Midterm Exam

Instructions: (1) Please note that the exam has two pages. (2) You have 120 minutes to complete the exam and submit your answers. (3) You may use any non-human resource (e.g. book, notes, internet), but you may not discuss your exam with any other human being — except that you may ask instructors practical questions. (4) There are 38 possible points.

Translation

Translate the following sentences into propositional logic. (2 points each)

1. If Alice gets an A, then Bob gets a B only if Catherine gets a C.
2. Catherine gets a C if and only if either Alice gets an A or Bob gets a B.
3. Alice's getting an A is neither a necessary nor sufficient condition for Bob getting a B. [Here "Alice's getting an A" becomes the atomic sentence "Alice gets an A."]

Truth tables

1. Determine whether the following sequents are valid. If they are not, then provide a counterexample. (2 points each)
   (a) $(P \to Q) \to (\neg P \to Q) \vdash Q$
   (b) $(P \vee Q) \to (P \vee R) \vdash Q \to R$
2. For each of the following sentences, say whether it is a tautology, contingency, or inconsistency (self-contradiction). (2 points each)
   (a) $\neg(P \leftrightarrow Q) \to (P \vee Q)$
   (b) $(P \to Q) \vee (R \to Q)$

Proofs

1. Prove the following sequent using only the basic rules. (4 points)

   $$P \vee Q, P \vee \neg Q \vdash P$$

2. Prove the following sequent. Besides the basic rules, you may cut in the sequents $\psi \vdash \phi \to \psi$ (positive paradox) or $\neg\phi \vdash \phi \to \psi$ (negative paradox). (4 points)

   $$(P \to Q) \to (\neg P \to Q) \vdash \neg P \to Q$$

3. Prove the following sequent. Besides the basic rules, you may use cut or replacement with the sequents on pages 233–34 of the textbook and/or any other sequent you prove on this exam. (4 points)

   $$(\neg P \vee Q) \to (P \vee Q) \vdash P \vee Q$$

4. Prove the following sequent. Besides the basic rules, you may use cut or replacement with the sequents on pages 233–34 of the textbook and/or any other sequent you prove on this exam. (4 points)

   $$\neg(Q \leftrightarrow P) \vdash Q \to (P \to R)$$

Conceptual

1. True or False (explain your answer): There is a correctly written proof with the following lines. (4 points)

1	(1)	$\neg(P \to Q)$	A
2	(2)	$\neg P$	A
⋮	⋮		
1, 2	(n)	R	

2. True or False (explain your answer): If ψ is an inconsistency (self-contradiction), then the sequent $\phi \vdash \psi$ is invalid, no matter what the sentence ϕ is. (4 points)

THE END