Bohmian Mechanics

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In 1952, a young Princeton physicist named David Bohm discovered a new theory whose predictions match, for all practical purposes, those of standard quantum mechanics. (I add "for all practical purposes" for those who doubt that standard QM makes clear predictions.) What's more, defenders of Bohmian mechanics claim that this new theory has none of the drawbacks of standard QM: it doesn't treat "measurement" as a primitive, it doesn't require violations of Schrödinger's equation, it has a clear ontology (i.e. local beables), it has a deterministic equation of motion, etc.

In the past, opponents of Bohmian mechanics often dismissed it on the basis that it is a "metaphysical addition" to QM, i.e. it adds new entities without adding predictive power, therby violating Ockham's razor. However, that style of criticism has fallen out of fashion along with other radical forms of empiricism that were popular in the twentieth century. Defenders of Bohmian mechanics will say that it's not an addition to an already well defined theory (QM), but that Bohm finally gave us a theory does the explanatory work that QM was supposed to do. What's more, they'll say, Bohm's theory doesn't call for a radical reconceptualization of the human epistemic predicament (as Bohr suggests), nor does it call for a radical rethinking of the nature of personal identity (as Everett requires), nor does it call for rejection of the Schrödinger equation (as GRW requires).

The purpose of this chapter is to assess the extent to which Bohmian mechanics can live up to these promises. Now that radical empiricist critiques of Bohm are out of fashion, the next main complaint about Bohm is that it's nonlocal, and hence conflicts with relativity theory. Bohmians have an answer. They claim that Bell's theorem shows that any empirically adequate theory will be nonlocal. They have also argued (persuasively, I think) that Bohmian mechanics is consistent with the empirical predictions of relativity theory. So, there's no easy argument from locality to not-Bohm.

In this chapter, I'll focus primarily on other issues with Bohmian mechanics. In particular, I will focus on the motivation behind Bohmian mechanics, and whether what it gives us in the end is really better than the alternatives.

1 The preferred quantity

Consider again the example from earlier in the course. There are two boxes, one labelled L and one labelled R, and there is a marble that we place into one of the boxes. Let $|L\rangle$ be the state where the marble is in the left box and let $|R\rangle$ be the state where the marble is in the right box. In addition to these two states, we have the state $|0\rangle$, in which the marble is stationary, and the state $|1\rangle$ in which the marble is moving. We stipulate the following relations (which are the structural relations between position and momentum in any quantum system):

$$\begin{aligned} |0\rangle &= \tfrac{1}{\sqrt{2}}\left(|L\rangle + |R\rangle\right), \\ |1\rangle &= \tfrac{1}{\sqrt{2}}\left(|L\rangle - |R\rangle\right). \end{aligned}$$

The basic idea behind Bohmian mechanics, in a quick snapshot, is that we described the states $|0\rangle$ and $|1\rangle$ in the wrong way. We said that $|0\rangle$ is the state in which "the marble is stationary." But why say that? After all, the marble is stationary just in case it stays in the same box from one moment to the next. So we don't need any additional states besides $|L\rangle$ and $|R\rangle$. Sure, the vectors $|0\rangle$ and $|1\rangle$ are in the Hilbert space, but the question is what they mean *qua* states. The Bohmian proposal is that $|0\rangle$ and $|1\rangle$ should be interpreted as probability distributions over the space $\{|L\rangle, |R\rangle\}$. In particular, both $|0\rangle$ and $|1\rangle$ correspond to the flat distribution that assigns 0.5 to both $|L\rangle$ and $|R\rangle$. (That might make $|0\rangle$ and $|1\rangle$ seem like the same state. But we will see later that $|0\rangle$ and $|1\rangle$ have different dynamical properties.)

The move we just made can be generalized. In any quantum system where there is a well-behaved position operator Q, the Hilbert space $\mathcal{H}$ is isomorphic to a space of functions $L(X)$, where X is the set of values that Q can take.[1] Thus, any state $\psi \in \mathcal{H}$ can be *interpreted* as a probability distribution over X, and that's exactly what the Bohmian does. The Bohmian treats the

[1] I am oversimplifying here. Typically there will be several position operators, three for each particle.

quantity Q as privileged in the sense that (1) Q always has a definite value, and (2) every state should be interpreted as a probability distribution over Q values.

