Redutio ad absurdum

Prove the following sequents.

1. $\neg P \vdash \neg(P \wedge Q)$
2. $\neg(P \vee Q) \vdash \neg P$
3. $\vdash P \vee \neg P$
4. $\neg(P \wedge Q) \vdash \neg P \vee \neg Q$

Truth tables

1. Let ϕ be a sentence. How can you tell whether or not ϕ can be proven?

2. Let $\phi_1, \ldots, \phi_n$ and ψ be sentences. How can you tell whether the sequent $\phi_1, \ldots, \phi_n \vdash \psi$ can be proven?

3. Write the truth table for the following sentence:

   $$(P \to Q) \wedge \neg Q$$

4. Determine the truth values of the following statements when P and Q are true and R is false

   (a) $P \to \neg(\neg P \wedge (Q \vee \neg R))$

   (b) $\neg Q \vee ((\neg P \wedge R) \to Q)$

5. Determine whether the following sequents are provable, and briefly explain your answer.

   (a) $\neg(P \wedge Q) \vdash \neg P$

   (b) $\neg(P \wedge Q) \vdash \neg P \vee \neg Q$

   (c) $(P \to Q) \to R, \neg Q, \neg R \vdash P$

   (d) $(P \vee Q) \to (R \wedge D), R \to D \vdash \neg P$