Logic precept: Week 8

Translation

1. Mary is the only student who didn't miss any questions on the exam.
2. All professors except a are boring.
3. There is no greatest prime number.
4. The smallest prime number is even.
5. For each natural number, there is a unique next-greater natural number.
6. There are at least two Ivy League universities in New York state.

Proofs with equality

1. $Fa \vdash \forall x((x = a) \to Fx)$
2. $\forall x((x = a) \to Fx) \vdash Fa$
3. $\exists x \forall y(x = y) \vdash \forall x \forall y(x = y)$

Partial order

In real life, rigorous proofs are rarely written with line numbers, dependencies, or named justifications. But the idea is to give the reader enough information so that s/he could reconstruct such a proof.

1. Write down a predicate logic sentence that expresses the claim that every two elements have a least upper bound.
2. Give an example of a partially ordered set in which that sentence is false.

3. Prove (informally) that if any two elements have a least upper bound, then so do any three elements.
4. We say that $\leq$ is a serial relation just in case $\forall x \exists y(x \leq y \wedge x \neq y)$. Is there a *finite* partially ordered set that satisfies the serial axiom?

Set theory

For sets a and b, we write $a \subseteq b$ for the claim that $\forall x(x \in a \rightarrow x \in b)$.

We let $a \cap b$ be the set defined by $\forall x((x \in a \cup b) \leftrightarrow (x \in a \wedge x \in b))$.

1. Show that if $a \subseteq b$ and $b \subseteq c$ then $a \subseteq c$.
2. Show that $a \subseteq b$ if and only if $a \cap b = a$.