,

Abstract

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

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And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

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Preface

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Part I

Introduction

Chapter 1
Introduction to Cyber Insurance

Cyber-insurance is an insurance product used to transfer financial risk associated with computer and network related incidents over to a third party. Coverages provided by cyber-insurance policies may include property loss and theft, data damage, cyber-extortion, loss of income due to denial of service attacks or computer failures [PD12]. Traditional coverage policies rarely cover these incidents, therefore cyber-insurance is seen as a huge potential market. Although the concept of cyber-insurance has been around since the 1980s, it has failed to reach its promising potential. There might be several reasons for this slow development, however, it is believed that the main reason so far, is that no model deals with all the unique problems of cyber-insurance at once. In addition to the known difficulties of insurance, cyber-insurance has to deal with the problem of nodes asymmetric information, correlated risk and interdependent security [GGJ+10]. These three problem areas will be discussed in detail later in 1.2.1. First let's have a look at the similarities of normal insurances and cyber-insurance.

The basics principles of cyber-insurance relates to traditional insurance, where the insurance contract (policy) binds the insurance company to pay a specified amount to the insurance holder in case certain incidents occurs. In return, the insurance holder has to pay a fixed sum (premium) to the insurance company [Rob12]. As with other insurances, the cyber-insurance contract is signed between the insurance company and the insurer. The contract clearly specifies the type of coverage of the different risks, a risk assessment of the companies vulnerability and also an evaluation of the companies security systems. These assessments are used to calculate the companies premium [Rob12]. Generally, this means that the security is negatively correlated with the premium costs. Better security means lower price on the insurance premium.

1.1 General insurance

Generally, from the perceptive of the insurance company an insurable risks possesses seven distinct characteristics [MCR80]:

1. Large number of similar exposure units: Insurance companies is based on the principle of pooling resources, where insurance policies are offered to individual members of a large class, meaning the more insurers the predicted losses is closer to the actual losses.

2. Definite loss: A loss should take place at a known time, in a known place and from a known cause. Incidents such as a fire or car crash, are examples where these terms are easy to verify.

3. Accidental loss: The event that triggers a claim should not be something the insurer has discretion or control over.

4. Large loss: The size of the loss must be meaningful from the perspective of the insured. Insurance premiums need to cover both the expected cost of the loss, in addition, cover all the expenses regarding issuing and administrating policies, adjusting losses and supplying the capital needed to be able to pay claims.

5. Affordable premium: The premium must be proportional to the security offered, otherwise no one will offer/buy the insurance. In the situation where the likelihood of the insured event is high, and the cost is large, it is unlikely that the insurance company will offer the insurance, or at least the premium would be too high for anyone to consider buying it.

6. Calculable loss: Both the probability and the cost of an insurable event, has to atleast be possible to estimate.

7. Limited risk of catastrophically large losses: If losses happen all at once the likelihood of the insurance company getting bankrupt is high. Therefore, losses are ideally independent and non-catastrophic.

1.2 Cyber-insurance

When facing risk, there are typically four options available [A new perspective on internet security using insurance.. Bolot Lelarge]:

1. Avoid the risk

2. Retain the risk

3. Self protect and mitigate the risk

4. Transfer the risk

So far the risk management for computer networks have introduced methods to reduce the risks, a mixture of option 2 and 3. This has lead to creation of systems and software trying to detect threats and anomalies and to protect the users and the structure from these threats. Anti-virus software is also a good example of a system which perform self protection and hence mitigate the risk of becoming a victim of malicious attacks.

Unfortunately these types of systems does not eliminate the risk. Threats evolve over time, and there will always be accidents and security flaws. Cyber-insurance acts in the domain of the fourth option, and seeks to answer the question; -how can one handle this residual risk. The basic idea for cyber-insurance and insurance in general is to transfer the risk to a party who willingly accept it in exchange for a predictable periodical fee, namely premiums [BL08a].

1.2.1 Obstacles in cyber-insurance

As we have seen, cyber-insurance fit relatively well to the general insurance model, however there are some identifiable obstacles. These obstacles can be divided in to three categories, information asymmetry, interdependent security and correlated risk.

Information asymmetry Information asymmetry arises when one side in a transaction or a decision has more or better information than the other party. There are two different cases of information asymmetry, the first one is called adverse selection, where one party simply has less information regarding the performance of the transaction. A good example is when buying health insurance, if a person with bad health purchases insurance, and the information about her health is not available to the insurer, we have a classical adverse selection scenario. A similar case for the security industry occur when buying insurance for your computer, and the insurance company has no way of confirming whether your computer is "healthy", i.e. not contaminated, or if it is infected. The other information asymmetry scenario is called moral hazard. It occurs when after the signing of the contract, one party deliberately takes some action that makes the possibility of loss higher, i.e. choosing not to lock your door, since you have insurance. Or in the computer setting, deliberately visiting hostile web-pages, or not using anti virus software, firewalls or other self-protection software. [Pal12]

As we will see the information asymmetry problem is highly relevant regarding cyber insurance. Measuring the level of security is very hard, in addition will often people have an incentive for hiding information about their security strength. Because they might end up in a scenario where they describes what their weaknesses are, and thus the difficulty of successfully attacking them are lowered. Another problem arising due to information asymmetry, is the so called lemons market [1]. It is difficult for a security software buyer to distinguish the performance(bad vs good) of different software products, and thus the reasonable thing to do, is to buy the cheapest. From this we see that every security software has to be sold at approximately the same price, and there is no way to distinguish good and bad software. If the cost of producing good security software is to high, the problem can even result in abandoning the production of good software, because it would not be profitable.

Correlated risk Another big concern regarding cyber-insurance, is the correlated risk. Among others the problem occurs due to the need of standards. Standardization is an important part of computers and computer networks, it enables computers to communicate, install and use different software. A good example is the operative systems for personal computers, today we only have a small set of operative systems available for use, and these systems have been standardized, such that they can communicate over the same communication channels, such as HTTP/IP. The standards are what makes the ICT-industry valuable, but also what makes the possible extent of the threats so large. All these systems that uses the same standards, creates a large number of similar exposure units, they share common vulnerabilities, which can be exploited at the same time.

A different scenario is natural disasters, If the backbone network is down for numerous reasons, every operator connected will loose the Internet connection, hence be entitled to receive compensation for the lost income.

This creates a significant difficulty for the cyber-insurance industry, because when a security breach occurs there is a high probability that it will occur to a large number of people, i.e catastrophic and extreme events occurs more likely, resulting in extremely high expenses. If the security breach is large, it could potentially cause so much damage, that the insurers will not be able to pay all of the customers who suffered, i.e. they go bankrupt.[BS10]

Interdependent security Investment in security generates positive externalities, and as public goods, this encourages to free riding. Why should I pay for security when I can just free ride on security invested by others. The problem is that the reward for a user investing in self-protection depends on the security in the rest of the network, i.e. The expected loss due to a security breach at one node, is not only dependent on this nodes level of investment in security, but also on the

[1] Lemon market, the problem of quality uncertainty, was first introduced in a paper [Ake97] by the economist George Akerlof in 1970, and used the market for used cars as an example.[Wik] The conclusion of the paper is that since the buyers lack information to distinguish a bad car(lemon) from a good one(cherrie), the buyer will not pay the price the seller wants for a cherrie, and the seller will not sell a cherrie for the price of a lemon, and thus the lemons drives the cherries out of the market.

security investment done by adjacent nodes, and theirs adjacent nodes and so forth. A good example of this is the amount of spam sent every day, which is dependent on the number of compromised computers. Meaning if you have invested in security software of some kind, you still receive lots of spam due to the fact that there are a variety of people who have not invested [Böh10].

Calculating loss Another concern regarding cyber-insurance relates to characteristic of calculating loss from [MCR80]. When facing a security breach there are to potential loss classes:[BMR09]

– primary losses or first-degree losses: direct loss of information or data and operating loss. These arises from disuse, abuse or misuse of information. And the cost of these arise from recovering, loss of revenue, PR and information sharing costs, hiring of IT-specialists etc.

– Secondary losess are indirectly triggered. These are the loss of reputation, goodwill, consumer confidence, competitive strength, credit rating and customer churning.

The value of the loss from both these classes can be difficult to determine, although the second one is probably the most difficult. Because it is challenging to put a value on i.e. how many potential customers did they loose due to the reputation loss, how many customers churned, and what was their value etc.

Cyber-insurance instead of security One problem with cyber-insurance is actors seeing it as a solution to the problem of being secure. Instead of investing in security, they now have a way of buying their way out. However, this problem might solve it self with the right pricing options. Meaning that the insurance companies can create pricing models which makes it economical beneficial to invest in security. Such model will also make sense for the insurance company, since better security systems yields lower probability for incidents. Similar pricing models are common through out the insurance industry, e.g. the bonuses a car driver might be offered due to no accidents for some time or being above a certain age etc. will lower the price the insurance premium.

1.3 Insurable topology

THIS IS WHAT WE MEAN BY AN INSURABLE TOPOLOGY

1.4 A small summary

Add a small summary here.. summary of related work short presentation of what to come. "glidende overgang til current market".

Chapter 2

The cyber-insurance market

The market for cyber-insurance emerged in the late 90's when security software companies partnered with insurance companies and started offering insurance policies together with their security products. From a marketing perspective, adding the insurance helped highlighting the supposedly high quality of the security software. Regardless, the new product was a comprehensive solution, which dealt with both risk reduction and residual risk [BL08b]. Continuing into the beginning of the new millennium, several companies started offering standalone cyber-insurance, which sat the frame for the current insurance product. In Norway, starup companies, such as Safensure AS where established with respect to deliver cyber-insurance to the Norwegian and European market [dig]. Also established insurance companies such as Gjensidige Nor, started offering insurance products aimed for Internet web-sites. These insurances where created to insure lost income due to malicious hacker attacks, denial of service and other well know cyber-attacks at that time. E.g. in 2001 Gjensidige Nor in cooperation with the German company Tela Versicherung offered businesses insurance against financial losses due to hacker attacks and sabotage for up to 5 million NOK, given that specified security measures were taken by the company [it].

2.1 Current market state

Despite the fact that cyber-insurance has been around for over a decade, the market still struggles to gain a foothold. Safensure AS does not exist anymore and Gjenside Nor does not advertise a cyber-insurance product. It seems to be lots of challenges for both buyers and sellers. Buyers face tremendous confusion about cyber risks and their potential impacts on business. In general, [PpD12] points out that people do not know or understand what kinds of risks the cyber space involves, and how large the losses can be especially due to network externalities. Even when companies have decided to purchase a cyber-insurance, they are confused with what kind of insurance they should purchase. The market of cyber-insurance tend to become a lemons market, where the buyer have little knowledge to choose between the different insurances. Hence, people will buy the cheapest insurance, although it sometimes does not satisfies their requirement.

2.1.1 The UK and US market

The media coverage on corporate threats such as Stuxnet[1] and the attacks on Playstation, which lead to a compromise of 77 million user accounts including credit card numbers [Chu], shows that the cyber-threats is growing. There are several different results and opinions regarding the health of the global cyber-insurance market. Companies studied in [Ins11] experienced successful attacks every week, and showed that successful cyber attacks could result in serious financial consequences. They found that the median cost of cyber crime in the U.S is $5.9 million per year, ranging from $1.5 million to $36.5 million per company, which is an 56 percent increase from last year.

Another paper [Ris12] collected statistics about cyber attacks in the UK, and the result claims that the costs is expected to be £27 billion a year, and that it is one of UKs biggest emerging threats. In addition, they pointed out that the victims is not only large companies like Google and Playstation, but also small businesses. Despite these numbers only 35 % of the companies in the survey had purchased cyber-insurance. Although there is no shortage of providers,-they found that there are 9 insurers with specialists in cyber-insurance in the UK, and in the US around 30-40 actors.

An article from CFC underwriting [New], a UK firm offering insurance to small and medium sized businesses, claims promising numbers for the US cyber-insurance market. On US soil, 20-50% of businesses purchased either standalone cyber-insurance or benefits from coverage provided in their already exciting insurance. However, despite recent years focus on the increasing cyber-crime activity and the catastrophic consequences of having weak security, only 1% of European businesses are enrolled in an insurance program covering cyber-threats. A more optimistic survey pointed out that more and more insurance companies offered cyber-insurance. Yet, of the 13000 companies, only 46 percent said they where insured against cyber-attacks [Pra].

The numbers vary between the different surveys. However, all of them concludes that a large share of the companies are not protected against the residual risk of cyber attacks.

[1] Stuxnet, SKAL VI BESKRIVE HVA DET ER? ??

2.1.2 The Norwegian market

In comparison, our survey of the Norwegian insurance market relieved that specialized cyber-insurance companies such as Safensure AS does not exist anymore. Additionally, only one out of the five biggest actors[2] offer something similar to a cyber-insurance. Gjensidige Nor offers something they call operation-loss-insurance which covers expenses due to reconstruction of files and reinstalling software and denial of service attacks. In addition, it is also possible to insure against hacking and sabotage [Nor]. From mail correspondence with Gjensidige Nor it was clear that they needed information to be able to calculate the insurance premium. They required extensive information about the economic health of the company, and a model of what kind of software and hardware where used with estimated values on each component. [Email from: Arild Hjelde, Gjennsidige Nor.] Unfortunately we were not able to obtain the cost of such insurance. However, a similar insurance is offered by RTM Insurance Brokers, a Danish company, with premiums ranging from DKK3400 for insuring a loss up to DKK2.5 million , to DKK12900 for insuring a loss valuated to DKK25 million [Bro]. This gives an indication of the cost of the current cyber-insurance in the Norwegian market.

2.2 Future market

The survey from [New] claimed that the US cyber-insurance market was much more mature compared to the European. A possible reason is the breach notification laws. In the US, 46 states have mandatory breach notification laws, combined with significant penalties for companies failing to protect sensitive data. This means that the US government are creating incentives for firms to buy cyber-insurance. In Europe, only Germany and Austria have similar laws, forcing companies to notify affected customers of data leakage. A recent proposal of the EU wants to introduce the notification law in Europe, and also include penalties for serious data breaches, these could be as high as 2 % of a companies global revenue [New]. It is proposed that the law should take effect in 2014, although this is highly unlikely regarding the complexity of the effects of this law. Undoubtedly this law would be a health injection to the rise of the cyber-insurance market, however, a market based on fear of the consequences of not being insured is not desirable. The ultimate goal for cyber-insurance, is to correlate the purchase of cyber-insurance with companies growing desire to invest in more security, and hence lower the risk of being a victim of cyber-crimes. The article claims that the way to meet this goal, is to the focus on the serious brand damage a company will experience and not just the financial loss.

[2] Gjensidige, If Skadeforsikring, DNB, TRYG, Storebrand

notes... One reason for why the number of insured companies are low could be the fact that a lot of companies are trusting their own IT-department to handle cyber risk. Hence they believe that they would not need a cyber-insurance [Wat11].

Chapter 3
Graph Theory

In nature and human societies there are lots of scenarios that can be described by using graphs and graph theory, from infrastructure, such as railroads, water pipelines and electricity grid, to societal relationships, disease epidemics and much more. Additionally computer networks, such as peer-to-peer networks, number of links to/from web-sites etc, is formed and evolves according to the laws of random graphs. When one can describe a phenomenon with graphs, it is much easier to analyze and find characteristics about the phenomenon, the graph serves as an analytical tool [Aud]. Our goal is to identify insurable graphs, such as graphs which yields higher security or graphs where the risk is calculable. This section will provide background information on how different graphs can be created and how they evolve.

There are some basic properties of graphs which is important to be familiar with. Figure 3.1 depicts the basics of an unweighted graph, the edges are not assigned any value. Weighted edges can be useful to e.g. reflect capacity constraints such as a link's maximum bandwidth, or the length of a road(edge). Other common definition used when describing graphs are listed below [Aud]:

- Edge degree: Number of edges connected with a node.
- Hub: Node with high edge degree.
- Cycle: A chain originating and terminating at the same node.
- Cluster: Subgraph of highly connected nodes.
- Cluster coefficient: Probability that two nodes that are adjacent to a third node are also adjacent.
- Clique: Subgraph where all nodes are adjacent (cluster coefficient = 1).
- Small world graph: Graph with small diameter and large cluster coefficient (e.g. the Internet and A-B graphs, described in section 3.1).

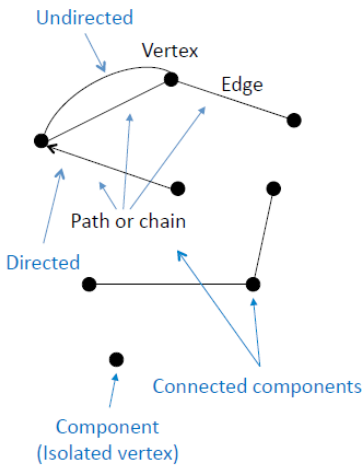


Figure 3.1: General graph [Aud].

3.1 Random Graphs

Cyber-insurance cover many fields, from financial transactions and outsourcing of tasks to computer networks, many of these fields share a common characteristic, they can all be described as a graph, and often a random graph. Therefore the study of random graphs are of special concern. The research on random graphs are fairly new compared to other mathematical discoveries. E-R graphs were first studied in 1959 by Erdös and Rényi, later and probably with more promising results was the graphs studied by Albert-Barabási in 1999 [Aud].

Erdös-Rényi Graphs E-R graphs is a network created between a fixed number of n-nodes, where each node connects to another of the $n-1$ nodes with probability p. The resulting graph will on average contain $n(n-1)p/2 \approx n^2p/2$ edges [Bol85]. By analysing the graph, the authors found some interesting properties:

- If $p < n^{-2}$ then there is no edges in the graph.
- If $p = c/n$ where c is a constant between $1 < c < log\, n$, the graph will provoke a single large component to grow within the graph.
- If $p > (ln\, n)/n$ then the graph is completely connected.
- If $p = 1/n$ triangles start forming in the graph.

A fully connected E-R graph will have a short diameter similar to the Internet, and thus could be a very good desrcription of the internet. However, the edge degree follows a Poisson distribution, which means that the edge degrees are peaking around the average value [Aud]. Consequently E-R graphs does not capture the immense clustering coefficient which is present in networks such as the Internet. In other words, E-R graphs are not small world graphs, and another graph structure is needed to model computer networks. A interesting fact about these graphs are their vulnerability, these graphs are very vulnerable against random attacks, such as nature disasters, but robust against directed attacks. Due to the fact that if you remove all edges from one node, it does little damage, since the network is not dependent on single nodes, every node has approximately the same node degree, and it is the sum of all the nodes connections that creates the network.

