# Chapter 5: Relational Theories of Motion

1. "Can there be interesting theories of motion based on classical space-times that do not involve *absolute quantities of motion*, whether absolute velocity, acceleration, or rotation?" (p 91)

   What do you think Earman means by an **absolute quantity**?

   (a) Proposal: A necessary condition for a quantity to be absolute is that it corresponds to a monadic predicate of trajectories that is (a) non-trivial, and (b) preserved by all symmetries of the relevant type of spacetime.

   (b) Example: "$x$ is an inertial trajectory" is a monadic predicate for any theory whose models consist of manifold, covariant derivative operator (and possibly other structure).

   (c) Note: Some such global properties might be specifiable locally, i.e. some local property holds at all points along the trajectory. In this case: $\nabla_{\dot{\gamma}}\dot{\gamma} = 0$.

   (d) Example: "$x$ is a stationary trajectory" is *not* a monadic predicate in Galilean spacetime. If we treated one inertial trajectory as stationary, we'd have to treat them all as such.

      The key fact is that the inertial trajectories form an invariant subset of all trajectories under the induced action of the Galilei group on trajectories: $\gamma \mapsto \varphi \circ \gamma$.

2. HH: It's misleading to say that these theories lack absolute velocity. It would be more correct to say that they lack an absolute notion of rest, or perhaps even better, that they do not distinguish bewteen rest and inertial motion. I might even say: the standard of inertia is local to an object itself, and does not depend on its relation to some fixed resting point. In particular, $x$ is inertial just in case $x$ stays on the trajectory determined by its own velocity vector (as opposed to being pushed off it by some external force).

   Every theory with a covariant derivative operator $\nabla$ has a predicate $\mathsf{Iner}(\gamma, s)$ that says a curve $\gamma : \mathbb{R} \to M$ is inertial at a point $s \in \mathbb{R}$. *Does this mean that a theory with a covariant derivative operator is automatically substantivalist?*

3. A relativistic spacetime is a smooth manifold $M$ with a Lorentzian metric $g$.

   (a) The metric defines lightcones.

   (b) The metric defines a covariant derivative operator $\nabla$.

   (c) Einstein's field equations establish a relation between spacetime metric $g$ and matter distribution $T$.

4. Earman: Relativity is less friendly to relationalism than classical spacetime theories.

   (a) A relativistic spacetime has a metric, which defines a standard of acceleration (via the covariant derivative) and of rotation. Contrast with Machian and Leibnizian spacetime

   (b) What about the idea that relativity theory entails the following order of explanation:

      matter $\Rightarrow$ metric $\Rightarrow$ motion

   (c) Fact: There is more than one solution to EFE with $T = 0$.

5. Acceleration in STR and GTR

   Given a timelike curve $\gamma$, the acceleration vector field $\nabla_{\dot{\gamma}}\dot{\gamma}$ along it is a measure of how much $\gamma$ differs from a geodesic. The metric $g$ determines length of vectors, and hence, gives an absolute magnitude of acceleration.

   Fact: If $X$ is a vector field along a curve such that $g(X,X)<0$ is constant, then $\nabla_X X$ is spacelike.

   Since $\nabla$ is the Levi-Civita connection for $g$, and $g$ is symmetric, we have

   $$X(g(X,X)) = 2g(\nabla_X X, X).$$

   Since $g(X,X)$ is constant, it follows that $g(\nabla_X X, X) = 0$. The conclusion follows by the reverse Cauchy-Schwartz inequality.

6. Einstein's challenge

   "What is the reason for this difference in the two bodies? No answer can be admitted as epistemologically satisfactory, unless the reason given is an *observable fact of experience.* ... But the privileged space $R_1$ of Galileo ... is a merely *facticious* cause, and not a thing that can be observed."

7. Does spacetime structure *cause* acceleration effects, e.g. the concavity of the water in the bucket, the tension in the string between the spheres, the Lorentz contraction?

   Does the distinction between accelerated and non-accelerated motion need to be grounded in the existence of some thing?

8. The Mach-Einstein critique of absolute space

   (a) "It is clear from the foregoing that if Newtonian mechanics and STR are unsatisfactory because they employ the 'factitious cause' of inertial frames, then GTR is equally unsatisfactory." (p 102)

   (b) HH: Consider the Newtonian explanation of why one bucket of water is concave and the other is flat, or why one string between spheres is taut and another is not

      i. The causal differentia are not observable
      The effect cannot be predicted from observing some other fact. There is no "constant conjunction".

      ii. The causal differentia are not manipulable
      We can give a shove, but we don't know whether we are starting absolute motion or stopping it!