# Reduction Redux

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# Prescript

This topic is too big to be dealt with in a one hour lecture, especially because I intend to cause trouble for the dominant picture, and to replace it with something quite different. The dominant picture is that reality (or our description of it) is like a tower, with microphysics at the bottom, and where "reduction" breaks the symmetry of the spatial metaphor. (If $B$ reduces to $A$, it's as if $A$ bears weight of $B$.)

The first stage of this project is to try to find an explication of "$B$ reduces to $A$". In this talk, I point out that "$B$ is a Morita extension of $A$" is a useful generalization of classical, Nagelian reduction.

We could go on and do lots of other fancy formal work, e.g. with category theory. But I don't think it will help the dominant picture. Scientific theories form an eco-system, not a tower.

# Outline

# Reducible or not: Who cares?

# Reduction as religious stance

1. The church of reduction
    - Lewis (Albert, Loewer, Butterfield)
    - Dennett (Wallace)
2. The church of anti-reduction
    - Chalmers
    - Thomas Nagel
    - Ellis, Drossel, Koons, Simpson
3. Non-practicing reductionists
    - Davidson

# Where reduction pops up

- within physics
    - thermodyanmics to statistical mechanics
    - classical world from quantum
- between (empirical) sciences
- manifest image to scientific image
    - mind to brain
- within mathematics

# The forward march of physics?

# Success?

- The mechanical worldview: Galileo, Descartes, etc.
    - Primary and secondary qualities
- Spacetime
- Reduction of thermodynamics to statistical mechanics?

# Trouble on the horizon?

- The reversibility problem in thermodynamics
- It is *not* as if the *user* of quantum mechanics has any immediate reason to believe that he has run up against a non-material element of reality.
- So if QM poses a problem for physicalism, it is a different sort of problem then the one encountered in psychology or biology.
- It is the *meta-theory* of QM that causes trouble: it doesn't seem possible to use QM to describe someone using QM to describe reality.

This last problem is often called “the measurement problem”. But I prefer to frame it as “the description problem”, as it crosses over into the space of reasons.

The problem only arises if one thinks of the first theorist as able to stand in “aboutness” relations to a state of affairs.

# Two kinds of solutions

- Mono solutions
  - Bohm
  - Everett
- Duo solutions
  - Heisenberg cut — old and much derided, e.g. by Bell
  - Quantum hylomorphism — new

# The religion in reduction

- This debate reminds me of philosophy of religion with its countless arguments for and against God's existence.
- After surveying a dozen or so of these (inconclusive and conflicting) arguments, the returns start to diminish (for me at least).
- Let's seek insight rather than an existential archimedean point.

# Explications of reduction

# From emotion to analysis

- The previous one-hundred years in philosophy have been genuinely different — because of its connection with symbolic logic and mathematics.
- Carnapian explication: where we take a vague concept or thesis and provide a (mathematically) precise counterpart
- Example: inference rules for classical predicate calculus

# Preliminaries

- What kinds of things stand in the "reduction" relation?
- I will take the relata to be **theories**, and I will take theories to be represented by pairs $L, T$, where $L$ is a language and $T$ is a set of $L$-sentences.
- I do *not* assume that "same theory" means $L = L'$ and $T = T'$.

# What reduction could not be?

- Just as some have complained that equivalence (of theories) is not a purely formal matter, they might also complain that reduction is not a purely formal matter.
- Is reduction a "worldly relation"?

# Criteria for success

- What kind of understanding will such an account give?
- What kind of answers will such an account give?
- Is it a factual question whether one theory is reducible to another?

# Classical reduction

**Definition:** $T$ explicitly defines $R$ in terms of $\Sigma_0$ just in case:

$$T \vdash \forall x(R(x) \leftrightarrow \theta(x))$$

**Proposition:** For every $\Sigma$-formula $\varphi(x)$, there is a $\Sigma_0$-formula $\varphi^*(x)$ such that $T \vdash \forall x(\varphi(x) \leftrightarrow \varphi^*(x))$.

# Identity theory

- The **identity theory** is stronger than explicit definition.
- Hence, if explicit definition is too restrictive, so is the identity theory.

# Challenges to explicit definability

- Ramsey: "[I]f we proceed by explicit definition we cannot add to our theory without changing the definitions, and so the meaning of the whole."
- Carnap: "It is, in general, not possible to give explicit definitions for theoretical terms on the basis of $\Sigma_0$." (1956, p 42)
- Putnam: Multiple realizability
- Fodor: The special sciences
- Jackson: Explanatory gap

- I'm not primarily interested in whether mind is reducible to brain.
- I want to know what kind of thing *reduction* is, and whether *classical reduction* admits of fruitful generalization.

# Open the semantic floodgates!

- 1970s: Buzzwords about relations binding the mental to the physical: determination, supervenience, functionalism, etc.
- ca. 1965–2020 common wisdom: you want some kind of root in the physicalist basis (determination, supervenience, emergence, . . . ), but *not* reducibility.
- These philosophers **failed** to enable interdisciplinary discussion
- **Functional definition** is one of the more promising ideas. (See especially Lewis, 1970; Lewis, 1972)
    - Popular in recent philosophy of science, e.g. spacetime functionalism (see Butterfield and Gomes, 2023)

"Accounts of inter-theoretic reduction differ between the language-first and math-first views of theories in quite similar ways to their respective accounts of theoretical equivalence." (Wallace, 2022, p 356)

"On the math-first view, reduction is something like instantiation: the realizing by some sub-structure of the low-level theory's models of the structure of the higher-level theory's models." (Wallace, 2022, p 357)

# Semantic strategies

- After the invention of model theory, formal philosophy fell into confusion about what could be achieved by semantic methods.
- Many philosophers claimed that old problems were artifacts of the syntactic approach.
- The confusion was increased by an ambiguity about whether semantics is about mathematical models or about concrete existents.

