Equivalence Results in the Foundations of Mathematics

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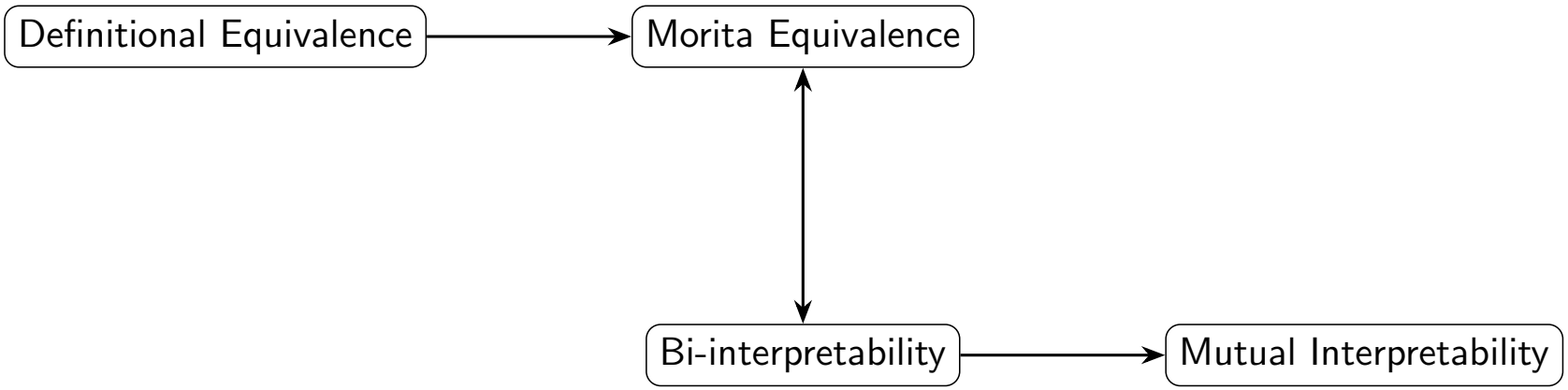
- The old consensus: Zermelo-Frankel set theory won the early 20th century battle about the foundations of mathematics
- Main competitors:
  - Finitism
  - Intuitionism
  - Type theory

New developments

- Category theory and the use of topos theory in various branches of pure mathematics (Grothendieck, Mac Lane, Lawvere)
- Martin-Löf type theory
- Computation
- Homotopy type theory (HoTT)

- In the 1970s and 80s, Sol Feferman argued against category-theoretic foundations for principled (philosophical) reasons
- The idea that **Set** and **Cat** are incommensurable foundations was challenged via results of Mitchell, Osius, and Mathias
    - But what exactly did they prove?
- Steve Awodey: **Set**, **Cat**, and **Typ** are equivalent

What do we mean by equivalent?



Bi-interpretability: syntax and semantics

Shulman's equivalence proof

Questions about Shulman

Type theory: Kemeny or Awodey?