PHI 201, Practice Final Exam

Instructions: Write your name, preceptor's name, and pledge on the exam booklet. Write your answers *legibly* in the exam booklet. While you may take up to three hours to complete the exam, it was designed to take no more than two.

1. Translate the following into predicate logic. You can assume that the domain is people, and so you don't need an additional predicate symbol for "x is a person".

   (a) There is a person who loves all people who love her. (Use Lxy for "x loves y".)

   (b) Every lover loves herself.

   (c) There are exactly two people.

2. Could the following sentence be true? Explain your answer.

   $$(\neg P \vee Q) \wedge ((Q \to (\neg R \wedge \neg P)) \wedge (P \vee R))$$

3. Explain what's wrong with the following attempted proof:

1	(1)	Fa	A
$\varnothing$	(2)	$Fa \to Fa$	1,1 CP
$\varnothing$	(3)	$\forall y(Fy \to Fa)$	2 UI
$\varnothing$	(4)	$\exists x \forall y(Fy \to Fx)$	3 EI

4. Prove the following sequent. You can use "cut" or "replacement", but only if you prove the relevant sequents in your exam booklet.

   $$\vdash \exists x \forall y(Fy \to Fx)$$

5. Prove the following fact of set theory:

   $$C \setminus (A \cap B) \subseteq (C \setminus A) \cup (C \setminus B),$$

   where $C \setminus X$ is defined by

   $$\forall x((x \in (C \setminus X)) \leftrightarrow (x \in C \wedge x \notin X)).$$

   Your proof should be rigorous, but it can (preferably) be written in English prose.

6. Let Γ be the set of sentences defined inductively by:

   - $P \in \Gamma$
   - If $\varphi \in \Gamma$ then $\neg\varphi \in \Gamma$.
   - If $\varphi \in \Gamma$ and $\psi \in \Gamma$ then $\varphi \to \psi \in \Gamma$.

   Show that for every $\varphi \in \Gamma$, either $P \vdash \varphi$ or $P \vdash \neg\varphi$.

7. Provide a countermodel to show that the following sequent cannot be proven.

   $$\exists x(Fx \to \exists y Gy) \vdash \exists x Fx \to \exists y Gy$$