Albert-Barabási Graphs The structure which is believed to be most accurate regarding modeling computer networks are A-B graphs. A-B graphs are different from E-R graphs since they are scale-free, meaning that the vertices does not have an constant value throughout the entire graph. The formation of an A-B graph results in multiple hubs with a high edge degree. Albert and Barabási found that the edge degree of each vertex follows a power law distribution; meaning that the probability that the edge degree is g is proportional to $g^{-\gamma}$ where γ usually is a number between 2 and 3 This distribution is called a thick-tail distribution, because there is a significant probability that a node may have a very high degree. [Aud] These graphs are in contrast to E-R-graphs, very vulnerable to directed attacks, because if you take out a hub, you suddenly destroyed the whole graph. But the graph is very robust against random attacks, this is why most of the networks we observe in the nature can be depicted as A-B-graphs. A-B graphs can grow and become scale-free if every new vertex is connected to one or more already existing node with a probability proportional to the edge degree of that node . The paper presents an algorithm that creates A-B graphs and Figure 3.2 shows one graph that evolved from this algorithm:

- A new single vertex is added to the graph.
- This vertex is connected to exactly two other vertices in the graph.
- The probability that the new vertex connects to another vertex is dependent on the edge degree of the other vertex, higher edge degree meaning higher probability
- There is only one edge between two vertices.

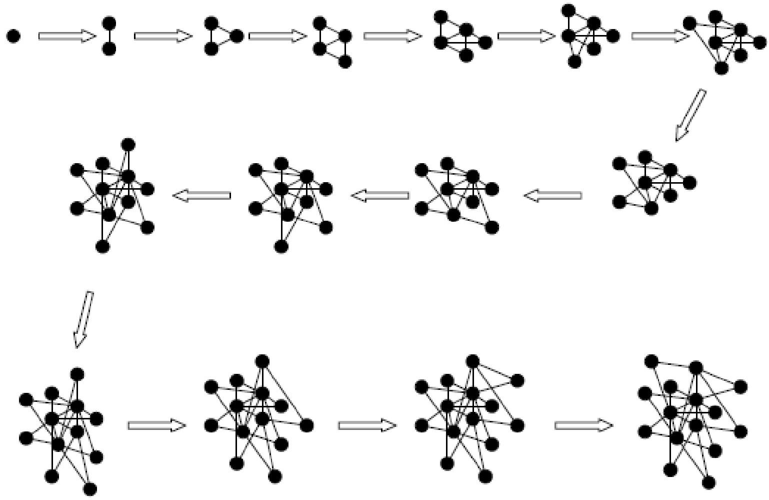


Figure 3.2: Forming a A-B graph in 15 generations [Aud].

In addition to the high clustering coefficient they showed that A-B-graphs have a fairly small diameter, which can be seen in Figure 3.2. A-B graphs are therefore comparable to the network formation of the Internet and other computer networks.

3.2 Real world graph structures

The internet, the World Wide Web, neural networks, scientific referencing and co-authorship, stock markets, airline routes, food webs, and modular software systems, all tend to evolve in a way similar to that described in the examples above. This section will provide some real world examples of how complex systems with huge amount of data can be described as network structures having the same characteristics as A-B graphs.

Stock markets The research paper: [Gar07], analyzes the correlation between different stocks in the Greek stock market in year 1997. They compared the daily closing price of stock i at day t, and compared the similarity of a pair of stocks i and j by using the correlation coefficient. The idea is that the correlation coefficient between a pair of stocks can be expressed using different distances in a graph structure. A short distance means high correlation and long distance means low correlations between the stocks. Normally this network would be shown as a fully connected graph, which will consist of $\frac{n(n-1)}{2}$ edges, and would be difficult to analyze. However the approach taken in the paper will present a clear understandable graph consisting of $(n-1)$ edges.

The resulting graph can be seen in Figure 3.3, and show a network consisting of several clusters linked together. Instead of having to analyze a complex system with huge amount of data, this stock market can be analyzed by its topological properties, such as the high clustering coefficient, i.e a star-topology, which will among others point out which stocks have the most influence on others.

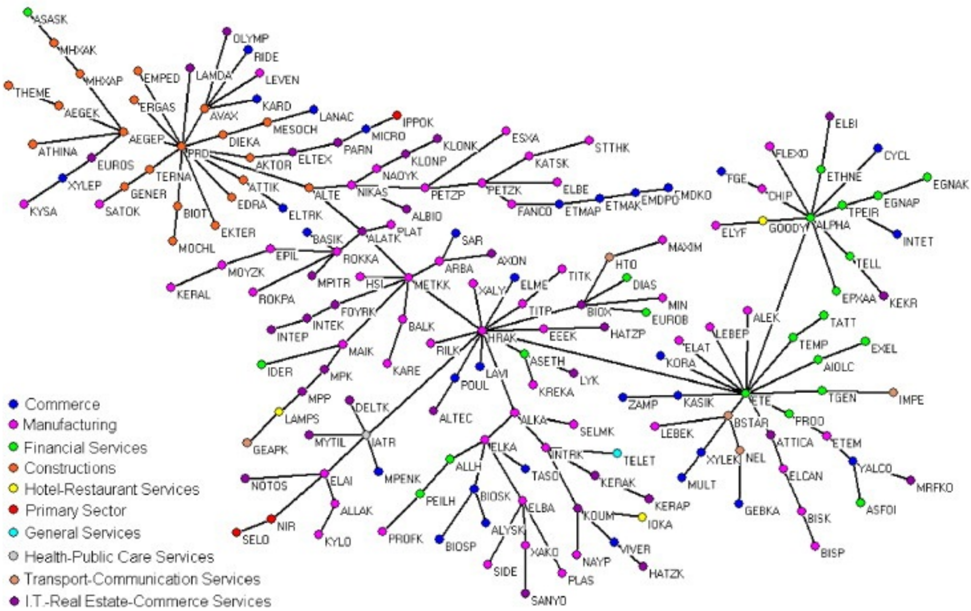


Figure 3.3: Network obtained by comparing two stocks correlation coefficient in the Greek stock market (Athens Stock Exchange, ASE) in year 1997. The different colors represent the different sectors of economic activity [Gar07].

Airline routes Another real world network which shows the same characteristics as scale-free graphs is the map of airline routes. Figure 3.4 shows the US route map of the American airline company, SkyWest. The characteristic clustering emerges in the figure, where a majority of the flights departs from either Denver, Chicago or San Francisco. Not surprisingly, these airports are all in the top 7 busiest airports in the US [Faa], and serves as hubs for many of SkyWest flights. In the airline industry some airports are called hubs, because that's what they are, - a connection point for major parts of the network of flights. The network of flights, as depicted in Figure 3.4 follows the characteristics for A-B graphs. From the graph, we see that the network are vulnerable against direct attacks, meaning if an low edge degree airport is shut down, there will be little consequence for the rest of the network. However, if one of the hubs is forced to close, it will provoke huge delays through out the whole network of flights, because many of the destinations are interconnected via the hubs.

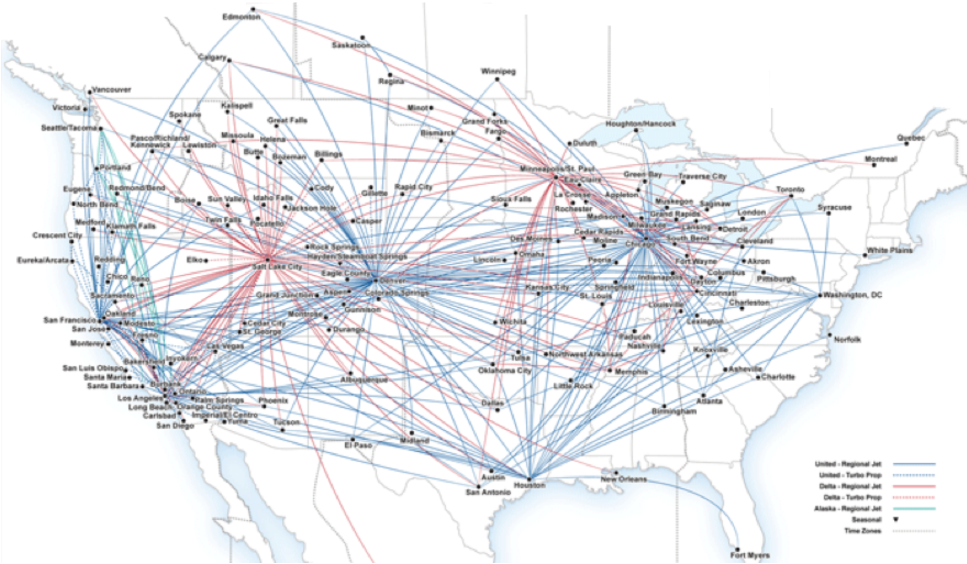


Figure 3.4: SkyWest Airline combined route map [CfAPA].

Similar findings will appear in the different networks mentioned earlier in this chapter, and all of them will experience large consequences if a hub in the network stop functioning. This is important for cyber-insurance because many of the networks we are analyzing tends to look and behave like A-B graphs. For example, transactions between companies, big companies probably have more transactions than small companies, and thus creates a hub, this can be compared with how the correlation between stocks in a stock market works. I.e. we can say that small firms correlate highly with big-firms.

3.3 Evolutionary dynamics on graphs

When investigating cyber insurance and insurable topologies, it is important not to only focus on standard risk networks, such as the internet. Our goal is to investigate all kinds of networks, or especially networks where players actions are influenced by their neighbourhood structure, i.e. the network connections will affect each individual players payoff. In this case there are several types of networks to consider, all social and economic interactions where an agents well being is dependent on externalities as well as her own actions, is a network worth considering network.

As mentioned earlier, the internet is a very good example, because on the internet we are "all" connected, the benefit we get from the internet is strongly dependent on this, and so is the risk we face when using the internet. Other examples could be the networks that are formed when a company are developing a software product, this development process is often done by several different firms, and thus creates a development network, where everyone is dependent on the result of the others. If one or more fail in some way, bankruptcy, failure to deliver at the expected time, higher cost etc. Then the whole network will be affected. Or in a cloud computing network, there are many different users and internet service providers, and the overall security is dependent on all of them. As we see all these networks are different from each other, some face direct connections, other consist of social and economical connections. But they all share some main characteristics, they are all experiencing network effects, externalities, information asymmetry, correlated risk and interdependent security. [GGJ+10]

In our paper an insurable topology, is an network structure which makes it feasible for both the insurer(supply side) to offer and the customer(demand side) to acquire insurance. For this to be possible there are many difficulties to overcome, one example are the correlated risks, from the insurers point of view, the problem is to be able to calculate the overall probability of casualty/infection, which can be very difficult without graph theory.

The paper [LHN05] is about evolutionary dynamics and how some structures can amplify or sustain evolution and drift[1]. Regarding security, knowledge of how viruses spread and how to use graph structures to prevent malicious hackers from entering your network is important. Evolutionary dynamics, and the research of how mutant genes spread though out a population is a very useful field when looking for an insurable topology. If we can determine some structures, where some nodes are advantageous/disadvantageous , then these structures will have properties, such as sustaining viruses from spreading, or amplify the incentive for obtaining cyber-insurance and protection software. And these could maybe be considered as insurable topologies.

In the [LHN05] paper, they show that mutants inserted in to a circulation graph, will have a fixation probability equal to

$$p_1 = \frac{(1 - \frac{1}{r})}{(1 - \frac{1}{r^N})} \tag{3.1}$$

Where r represents the relative fitness of the mutant, if it is advantageous it will have a certain chance of fixation, and disadvantageous mutants will have a chance of

[1] Drift is the opposite of selective evolution , it is when the network/structure evolve and change at random

extinction. A circulation graph is a graph that satisfy these two properties:

1. the sum of all edges leaving a vertex is equal for all vertexes
2. the sum of all edges entering a vertex i equal for all vertexes

I.e. a clique is a circulation graph, and the probability of fixation is as in Eq. (3.1). The fixation probability determines how probable it is that the whole network will eventually be "infected" by the mutant. It determines the rate of evolution, which relies on both the size of the network and the evolution speed. If the relative fitness of the nodes are high, then the probability of fixation will be low. A probability equal to one means that every node in the network eventually will be affected by the mutant. A circulation graph is not the best insurable topology, but since the probability of fixation can be calculated, it is an insurable topology. If we can find graphs with fixation probability that exceeds Eq.(3.1) they could possibly be considered as insurable topologies, because if we can find these graphs, then it will be possible to further suppress drift and amplify selection and visa versa. The paper shows that there exists such graphs, one example is the star topology, (see Figure 3.5). In this topology the fixation probability is as shown in Eq.(3.2), or for more general see Eq.(3.3).

$$p_2 = \frac{(1 - \frac{1}{r^2})}{(1 - \frac{1}{r^{2N}})} \tag{3.2}$$

. or more generall:

$$p_k = \frac{(1 - \frac{1}{r^k})}{(1 - \frac{1}{r^{kN}})} \tag{3.3}$$

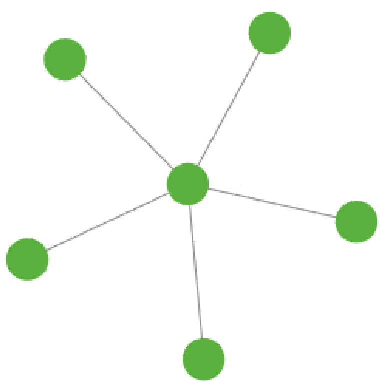


Figure 3.5: A star-topology.

When comparing the Eq.(3.1) and Eq.(3.2), we see that the selective difference is amplified from r to r^2, i.e. a star act as an evolutionary amplifier, favouring advantageous mutants and inhibiting disadvantageous mutants.

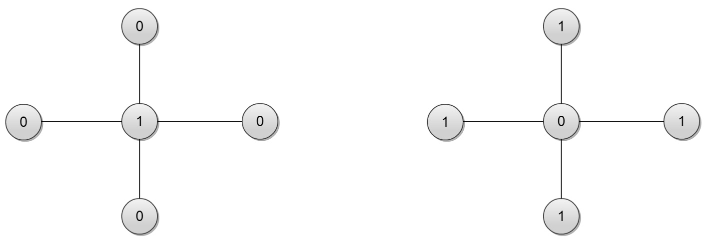
There exists other graphs where the fixation probability is equal to 3.3, examples are super-stars, such as funnels and metafunnels. These are just more complex star networks. This paper shows, that as N get large, the super-stars will have fixation probability, for an advantageous mutant, that converges to 1, and for disadvantageous converges to 0. As we know from chapter 3, there are many topologies in our society that are so called scale-free. Scale-free networks have most of their connectivity clustered in a few verices, very similar to a star, and these networks can also be considered as potent selection amplifiers.

Benefits of cliques The paper [Blu11] find some interesting results regarding network formation games. They set up a game where the nodes benefit from direct links, but these links also expose them for risk. Each node gains a payoff of a per link it establishes, but it can establish a maximum of δ links. A failure occur at a node with probability q, and propagates on a link with probability p. If a node fail, it will receive a negative payoff of b, no matter how many links it has established.

The results from their model shows a situation where clustered graphs achieve a higher payoff when connected to trusted agents, compared to when connecting with random nodes. Unlike in anonymous graphs, where nodes connect to each other at random, nodes in these graphs share some information with their neighbors, which is used when deciding whether to form a link or not. To further explain these results, they show that there exists a critical point, called *phase transition*, which occurs when nodes have a node degree of $\frac{1}{p}$. At this point a node gets a payoff of $\frac{a}{p}$, and to further increase the payoff the node needs to go into a region with significantly higher failure probability. Because once each node establish more than $\frac{1}{p}$ links, the contagious edges, will with high probability form a large cluster. Which results in a rise in probability of node failure, and reduces the overall welfare. From this the paper say that when the minimum welfare exceeds $(1 + f(\delta) * \frac{a}{p})$ we have reached super critical payoff. Otherwise it is called sub-critical payoff. Further they show that the only possible way of ending up with supercritical payoff, is by forming clustered networks consisting of cliques with slightly more than $\frac{1}{p}$ nodes. If the nodes form an anonymous market, random linking, they can only get sub-critical payoff. In other words, if the nodes can choose who they connect with, and by doing so, creating trusted clustered markets, they can achieve a higher payoff, by exceeding the critical node degree point. But in random graphs, this is not possible.

Star-network as an insurable topology The paper [GGJ$^+$10] shows how network games evolve when the payoffs are determined not only by your own decisions, but also by your neighbours. This can be used to analyze the star network further. One of the games they analyzes is simple but highly relevant for our paper, a public goods game. A good example of a public goods is security product. A security product suffers from strategic substitutes, i.e. if your neighbour acquire the security product, you have less incentive of also acquiring the security product. This is because when he acquire it, he gets more secure, but so do you, due to the positive externalities of the product.

Lets consider a simple game shown in this paper, We have an action space: $X = \{0, 1\}$, where 1 can be considered as acquiring information, take vaccine, buy security software etc. And 0 is not doing so. Each node i has a set of neighbours: N_i, and a payoff function $y_i = x_i + \bar{x}N_i$. The gross payoff to player i is 1 if $y_i >= 1$ and 0 otherwise. But each player also suffer from a cost of $0 < c < 1$ if they choose action 1. When looking at Figure 3.6, we easily see that there is two equilibriums.



(a) Socially Optimal equilibrium, center node choose action 1

(b) Non Socially Optimal equilibrium, leaf nodes choose action 1

Figure 3.6: Figure 3.6a shows the socially optimal equilibrium, and Figure 3.6b shows the non optimal equilibrium.

One where the center node choose action 1 and the rest of the nodes choose action 0, and a second equilibrium where all the leaf nodes chooses 1 and the center choose 0. The overall payoff in these two differ from each other, the latter is not socially optimal because it suffers from a cost equal to: $\#leafnodes * c$, the first equilibrium have a total cost of only c. It would have been very good if we where able to force the game to end up in the social optimal equilibrium.

