

Truth tables

1. What does it mean (in terms of truth tables) to say that two sentences are *logically equivalent*?
2. Show that $P \rightarrow Q$ and $\neg P \vee Q$ are logically equivalent.
3. If two sentences ϕ and ψ are logically equivalent, what do we know about the sequents $\phi \vdash \psi$ and $\psi \vdash \phi$?
4. If $\phi \vdash \psi$, then what do we know about the truth tables of ϕ and ψ ?
5. Find a sentence whose only connectives are \wedge and \neg , and that is logically equivalent to $P \rightarrow Q$.
6. Can there be a correctly written proof with the following lines?

$$1 \quad (1) \quad P \rightarrow Q \quad \text{A}$$

$$1 \quad (n) \quad Q \rightarrow P$$

Biconditional

1. Show that $\neg(P \leftrightarrow Q)$ and $P \leftrightarrow \neg Q$ are logically equivalent.
2. Can there be a correctly written proof with the following lines?

$$1 \quad (1) \quad \neg P \vee Q \quad \text{A}$$

$$2 \quad (2) \quad \neg(Q \wedge \neg P) \quad \text{A}$$

$$1, 2 \quad (n) \quad P \leftrightarrow Q$$

3. Prove that $P \leftrightarrow \neg P \vdash P \wedge \neg P$
4. Prove that $P \wedge Q \vdash P \leftrightarrow Q$
5. Prove that $\neg P \wedge \neg Q \vdash P \leftrightarrow Q$

New proofs from old

1. Prove that $(P \wedge Q) \vee (\neg P \wedge \neg Q) \vdash P \leftrightarrow Q$
2. Prove that $(P \rightarrow Q) \rightarrow R \vdash (\neg P \vee Q) \rightarrow R$
3. Show that if $\phi \vdash \psi$ then $\chi \rightarrow \phi \vdash \chi \rightarrow \psi$