

## Logic pset 6

Resources: HLW [Ch 7](#) pp 116-127 and

1. Represent the form of the following sentences in predicate logic using the  $=$  symbol where necessary.
  - (a) There is one and only one Princeton University. (Use  $Px$  for “ $x$  is a Princeton University”)
  - (b) There is at most one Ivy League university in New Jersey. (Use  $Ix$  for “ $x$  is an Ivy League university”, and use  $Nx$  for “ $x$  is in New Jersey.”)
  - (c) There is a smallest prime number. ( $Px, Sxy$ , variables are restricted to numbers.)
2. Prove the following sequents using any of the rules, including  $=E$  and  $=I$ .
  - (a)  $\exists x(Px \wedge \forall y(Py \rightarrow x = y)) \vdash \forall x\forall y((Px \wedge Py) \rightarrow x = y)$
  - (b)  $\vdash \forall x\forall y((x = y) \rightarrow (y = x))$
3. Let  $Rxy$  be a binary relation symbol that satisfies the transitivity axiom (page 126). Suppose that  $Rxy$  satisfies two other axioms: serial  $\forall x\exists yRxy$  and irreflexive  $\forall x\neg Rxx$ . Show that there are at least three distinct things, i.e.,

$$\exists x\exists y\exists z((x \neq y \wedge x \neq z) \wedge y \neq z).$$

It would also suffice to show that the claim “there are at most two things” contradicts the assumptions. You may write your proof in English prose (not our formal system), but you need to convince the reader that you would be able to write a full formal proof.