From a insurers point of view If a insurance company could identify these star-structures, and force them to end up in the social optimal equilibrium it would have been very beneficial for both the insurer and the customers. First of all if the insurer could identify these structures, he could calculate the overall probability of fixation by a diseased mutant(virus, worm, trojan or other failures) as shown earlier. And if they could ensure that the center node is protected they could also calculate the probability of the diseased mutant being extinguished from the network. One possibility of achieving this could be by offering very cheap insurance to the leaf nodes, and giving the center node an incentive to acquire security product, by informing the center node about the probability of failure unless he acquires security. And offer him a very good rebate if acquire the security product, and a very expensive insurance if not. In this way the insurer could force a rational center node to getting both insurance and security product, and thus securing the whole network.

This is a simple scenario, analyzing an exogenous network formation [2], but it shows how a insurer can, by using the results from [LHN05], force the game to end up in the social optimal equilibrium, and also how the insurer can calculate the probabilities of failure. The contributes significantly to solving some of the problems with cyber-insurance. The problems with information asymmetry and interdependent risk problem has been reduced, since if the insurer knows the network structure, he can calculate the probabilities of failures and catastrophic events, the most important information he needs is how secure the center node is. If he also can ensure that the center node is secure, the interdependent risk problem is limited to only one node, the center node. All this result in a simple but insurable network topology.

[2] Exogenous: The network formation is given. Endogenous: The structure originates from within the network, i.e. the oposite of exogenous

Chapter 4
Relatedwork

4.1 Cyber-Insurance

4.1.1 Paper from Bohme - SKAL FJERNES ETTERHVERT

While several authors have expressed doubts about the future of cyber-insurance, the authors of [BS10] still have faith in the prevalence of cyber-insurance. The paper describes the three main problems of cyber-insurance; information asymmetry, correlated risk and interdependent agents. They argue that a model for cyber-insurance has to encounter each of these obstacles. Instead of presenting a solution they propose a framework to classify models of cyber-insurance.

The framework breaks the modeling down to five key components:

- network environment(nodes controlled by agents, who extract utility. Risk arises here.)
- demand side(agents)
- supply side(insurers)
- information structure, distribution of knowledge among the players.
- organizational environment. Public and private entities whose actions affect network security and agents security decisions.

The goal is that this unifying framework will help navigating the literature and stimulate research that results in a more formal basis for policy recommendations involving cyber-risk reallocation. They encourage to answer questions such as; under what conditions will a cyber-insurance market thrive? What is the effect of an insurance market, -will the Internet be more secure? Does it contribute to social welfare? The paper studies other existing models, and reveals a discrepancy between informal arguments in favor of cyber-insurance and analytic results questioning the viability of a cyber-insurance market.

4.1.2 A novel cyber-insurance Model - FJERNES ETTERHVERT

The paper [PGP11] presents a cyber-insurance model which handles both risks due to security (e.g virus) and non-security related features such as power outage and hardware failure. Their model, Aegis, is a simple model in which the user accepts a fraction of loss recovery to himself and the rest is transferred to the insurance company. They show that when it is mandatory to purchase insurance, risk averse agents would prefer Aegis contracts over traditionally cyber-insurance products. The model also give users incentive to take a greater responsibility in securing their own systems. Hence this answers one of the questions from [BS10]: The overall security of the Internet will increase if the Aegis is offered to the market. An interesting result from their analysis is the fact that a decrease/increase in the insurance premium may not always lead to increase/decrease in demand. From the insurers point of view, this features means that one can increase the margins without loosing possible customers. Hence it will be easier to create a market for cyber-insurance. This is good, because in our findings, we show that an increase in insurance premium is needed to make the insurance product attractive to the customers, because if the price is to low, it will encourage making risky decisions.

4.1.3 Cyber-insurance for cyber-security, A Topological Take on Modulating Insurance Premiums - FJERNES ETTERHVERT

[PH12] adopts a topological perspective in proposing a mechanism that accounts for the positive externalities (due to purchase of security mechanisms)and network location of users. In addition they provide an appropriate way to proportionally allocate fines/rebates on user premiums. This feature relates to our model, where a central node in the network receives a bulk insurance discount, in order to facilitate creation of insurable star topologies.

4.1.4 Differentiating Cyber-insurance Contracts, a topological Perspective - FJERNES ETTERHVERT

[PH] present the importance of discriminating network users in insurance contracts. This is done to prevent adverse selection, partly internalizing the negative externalities of interdependent security, achieving maximum social welfare, helping a risk-averse insurer to distribute costs of holding safety capital among its clients, and insurers sustaining a fixed amount of profit per contract. The paper proposes a mechanism to pertinently contract discriminate insured users when having complete network information. This is important since almost every node in the network is different from each other. Hence we need a way of distinguish good nodes from bad ones by the means of the premium price.

4.2 Cyber insurance as an Incentive for Internet Security-PAPER fjern overskrift..

In the paper [BL08a], they talk about how risk management on the internet only have involved methods to reduce the risks, such as firewalls, intrusion detection systems, anti virus etc. But none of these have managed to remove the risk completely. In general there are four possible ways of removing risk: avoid it, retain it, self protect and mitigate it or transfer the risk. And most entities on the internet have chosen a mix of of retaining and mitigate by self protecting.

Unfortunately, these solutions does not eliminate risk completely, and threats evolve over time. Thus, the only option for completely removing the risk, is to transfer it to a party who willingly accepts it, in exchange for a fee. The keyresult of this paper is that they show that cyber-insurance will result in overall higher payoff. Because when the premiums discriminates users based on the investment in self protection, it will act as an strong incentive to acquire self protection.

The paper [DS06] describes an interesting network formation games. Although the paper tries to observe suseptibility to sybil attacks in peer-to-peer networks, their approach on network formation can be related to our thesis. In the game they come up with, nodes are either friends or strangers. And the goal of the nodes is to selfishly try to fulfill their communication needs. Their needs is to communicate with as many as possible of their friends. This can be achieved by either direct or indirect connections. Every node has a link budget, i.e. a maximum number of links they can establish, and a set of friends they want to connect to. They proposes two random games where nodes might have to take the risk of connecting to non-insured nodes.

1. Random model: Every node in the network initiate a set for friendships with other nodes, denoted F. All nodes have the same link budget $L < F$.
2. Unbalanced Random Mode. The same friendship graph as in the random model is created. However one of the nodes have a significantly larger link budget $(L_0 > 2F)$

The first model does not result in any equilibrium, except the one where friends only connect to other friends. The other model shows some new insights, when the link budget is comparable with their number of friends, most still choose to only connect to friends. However, when the link budget is set to only one link, except for the rich node. Then the resulting equilibrium is a star topology.

Chapter 5

Methodology

5.1 Game Theory

Here we will present some of the game theory concepts we use in our models, for more thoroughly explanation of game theory, see: [NRTV07].

One shot game This type of game assumes that players act at the same time instant, therefore there is no causality. A game in strategic (normal) form can be described by three elemetns:

- the set of players $i \in I$, which we take to be the finite set $1, 2, ..., I$.
- the pure-strategy space $s_i \in S_i$ for each player i, where s_i is a possible action of player i.
- and payoff functions U, which gives the players utility functions for each profile $s = (s_1, s_2, ..., s_I)$ of strategies.

A general solution concept for games of economic interest is the Nash Equlibrium solution. A Nash Equilibrium is a profile of strategies such that each players strategy is an best response to the other players strategies.

Nash Equilibrium A pure strategy profile s^* is a Nash eqilibirum if, for all players i

$$U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*) \in S_i \tag{5.1}$$

Stackelberg Also known as a leader-follower game, it introduces multiple stages. The leader commits itself first, chooses its strategy, then the followers respond sequentially. The Stackelberg model can be solved to find the subgame perfect Nash Equilibrium, i.e. the strategy profile that serves each player best, given the strategies of the other players and that entails every player playing in a Nash Equilibrium in every subgame.

Subgame-perfect equilibrium A strategy profile s is a subgame perfect equilibrium if it represents a Nash Equilibrium of every subgame of the original game.

Socially optimal A socially optimal outcome is the set of choices that maximizes the sum of all players payoffs.

Price of stability The price of stability (PoS) of a network game, is the ratio between the maximum sum of the players payoff, in a stable outcome, and the Socially optimal outcome.

5.2 Netlogo

In addition to analyzing the different models with game theory, we created a simulator for the models, in a program called Netlogo. Netlogo is a programmable modeling environment for simulating natrual and social phenomena. It is well suited for modeling complex systems developing over time [Wil]. Netlogo where good to model our complex network formation games, and at the same time provided us with a good graphical user interface, that enabled us to see the result of the games, and also to easily adjust the different parameters. In Figure 5.1 we see the user interface, which are used to setup the parameters, start the modeling, and showing the resulting network formation. Figure 5.2 shows how the coding interface looked like. For detailed overview of the code used in our different models, see the appendix.

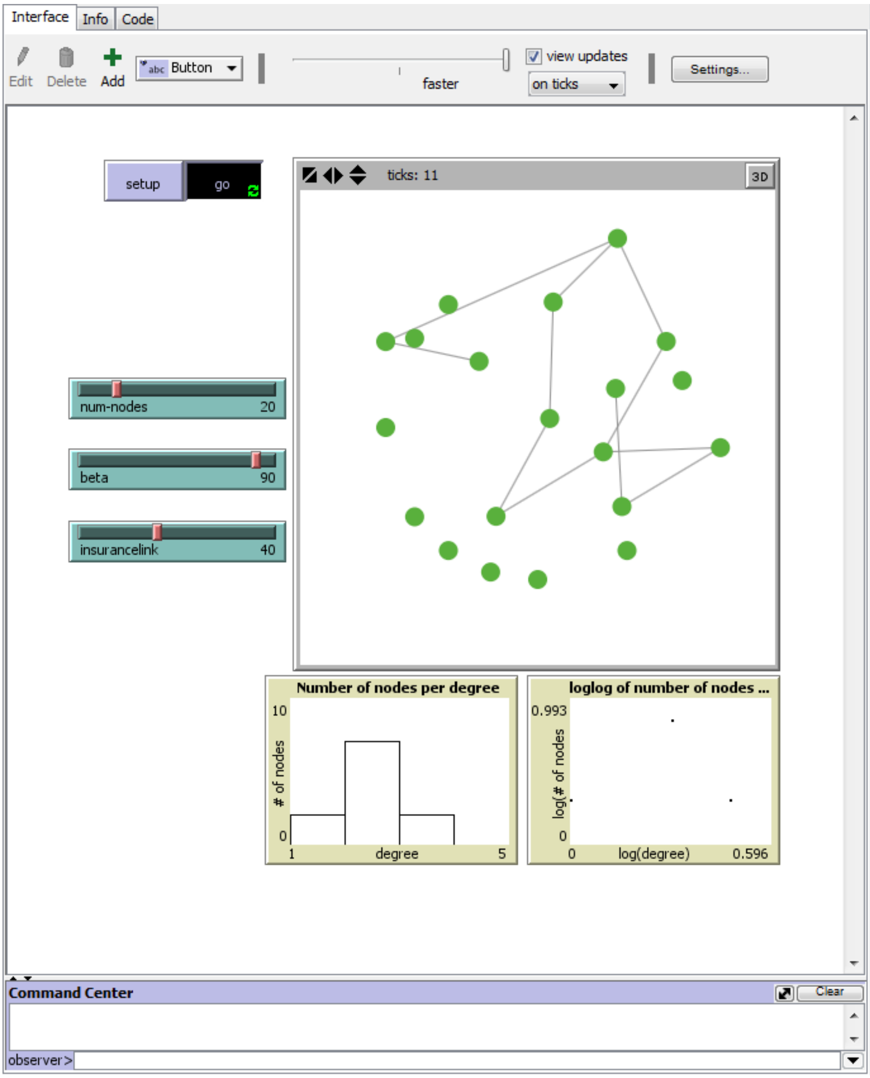


Figure 5.1: The figure shows a screen capture of netlogo, while we are running one of our simulations.

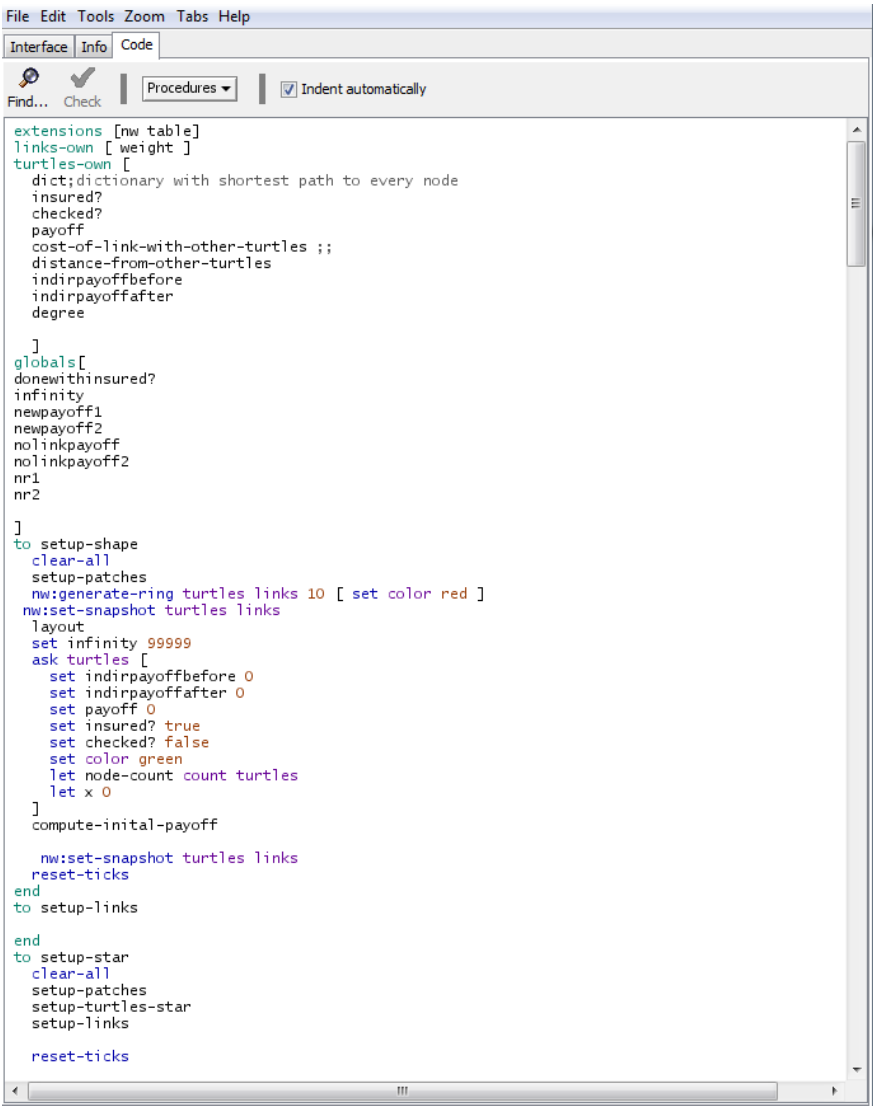


Figure 5.2: The figure shows how the code interface in netlogo looks like.

Part II

Own Contributions

Chapter 6
Modeling Cyber-Insurance

In many scenarios nodes seeks to create networks in order to directly benefit from each other. The established links might represent companies out sourcing part of their manufacturing, or cooperative agreements in the development of new software products. In addition to increase the trade-off, each of the established links represents risk of being a victim of cascading failures. The intuitive example is the spread of epidemic diseases, also node failures of a power grid and financial contagion such as the one back in 2008 was a result of cascading failures. Strategic network formation using cyber-insurance can be used to prevent such situation in addition to increase the overall payoff of participants in a clustered network.

When deciding whether to establish connection to a neighbor agent, the payoff has to be higher in the balance between the expected earnings and the risk of the other party failing to complete the transaction. This is the reason why we seek to only download content from trusted peers and outlaw MC-gangs are consistently skeptical to enter into new agreements despite promising increased earnings, since the risk of undercover police are too high.

The model from [Blu11] described in the related work chapter introduces a model where each node benefit from direct links to other nodes. However establishing a link will expose them to a risk if the node is not protected. Information about other nodes status (in our case, insured or not) will help nodes to guarantee cliques consisting of only the insnured nodes. According to the paper, such cliques will result in super critical payoff for every node connected.

Insurable topology One of the problems with cyber-insurance is to define and calculate the risks, because the network structure is undefined. If an insurer where able to predict the network structure, the calculations of overall risk would be realizable, and even better if the insurer where able to force some more robust network structures to evolve. Examples of such structures are as described in the graph theory chapter, scale-free network, which have been proven to be very robust against random attacks. Star-topologies, or star-like topologies such, which have a fixation probability that exceeds the fixation probability of circulation graphs. Star structures also have the nice property of minimizing the average path length, i.e. minimizing the cost spent on establishing links. In our thesis we define an insurable topology as a structure which enables the insurer to calculate the overall risk, i.e. a structure who can be analyzed by the insurer. The two main types of insurable topologies we will focus on are the clique and star-like structures. Since both of these have been identified to have calculable fixation probability. They also have other properties that are desirable for the insurer, such as the possibility of amplifying or suppress selection and drift. This is very desirable, because if the insurer is able to ensure that the nodes have a certain security level, especially the center node, then he can ensure that viruses does not spread. The clique has the nice property of being able to achieve super critical payoff, as showed in [Blu11].

Introduction to the modeling. There are many examples of nodes needing to establish connections, one example is a company needing to out-source certain tasks to remain competitive. This outsourcing involves some risks, such as, will the company deliver at the reported time, to the reported costs, what happens if they fail to deliver, what if they go bankrupt etc. If the companies that are going to establish links(cooperative contracts), know that the other firms are insured, it will be more secure and reliable to enter into an cooperative agreement. In this way trusted cliques can evolve. The firms benefit from connecting to other insured firms, and the insurance company can offer fair prices to the insured companies, because the risk is calculable in a trusted clique.

So our goal with the models are to find out how and when different networks evolve, and how the insurer can ensure that this will happen, by adjusting the parameter he can control, the insurance cost.

Inspired by this model, we are step wise building our own model which shields light on how cyber-insurance can be used in network formation games to prevent cascading failures and increase an agents payoff. Through out this chapter new features will be added to the models, starting at the simple model-1, to make it more realistic and applicable, an overview of the different models can be seen in Figure 6.1.

