Here's an alternate solution to 1.6.2.

Using the Leibniz rule and the fact that L_ξ commutes with index substitution, we have:

$$\begin{aligned} L_\xi(\eta^a\eta^b) &= \eta^a L_\xi(\eta^b) + \eta^b L_\xi(\eta^a) \\ &= \eta^a \delta^b{}_a L_\xi(\eta^a) + \eta^b L_\xi(\eta^a) \\ &= \eta^b L_\xi(\eta^a) + \eta^b L_\xi(\eta^a). \end{aligned}$$

Thus, if $L_\xi(\eta^a\eta^b) = 0$ then $\eta^b L_\xi(\eta^a) = 0$. If η is nonvanishing, then it can be cancelled, leaving $L_\xi(\eta^a) = 0$.

One thing I would prefer to confirm: how do we know that there is a smooth field λ_a that is inverse to η^a?

Question: Is there an example of a smooth field η such that both $L_\xi(\eta^a\eta^b) = 0$ and $L_\xi(\eta^a) \neq 0$? Obviously it would have to vanish at some point.