- Supervenience: No change to the new objects without a change in the old objects.
- Any elementary embedding $h : M|_{\Sigma_0} \to N|_{\Sigma_0}$ lifts uniquely to an elementary embedding $\overline{h} : M \to N$.

# From semantics to syntax

- Hellman and Thompson (1975) point out that supervenience (determination) is just implicit definition, and it implies (via Beth's theorem) explicit definition. But they claim that the result is irrelevant since they aren't talking about models of a theory!
- Bealer (1978) argues that functional definition is just implicit definition and so implies explicit definition.

# Formalizing functionalism

$$t = \iota x \phi(x)$$

Lewis: "This is what I have called **functional definition**. The $\Sigma$-terms have been defined as the occupants of the causal roles specified by the theory $T$; as *the* entities, whatever those may be, that bear certain causal relations to one another and to the referents of the $\Sigma_0$-terms." (1972, p 254)

# Reduction of domains

- Classical reduction only permits relations (on a fixed domain) to be reduced to relations (on that same domain).
- If reduction is inverse to definition, then what we need is a method of defining new domains out of old ones.

# Sort Constructions in First-Order Logic

**Quotient**

$$\begin{array}{c} \sigma \\ \downarrow e \\ \sigma' \end{array}$$

**Subsort**

$$\begin{array}{c} \sigma \\ \uparrow i \\ \sigma' \end{array}$$

**Product**

$$\begin{array}{ccc} & \sigma_0 \times \sigma_1 & \\ \swarrow & & \searrow \\ \sigma_0 & & \sigma_1 \end{array}$$

**Coproduct**

$$\begin{array}{ccc} & \sigma_0 \coprod \sigma_1 & \\ \nearrow & & \nwarrow \\ \sigma_0 & & \sigma_1 \end{array}$$

- **Definition:** $T^+$ is a **Morita extension** of $T$ just in case $T^+$ is at the end of a finite chain of extensions by the sort constructions and explicit definitions on $T$.
- **Proposition:** If $T^+$ is a Morita extension of $T$, then there is reduction map $G : T^+ \to T$.

# Example: The reducibility of universalism to atomism

- The nihilist theory $T$ defines a domain of mereological $n$-sums (product, quotient by permutation symmetry) and so a universalist theory $T'$.
- For each formula $\phi(x)$ in $T'$, there is a corresponding formula $\phi^*(y_1, y_2)$ in $T$.
- $T' \vdash \phi$ iff $T \vdash \phi^*$

# Nota bene

The previous two theories are actually equivalent.

A more natural example might be:

- $T$ is the theory of the integers.
- $T'$ is the theory of the rational numbers.

# Conclusion

# The fundamental dilemma for reduction

- If the $N$ to $O$ relation is too close, then $N$ seems like another name for $O$.
- If the $N$ to $O$ relation is too far apart, then $N$ seems "ontologically problematic".

# Manifesto

- Carnap the existentialist: freedom to develop new theoretical concepts.
- Reduction should give "inference tickets" in both directions.
- Compare (HH view of theoretical equivalence): if two theories are equivalent, you can switch between them.
- Against absolutism

# References

# References i


- Bealer, George (1978). **"An inconsistency in functionalism".** In: *Synthese* 38.3, pp. 333–372. DOI: 10.1007/BF00486635.
- Butterfield, Jeremy and Henrique Gomes (2023). **"Functionalism as a species of reduction".** In: *Current debates in philosophy of science: In honor of Roberto Torretti*. Springer, pp. 123–200.
- Carnap, Rudolf (1956). **"The methodological character of theoretical concepts".** In: *The Foundations of Science and the Concepts of Psychology and Psychoanalysis*. Ed. by Herbert Feigl and Michael Scriven. Minneapolis: University of Minnesota Press, pp. 38–76. URL: https://hdl.handle.net/11299/184284.

# References ii


- Hellman, Geoffrey and Frank W. Thompson (1975). **"Physicalism: Ontology, determination, and reduction".** In: *The Journal of Philosophy* 72.17, pp. 551–564. DOI: 10.2307/2024646.
- Lewis, David (1970). **"How to define theoretical terms".** In: *Journal of Philosophy* 67.13, pp. 427–446. DOI: 10.2307/2023861.
- — (1972). **"Psychophysical and theoretical identifications".** In: *Australasian Journal of Philosophy* 50.3, pp. 249–258. DOI: 10.1080/00048407212341301.
- Wallace, David (2022). **"Stating structural realism: Mathematics-first approaches to physics and metaphysics".** In: *Philosophical Perspectives* 36.1, pp. 345–378. DOI: 10.1111/phpe.12172.

# Postscript

I predict that classical Nagelian reduction is going to to turn out to be too restrictive, no matter how sophisticated we get with the formal machinery, e.g. generalized translations. But it's still interesting to get a handle on the possible formal relationships between formal theories. In the case of propositional theories, translations are naturally classified by: **conservative**, **essentially surjective**, and of course, as being one half of an equivalence. Nagelian reductions are inverse of definitional extensions, and that entails that they are one half of an equivalence. Thus, the paradigm example of a reduction of formal theories would be both conservative and essentially surjective. That doesn't seem like a great model for examples we know from physics. For example, it does not seem that thermodynamics can define all of the concepts of classical mechanics.