6.1 Model 1 - Initial Model

As a starting point the model is highly simplified in order to show the concept of how cyber-insurance can be used to create an insurable topology. We assume that every node has complete network information, i.e. it knows how many nodes that exists, and whether they are insured or not. The link establishment process is bidirectional, meaning both nodes must agree to establish the connection.

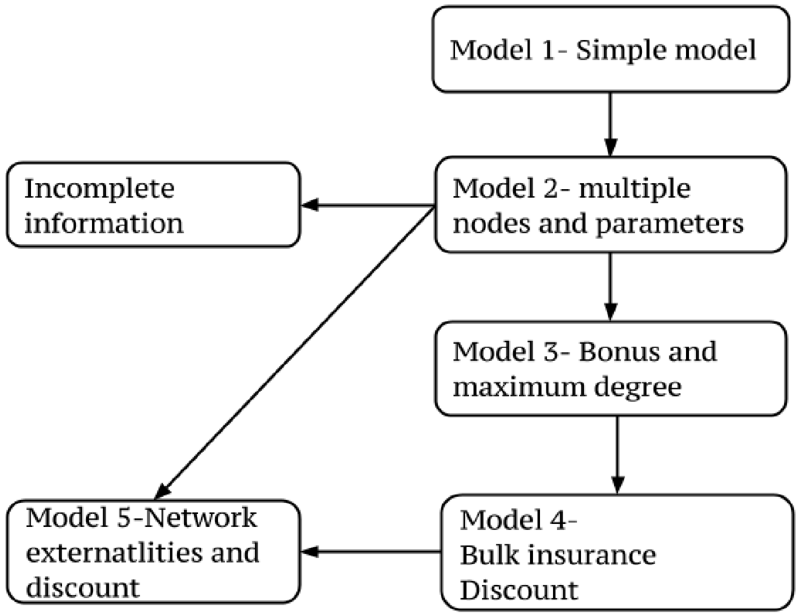
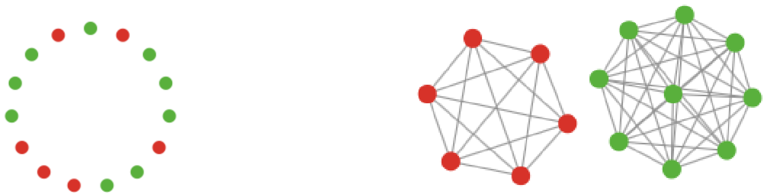


Figure 6.1: The figure show an overview of the different models we have created, and how they relate to each other. For every step, there are added some new features to the model.

For the first model, we assume a set of n nodes that are randomly chosen to be insured or not, as depicted in Figure 6.2a. They all get their own fixed income, and by connecting to other nodes they can increase their payoff. Non-insured nodes will have a risk of failure i.e. an expected cost of failure. Therefore if an insured agents chooses to connect to a non-insured nodes they will also suffer from this expected cost of failure. To simplify the decision process, the model follows a rule that only allows insured to connect to other insured agents and non-insured agents can only connect with each other. The resulting graph will be two fully connected cliques, one consisting of insured agents and the other of non-insured agents, as shown in Figure 6.2b.

This dichotomy represents a trusted environment for the insured nodes, because they know that each node in the clique is insured against risk. These nodes will benefit from each connection without having to worry about contagious risks from the connected nodes. A node in the non-insured clique will also experience a change in payoff from the links it has established, however each of the links has a probability of failure. Hence this environment is not trusted, and a link establishment will always involve some risk.

The first model, although very simple, shows an insurable topology where insured agents benefit from being insured, and are candidates to receive super critical payoffs as described in [Blu11].



(a) 15 Agents randomly choosen to be either insured (green) or non-insured (red).

(b) Two clustered networks. One consisting of insured agents the other consists of non-insured.

Figure 6.2: Shows how agents connects to eachother according to model described in section 6.1.

fjern dette? This model is very simplified and suffer from many limitations, among others it is too simple to reflect the dynamics of a real world scenario, where each node will have different variables with different values. Although it tries to deal with the problem of correlated risks and preventing free riders from entering the trusted clique (interdependent security problem), each node have a complete network information i.e. the problem with information asymmetry is not taken into account. **slutt paa fjern dette**

6.2 Model 2 - Including Parameters

The first model is highly simplified and suffer from many limitations, among others it is too simple to reflect the dynamics of a real world scenario, where each node will have different variables with different values. To improve this model we have to introduce parameters, that can be adjusted and reflects real world scenarios. It is fair to assume that the insured nodes must pay an insurance premium, and this premium should be dependent on the number of links the node establishes. When two insured nodes establish a link between each other, they both have to pay a premium, this is to make the game more fair, and more realistic. For example if the two nodes had different insurance companies, then both companies would charge them for insuring the link. When a node, insured or not, establish a link to a non-insured node, this involves a risk, and this risk will be represented as a expected risk cost. However if the changes in payoff when establishing a link is only negative, then no node would want to establish links. Thus the nodes will also receive a positive change in payoff when establishing different links.

Characteristics of the model

The type of the nodes are given in advance, i.e. they are chosen to be insured or non-insured. The process of establishing link is a bidirectional decision. The insured nodes have to pay an insurance cost I_0, which represents the cost of signing a contract with an insurance company. I.e this could be an actual fee or a cost reflecting the work a player has to do to get a contract. The insurance premium is I_l, the expected risk cost is represented by r. β represents the benefit of establishing a link. Table **??** presents an overview of the parameters.

β - income from establishing a direct link
I_o - cost of having insurance.
I_l - increased insurance cost per link the node establishes
r - expected risk cost

6.2.1 Two nodes scenario

As a starting point lets look at the scenario involving only two nodes. In this game the strategy space of both players consist of four different strategies. A node can be insured or not, and choose whether to establish a link to the other node. I.e. the different strategies are: Be insured and establish link noted as: IL, be insured and not establish link: $I\overline{L}$. Not insured and establish link: $\overline{I}L$, and not insured and not establish link: $\overline{IL}$. It should be noted that since the decision to establish a connection is bidirectional, both have to choose a strategy where they want to establish a link, for the link establishment to be successful. Hence we end up with the game as shown in Figure 6.3.

As long as both I_l and r is less than β, the only nash equilibrium in Figure 6.3 is when both nodes chooses $\overline{I}L$. If we first look at node A, we see that when node B chooses IL, the best response is $\overline{I}L$, because $\beta > \beta - I_l$. And since the game is symmetric, the same holds for node B. When one of the nodes chooses $\overline{I}L$, the best response will be $\overline{I}L$, because $\beta - r > \beta - I_l - r$. And thus the only nash equilibrium is when both nodes play $\overline{I}L$.

This means that two nodes will end up in a classic prisoner's dilemma[1], where

[1] Prisoner's dilemma was originally framed by Merrill Flood and Melvin Dresher in 1950. The dilemma expresses a situation where two players each have two options whose output depends on the simultaneous choice made by the other. The original dilemma concerns two prisoners which separately decides whether to confess to a crime [Dic]. It is a paradox in decision analysis which shows why two individuals might not cooperate, even if it is in their best interest to do so.

Firm A \ Firm B	IL	$I\overline{L}$	$\overline{I}L$	$\overline{IL}$
IL	$\beta - Il$, $\beta - Il$	0, 0	$\beta - Il - r$, β	0, 0
$I\overline{L}$	0, 0	0, 0	0, 0	0, 0
$\overline{I}L$	β, $\beta - Il - r$	0, 0	$\beta - r$, $\beta - r$	0, 0
$\overline{IL}$	0, 0	0, 0	0, 0	0, 0

Figure 6.3: Normal form game, showing the different strategies and the payoffs for the different outcomes. The payoff are written in this order, A then B's. An agent has a strategy space of size 4. Maa ENDRES, FIKS NAVN IKKE FIRM, MEN NODE

the best response is actually worse than the social optimal. In this case it is trivial to see that the social optimal scenario is for both nodes to choose IL, as long as $I_l < r$. However, the nodes will choose not to buy insurance. Or else they could risk ending up in a case where they pay I_l without receiving any other benefit.

Solving the prisonersdilemma

One possibility for solving the problem that the two nodes end up choosing not to acquire insurance is to introduce a leader follower game. In this game the players does not act at the same time, but in order, and they can observe the other players action. If we consider a game with only two players, player one are the first to select an action. He chooses to insure or not. Then after observing this action player two chooses if he would like to insure or not. Then they choose if they would establish link or not, in the same order. In this type of game the leader, will benefit from a first mover advantage, because he can now force the game in a direction he prefers, as long as:

$$I_l < \beta \text{ and } I_l > \beta - r \text{ and } r < \beta \tag{6.1}$$

By finding all subgame equilibria in Figure 6.4 except the last one, i.e. the subgame where player one chooses to Insure or not, we get this subgame equilibria: $(L, \overline{L_1^I}, \overline{L_1^{II}}, L_1^{III}), (I_2, \overline{I_2^I}, L_2 L_2^I, \overline{L_2^{II}}, L_2^{III})$ We have now analyzed the two different outcomes of player 1 choosing insure or not, thus he can now see what find his best response. The two options he can choose between are: Insure and get payoff $\beta - I_l$ or not insure and get payoff $\beta - r$. I.e. if $I_l < r$ player one will chose to insure, and thus forcing the game to end up in a equilibrium where both players insure and establish link. If the cost of insuring is higher than the expected risk cost r, then obviously there is no reason to choose insurance. From this we see that if the insurance price are set to the right amount, the first player can force the outcome of the game to be the socially optimal outcome. The problem with this way of solving the problem is that it is very hard to solve for multiple nodes, because the extensive form game becomes extremely complicated.

6.2.2 Multiple nodes

Assumptions

To improve the second model we now introduce a scenario with multiple nodes. As before the type is give, i.e each node is chosen to be either insured or not. The objective of this model is to find characteristic network formations that will evolve endogenously when the parameters are within certain conditions. Examples of characteristic networks of interest are cliques, scale-free and star networks. We assume that every node has complete information of the network, i.e. every node knows the type of the other players. This is a very strong assumption, however

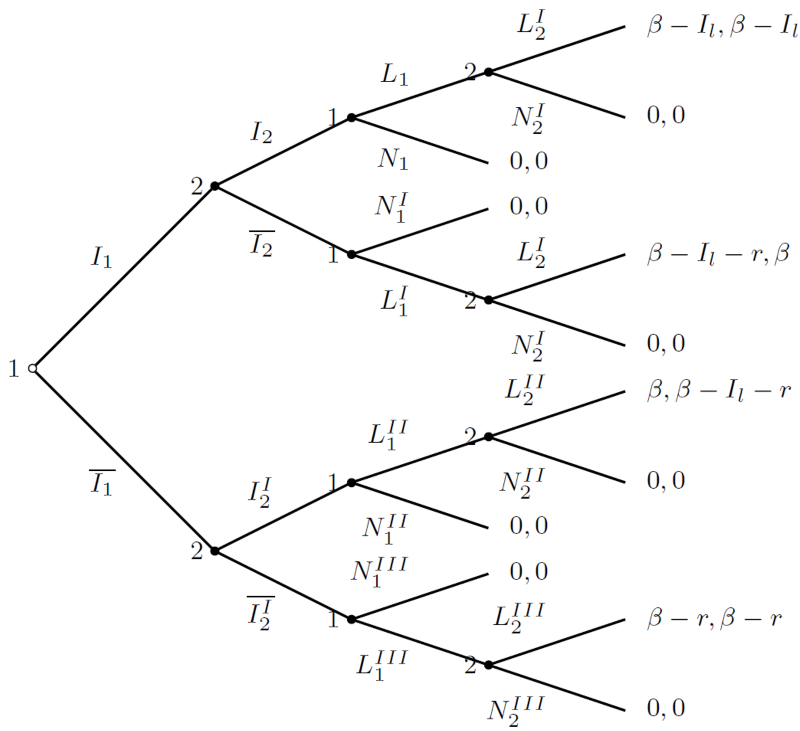


Figure 6.4: Leader follower game, first player 1 chooses to insure or not, then player 2, and then they choose to establish link or not in the same order.

in financial transactions and in cooperative software development networks, it is reasonable to assume that the parties can acquire this type of information prior to establishing a financial contract with each other.

Analyzis

As mentioned our goal is to find how and when certain network formations evolve. We know that if a node can increase his payoff by establishing a link, he will do so. Thus we can start analyzing the four possible link establishment scenarios, insured to insured, insured to non-insured, non-insured to insured, and non-insured to non-insured. Let U_i denote the payoff of a node with degree i, and let U_{i+1} be the payoff a node will receive if it establishes a new link.

Insured to insured When two insured nodes are considering establishing a link, they will do so, if and only if both receive a higher payoff. In this scenario the

the payoff function of adding a link is as shown in Eq.(6.2).

$$U_{i+1} = \begin{cases} \beta - I_l, & \text{if } i = 0 \\ U_i + \beta - I_l, & \text{if } i > 0 \end{cases} \tag{6.2}$$

In this scenario the condition shown in Eq.(6.3) has to hold.

$$I_l < \beta \tag{6.3}$$

Non-insured to insured The payoff a non-insured receives by connecting to a insured is as described in Eq. (6.4). As we see this will allways be a positive change in payoff, and thus an non-insured node will always choose to connect to an insured node.

$$U_{i+1} = \begin{cases} \beta, & \text{if } i = 0 \\ U_i + \beta, & \text{if } i > 0 \end{cases} \tag{6.4}$$

Insured to non-insured The payoff an insured node receives in this scenario is as follows:

$$U_{i+1} = \begin{cases} \beta - I_l - r, & \text{if } i = 0 \\ U_i + \beta - I_l - r, & \text{if } i > 0 \end{cases} \tag{6.5}$$

For this to happen Eq.(6.6) has to hold, a non-insured node will allways want to connect to a insured one, so this is the only condition that is needed for this to happen.

$$I_l + r < \beta \tag{6.6}$$

Non-insured to Non-insured The payoff a non-insured nodes receives when connecting to another non-insured node is as follows:

$$U_{i+1} = \begin{cases} \beta - r, & \text{if } i = 0 \\ U_i + \beta - r, & \text{if } i > 0 \end{cases} \tag{6.7}$$

For this link-establishment scenario to happen Eq.(6.8) has to hold.

$$\beta > r \tag{6.8}$$

Forming a trusted clique We want to find the conditions for when different network structures will evolve, for example a clique of only insured nodes. For this to happen, all insured nodes must connect to each other, i.e. Eq.(6.3) has to hold. But we also need to ensure that insured nodes do not establish links with non-insured nodes. I.e. this has to hold:

$$I_l + r > \beta \tag{6.9}$$

This gives us the limitation shown in Eq. (6.10) on the insurance link cost.

$$\beta - r < I_l < \beta \tag{6.10}$$

As we see from the condition, if the link insurance cost is between the two boundaries all the insured nodes will connect with each other, and no other nodes. If the link insurance cost is greater than β, then no insured node will establish any links. And if it is below $\beta - r$, then the insured nodes will also connect to the non-insured ones. It should also be noticed that as long as $r < \beta$, then the non-insured nodes will connect to each other.

6.2.3 Result and findings

From the analyzis we found different conditions on the link establishment process. If Eq.(6.10) is fulfilled, then the network will end up with one clique of only insured nodes. The non-insured nodes will end up in another clique if the risk of connecting to another non-insured node is less than the benefit of establishing link($r < \beta$). If the link insurance cost and risk of connecting to non-insured nodes is less than the benefit($I_l + r < \beta$), then insured nodes will also connect to non-insured nodes. And the network will end up in one giant clique.

These findings is independent of number of players, because we only consider one link at a time, and the change in payoffs is linear an independent of the nodes degree.

Stability versus efficiency When measuring stability in this model, it is easily seen that since the change in payoff when adding links is linear, and non-dependent on the nodes degree, the resulting network will be pairwise-stable. It also follows from the defintion of a nash equilibrium, that the resulting network is a equilibrium, since every player have best responded to the other players best responses, and no node can increase its payoff by single handedly changing a strategy. To calculate the efficiency we need to sum up the overall payoff, and compare it with the maximum possible payoff. The total payoff can be calculated as in Eq.(6.11), where $\sum I \times I$ represents the sum of payoffs achieved from links between insured nodes. $\sum I \times \overline{I}$ the sum of payoffs achieved from links between non-insured and insured, and $\sum \overline{I} \times \overline{I}$, the sum of payfoss achieved from links between non-insured and non-insured nodes.

$$U_{total} = \sum I \times I + \sum \overline{I} \times \overline{I} + \sum I \times \overline{I} \tag{6.11}$$

When the parameters are inserted in Eq.(6.11), we get the Eq.(**??**), where N_I and $N_{\overline{I}}$, represents the number of insured and non-insured nodes in the network.

$$U_{total} = N_I(N_I - 1)(\beta - I_l) + N_{\overline{I}}(N_{\overline{I}} - 1)(\beta - r) + N_I N_{\overline{I}}(2\beta - r - I_l) \tag{6.12}$$

If we calculate the overall payoff for a network with one-clique of insured and another with non-insured, i.e. Eq. (6.10) has to hold and $r < \beta$. The total payoff of this condition is as shown in Eq. (6.13).

$$U_{total} = N_I(N_I - 1)(\beta - I_l) + N_{\overline{I}}(N_{\overline{I}} - 1)(\beta - r) \tag{6.13}$$

However, this is not the socially best outcome, because in this scenario, $2\beta > r + I_l$, will always be true. Thus the socially best outcome would have been one clique, with both insured and non-insured nodes. The price of stability is shown in Eq. (6.14).

$$PoS = \frac{N_I(N_I - 1)(\beta - I_l) + N_{\overline{I}}(N_{\overline{I}} - 1)(\beta - r)}{N_I(N_I - 1)(\beta - I_l) + N_{\overline{I}}(N_{\overline{I}} - 1)(\beta - r) + N_I N_{\overline{I}}(2\beta - r - I_l)} \tag{6.14}$$

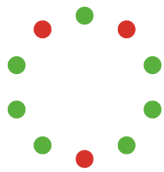
From this we see that the only scenario where the insurer are able to separate the two types of nodes, and have an efficient and stable outcome, is when there are only links between insured, or between non-insured, or no links at all. This can only happen when $2\beta < I_l + r$, and $I_l > \beta + \beta - r$ or $r > \beta + \beta - I_l$ or if both I_l and r is larger than β.

