logic pset4

Resources: Lecture 4 and Chapter 6 (pp 84-99) of *How Logic Works*.

A. Translation

Represent the form of the following sentences in predicate logic. We've suggested appropriate symbols. (We assume that quantifiers are restricted to persons, so you don't need to add an extra predicate for "x is a person.")

1. Only students who do the homework will learn logic. (Sx, Hx, Lx)

   $$\forall x(Sx \to (Lx \to Hx))$$

2. All students and professors get a discount. (Sx, Px, Dx)

   $$\forall x((Sx \lor Px) \to Dx)$$

3. Every student respects every professor who respects some student. (Sx, Px, Rxy)

   $$\forall x(Sx \to \forall y(Py \to (\exists z(Sz \land Ryz) \to Rxy)))$$

   $$\forall x \forall y((Sx \land Py \land \exists z(Sz \land Ryz)) \to Rxy)$$

4. There is some student who respects only those professors who respect all students. (Sx, Px, Rxy)

   $$\exists x(Sx \land \forall y(Py \to (Rxy \to \forall z(Sz \to Ryz))))$$

   $$\exists x(Sx \land \forall y((Py \land Rxy) \to \forall z(Sz \to Ryz)))$$

B. Proofs

Prove the following sequents with the propositional logic rules plus UE and UI. You may also use cut and replacement with any of the "useful sequents" from the back of the textbook.

1. $\forall x(Fx \to \forall y Gy) \vdash \forall x \forall y(Fx \to Gy)$

1	(1)	$\forall x(Fx \to \forall y Gy)$	A
2	(2)	Fa	A
1	(3)	$Fa \to \forall y Gy$	1 UE
1,2	(4)	$\forall y Gy$	3,2 MP
1,2	(6)	Gb	4 UE
1	(7)	$Fa \to Gb$	2,6 CP
1	(8)	$\forall y(Fa \to Gy)$	7 UI
1	(9)	$\forall x \forall y(Fx \to Gy)$	8 UI

2. $\forall x \forall y(Fx \to Gy) \vdash \forall x(Fx \to \forall y Gy)$

1	(1)	$\forall x \forall y(Fx \to Gy)$	A
2	(2)	Fa	A
1	(3)	$\forall y(Fa \to Gy)$	1 UE
1	(4)	$Fa \to Gb$	3 UE
1,2	(5)	Gb	4,2 MP
1,2	(6)	$\forall y Gy$	5 UI
1	(7)	$Fa \to \forall y Gy$	2,6 CP
1	(8)	$\forall x(Fx \to \forall y Gy)$	7 UI

3. $\vdash \forall x(\forall y Rxy \to Rxx)$

1	(1)	$\forall y Ray$	A
1	(2)	Raa	1 UE
$\varnothing$	(3)	$\forall y Ray \to Raa$	1,2 CP
$\varnothing$	(4)	$\forall x(\forall y Rxy \to Rxx)$	3 UI

C. Conceptual

It can be proven that $\forall x Fx \to \forall x Gx \vdash \forall x(Fx \to Gx)$, but the following attempt at a proof has a mistake. What is the mistake? A good answer can be as short as one sentence.

1	(1)	$\forall x Fx \rightarrow \forall x Gx$	A
2	(2)	Fa	A
2	(3)	$\forall x Fx$	2 UI
1,2	(4)	$\forall x Gx$	1,3 MP
1,2	(5)	Ga	4 UE
1	(6)	$Fa \rightarrow Ga$	2,5 CP
1	(7)	$\forall x(Fx \rightarrow Gx)$	6 UI

Line 3 is not a valid application of UI, since "a" occurs in the dependencies of line 2.