Logic pset 1

1. For each of the following paragraphs: (a) State whether or not that paragraph contains an argument. (Note: an argument does not need to be a *good* argument.) (b) If there's an argument, identify its conclusion and premises. (c) If there's no argument, explain what's lacking.

   (a) Professor Plum was in the drawing room. Miss Scarlet was in the kitchen. The murderer used the knife and the evil act was committed in the hall.

   (b) If Professor Plum was in the drawing room then Colonel Mustard was the murderer. Professor Plum was in the drawing room. So, Colonel Mustard was the murderer.

   (c) Every student of logic is wise and knowledgeable. Anyone attempting this exercise is a student of logic. Therefore, anyone attempting this exercise is wise and knowledgeable.

   (d) I am absolutely sick and tired of getting wet every time it rains. From now on I will never forget to take my umbrella with me in the morning. Even if the weather looks fine when I leave I will certainly make a point of taking that umbrella.

   (e) All human beings are mortal. So, it stands to reason that Socrates is mortal. After all, he is a human being.

   (f) Professor Plum was obviously the murderer in this instance. For the murderer used the knife and Professor Plum had the knife. And the murder was committed in the hall and Professor Plum was in the hall.

2. Short answer

   (a) What are the components of an argument?

   (b) Which kinds of sentences can be premises or conclusions of an argument?

   (c) Is a valid argument necessarily a "good" argument? What might a good argument have that a valid argument doesn't?

   (d) What's the point of studying the validity of arguments, as opposed to their goodness?

   (e) If an argument has true premises and a true conclusion, then is it valid?

   (f) If you disagree with the conclusion of an argument, might you still say that it's a valid argument?

   (g) According to the definitions given in lecture, which of the following sentences makes sense, and which do not?

      i. That's a true argument.

      ii. That's a true statement.

   iii. That's a valid point.
   iv. That's a valid argument.
   v. That's a valid reason.

(h) Give an example of a valid argument with false premises and a true conclusion.

(i) Give an example of an invalid argument with true premises and a true conclusion.

3. True or False. Discuss.

   (a) If an argument is valid, then you might be able to make it invalid by adding further premises.

   (b) You can make an invalid argument valid by removing premises.

   (c) If a sentence doesn't follow from another, then its denial must.

Logic pset 2

Resources: Ch 2 of HLW and Lecture 3

Instructions: Represent the propositional structure of each of the following sentences. First identify the atomic component sentences (i.e. sentences that do not contain connectives) and abbreviate each with a distinct capital letter. We have suggested letters after the sentences. Then represent the form of the original sentence using the symbols $\vee, \wedge, \neg, \rightarrow$ for the connectives "or", "and", "not", "if…then…". Make sure to include parentheses, if necessary to disambiguate.

1. It's not true that if Ron doesn't do his homework then Hermione will finish it for him. (R,H)
2. Harry will be singed unless he evades the dragon's fiery breath. (S,E)
3. Aristotle was neither a great philosopher nor a great scientist. (P,S)
4. Mark will get an A in logic only if he does the homework or bribes the professor. (A,H,B)
5. Dumbledore will be killed, and either McGonagall will become head of school and Hogwarts will flourish, or else it won't flourish. (D,M,H)
6. Harry and Dumbledore are not both right about the moral status of Professor Snape. (H,D)

Logic pset 3

Resources: HLW Ch 2, Lecture 4 and Lecture 5

1. Prove that the following argument forms are valid. The premises are to the left of the $\vdash$ symbol, the conclusion is to the right. You should number the lines of your proof, and each line must either be a premise (i.e. an assumption) or be justified by one of the following rules of inference: $\wedge$I, $\wedge$E, $\vee$I, MP, MT, or DN.

   (a) $P \to (Q \to R),\ P \to Q,\ P \vdash R$

   (b) $P \vdash (P \vee R) \wedge (P \vee Q)$

   (c) $P \vdash Q \vee (\neg\neg P \vee R)$

   (d) $\neg\neg Q \to P,\ \neg P \vdash \neg Q$

   (e) $Q \to (P \to R),\ \neg R \wedge Q \vdash \neg P$

2. Explain what is wrong with the following "proof".

(1)	$P \vee (Q \wedge R)$	A
(2)	$P \vee Q$	$1 \wedge$ E

Logic pset 4

Resources: HLW Ch 3 and Lecture 6

Use Conditional Proof (and possibly the previous rules) to prove the following sequents. Be sure to include dependency numbers in the leftmost column of your proof.

1. $P \to Q \vdash P \to (Q \vee R)$
2. $P \vdash (P \to \neg P) \to \neg P$
3. $P \to (Q \to R) \vdash Q \to (P \to R)$
4. $\vdash P \to (P \vee Q)$
5. $\vdash P \to ((P \to Q) \to Q)$

Logic pset 5

Resources: HLW Ch 3 and Lecture 7

Use $\vee$-elimination (and possibly the previous rules) to prove the following sequents. Do *not* use reductio ad absurdum for any of these proofs.

1. $P \vee (Q \wedge R) \vdash P \vee Q$
2. $P \wedge (Q \vee R) \vdash (P \wedge Q) \vee (P \wedge R)$
3. $P \vee Q, \neg P \vdash Q$
4. $(P \to R) \wedge (Q \to R) \vdash (P \vee Q) \to R$

Logic pset 6

Resources: HLW Ch 3 and Lecture 8

Problems: You may use any of the rules of inference, including reductio ad absurdum, to prove the following sequents.