Simulation of the results

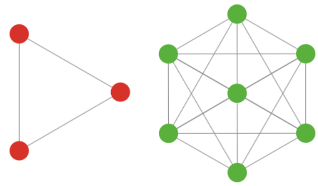
To verify that our calculations of the network formation where consistent with the assumptions our, we performed different simulations using NetLogo. The network formation is performed by selecting two random nodes, not neighboring each other, then both nodes checks whether they would prefer to establish a connection or not. The rules are as described earlier, when a node is considering establishing a link it chooses to do so if the payoff received is larger than the payoff he already poses, and the decision is bilateral. In the simulator a node is insured with a probability, p. This selection is repeated until the network are fully connected or no more nodes are willing to establish new connections. By selecting nodes at random and checking if both of them would like to connect to each other, we relax the assumption of full network information, because now nodes only get to know if another node is insured or not, by asking them.

In Figure 6.5 we see the result of a simulation with the parameters: $\beta = 0.9$, $I_l = r = 0.5$. With these parameters the Eq.(6.10) holds, and $r < \beta$. Thus the network formation game ends up in two cliques, one with insured nodes and another with non-insured. The result are shown in Figure 6.5b, and confirms our calculations. In this figure there are only included $n = 10$ nodes, this is done to make the figure readable and easy to understand. The same results where obtained when performing the simulation with larger values of n, however the resulting printouts was very complex and chaotic.

Secound Simulation In the next simulations, the parameters where chosen to violate the Eq.(6.10). The result can be seen in Figure 6.6. In figure 6.6a we see the result when $I_l < \beta - r$, the result is one clique of both insured and non-insured



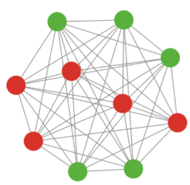
(a) Ten nodes randomly, with probability 0.5, chosen to be either insured (green) or non-insured (red).



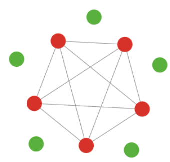
(b) Two clustered fully connected networks. One consisting of insured agents the other consists of non-insured. The link establishment is done by following the rules described above

Figure 6.5: The figure shows the resulting network from a simulation with parameters: $\beta = 0.9$, $I_l = r = 0.5$.

nodes. In figure 6.6b the insurance cost is $I_l > \beta$, and as we see only non-insured nodes connect to each other, because the insurance cost per link cost more than the benefit given from connecting to a new node, i.e. the insured ones choose not to establish any connections.



(a) Ten nodes insured with probability 0.5, the parameters where: $\beta = 0.9$, $I_l = 0.3$ and $r = 0.5$. I.e. the link insurance cost, I_l, is violating the condition in Eq.(6.10), and the resulting network is one clique of both insured and non-insured nodes.



(b) Ten nodes insured with probability 0.5, with parameters as before, except for the link insurance cost:$I_l = 0.95$. This resulted in a clique of only non-insured nodes.

Figure 6.6: The figure shows the two possible scenarios that violates the Eq.(6.10), 6.6a shows the result when $I_l < \beta - r$ and 6.6b shows the result when $I_l > \beta$.

6.3 Forcing non-insured nodes to buy insurance(FIX!!!!!!!!)

In this section we try to find a condition which gives all nodes incentive to buy insurance. The basic idea is to create a scenario where it is beneficial for a node to be insured, i.e the non-insured nodes wants to to purchase insurance. This scenario will also benefit the insurer, obviously because more nodes purchases insurance. In addition, the insurer now have incentive to handle the problem with asymmetry. In previous models, the insurer would have difficulty obtaining sufficient information to calculate a node's risk. Because the nodes would did not have incentive to provide information to the insurance company. Hence the insurance company could enter into risky contracts. Now, we have a different scenario. Since non-insured nodes want to be insured, they can be forced to give up information about their current condition, both financial and list possible risks. From the market survey, we found that companies offering cyber-insurance actually required this information. The information received can be analyzed in different ways. If the nodes provide enough information, the insurer are able to calculate the risk, and offer a premium. On the other hand, if a node acts suspiciously and tries to hide information from the insurer, the insurer have reasonable cause to not chose to insure the node. Either way, it is a seller's market, where the insurance company can dictate the outcome of the network by pricing the insurance according to equations provided in this section.

Initial conditions are equal to the previous, where every node are randomly chosen to be either insured or not. Hence we could use the same payoff matrix as shown in Figure 6.3 to analyze how we could force the non-insured nodes to purchase insurance. In order to give incentive for a non-insured node to purchase insurance, the payoff has to always be higher, i.e.

$$\textit{Utility insured node} > \textit{Utility non-insured node} \tag{6.15}$$

This means that we need to make sure that a non-insured node in any circumstances will benefit from purchasing insurance. From the payoff matrix, Figure 6.3 we find the different conditions. When a connection is established, we need the payoff for insured nodes to be higher than non-insured nodes:

$$\begin{aligned} & \beta - I_0 - I_l > \beta - r \\ \rightarrow \quad & I_0 + I_l < r \end{aligned} \tag{6.16}$$

For the other case, when the nodes have not established any connections the following has to hold:

$$\begin{aligned} & -I_0 > -r \\ \rightarrow \quad & I_0 < r \end{aligned} \tag{6.17}$$

In addition we need to make sure that it is not beneficial for insured nodes to connect to non-insured nodes:

$$I_l + r < \beta \tag{6.18}$$

If these conditions are met, we are guaranteed to get a network consisting of only insured nodes. Because in any case, the non-insured nodes will get a higher payoff from purchasing insurance. It is interesting to see that both conditions are completely dependent upon how the insurance company chooses to price their products. If the insurer collects enough information to calculate an accurate risk, he could price both I_0 and I_l to meet the conditions. Hence he forces the network to end up with every node having incentive to purchase insurance.

6.3.1 Violating the conditions

Since the risk are difficult to calculate, there is a possibility of ending up in states where a node would actually benefit from doing the opposite. If the actual scenario ends up with the following conditions:

$$\begin{aligned} I_0 &< r \\ I_l &> r \end{aligned} \tag{6.19}$$

Now we will have a situation where it first looks beneficial to be purchase insurance. However, as the nodes adds more connections, and pays I_l pr connection, the node would actually be better of with not being insured. This demonstrates the importance of being able to accurately calculate the risk.

6.4 Model 3 - Including maximum node degree and bonus

In real world networks, such as in the manufacturing industry, software development firms and many other types of business, a product can not be completed without outsourcing some of the task needed. For the manufacturer, it could be beneficial to buy certain parts from others instead of producing them on their own. A software product might need the combined knowledge from different firms. Thus the firm that outsource tasks are dependent on the other firms, and will not reach their goal before the other firms deliver their contribution. For example, lets consider a software company who want to develop a new product. However, they do not have the required resources or knowledge to complete the product, and will therefore need help from other companies with the desired knowledge or resources. When the product is finished the company get paid, but not before, to finish the product they need to cooperate with others. This process of outsourcing introduces a risk of failure due to other parties. To model this scenario we introduce a maximum node degree per node, and a bonus γ, which represents the payoff when a node reach their desired number off established connections, i.e. their maximum node degree(m). Except from this the game is as before.

6.4.1 Analyzis

This model is very similar to the earlier model, for nodes to connect to each other, the change in payoff still has to be positive: $U_{i+1} > U_i$. However, we also need to consider the bonus received when reaching the maximum node degree, m. To model this we add, the possible bonus divided on the number of links required to reach the bonus($\frac{\gamma}{m-i}$), every time a node is considering a link establishment. In this way the model will change from the former models, because now the nodes have more incentive to connect to other nodes, and for every step closer to the goal, the nodes are more willing to accept risk than before. For example, an insured node is more likely to accept a risky link when it only need one more link to reach the goal. Compared to when it needs many more links to reach the goal.

The model now introduces a risk factor, because it is not certain that the nodes will obtain enough links, and if not, they will not receive their bonus, and they are stuck with the already established connections.

To analyze this model, lets take a closer look on the four different scenarios of the game.

Insured to insured When establishing a link between two insured nodes, the payoff the nodes will receive is as described in Eq. (6.20).

$$U_{i+1} = \begin{cases} \alpha + \beta - I_0 - I_l, & \text{if } i = 0 \\ U_i + \beta - I_l, & \text{if } i > 0 \\ U_i + \beta - I_l + \gamma, & \text{if } i = m \end{cases} \tag{6.20}$$

As described earlier we need to include the possibility of reaching the goal in the decision, and thus for insured nodes to connect to each other, Eq. (6.21) has to hold.

$$\begin{aligned} U_i + \beta - I_l + \frac{\gamma}{m-i} &> U_i \\ \beta - I_l + \frac{\gamma}{m-i} &> 0 \\ \rightarrow \qquad \beta + \frac{\gamma}{m-i} &> I_l \end{aligned} \tag{6.21}$$

Insured connect to non-insured The payoff an insured node received in this scenario is as follows:

$$U_{i+1} = \begin{cases} \alpha + \beta - I_0 - I_l - r, & \text{if } i = 0 \\ U_i + \beta - I_l - r, & \text{if } i > 0 \\ U_i + \beta - I_l - r + \gamma, & \text{if } i = m \end{cases} \tag{6.22}$$

To establish a connection from an insured node to a non-insured one, the following has to hold:

$$\begin{aligned} U_i + \beta - I_l - r + \frac{\gamma}{m-i} &> U_i \\ \beta - I_l - r + \frac{\gamma}{m-i} &> 0 \\ \rightarrow \qquad \beta + \frac{\gamma}{m-i} - r &> I_l \end{aligned} \tag{6.23}$$

Non-insured to non-insured When a non-insured node connect to another not-insured node this is the payoff they receive:

$$U_{i+1} = \begin{cases} \alpha + \beta - r, & \text{if } i = 0 \\ U_i + \beta - r, & \text{if } i > 0 \\ U_i + \beta - r + \gamma, & \text{if } i = m \end{cases} \tag{6.24}$$

To establish the connection the following equation has to hold:

$$\begin{aligned} U_i + \beta - r + \frac{\gamma}{m-i} &> U_i \\ \beta - r + \frac{\gamma}{m-i} &> 0 \\ \rightarrow \qquad \beta + \frac{\gamma}{m-i} &> r \end{aligned} \tag{6.25}$$

Non-insured to insured

$$U_{i+1} = \begin{cases} \alpha + \beta, & \text{if } i = 0 \\ U_i + \beta, & \text{if } i > 0 \\ U_i + \beta + \gamma, & \text{if } i = m \end{cases} \tag{6.26}$$

As we see, this is a strictly increasing function, and thus a non-insured will always connect to an insured node if possible.

6.4.2 Result and findings

If we want an clique of only insured nodes, we have to ensure that insured nodes connect to each other, and that they do not establishes connections to non-insured nodes. We know that an insured node would want to connect to another insured node if Eq.(6.21) is satisfied. In the equation we see that the expected bonus per established link is increasing, i.e. if an insured node of degree zero is willing to connect to another insured node, then every node with a degree higher than zero also would like to connect to another insured node. Thus to ensure that insured nodes connect to each other this equation has to hold:

$$\beta + \frac{\gamma}{m} > I_l \tag{6.27}$$

We also want to ensure that insured nodes never establishes links with non-insured nodes, from 6.22 we see that this has to hold:

$$\beta + \frac{\gamma}{m-i} - r < I_l \tag{6.28}$$

This can be simplified, if one can ensure that the least risk averse insured node, i.e. the node with degree $m-1$, do not establish links with non-insured nodes. Then we know that no insured node with degree less than $m-1$ will establish link with non-insured nodes. From this we get the equation Eq. (6.29).

$$\begin{aligned} \beta + \frac{\gamma}{m-(m-1)} - r &< I_l \\ \rightarrow \qquad \beta + \gamma - r &< I_l \end{aligned} \tag{6.29}$$

To summarize, Eq.(6.27) and Eq.(6.29) gives the final limitation on the link insurance cost, Eq.(6.30). If this equation is satisfied the resulting network will contain a clique of only insured nodes.

$$\beta + \gamma - r < I_l < \beta + \frac{\gamma}{m} \tag{6.30}$$

For this to even be possible $\beta + \gamma - r < \beta + \frac{\gamma}{m}$, i.e. Eq.(6.32) has to hold. This equation reflects that as the risk to bonus ratio gets smaller, it gets more and more unlikely to ensure a clique of only insured nodes. When the risk to bonus ratio is less than $1 - \frac{1}{m}$, such a clique will never occur. The equation really expresses a scenario in which you would be more willing to take a risk if the reward of doing so is large. On the other side it is also useful to know when non-insured nodes connect to each other, this happens when Eq.(6.24) is satisfied. This equation is dependent on the node degree, and thus for the first link to be established from a non-insured node the expected payoff has to be higher than the risk($\beta + \frac{gamma}{m} > r$). If the risk is to high, then the non-insured node must wait for one or more insured node who are willing to establish links with non-insured nodes. With these findings, an insurer can easily determine the outcome of the network formation game, by adjusting the insurance cost parameter. If he want a clique of only insured nodes Eq.(6.30) has to hold. However, it is easy to relax the condition, such that a insured node only connect to, $j = 1, 2, 3..m$ nodes, this is done by changing Eq.(6.29) to $\beta + \frac{\gamma}{m-(m-j)} - r < I_l$, which gives us Eq. (6.31). An interesting result in this model is that due to the risk willingness among the nodes, the lower boundary on the link insurance cost has increased compared to the one in model 2.

Consequences of not reaching required number of edges When a node establishes a link, it does not know whether it will reach the maximum node degree, unless the current node degree is $m-1$. Hence the node might end up not reaching the desired goal. This can happen if there is not enough nodes willing to establish links. If the bonus to risk ratio is very high, or nodes need lots of links, then this is a likely scenario. Resulting in nodes who do not reach their goal will end up with a payoff less than U_0.

$$\beta + \frac{\gamma}{j} - r < I_l \tag{6.31}$$

$$\begin{aligned} \gamma - r &< \tfrac{\gamma}{m} \\ 1 - \frac{r}{\gamma} &< \tfrac{1}{m} \\ \rightarrow \qquad 1 - \frac{1}{m} &< \tfrac{r}{\gamma} \end{aligned} \tag{6.32}$$

Efficiency and Stability(SKRIVE NOE OM DETTE, selvom det er vanskelig å analysere In this model, the incentive for establishing links has been increased. Thus to maintain a stable network with two cliques, the cost of link establishment has to be increased, compared to model 2. This increased incentive may result in a higher price of stability.

Simulation of the results

For the first simulation the parameters are set to the following: $\beta = 0.9, I_l = 0.7, r = 0.5, \gamma = 0.2$ and $m = 5$, in order to satisfy the condition Eq.(6.30).

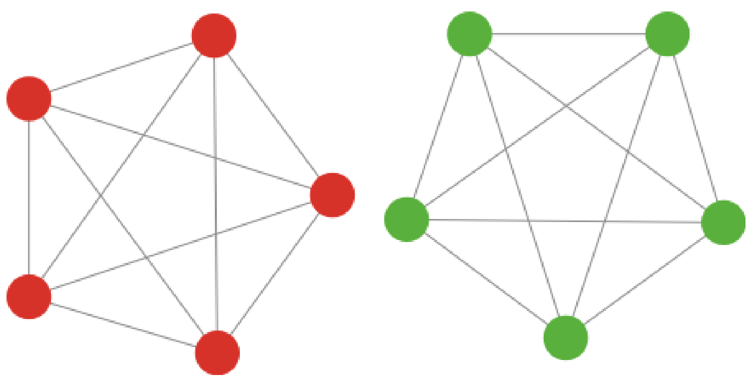


Figure 6.7: Two cliques, one consisting of insured agents the other consists of non-insured. All nodes have reached their goal.

As we see in Figure 6.7 the results where as expected, the cost of insuring a link satisfied the conditions found earlier and thus the result where two cliques, one consisting of only insured and the other of non-insured nodes. An interesting thing to notice is that β and r is the same as in model 2, but to ensure that only insured connect to each other, the link insurance cost needs to be higher. This is to compensate for the risk the nodes now are willing to take.

Second simulation If we change the link insurance cost to the same value as in model 2, $I_l = 0.5$, the result is as depicted in Figure 6.8. Here we see that allmost every insured node has taken the risk of connecting to a non-insured node in order to achieve their goal. By inserting the numbers in to Eq.(6.31) we see that the insured nodes are willing to connect to up two non-insured nodes to reach their goal.

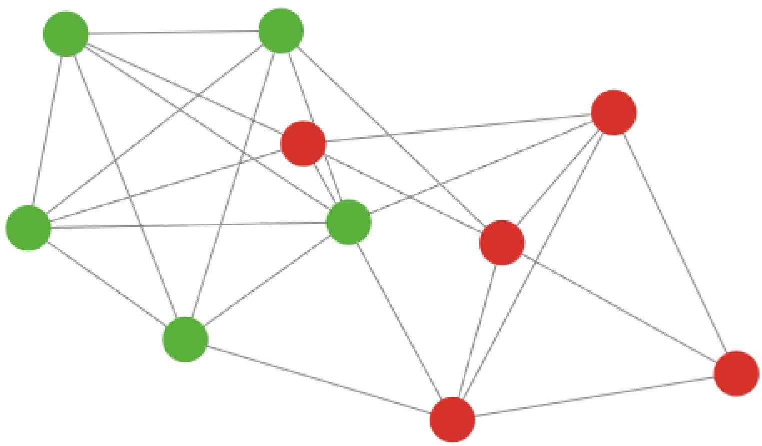


Figure 6.8: Simulation when the cost of insuring a link is just below the limits.

6.5 Model 4 - Including bulk insurance discount

Putte det som starter her i background? Insurance companies often interpret a quantum discount when purchasing multiple products. From convenience stores we are used to the slogan "buy one get one for free". It seems to be common for insurance companies to offer discount to their customers if they choose to collect some or all of their insurances with them. Several insurance companies in Norway, such as Sparebank 1 offers customers up to 25 % discount according to the following rules [Spa].

