PHI 201, Final Exam 2025

Instructions: Write your name, preceptor's name, and pledge on the exam booklet. Write your answers **legibly**. You have up to three hours to complete the exam, although it was designed to take no more than two. You may use one letter-sized page of notes during the exam. When finished, place everything in the box at the front of the room, and leave quietly.

A. Engagement

Please write the following if and only if you were in lecture on December 1st: "I attended the lecture on December 1st, and the coupon code is x", where you replace x with the coupon code. (2 points)

B. Translation

Please translate the following into the symbolism of predicate logic. Use Gx for "x is a god" and Hxy for "x helps y". You may assume that quantifiers and variables apply only to people and gods. (4 points each)

1. There are at most three gods.
2. There is a god who helps nobody.
3. The gods help those who help themselves.

C. Proofs

Prove the following sequents. You may simply cite "prop taut" for any substitution instance of a propositional tautology — no need to include a proof. You may also substitute quantifier validities and equivalences, but only if you prove them in your exam booklet. (6 points each)

1. $\neg\forall x Fx \vdash \exists x \neg Fx$
2. $\forall x Fx \leftrightarrow \neg\exists x \exists y Rxy \vdash \exists x \forall y \forall z (Fx \rightarrow \neg Ryz)$

Please turn over

D. Models

Show that the following sequents are invalid by providing countermodels. (6 points each)

1. $\forall x(Fx \to Gx), \exists x Gx \vdash \exists x Fx$
2. $\forall y \exists x Rxy, \forall x \forall y \forall z((Rxy \wedge Rzy) \to x = z) \vdash \exists x \forall y Rxy$

E. Conceptual

Note: This problem is *extra credit*, and will only be used to raise grades from A$^-$ to A, or from A to A$^+$.

Let Γ be the set of sentences built from P using $\neg$ and $\wedge$. Show that for every sentence φ in Γ, either $P \vdash \varphi$ or $P \vdash \neg\varphi$.

The end