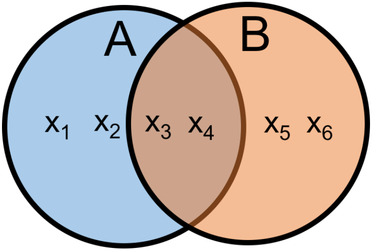

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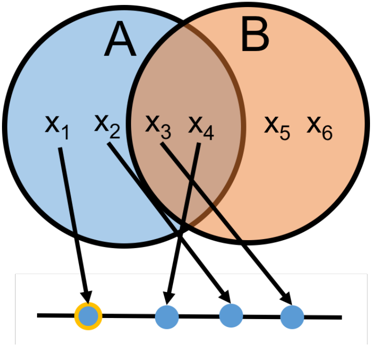

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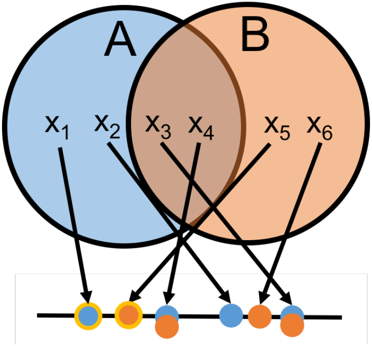

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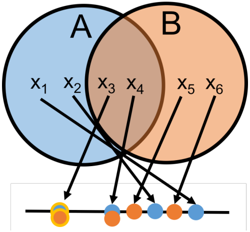

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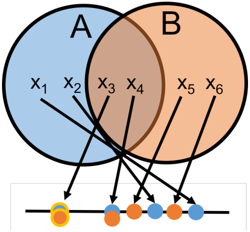

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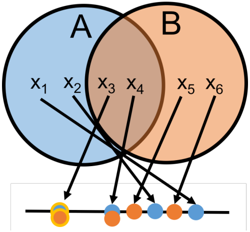

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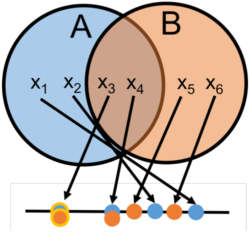

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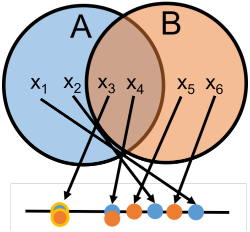

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