

## Logic final exam

Time limit: 2 hours

Instructions: (1) Read the exam all the way through before you begin.  
(2) Write your preceptor's name on your exam booklet. (3) When you finish your exam, place it in the box at the front of the room, and leave quietly.

1. Translate the following sentences into predicate logic. Use a relation symbol  $Lxy$  for " $x$  loves  $y$ ," and use the equality symbol  $=$  when appropriate. Use  $a, b, c$  for names of the relevant people. (You don't need a separate predicate symbol for " $x$  is a person.")
  - (a) Alice loves all people who do not love themselves.
  - (b) There is a person who does not love herself.
  - (c) Everybody loves Bente.
  - (d) Bente loves nobody but Cordelia.
2. Provide a model to show that these four sentences are consistent. (It would suffice to draw an arrow diagram, or to write a domain set and extensions of the relevant symbols.)
3. Show that the first sentence implies that Alice loves herself. (You can use any meta-rules such as cut, so long as long as you include proofs of the cited results.)
4. Show that the first sentence implies that everyone is loved.
5. Show that the last two sentences imply that Bente and Cordelia are the same person.
6. Symbolize "there are at least three people," and prove that it follows from the four sentences above.