- 10% discount if the person has signed three different insurances
- 15% discount if the person has signed four different insurances

– 20% discount if the person has signed five or more different insurances

– Plus additional 5% discount if the person is a customer of the bank.

The insurance offered is intended to the individual market and includes among others: travel insurance, household insurance, car insurance, house insurance, insurance of valuable items and yacht insurance. **til hit!!**

So far our model reflects that a company have to insure each of the links to other nodes, inspired by other insurance products, we would like to introduce a discount rate following the degree of the nodes. This will make it more attractive for nodes with high degree to acquire insurance, and this could act as a incentive for other nodes to also acquire insurance. Thus this seems like a reasonable model to include.

How insurance companies choose to formulate their discount rate might vary. One solution might be to follow a strict 5% discount per new connection, similar to the one from Sparebank 1(REFERER TIL CHAPTER BACKGROUND), or let the discount follow a power law. However, we choose to follow a discount rule which directly reflects the number of links the node has established.

6.5.1 Analyzis

The price for adding a new link follows the equation:

$$\frac{I_l}{i+1} \tag{6.33}$$

Here, i is the current number of established connections. This means that the more connections a node acquire the cheaper the links will be.

Discount model

We start our analyzis by considering a model where only the discount are included, not the bonus, and as before we analyze the four different connection scenarios. However, it is only the scenario where insured connects to other insured nodes and insured to non-insured nodes, that has changed compared to model 2.

Insured to insured When we add the discount to the conditions found in model 2 we find the condition shown in Eq. (6.34).

$$\frac{I_l}{i+1} < \beta \tag{6.34}$$

Insured to non-insured For this scenario to be possible Eq. (6.35) has to hold.

$$\frac{I_l}{i+1} + r < \beta \tag{6.35}$$

Result and findings For an insurer to be able to ensure that the network ends up in a clique with only insured nodes, we must ensure that the most expensive link establishment, i.e. the first, to another insured node can be achieved. This gives us the same condition as in model 2, i.e. $I_l < \beta$. We also need to ensure that insured nodes does not connect to non-insured, thus we get the final condition in Eq. (6.36), where N_I is the number of insured nodes in the network.

$$(N_I - 1)(\beta - r) < I_l < \beta \tag{6.36}$$

This is condition is very strong, because it says that $\beta - r < \frac{1}{N_I - 1}$, and as the number of insured nodes gets higher this get more and more unlikely. Thus by including bulk-discount, the insurer are making it harder for himself to constrain the network formation. This is because the incentive for establishing links are higher than without discount, and thus more links will be established.

Price of Stability versus efficiency If we compare the total payoff equation in this model, see Eq.(6.37), with the one in model 2 (Eq.(6.12)). We see that the cost for insured nodes has changed, and therefore the payoff generated from links between insured nodes has increased, and so have the payoff received from links between insured and non-insured nodes. As we know, in a scenario where the insurer sets the cost, such that the network will end up in two cliques, the payoff received from links between insured and non-insured are zero. This potential payoff, in a scenario where there are two cliques, can be described like this: $(N_I N_{\overline{I}}\beta + N_I(-\sum_{i=N_I}^{N_{\overline{I}}-1} \frac{I_l}{i}))$, and as long as $(N_I N_{\overline{I}}\beta > N_I(-\sum_{i=N_I}^{N_{\overline{I}}-1} \frac{I_l}{i}))$ it would have been socially optimal to have one-clique of both insured and non-insured nodes. When the cost of establishing links decreases and the insurer forces the network formation to end up in two cliques, the price of stability will increase compared to the price of stability in model 2. This is because the incentive for establishing links has increased, and thus for the insurer to be able to constrain the network formation, the cost of establishing links has to be higher.

$$U_{total} = (N_I(N_I-1)\beta - N_I \sum_{i=1}^{N_I-1} \frac{I_l}{i}) + (N_{\overline{I}}(N_{\overline{I}}-1)(\beta - r)) + (N_I N_{\overline{I}}\beta + N_I(-\sum_{i=N_I}^{N_{\overline{I}}-1} \frac{I_l}{i})) \tag{6.37}$$

Discount and Bonus model

We also need to apply the discount to the model where the bonus is included. We can see that only the scenario where insured nodes connects to either other insured nodes or non-insured nodes are affected. The analysis will therefore cover these two scenarios.

Insured to insured If we add the new rule to the Eq.(6.20) which shows the connection between two insured nodes, we get the following equations:

$$U_{i+1} = \begin{cases} \beta - I_l, & \text{if } i = 0 \\ U_i + \beta - \frac{I_l}{i+1}, & \text{if } i > 0 \\ U_i + \beta - \frac{I_l}{i+1} + \gamma, & \text{if } i = m \end{cases} \tag{6.38}$$

For insured to connect to each other Eq.(6.39) has to hold.

$$\begin{aligned} U_i + \beta - \frac{I_l}{i+1} + \frac{\gamma}{m-i} &> U_i \\ \beta - \frac{I_l}{i+1} + \frac{\gamma}{m-i} &> 0 \\ \rightarrow \qquad \beta + \frac{\gamma}{m-i} &> \tfrac{I_l}{i+1} \end{aligned} \tag{6.39}$$

Insured to non-insured When insured nodes are considering connecting to non-insured, they will receive the following payoff.

$$U_{i+1} = \begin{cases} \beta - I_l - r, & \text{if } i = 0 \\ U_i + \beta - \frac{I_l}{i+1} - r, & \text{if } i > 0 \\ U_i + \beta - \frac{I_l}{i+1} - r + \gamma, & \text{if } i = m \end{cases} \tag{6.40}$$

For this scenario to happen Eq.(6.41) has to hold.

$$\begin{aligned} U_i + \beta - \frac{I_l}{i+1} + \frac{\gamma}{m-i} - r &> U_i \\ \rightarrow \qquad \beta + \frac{\gamma}{m-i} &> r + \tfrac{I_l}{i+1} \end{aligned} \tag{6.41}$$

6.5.2 Result and findings

If we analyze the same scenario as in the other models, namely a clique of only insured nodes. The first step is to ensure that insured nodes connect to each other when the expected payoff is lowest, i.e. at node degree zero. If they are willing to establish link at this point, then they will also be willing at all degrees higher than zero. At degree zero there is no discount on the insurance link cost, and thus if Eq.(**??**) from model 3 holds, insured nodes will connect to other insured nodes.

However the condition for ensuring that insured nodes do not connect to non-insured has changed, we know if an insured node do not want to establish a link with a non-insured at degree $m-1$, then no insured node with degree lower than $m-1$ will either do so. From this we find the condition, see Eq.(6.42)

$$\begin{aligned} U_i + \beta - \frac{I_l}{m} + \frac{\gamma}{m-(m-1)} - r &< U_i \\ \beta + \gamma - r &< \tfrac{I_l}{m} \\ \rightarrow \qquad m(\beta + \gamma - r) &< I_l \end{aligned}$$

This is a very strong condition, because the only way this can happen is if $\beta+\gamma-r < \frac{1}{m}$. This shows us that when the incentives for establishing links increases, it gets more and more difficult for the insurer to ensure a clique of only insured nodes. The final condition for ensuring a clique of only insured nodes is shown in Eq. (6.42).

$$m(\beta + \gamma - r) < I_l < \beta + \frac{\gamma}{m} \tag{6.42}$$

Similar calculation can be done for the other three scenarios in the game, and they all show the same. The quantum discount results in a overall higher payoff for the nodes, since the cost of insuring a new link becomes cheaper. This means that the nodes will have a higher incentive to create links to each other. Which makes it harder for the insurer to separate insured and non-insured nodes.

6.6 Model with incomplete information

An interesting scenario to model is when the nodes lack information about the other nodes type. The way we model this is by letting nature selecting whether a player is insured or not, a node is insured with probability p, and not insured with probability $1-p$. All nodes know their own type, but in the link establishment process there are only one node who knows the type of the other. The other node only know the probability of the other node being insured or not. What we want to find is if it possible for the nodes with incomplete information to distinguish a insured node from a non-insured one. In order to form insurable topologies although we have a scenario with incomplete information.

6.6.1 Analyzis

When facing a game like this, there exists two types of equilibriums, one where node 2 is able to seperate node 1's type, seperating equilibrium. The another where he can seperate them, pooling equilibrium. In this game we have two types of node, type 1 ($t1$): insured and type 2 ($t2$): not insured.

Node 2 is insured Since every node knows their own type, there are two different games to model, one where node 2 is insured, and the other where he is not insured. We start with the one where he is insured. Node 1's type is chosen randomly by nature, with probability p of being type 1 and $1-p$ of being type 2.

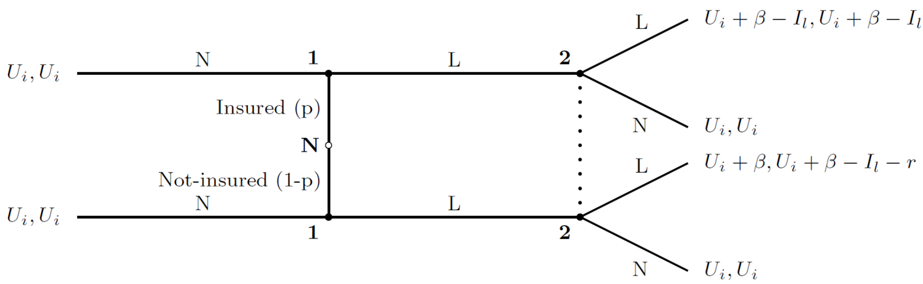


Figure 6.9: Signalling game with two nodes, node 1's type choosen by nature, node 2 is insured. Node 1 have complete information, node 2 suffer from incomplete information, and act on best response functions based on beliefs.

In the extensive-form shown in Figure 6.9, we see that $t2's$ strategy L dominates N, and thus $t2$ will never play N.

Separating equilibrium Since node 1 will never play N as type 2, there are only one possible separating equlibrium, type 1 plays L and type 2 plays N. Hence node 2's beliefs are as in Eq.(6.43).

$$\sigma_1(t_i) = \begin{cases} N, & \text{if } t1 \\ L, & \text{if } t2 \end{cases} \tag{6.43}$$

Let $\mu_1(t_i|N)$, denote the probability that node 1 is of type t_i. By using bayes rule we get this equation:

$$\mu_1(t_1|N) = \frac{P(N|t_1)P(t_1)}{P(N)} = \frac{P(N|t_1)P(t_1)}{P(N|t_1)P(t_1) + P(N|t_2)P(t_2)} \tag{6.44}$$

With node 2's belief, we get that $\mu_1(t_1|N) = 1$ and $\mu_1(t_2|L) = 1$. We can now calculate node 2's expected utility from playing L and N:

$$\begin{aligned} EU_2(L,L) &= \mu_1(t_1|L)U_2(L,L;t_1) + \mu_1(t_2|L)U_2(L,L;t_2) \\ \rightarrow \quad & EU_2(L,L) = U_i + \beta - I_l - r \qquad (6.45) \\ EU_2(N,L) &= \mu_1(t_1|L)U_2(N,L;t_1) + \mu_1(t_2|L)U_2(N,L;t_2) \\ \rightarrow \quad & EU_2(N,L) = U_i \qquad (6.46) \end{aligned}$$

From these two equations we see that the best response of node $2(BR_2)$ when he observes the other node choosing action L is:

$$BR_2(L) = \begin{cases} L, & \text{if } \beta - r \geq I_l \\ N, & \text{if } \beta - r < I_l \end{cases} \qquad (6.47)$$

Node 2's expected utility when type 1 chooses N, is easily seen to be U_i. To confirm if this is a separating equilibrium we must see if node 1 has any incentive to deviate from the strategies in node 2's belief. Type 2 will never deviate, so lets investigate type 1. In order to get node 1 to be willing to play N when he knows node 2's best response function, the following must hold: $\beta < I_l$. If this is true, then node 2's best response is to play N. I.e. the only separating equilibrium is the following:

$$\beta < I_l \qquad (6.48)$$

$$\sigma_1 = \begin{cases} N, & \text{if } t1 \\ L, & \text{if } t2 \end{cases} \qquad (6.49)$$

$$BR_2(\sigma_1) = N \qquad (6.50)$$

This means that in a separating equilibrium, the game will end up with no link establishment.

Pooling equilibrium In a pooling equilibrium node 2 will not be able to distinguish the two types, and since $t1$'s strategy L dominates N, i.e. there is only one possible equilibrium, the one where both types of node 1 plays L.

$$\sigma_1(t_i) = \begin{cases} L, & \text{if } t1 \\ L, & \text{if } t2 \end{cases} \qquad (6.51)$$

By using bayes rule we get that $\mu(t_1|L) = p$ and $\mu(t_2|L) = 1 - p$. Node 2's expected utility is then:

$$\begin{aligned} EU_2(L,L) &= p(U_i + \beta - I_l) + (1-p)(U_i + \beta - I_l - r) \\ \rightarrow \quad & EU_2(L,L) = U_i + \beta - I_l - r + pr \qquad (6.52) \\ & EU_2(N,L) = U_i \qquad (6.53) \end{aligned}$$

From this we get node2's best response:

$$BR_2(L) = \begin{cases} L, & \text{if } \beta + rp - r \geq I_l \\ N, & \text{if } \beta + rp - r < I_l \end{cases} \tag{6.54}$$

By using this best response function, node 1 sees that as long as $\beta > I_l$ he will never deviate from node 2's beliefs. And it is a pooling equilibrium where both nodes choose L, as long as $\beta > I_l$ and $\beta + rp - r > I_l$. We also know that: $rp - r \leq 0$ is allways true, and thus there also exists a pooling equilibrium where node 1, plays L, and node 2, plays N. This equilibrium will occur when $\beta > I_l$ and $\beta + rp - r < I_l$.

Node 2 not insured Here we will analyze the game when node 2 is not insured. The rules of the game are as before, the only thing that has changed is the type of node 2, and thus the payoffs are different and we need to see if there exists separating and pooling equilibrium in this game as well.

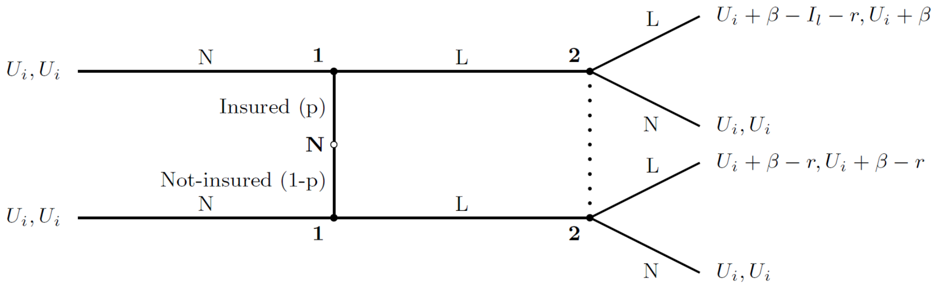


Figure 6.10: Signalling game with two nodes, node 1's type choosen by nature, node2 is not insured. Node 1 have complete information, node 2 suffer from incomplete information, and act on best response functions based on beliefs.

Separating equilibrium In this game there is no dominant strategy for node 1, thus we have to check for the two possible separating equilibriums. We start with the separating equilibrium with the beliefs shown in Eq.(6.55).

$$\sigma_1(t_i) = \begin{cases} L, & \text{if } t1 \\ N, & \text{if } t2 \end{cases} \tag{6.55}$$

With the beliefs in Eq.(6.55), this is node 2's expected payoffs:

$$EU_2(L, L) = (U_i + \beta) \tag{6.56}$$

$$EU_2(N, L) = (U_i) \tag{6.57}$$

From this we see that his best response when node 1's action is L, is to allways play L:

$$BR_2(L) = L \tag{6.58}$$

To see if this is an equilibrium, we have to see if node 1 has any incentive to deviate. We need to check for the two types of node 1: If $\beta > r$ then type 2 would deviate, because he could achieve a higher payoff by playing L, given the beliefs of node 2 in Eq.(6.55). So we know that for this to be an equilibrium,

$$\beta < r \tag{6.59}$$

When analyzing from node 1 type 1's perspective, for him to play L, this has to hold: $U_i + \beta - I_l - r > U_i$. The only way this can hold is if $\beta > I_l + r$. We see that Eq.(6.59) is violating this condition, and thus we have no separating equilibrium with the beliefs in Eq.(6.55).

Now lets look at the other possible separating equilibrium, see Eq.(6.60).

$$\sigma_1(t_i) = \begin{cases} N, & \text{if } t1 \\ L, & \text{if } t2 \end{cases} \tag{6.60}$$

Node 2's expected payoffs are as follows:

$$EU_2(L, L) = U_i + \beta - r \tag{6.61}$$

$$EU_2(N, L) = U_i \tag{6.62}$$

From this we get the best response function:

$$BR_2(L) = \begin{cases} L, & \text{if } \beta \geq r \\ N, & \text{if } \beta < r \end{cases} \tag{6.63}$$

For this to be a separating equilibrium, we need to see if node 1 would deviate from node 2's beliefs. Type $t1$ will not deviate as long as $\beta < I_l + r$. Type $t2$ will not deviate if $\beta \geq r$, if this condition is true, we see that node 2 will play L. I.e. the only separating equilibrium that exists is when node 2 plays L, node 1 of type $t1$ plays N and node 1 of type$t2$ plays L. For this to happen we get this condition on β.

$$I_l + r > \beta > r \tag{6.64}$$

Pooling equilibrium Two possible, one where both types of node 1 plays L, and one where both types plays N. Lets first analyze the one where both types of node 1 plays L.

$$\sigma_1(t_i) = \begin{cases} L, & \text{if } t1 \\ L, & \text{if } t2 \end{cases} \tag{6.65}$$

With the beliefs shown above, node 2's expected payoffs are:

$$EU_2(L) = p(U_i + \beta) + (1-p)(U_i + \beta - r)$$
$$EU_2(L) = U_i + \beta - r + pr \tag{6.66}$$
$$EU_2(N) = U_i \tag{6.67}$$

From this we get the best response function :

$$BR_2(L) = \begin{cases} L, & \text{if } \beta \geq r - pr \\ N, & \text{if } \beta < r - pr \end{cases} \tag{6.68}$$

Will node 1 deviate knowing this? Type $t1$ will not deviate as long as: $\beta - I_l \geq r$, and type $t2$ will not deviate as long as $\beta > r$. From this we get this final condition, if $\beta - I_l \geq r$ then there exists a pooling equilibrium where both types of node 1 plays L and node 2 also play L. From this we see that the other pooling equilibrium where both types of node 1, plays N, will only occur when $\beta < r$ and $\beta < I_l + r$.

