Intro Logic: Midterm Exam 2025 — Answer Key

Instructions: (same as on exam)

A. Translation

Translate the following sentences into propositional logic. In each case, clearly indicate what letters you are assigning to atomic sentences. (2 points each)

1. If Bob does not exercise regularly or does not eat healthy meals, then Bob will not maintain good health.

   Answer: Let E = Bob exercises regularly; H = Bob eats healthy meals; G = Bob maintains good health. Translation: $(\neg E \vee \neg H) \to \neg G$.

2. Carla will increase her chances of admission only if Carla submits her application early and asks for strong recommendation letters.

   Answer: Let I = Carla increases her chances of admission; S = Carla submits her application early; R = Carla asks for strong recommendation letters. Translation: $I \to (S \wedge R)$.

3. Either David gets a front-row seat, or if David does not go with friends then David will not enjoy the concert.

   Answer: Let F = David gets a front-row seat; K = David goes with friends; J = David enjoys the concert. Translation: $F \vee (\neg K \to \neg J)$.

B. Semantics (truth tables)

1. For each of the following sentences, state whether it is a tautology, contingency, or inconsistency, and justify your claim in terms of truth tables. (3 points each)

   (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
   Answer: Contingency. True when P and Q have the same truth value (true/true or false/false). False otherwise.

   (b) $P \to (Q \to (R \to (S \to P)))$
   Answer: Tautology. If P is true, all nested implications yield true; if P is false, the outer conditional is true. No false rows.

2. For each of the following arguments, state whether it is valid or invalid, and justify your claim in terms of truth tables. (3 points each)

   (a) $P \to Q \vdash P \to (Q \wedge R)$
   Answer: Invalid. Counterexample: $P = 1, Q = 1, R = 0$ makes the premise true and conclusion false.

(b) $Q \to R \vdash (P \vee Q) \to (P \vee R)$

Answer: Valid. If the conclusion is false, then $P \vee Q$ is true and $P \vee R$ is false. The only way for this to be the case is if Q is true and R is false, which means that the premise $Q \to R$ is false.

C. Proofs

Prove the following. Besides the basic rules, you may also use cut and replacement, but only if you include a proof of the relevant "lemmas" in your exam booklet. (4 points each)

1. $P, \neg P \vdash Q$

1	(1)	P	A
2	(2)	$\neg P$	A
3	(3)	$\neg Q$	A
1,2	(4)	$P \wedge \neg P$	1,2 $\wedge$I
1,2	(5)	$\neg\neg Q$	3,4 RA
1,2	(6)	Q	5 DN

2. $P \vee Q, \neg P \vdash Q$

1	(1)	$P \vee Q$	A
2	(2)	$\neg P$	A
3	(3)	P	A
2,3	(4)	Q	3,2 problem 1
5	(5)	Q	A
1,2	(6)	Q	1,3,4,5,5 $\vee$E

3. $\neg P \to Q \vdash P \vee Q$

1	(1)	$\neg P \to Q$	A
2	(2)	$\neg(P \vee Q)$	A
3	(3)	P	A
3	(4)	$P \vee Q$	3 $\vee$I
2,3	(5)	$(P \vee Q) \wedge \neg(P \vee Q)$	4,2 $\wedge$I
2	(6)	$\neg P$	3,5 RA
1,2	(7)	Q	1,6 MP
1,2	(8)	$P \vee Q$	7 $\vee$I
1,2	(9)	$(P \vee Q) \wedge \neg(P \vee Q)$	8,2 $\wedge$I
1	(10)	$\neg\neg(P \vee Q)$	2,9 RA
1	(11)	$P \vee Q$	10 DN

4. $(\neg P \vee Q) \to (P \vee Q) \vdash P \vee Q$

1	(1)	$(\neg P \vee Q) \to (P \vee Q)$	A
2	(2)	$\neg P$	A
2	(3)	$\neg P \vee Q$	2 $\vee$I
1,2	(4)	$P \vee Q$	1,3 MP
1,2	(5)	Q	4,2 problem 2
1	(6)	$\neg P \to Q$	2,5 CP
1	(7)	$P \vee Q$	6 problem 3

D. Conceptual

1. Is there a correctly written proof with the following line fragment? Justify your answer by showing that the relevant argument is valid or invalid, and by invoking soundness or completeness. (4 points)

   $$\varnothing \vdash ((P \to Q) \to \neg P) \to \neg P$$

   Answer: No. The sentence is not a tautology. Counterexample: $P = 1, Q = 0$ makes the formula false. By soundness, no correct proof from $\varnothing$ exists.

2. Suppose that φ and ψ are contingencies. Can $\varphi \to \psi$ be a tautology, contingency, or inconsistency? Justify your answers. (4 points)

   Answer:

   - **Tautology:** Yes. Example: φ and ψ are identical.
   - **Contingency:** Yes. Example: $\varphi = P$, $\psi = Q$.
   - **Inconsistency:** No. Since φ is a contingency, it is false on some row of its truth table. On this row, $\varphi \to \psi$ is true. Hence $\varphi \to \psi$ cannot be an inconsistency.