logic pset2

A.

Use Conditional Proof (and possibly the previous rules) to prove the following sequents. Be sure to include dependency numbers in the leftmost column of your proof.

1. $P \to Q \vdash P \to (Q \vee R)$

```
1   (1) P→Q       A
2   (2) P         A
1,2 (3) Q         1,2 MP
1,2 (4) QvR       3 vI
1   (5) P→(QvR)  2,4 CP
```

2. $P \to (Q \to R) \vdash Q \to (P \to R)$

```
1     (1) P→(Q→R)   A
2     (2) Q         A
3     (3) P         A
1,3   (4) Q→R       1,3 MP
1,2,3 (5) R         4,2 MP
1,2   (6) P→R       3,5 CP
1     (7) Q→(P→R)   2,6 CP
```

3. $\neg P \vdash \neg(P \wedge Q)$

```
1  (1) ¬P        A
2  (2) P∧Q       A
2  (3) P         2 ∧E
   (4) (P∧Q)→P  2,3 CP
1  (5) ¬(P∧Q)   4,1 MT
```

4. $\neg(P \vee Q) \vdash \neg P$

```
1  (1) ¬(PvQ)   A
2  (2) P        A
2  (3) PvQ      2 vI
```

```
    (4) P→(PvQ)  2,3 CP
1  (5) ¬P       4,1 MT
```

5. $P \vdash (P \to \neg P) \to \neg P$

```
1   (1) P           A
2   (2) P→¬P        A
1,2 (3) ¬P          2,1 MP
1   (4) (P→¬P)→¬P  2,3 CP
```

6. $P \vdash \neg(P \to \neg P)$

```
1   (1) P           A
2   (2) P→¬P        A
1,2 (3) ¬P          1,2 MP
1   (4) (P→¬P)→¬P  2,3 CP
1   (5) ¬¬P         1 DN
1   (6) ¬(P→¬P)    4,5 MT
```

B.

Use $\vee$-elimination (and possibly the previous rules) to prove the following sequents. Do *not* use reductio ad absurdum for any of these proofs.

1. $P \vee (Q \wedge R) \vdash P \vee Q$

```
1 (1) Pv(Q∧R)   A
2 (2) P         A
2 (3) PvQ       2 vI
4 (4) Q∧R       A
4 (5) Q         4 ∧E
4 (6) PvQ       5 vI
1 (7) PvQ       1,2,3,4,6 vE
```

2. $P \wedge (Q \vee R) \vdash (P \wedge Q) \vee (P \wedge R)$

```
1   (1)  P∧(QvR)       A
1   (2)  P             1 ∧E
1   (3)  QvR           1 ∧E
4   (4)  Q             A
1,4 (5)  P∧Q           2,4 ∧I
1,4 (6)  (P∧Q)v(P∧R)   5 vI
7   (7)  R             A
1,7 (8)  P∧R           2,7 ∧I
1,7 (9)  (P∧Q)v(P∧R)   8 vI
1   (10) (P∧Q)v(P∧R)   3,4,6,7,9 vE
```

3. $P \vee Q, \neg P \vdash Q$

```
1   (1) PvQ     A
2   (2) ¬P      A
3   (3) ¬Q      A
2   (4) ¬Q→¬P   3,2 CP
5   (5) P       A
5   (6) ¬¬P     5 DN
2,5 (7) ¬¬Q     4,6 MT
2,5 (8) Q       7 DN
9   (9) Q       A
1,2 (10) Q      1,5,8,9,9 vE
```

4. $(P \rightarrow R) \wedge (Q \rightarrow R) \vdash (P \vee Q) \rightarrow R$

```
1     (1) (P→R)∧(Q→R)   A
1     (2) P→R           1 ∧E
1     (3) Q→R           1 ∧E
4     (4) PvQ           A
5     (5) P             A
1,5   (6) R             2,5 MP
7     (7) Q             A
1,7   (8) R             3,7 MP
1,4   (9) R             4,5,6,7,8 vE
1     (10) (PvQ)→R      4,9 CP
```