Result and findings When one player lack knowledge about the other player, we only found two scenarios where he could separate the two types of the other node. This is possible when player 2 is insured and $\beta < I_l$. He can then separate the insured and non-insured types of the other node, because it is only the non-insured node who would want to establish link. Since $\beta < I_l$ his best response is to not establish any link.

The other scenario where the node with incomplete information are able to seperate is when he is not insured, and $r < \beta < I_l + r$. In this scenario it is only the non-insured node who would want to establish a link, and this is beneficial for both. Thus in this scenario the game will end up with a link between two non-insured nodes.

We where also able to find some pooling equilibriums, if the node with incomplete information is insured, a link will be established if $\beta + rp - r > I_l$. However, if $I_l < \beta$ but $I_l > \beta + rp - r$, then the pooling equilibrium will be that node 1 wants to establish link, but node 2 rejects. A pooling equilibrium where both nodes want to establish a link, occur when node 2 is not insured and $\beta - I_l > r$. If $\beta < r$ there will be a pooling equilibrium where both players choose not to establish link.

What this shows us is that when one player suffer from incomplete information, it is no longer possible for the insurer to force a network to evolve into a clique of only insured nodes. It will also be harder to establish links, because one player must act on beliefs.

6.7 Model 5-The connection game

In the earlier model, the experienced network effects only arose from their neighbours. I.e. when a node established a connection the change in utility where only dependent on fixed variables, and not dependent of the rest of the network. In most real world scenarios there is more realistic that a node will also be strongly affected by the indirect connections to other nodes. Social relationships between nodes are good examples of such networks, where they offer benefits in terms of favors, information etc.

We apply the results from the paper from Jackson and Wolinsky [JW96] and uses a network formation game in [Jac05], to study indirect networks effects in our model.

The benefits a player receives in this game are calculated as follows. In addition to the benefit from the direct connection, a node will also benefit from "the friends of the friend", and "the friends of the friends of the friend" etc. This is achieved by letting the payoff be calculated relative to the distance between the nodes. β is now dependent on the minimum number of hops to the node e.g. the benefit of a direct connection is β, the benefit of a friend of a friend is β^2 etc. We want the benefit to decrease with the distance, therefore we need the limitation: $0 < \beta < 1$.

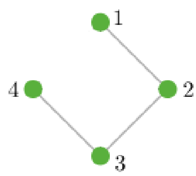


Figure 6.11: Four nodes interconnected with each other.

Example Lets consider the network shown in 6.11. Node 1 and node 4, in the network will receive a benefit of $\beta + \beta^2 + \beta^3$ by being connected with node 2 and 3. $\beta^2 + \beta^3$ represents the indirect benefits from node 3 and 4. Node 2 and 3 receives a benefit of $\beta + \beta + \beta^2$. For this network to make sense, it is important to also include some cost of having direct connections, or else the rational thing would be to establish a link with everyone. This is done as in earlier models, every node pay a cost for direct connections, but no cost for indirect connections. Thus the total payoff for a node is:

$$\sum_{j \neq i} \beta_{ij}^{d(ij)} - \sum_{j:ij \in g} c_{ij}, \tag{6.69}$$

where $d(ij)$ represents the shortest path between node i and node j, and c_{ij} represents node i's cost of establishing a link between the two nodes. To simplify the model we choose a symmetric connection process where β and c is set to a fixed global value.

In the paper [JW96], they analyze two different networks outcomes, one with the focus on efficiency and the other on pairwise stability. An efficient network means ending up with a network where the sum of every nodes payoff is maximized. The optimal network is of course both efficient and stable, but as we shall see there are some conflicts between efficiency and stability. In the paper they found that an efficient network is:

1. *a complete graph g^N if $c < \beta - \beta^2$,*
2. *a star encompassing every node if $\beta - \beta^2 < c < \beta + \frac{(N-2)}{2}\beta^2$,*
3. *an empty network(no links) if $\beta + \frac{(N-2)}{2}\beta^2 < c$.*

The most efficient structure is created in the intermediate cost of insuring links, and ends up in a star structure which encompasses every node. A star structure have the characteristics of minimizing the average path length and uses the minimum number of links($N-1$) required for including every node. Indisputable this structure provides the highest overall payoff for the network, but this network is not necessarily stable, as we will show later.

Pairwise stability:(HVERTFALL ISH) A graph is pairwise stable if:

1. *No node wishes to delete a link he is involved in.*
2. *If there exists a node who want to add a link, then the node on the other end of the link do not want to establish this link.*

The limitations of pairwise stability is that we only consider one link and one pair of nodes at a time.

When analyzing the stability of the network, by using the definition of pairwise stability, Jackson and Wolinsky found four different stability conditions:

1. *a pairwise stable network consists of at most one (non-empty) component,*
2. *if $c < \beta - \beta^2$, the unique pairwise stable network will be a complete graph g^N,*

3. *if* $\beta - \beta^2 < c < \beta$*, a star encompassing every node will be pairwise stable, although not necessarily the unique pairwise stable graph,*

4. *if* $\beta < c$ *, any pairwise stable network which is nonempty is such that each player has at least two links and thus be inefficient.*

We see that the stability condition 2, is the same as the efficiency condition 1, and thus if this condition is fulfilled, the network is both stable and efficient. Condition 3 shows us why the efficient star network is not necessarily stable. If $\beta \leq c < \beta + \frac{(N-2)}{2}\beta^2$ then the efficient network will be a star, but it is not stable.

It should be noticed that it is more beneficial for a node to operate as a leaf node compared to being a center node, due to the cost of direct connection. In a star structure, a leaf node will only have to pay the cost of the link to the center node, and will benefit indirectly for each node connected to the center node. The center node will benefit from each new connection, however, the payoff will only be $\beta - c$ for each connection.

6.7.1 Insurance and connection game

The findings about efficiency and stability are very useful for our model, because if one has knowledge of the different variables it is possible to determine how the network will evolve. If one are able to determine the variables one can actually determine the network structure. From the papers, we know that there exists different boundaries on the cost of establishing a new link, and how the resulting stable and efficient network will be. Our earlier models shows that the cost of establish a link is the insurance cost and the risk cost. From this we can show that if $\beta - \beta^2 < I_l < \beta$ and $r > \beta$ a star with only insured nodes, and no connections between non-insured nodes, are both a stable and an efficient network. If $\beta - \beta^2 < I_l + r < \beta$ and $\beta - \beta^2 < I_l$ and $\beta - \beta^2 < r$ the stable and efficient network is a star consisting of both insured and non-insured nodes. If $I_l < \beta - \beta^2$ all insured nodes will connect to every other insured node, and if $r < \beta - \beta^2$ all non-insured nodes will connect to every other non-insured node. In addition if $r + I_l < \beta - \beta^2$ the resulting network will be a clique of both insured and non-insured nodes. The insurer can thus determine the formation of the network by adjusting the cost parameters.

One important thing to notice is that even if the most efficient and the stable network is a star, we can not guarantee that the network formation game will end up in a star. This is because in this game we only consider one link at a time, and not the whole network.

6.7.2 Homogenous symmetric connection game

From this point and on, the game we will consider is a homogenous network setting where every node is considered to be insured. This is done because it will simplify an otherwise very complex model. We are analyzing the resulting network structure, which is easier when only considering one homogenous cost for every node. Lets look at an example, where the parameters are set to: $\beta = 0.9, I_l = 0.5$, the resulting network are shown in Figure 6.12.

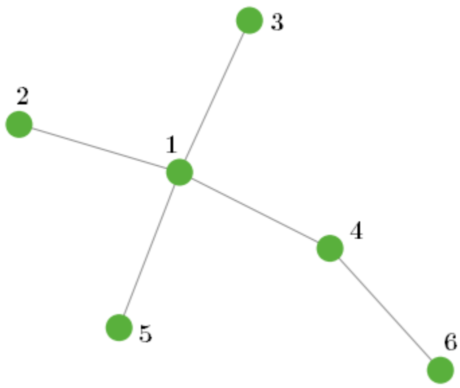


Figure 6.12: The resulting network after a simulation with the parameters $\beta = 0.9, I_l = 0.5$.

As we see this is not an efficient star, but the network is stable. The efficient network would be to delete the link $4, 6$ and adding the link $1, 6$. But since we only consider a link at a time this can not be done. To show this let U_i denote the payoff of node i, the payoffs of the nodes are as described in Eq.(6.73).

$$U_1 = 4\beta + \beta^2 - 4c \tag{6.70}$$

$$U_2 = U_3 = U_5 = \beta + 3\beta^2 + \beta^3 - c \tag{6.71}$$

$$U_4 = 2\beta + 3\beta^2 - 2c \tag{6.72}$$

$$U_6 = \beta + \beta^2 + 3\beta^3 - c \tag{6.73}$$

Node 6 would benefit from adding the link $1, 6$, but node 1 is not willing to do so because then he must pay an extra cost, and since $\beta^2 > \beta - c$. Thus the network is stable but not efficient.

Star not possible with high n In the paper [Jac05] they come up with the following proposition: Consider the symmetric connections model in the case where $\beta - \beta^2 < c < \beta$. As the number of nodes grows, the probability that a stable state (under the process where each link has an equal probability of being identified) is reached with the efficient network structure of a star goes to 0. But if a network reaches the efficient star structure, it is also pairwise stable, and will remain a star. We confirmed this when running multiple simulations, when we used few nodes the resulting network often became a star, but as the number of nodes increased the network rarely was a star.

However, the structure of the networks are very similar to a scale-free network. There are many nodes with low node degree, and few with a high node degree. One example of this is shown in Figure 6.13, there are only ten nodes, but the network have the properties of a scale-free. Two nodes with degree of 4, and the rest have a degree of one or two.

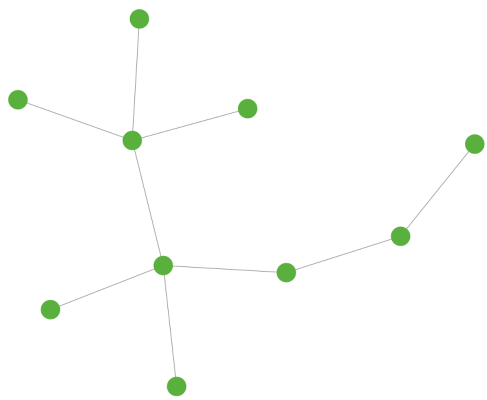


Figure 6.13: The resulting network after a simulation with the parameters described earlier and ten nodes.

Bulk insurance As noted before it is not preferable to be the center node, due to the cost of all the direct links. If we consider the model with bulk insurance discount, this would lower the extra cost for the center node significantly. This could be used to increase the probability of reaching a star formation.

Using the discount formula from the previous model, we end up with Eq.(6.74) to achieve a efficient and stable star topology. i represents the node degree.

$$\beta - \beta^2 < \frac{i_l}{i+1} < \beta \tag{6.74}$$

An interesting property of the discount model is that the conditions for a efficient networks will change. Because when the node degree increases, the insurance cost might reach the critical degree g, the best strategy for a node with degree g or higher, is to connect to every node, as shown in Eq.(6.75).

$$\frac{I_l}{g} < \beta - \beta^2 \tag{6.75}$$

This is possible when $g < n$, where n-represents the number of nodes in the network. The stability condition have changed for a node with a critical degree, the stable and efficient condition for this node is, as shown earlier, to have a direct connection to every other node. Thus if we have a star-topology both the leaf nodes and the center node are stable. Because the center node has been compensated for its role in the network.

Since the networks formed are similar to scale-free networks, we can calculate the probability of a node having degree g, see Eq.(6.76). γ is the power law parameter, as described in Chapter 3.

$$P(g) = g^{-\gamma} \tag{6.76}$$

When a node i reaches the critical degree g its optimal strategy is to connect to every node, since the payoff of direct connection is larger than any indirect connection. In general nodes prefer to connect to nodes with high connectivity [2], and will thus prefer to connect to this node compared to nodes with a degree lower than g. In this way nodes will connect to the node who have a degree greater or equal to g, and remove the links to their low-degree nodes which they can instead reach through i.

Lets consider a case with $n-$nodes, and two of these nodes, i and j, have an equal degree larger than g. The rest of the nodes has a degree of one or zero. If there exists a node with degree zero, it would prefer to be connected to i or j, and so will i and j, so this will eventually happen. If a node connected to i are considering connecting to j, or visa versa, it will do so because j can offer a higher connectivity than i. Now j has a higher degree than i, and thus every node would prefer to connect to j over i. This will eventually result in a star formation, with j as the center node. From this we get the propositions:

Conjecture 1. If the probability that there exists a node with a critical degree is high, the resulting network will with high probability end up in a network where the average degree is close to the max-degree, i.e. a clique or almost a clique.

Conjecture 2. If the probability that there exists a node with critical degree, is such that the expected number of nodes with critical degree($E(Nodeswithdegreeg) = g^-2)$ is less than 0.8. Then the resulting network will with high probability be a star-like structure.

[2] A node with high degree implies a node with high connectivity.

Results and findings

To see if the conjectures above where true, we created a simulator [REFERENCE TO APPENDIX?]. The rules of the simulator are as follows. Every round two random nodes, not neighbors, are selected, and asked if they would want to establish a link. The link establishment is a symmetric decision. If the link is added, we check if either of the nodes would prefer to delete some of their already existing links, this decision is asymmetric. And then we ask every node if they would like to delete any links. This procedure is repeated as long as there is possible to add new links. The payoff function of each node is as described earlier(see Eq.(6.69)), except that the cost is now dependent on the degree of the node. For the simulations to be realizable, we had to set the number of nodes to 20, or else the computational time was to high. For every critical degree, from three to nineteen, we ran 50 simulations, and noted the resulting network formation. We chose to start from critical degree equal three, since any number below would result in a clique, because it would be more beneficial to be directly connected to every node.

We know that if Eq.(6.74) is satisfied for all i, then the efficient and stable state is a star. But a more interesting scenario occurs when we have a graph where one or more of the nodes reaches the critical degree. -Will the final structure become a scale-free, a star or simply just unstructured? The results from the simulation can be seen in Figure 6.14, 6.17 and 6.15. As we see from the figure 6.14, the probability of the resulting network being a star, suddenly increases from zero to 42% at critical degree five to six, and then jumps from 42 to 70-, 86-,96-, 98% at critical degree six to nine. These results confirms our conjectures, and show that the discount can drastically increase the probability of the network ending up in a star.

From Figure 6.15 we can observe that the opposite is happening, as the critical degree is increased, the probability of the resulting network being a clique, drastically decreases. As we can see with a critical degree of seven or higher, it is very unlikely that we end up with a clique. These findings also supports our conjectures.

An interesting comparison can be made between the emergence of a star versus a clique. In Figure 6.16, we have draw the probability of a star and clique. As we can see, from a critical degree of five to seven, the resulting network structure, changes from almost certain ending up in a clique, to almost certain ending in a star structure. The reason is as mentioned before that when the critical degree is low, the likelihood of many nodes reaching it is high. And none of these would like to delete any links. Hence we end up with a clique. The reason why we end up with star structures is because it is less likely that many nodes end up reaching the critical degree, hence it most of the nodes still prefer to rely on indirect links, but the ones who reach the critical degree prefer to connect to every one. And since the nodes with critical degree, have high connectivity, nodes will prefer to be connected with

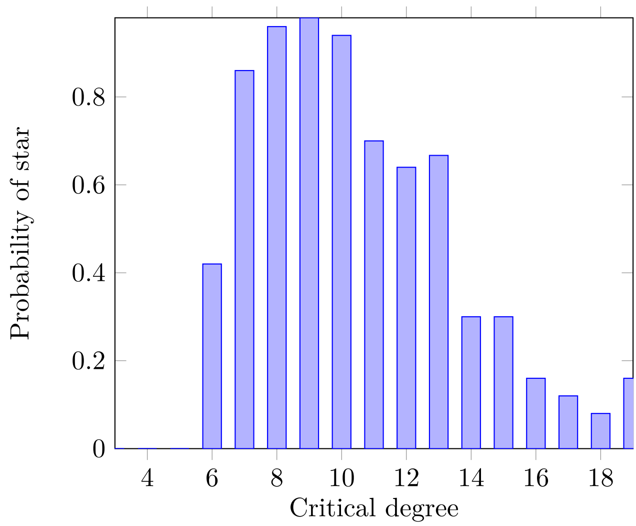


Figure 6.14: Shows the probability of the network ending up in a star given different critical degrees.

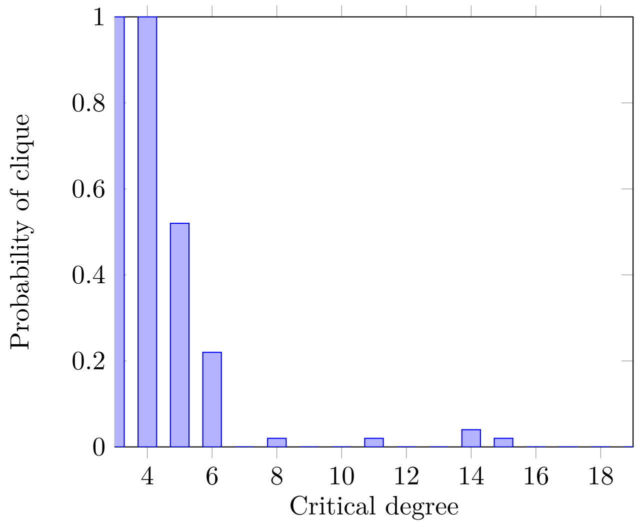


Figure 6.15: Shows the probability of the network ending up in a clique, given different critical degrees.

