PHI 201, Final Exam **Spring 2018**

1. Translate the following into predicate logic. You can assume that the domain is people, and so you don't need an additional predicate symbol for "x is a person".

   (a) There is a person who loves all people who love her. (Use Lxy for "x loves y".)

   (b) Every lover loves herself. (A "lover" is somebody who loves at least one person.)

   (c) There are exactly two people.

2. Could the following sentence be true? Explain your answer.

$$(\neg p \vee q) \wedge ((q \to (\neg r \wedge \neg p)) \wedge (p \vee r))$$

3. Explain what's wrong with the following attempted proof:

1	(1)	Fa	A
	(2)	$Fa \to Fa$	1,1 CP
	(3)	$\forall y(Fy \to Fa)$	2 UI
	(4)	$\exists x \forall y(Fy \to Fx)$	3 EI

4. Prove the following sequent. You can use cut/replacement, but only if you prove the relevant sequents in your exam booklet, and clearly cross-reference them.

$$\vdash \ \exists x \forall y(Fy \to Fx)$$

5. Give a rigorous, but informal, proof of the following fact of set theory:

$$C - (A \cap B) \ \subseteq \ (C - A) \cup (C - B)$$

   Here we use the definition:

$$\forall x((x \in (C - X)) \leftrightarrow (x \in C \wedge x \notin X)).$$

6. Let A be the set defined inductively by:

   - $p \in A$
   - If $\phi \in A$ and $\psi \in A$ then $(\phi \to \psi) \in A$.

   Show that for every $\phi \in A$, $\neg\phi \vdash \neg p$. [Spring 2019: we didn't do problems like this on a pset, so it's not likely that we would ask a question like this one.]

7. Provide a countermodel to show that the sentence on the left does *not* imply the sentence on the right. In your countermodel, you should explicitly specify a domain, and extensions for all the predicate symbols.

   $$\exists x(Fx \to \exists y Gy) \qquad\qquad \exists x Fx \to \exists y Gy$$