Bohmian mechanics is often described as a *hidden variable theory*, but that is misleading in a couple of ways. First, it's misleading from a mathematical point of view, because Bohmian mechanics does not add new states to the formalism of QM. Notice how we described the situation above: the states of the marble are just $|L\rangle$ and $|R\rangle$. There was no need to supplement with any further states. Second, it's misleading from an epistemic point of view to describe Bohmian mechanics as a hidden variable theory, because the variables aren't hidden. In fact, the states $|L\rangle$ and $|R\rangle$ are the opposite of hidden: they are what we see.

If interpreting states as distributions over positions were all there were to Bohmian mechanics, then it could have been discovered by anyone who understood Hilbert space. But there is more to Bohmian mechanics. The genuinely new thing that Bohm discovered is a "sub-dynamics" on the position eigenstates. Here's what I mean by a sub-dynamics:

Suppose that the quantum-mechanical time evolution is represented by a family of unitary operators U_t, where t is a real-number parameter. In other words, as time ticks from t to t', the quantum state changes from $U_t\psi$ to $U_{t'}\psi$. Then typically a position eigenstate such as $|L\rangle$ will be transformed by U_t to something that is not a position eigenstate, say

$$U_t|L\rangle \;=\; \tfrac{1}{\sqrt{2}}\left(|L\rangle + |R\rangle\right).$$

But that doesn't make any sense as a genuine change of the way things are, because the superposition state on the right is not a "way things are." The superposition state on the right represents our ignorance of the way things are.

Now Bohm responds to this challenge not by adding something new to the formalism of QM, but essentially by allowing there to be two states. The first state can be called the **value state**, and it must be an element of the set $\{|L\rangle, |R\rangle\}$. The second state can be called the **wavefunction**, and it can be any element of $\mathcal{H}$. The quantum dynamical evolution U_t is only applied to the wavefunction. Bohm's big discovery was finding a second dynamical law that applies to the value state and that meshes nicely with the first dynamical law.

The one tricky thing about Bohm's second dynamical law is that it depends on *both* the present value state and the wavefunction. In other words, the future value state $|j\rangle$ is a function of the present value state $|i\rangle$ and the present wavefunction ψ. This is the reason that the wavefunction is sometimes called the "pilot wave" and the corresponding dynamical law is called the "guiding equation."

2 Missing quantities

What then are the challenges for Bohmian mechanics? The first challenge is to explain the utility of the quantum-mechanical formalism, in particular the fact that operators (such as P) appear to represent quantities (such as momentum) that occasionally have values. Unfortunately, momentum itself is not the best example to start with. It's tempting to think that momentum is nothing but velocity times mass, and velocity is nothing more than a description of position over time, and hence, if one has positions at all times, then one automatically has velocities. This also might tempt you to think that there is nothing to explain vis-a-vis momentum, because to measure momentum one just measures a series of positions. So let's start with a different example.

Consider a two-dimensional Hilbert space with spin-z and spin-x operators. Suppose that we prefer spin-z in the way that Bohmians prefer position: at each time, the particle has value state either $|z+\rangle$ or $|z-\rangle$, and its quantum state (i.e. wavefunction) ψ happens to give the best guess (prior to measurement) of what this value state is. Now suppose that we "measure" spin-x and the particle comes out up. How are we supposed to understand what just happened? And how should we explain the fact that if we immediately measure spin-x again, we will get the same value?

Notice, in fact, that Bohm violates the EPR reality criterion at precisely this point. We can predict with certainty that spin-x will have a value, but there Bohm says that there is no corresponding element of reality! I myself don't take this to be a damning feature of Bohm, at least not if we can explain our predictive ability in terms of the behavior of the fundamental elements of reality (in this case, the values of spin-z).

References

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