1. $P \to Q \vdash \neg(P \land \neg Q)$
2. $(P \land Q) \to \neg Q \vdash P \to \neg Q$
3. $P \to \neg P \vdash \neg P$
4. $\neg(P \to Q) \vdash P \land \neg Q$

Logic pset 8

Resources: HLW Ch 4 and Lectures 10, 11, and 12

1. Prove that $\neg(P \leftrightarrow Q) \vdash P \rightarrow \neg Q$. Besides the basic rules, you may also cut in $\phi, \psi \vdash \phi \leftrightarrow \psi$ (biconditional).

2. Prove that $P \leftrightarrow Q, \neg(P \wedge Q) \vdash \neg P \wedge \neg Q$. You may only use the basic rules.

3. Prove that $P \leftrightarrow Q \vdash (P \wedge Q) \vee (\neg P \wedge \neg Q)$. Besides the basic rules, you may cut in sequents already proved in this pset and/or the sequent $\neg\phi \rightarrow \psi \vdash \phi \vee \psi$ (material conditional).

4. Prove that $\vdash (P \leftrightarrow Q) \vee (P \leftrightarrow \neg Q)$. You may cut in any of the "useful validities" from pp 233-4 in the book.

Logic pset 10

Resources: HLW Ch 6 and Lectures 13 and 14

1. Represent the form of the following sentences in predicate logic. We've suggested appropriate symbols. (For the sentences about people, you don't need to add an extra predicate for "x is a person.")
   (a) No logicians are celebrities. (Lx, Cx)
   (b) Only students who do the homework will learn logic. (Sx, Hx, Lx)
   (c) All students and professors get a discount. (Sx, Px, Dx)
   (d) Not all logicians are computer scientists. (Lx, Cx)
   (e) If there are rich logicians, the some logicians are computer scientists. (Rx, Lx, Cx)
2. Prove the following sequents using the propositional logic rules (including cut & replacement, if you want) and UE.
   (a) $\forall x \neg Fx \vdash Fa \to P$
   (b) $\forall x(Fx \to Gx), \neg Ga \vdash \neg \forall x Fx$
   (c) $\vdash \neg \forall x(Fx \wedge \neg Fx)$

Logic pset 11

Resources: HLW Ch 6 and Lectures 15 and 16

1. Represent the form of the following sentences in predicate logic. We've suggested appropriate symbols. (For the sentences about people, you don't need to add an extra predicate for "x is a person.")
   (a) Mary loves everyone who loves her. (m, Lxy)
   (b) Everyone loves their mother. (Lxy, Mxy)
   (c) Snape killed someone. (Kxy, s)
   (d) Some wizards only marry other wizards. (Wx, Mxy)
   (e) Anything that is greater or equal than both a and b is also greater or equal than c. $(a, b, c, x \leq y)$
2. Prove the following sequents using the propositional logic rules (including cut & replacement, if you want), plus UE and UI.
   (a) $\forall x(Fx \to Gx) \vdash \forall xFx \to \forall xGx$
   (b) $\forall xFx \land \forall xGx \vdash \forall x(Fx \land Gx)$
   (c) $\forall x\forall y(Fx \to Fy) \vdash \forall x(Fx \to \forall yFy)$
   Hint: you might first try proving $\forall y(P \to Fy) \vdash P \to \forall yFy$
   (d) $\forall x\forall yRxy \vdash \forall xRxx$

Logic pset 14

Resources: HLW Ch 7 pp 116-127 and Lecture 19

1. Represent the form of the following sentences in predicate logic using the $=$ symbol where necessary.
   (a) There is one and only one Princeton University. (Use Px for "x is a Princeton University")
   (b) There is at most one Ivy League university in New Jersey. (Use Ix for "x is an Ivy League university", and use Nx for "x is in New Jersey.")
   (c) The smallest prime number is even. ($Px, Ex, x < y$, variables are restricted to numbers.)
2. Prove the following sequents using any of the rules, including the $=$ intro and elim rules. You may write proofs in "sloppy mode", i.e. you may combine steps, cut in results proved elsewhere, etc., as long as you explain clearly and convincingly how the proof works.
   (a) $\exists x(Px \wedge \forall y(Py \to x = y)) \vdash \forall x \forall y((Px \wedge Py) \to x = y)$
   (b) $\vdash \forall x \forall y((x = y) \to (y = x))$
3. Let Rxy be a binary relation symbol that satisfies the transitivity axiom (page 126). Suppose that Rxy satisfies two other axioms: serial $\forall x \exists y Rxy$ and irreflexive $\forall x \neg Rxx$. Show that there are at least three distinct things. (Your proof need not be fully formal, but it needs to be clear that you understand why the moves you make are licensed by our system.)

Logic pset 15

Resources: HLW Ch 8 "Models" and Lecture 20

1. For each of the following sequents, provide a counterexample to show that it is invalid.
   (a) $\forall x Fx \to \forall x Gx \vdash \forall x(Fx \to Gx)$
   (b) $\forall x(Fx \to Gx) \vdash \exists x(Fx \wedge Gx)$
   (c) $\vdash \forall x Fx \vee \forall x \neg Fx$
   (d) $\exists x(Fx \to P) \vdash \exists x Fx \to P$

2. For each of the following sentences, provide one interpretation in which it is true and one interpretation in which it is false. An interpretation may be presented by giving a set M and a subset R^M of $M \times M$, or it may be presented as an arrow diagram.
   (a) $\forall x \forall y \exists z(Rxz \wedge Ryz)$
   (b) $\forall x(\exists y Ryx \to \forall z Rzx)$