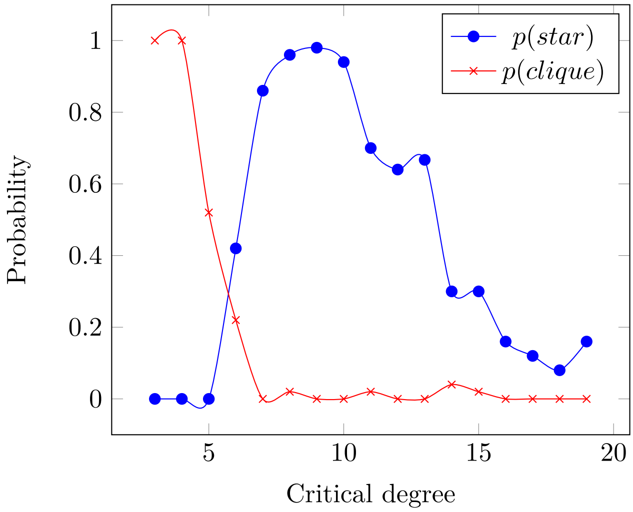


Figure 6.16: Shows the comparison between the probability of the network ending up in a star (blue) or clique (red), given different critical degrees.

these, compared to other nodes. Nodes prefer to be connect to the ones with critical degree, the nodes with critical degree would like to connect to every one, and thus the structure evolves into a star, with the critical degree node in the center.

As we see in Figure 6.14, when the critical degree gets closer to the number of nodes in the network, the probability of the network evolving into a star decreases. However, in Figure 6.17, we have plotted the probability of the network evolving into a network where only a few($twotofour$) nodes end up with a high degree, but not a critical degree. And as we see occurs with high probability from critical degree six and up. These networks are so called scale-free networks, because there are a few hubs, that connects the rest of the networks.

Example figures from the simulation. In Figure 6.18 we see two of the many possible outcomes when the critical degree is achieved at a low node degree. And as we see most of the nodes have reached the critical degree, and thus connected to every other node. In Figure 6.19 we see one example of a scalefree network, and the standard star network, both with twenty nodes and results from the simulations when the critical degree is above six.

Another possibility for solving the problem with unfair costs, could be to use a shared-network cost instead. i.e. every node pays a cost equal to the total cost of the network divided on the number of nodes. Similar value distribution has been analyzed by Jackson and Wolinsky in [JW96], and is called an egalitarian allocation

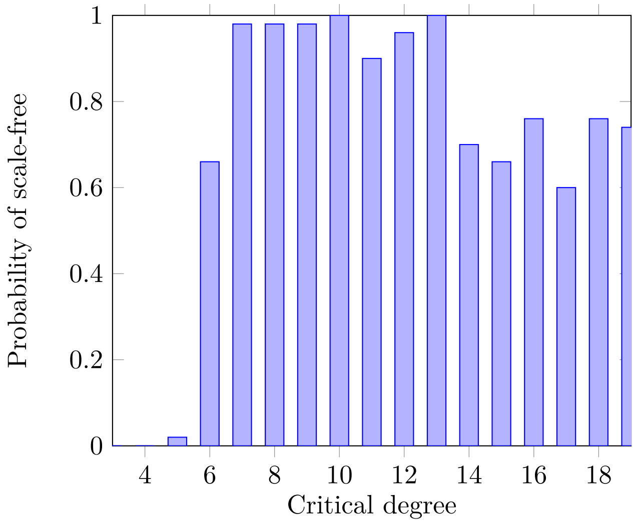
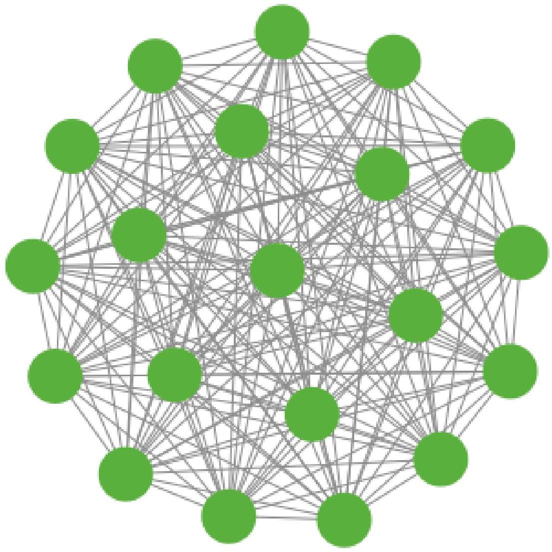
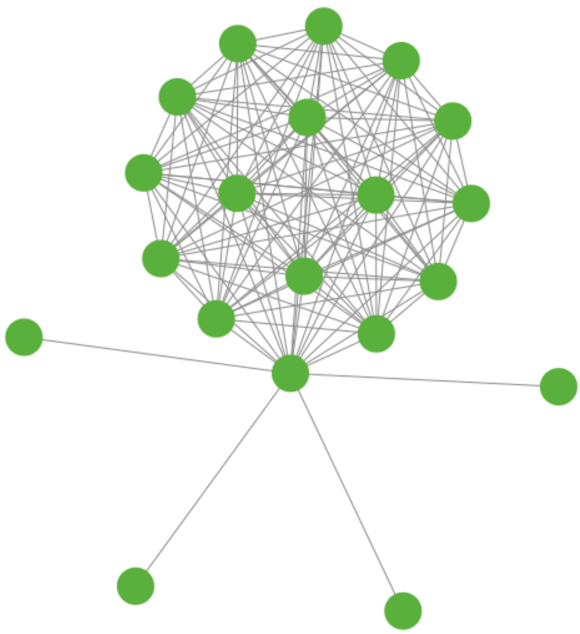


Figure 6.17: Shows the probability of the network ending up in a scale-free structure, given different critical degrees.

rule, this rule guarantees that any efficient network is also pairwise stable. But this is unfortunately a very extreme rule.

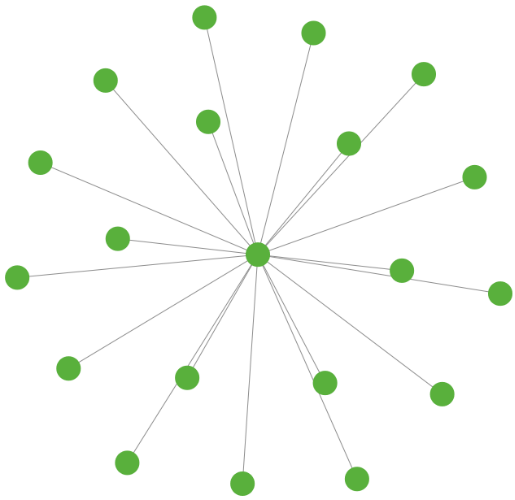


(a) A clique consisting of twenty nodes.

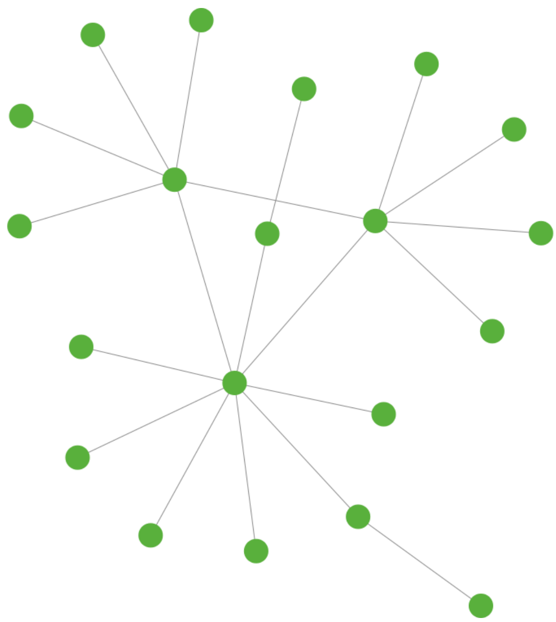


(b) A network with high average node degree, but not a clique.

Figure 6.18: Two different outcomes of the simulations where the critical degree is low



(a) A star consisting of twenty nodes



(b) A scalefree network with twenty nodes, where three nodes account for most of the connectivity.

Figure 6.19: Two different outcomes from running simulations with a high critical degree.

Part III

Summary

Chapter 7
Summary

7.1 Discussion

Skriv litt om current market, problemen med cyber insurance. **HUSK, maa faa med hva vi contributer til, altsaa hva loser vi** The first model we made, showed a very simple and naive way for the insurer to separate insured and non-insured nodes, into two cliques. By saying that insured only connects to other insured, and non-insured to non-insured.

In model-2 we made model-1 realizable, by including the parameters: expected risk cost,insurance cost and the benefit per link. We then analyzed the parameters and found out when and how different network structures would evolve. By adjusting the insurance cost to the right level, the insurer can make the network formation game end up in one clique of both insured and non-insured nodes, or a clique of only insured and another of only non-insured. We also showed that when the insurer sets the cost such that the network ends up in two cliques, it is not the socially optimal. Because the network will suffer from the lost benefits of connections between insured and non-insured nodes.

In model-3 we applied the model to certain real world scenarios, such as software development firms/chains, or other networks where the final product is dependent on the collaboration of several parts. This was done by including a bonus, which where received when you reached the desired number of links(called max-degree). This made the separation process of insured and non-insured nodes, more difficult for the insurer. In this model the nodes have more incentive to establish links, and are thus more acceptable towards risk. We found the conditions for the different network structures to evolve, and showed that these where strongly dependent on the max-degree. And when the max-degree increases, it gets harder and harder to guarantee two separate cliques.

In Model-4 we tried to make the model more comparable to other insurance products, by including a bulk-discount. The cost of insuring a link are now dependent on the nodes degree. We did this on both model 2 and 3. This resulted in even more incentive, or less disincentive, for insured node to establish links with non-insured nodes, since the cost of doing so decreases as the nodes degree increases. We found the different cost conditions for when the different networks will evolve, and showed when applying the discount to model 3, it is very hard for the insurer to ensure two seperate cliques. We also showed that the price of stability is even higher when applying discount to model 2. This is because the costs are decreasing, and thus the potential payoff that are missing, when we have two separate cliques, are increasing.

In our last model we applied our model-4(discount) to an already existing model, "the symmetric connection game". In this old game it had been shown that there exists three different efficient networks that arise under certain cost conditions, and that these also some times where stable. A clique is the most efficient if the cost of establishing links, is less than the benefit gained from indirect connections. A star is the most efficient if the cost is higher than the benefit from indirect connections, but less than the benefit of direct connections. It is also shown that as the number of nodes in the networks increase, the probability of the network ending up in star goes to zero. However, By applying our insurance discount to this model, we found a conjecture that says, by setting the cost to the right level, one can with high probability ensure that a star will evolve.

7.2 Conclusion

One paragraph stating what you researched and what your original contribution to the field is, after that brake into sections One of the many problems with cyber-insurance is to define the risk and calculate premiums, because the network structure is undefined. We have shown how insurers can, by adjusting the insurance cost, determine the result of different network formation games, and even force the network to end up in insurable topologies.

7.2.1 One section on what you researched and how you did it

We surveyed different literature on networks and risk, and found recent literature who showed how graphs like cliques, star, meta-star and funnel, has a calculable fixation probability. And that the star-structures act as an evolutionary amplifier. With these structures in mind, we created and analyzed different network formation games, and tried to find link-cost constraints, that enabled these structures to evolve. The analyzis where performed mathematically by using game-theory and confirming the results with a simulator created in netlogo. We analyzed five different models, started with a very simple model, and then made it more complex and realistic for every step.

7.2.2 One section on what are the main findings were... showinglinks across chapters (this explains why you chose thestructure you did)

In model one to four, we found cost constraints for different types of networks to evolve. And in all scenarios it is possible for the insurer to set the cost, such that the game will end up in two cliques, one consisting of insured and the other of non-insured. In every model we added some new features that made the model more applicable to real world scenarios, and for every feature added it became more difficult for the insurer to separate the two types of nodes. This is due to the increase in incentive of establishin links, and thus the nodes became more and more acceptable towards risk. From this findings we could see that the price of stability also increased. We also showed that to be able to separate the networks into two cliques, the nodes must know the other nodes types. Or else, the nodes will have incentive to pretend to be an insured node. This will result in risky links between insured and non-insured nodes.

In the last model we introduced the concept of bulk-insurance into an already existing networkformation game, the connection game, and showed that this would drastically increase the probability of the network ending up in star. This is because at a point, called critical degree, the nodes optimal strategy will change from being relaying on indirect connections to increase its payoff, to suddenly wanting to connect to everyone. We proved this by performing multiple simulations, and these simulations showed that in a network with twenty nodes, the network would result in a star with high probability, from critical degree six and up. Whats makes this a very interesting finding, is that it is earlier proven that in the connection game, the probability of the network reaching a star goes towards zero as the number of nodes increases. But by introducing a discount, that will subsidize the center node, the game will with high probability end up in a star, given the critical degree is reached at the right level. **MUST DEFINE WHERE THIS LEVEL IS**

7.2.3 One section on possible areas for future research

On suggestion for future work, is mapping our models and simulations to real world scenarios in a more convincing way. Because real world network are not random, nodes may prefer to talk to nodes with high degree or low degree, i.e. the payoff function has to be changed. We have assumed additive benefits and risk, so suggestions for future work could be to introduce different risk and benefit functions, that are more applicable to the real world. Another interesting thing to research, is the game of choosing insurance or not, in future work this could be applied to our models.

7.2.4 Final section reminding readers of the original contributionand significance of your research to your field

We have shown how an insurer can determine the resulting network structure, like separate insured and non-insured nodes into two cliques, by adjusting the insurance cost parameter. In this way the network can be considered as an insurable topology, since it enables the insurer to calculate the probability of fixation. We found these conditions for several models, with different aspects that relate them to real world and other insurance products.

TIPS og TRIKS til konklusjon Her oppsummerer en metoder, resultater og viktigste konklusjoner. Dette likner en del på sammendraget i begynnelsen, men konklusjonen er normalt fyldigere. Normalt skal ikke nytt stoff være med i konklusjonen, men ha vært beskrevet tidligere i rapporten.

To summarize –What you researched –Nature of your main arguments –How you researched it –What you discovered –What pre-existing views were challenged 2.To provide an overview of The new knowledge or information discovered • The significance of your research (where is it new?)-The limitations of your thesis (concepts, data)-Speculation on the implications of these limitations-Areas for further development and research(alternative data sets; links with other fields; differentmethod applied to same data

AVOID this in conclsuion Avoid claiming findings that you have not proven-throughout your thesis • Avoid introducing new data • Avoid hiding weaknesses or limitations in your thesis(make a virtue of showing strong analytical skills and self-critique by discussing the limitations–but don't gooverboard on this!) • Avoid making practical recommendations (e.g. for policy).If you must include them put them in an appendix. • Avoid being too long (repetitive) or too short (sayingnothing of importance)

7.3 Future work Notater

TA MED DETTE? As second important open problem is mapping our simple model to reality in a more convincing way. In particular real work networks are far from ran- dom: nodes want to talk to clusters of other nodes, and both friend ship and link capacity is distributed according to a power law. Furthermore what matters in real world networks is not a short path merely existing, but also being able to

nd it within some reasonable time (social networks in particular are navigable [7]). Modifying the friendship graph and link budgets to ful

l those requirements is an important next step

7.3.1 Risk

In our model we used an additive risk parameter, meaning that each connection to a non-insured node adds a fixed negative value r to the node's utility. It is reasonable to assume that the probability of failure increases if a node accepts more and more non-insured nodes. However, whether the risk parameter increases according to an additive distribution is difficult to confirm. Hence the decision of using additive risk was take due to the simplicity of the function and the fact that we do not know for sure how the distribution actually looks like. The probability might as well be multiplicative, exponential or logarithmic. Although it is highly uncertain, we believe that the risk parameter will follow an exponential distribution similar to the one in Eq.(7.1).

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases} \tag{7.1}$$

When λ have a value around 0.5, we get a curve 7.1 which captures how we believe the risk in a network will increase as more non-insured nodes are added. We believe that if one have a growing network consisting of insured nodes only, the first non-insured node added will contribute more risk than the consecutive non-insured nodes. When the 2.nd and 3.rd and so on, node are added there are already a probability that the network will be infected. It reasonable to believe that the overall risk wont increase additive, but at a lower rate. The risk added for each new non-insured node will decrease. Hence we believe that the accentual risk parameter will follow a exponential distribution.

read more in the paper: Uncertainty in Interdependent Security Games

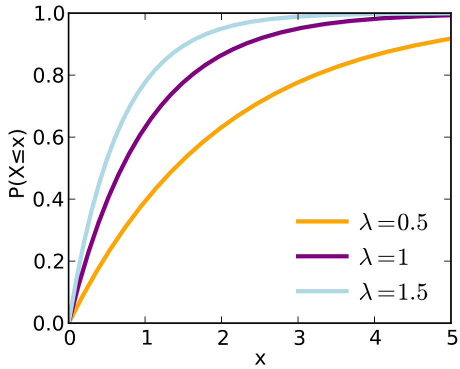


Figure 7.1: Figure showing the distribution of Eq.(7.1).

From this paper presents a description of how to measure risk within a local network. The idea is that for a cost c, you can protect yourself from threats outside your own LAN or corporation. This is analogous to purchase a firewall and anti-virus software. However, you can still be affected by threats from non-insured nodes inside your own local network. This means that as long as not every node is insured, the non-insured node will introduce a risk q to the local network. p reflects the probability of getting affected by a risk, and q represents the likelihood of spreading it to others in the local network. The paper presents a swift model for measuring the risk in you local network.

$$U_i = \begin{cases} -c + (1-q)^k, & \text{if not buying insurance} \\ (1-p)(1-q)^k, & \text{ifbuying insurance} \end{cases} \tag{7.2}$$

This model can be used to look at a the decision process of single node on whether to buy insurance or not. The paper presents certain conditions which creates scenarios where we end up with a network where either every node chooses to buy or every one chooses not to buy insured.

If $c < p$ then everyone will buy insurance, since this is cheaper than the expected loss. The other equilibrium where no one buys insurance, occurs when the cost of insurance is higher that the likelihood that a player fails to protect him selves, assuming that also fails to protect. i.e. $c > p(1-q)^{1-n}$

Our model's ultimate goal is to end up with insurable topologies which are able to measure risk in networks with a mix of insured and non-insured nodes. Therefore we will not take the same approach towards handling the problem with internal risk, i.e always forcing the network to either consist of insured nodes or not not-insured nodes. Instead we want to transfer this risk to the insurance company. Each node will have the opportunity to purchase a link insurance, which compensate from any financial loss from a specific node.

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