

1 Final Exam PHI 201 H. Halvorson

There are 150 possible points.

2 Part I: Propositional Logic (65 points total)

1. Translate the following argument into propositional logic. (5 pts.)
Neither Dolores nor Fred went to the movies last night.
Therefore, Dolores didn't go to the movies last night.
2. "It is true that ..." is a sentential operator. Is it truth functional? Explain your answer. (5 pts.)
3. Compute the truth value of the following statements, given that A and B are each true and X and Y are each false. (6 pts; 3 pts. each)
 - a) $\sim(A \vee \sim B) \rightarrow (X \vee \sim B)$
 - b) $(A \& \sim Y) \leftrightarrow (\sim X \rightarrow Y)$
4. Are the following true or false? (9 pts; 3 pts. each)
 - a) The negation of a contingent sentence is always an inconsistent sentence.
 - b) $(\sim(P \& Q) \& Q) \rightarrow \sim P$ is a tautology.
 - c) If a system of propositional logic is *complete*, then any tautologous sequent can be proved with the rules of that system.

Translate sentences (5) – (7) into propositional logic. Use the dictionary provided below. (15 pts.; 5 pts. each)

P \equiv the band performs M \equiv a Mingus record plays over the PA

T \equiv the manager threatens the band U \equiv the audience is unhappy

A \equiv the band is paid in advance

5. The band does not perform unless either they are paid in advance or the manager threatens them.

6. The manager threatens the band only if the band performs and a Mingus record plays over the PA.

7. The audience is unhappy, provided that the band is not paid in advance and a Mingus record plays over the PA.

8. Use the full truth table method to determine whether the following argument form is valid or invalid. Show your work. (10 pts.)

$(P \& \sim Q) \rightarrow R, \sim P \leftrightarrow Q : R \vee \sim P$

9. State the rule *v-Elimination*, including the method for tabulating the dependency numbers. (5 pts.)

10. Construct a proof of the following argument. Use only primitive rules of Lemmon's system. ***Do not use trees!*** (10 pts.)

$\sim(P \rightarrow Q) : P \& \sim Q$

3 TRIAL RESTRICTION

4 Part II: Quantifier Logic (85 points total)

Translate sentences (11) – (13) into quantifier logic. Use the dictionary provided below. (15 pts; 5 pts. each)

s = Sally $Lx \equiv x$ is a letter

j = Julia $Axy \equiv x$ is addressed to y

$Cx \equiv x$ is a courier $Px \equiv x$ is a person

$Gxyz \equiv x$ gives y to z $Wxy \equiv x$ is written by y

11. Every letter is written by someone.

12. Some couriers give everything to Julia.

13. If everything is written by Sally, then something is addressed to Julia.

TRIAL RESTRICTION

14. Translate the following quantifier logic sentence into English. (Let $Px \equiv x$ is a president of the US; $Tx \equiv x$ is a Texan) [5 pts.]

$(\exists x)[Px \ \& \ (Py \rightarrow (x = y)) \ \& \ Tx]$

15. State the restrictions that apply to the rule *U-Introduction*. (5 pts.)

16. Construct proofs of the following arguments using Lemmon's system of quantifier logic. You may also use quantifier equivalence. ***Do not use trees!*** (20 pts; 10 pts. each)

a) : $[(\exists x)Fx] \vee [(\forall y)\sim Fy]$

b) $(\forall x)(\forall y)(Rxy \rightarrow \sim Ryx) : (\forall x)[\sim Rxx]$

17. In what way does the following proof violate Lemmon's rules? (5 pts.)

- {1} 1. $(\exists x)Fx A$
- {2} 2. $Fa A$
- {3} 3. $\sim Fa A$
- {2,3} 4. $Fa \& \sim Fa 2,3 \& I$
- {2} 5. $\sim \sim Fa 3,4 RAA$
- {2} 6. $Fa 5 DN$
- {1} 7. $Fa 1,2,6 EE$
- {1} 8. $(x)Fx 3 UI$

18. Complete the following definitions. (10 pts; 5 pts. each)

- a) A quantifier logic sentence is *consistent* if and only if ...
- b) A quantifier logic sentence is *logically true* if and only if ...

19. Provide interpretations which show that the following argument forms are invalid. (20 pts; 10 pts. each)

- a) $(x)(Lx \rightarrow \sim Mx), (\exists x)(Nx \& Mx) : (\exists x)(Lx \& \sim Nx)$
- b) $(\exists x)Gx \& (y)[Gy \rightarrow (\exists x)(Fx \& Rxy)] : (\exists x)[Fx \& (y)(Gy \rightarrow Rxy)]$

20. Determine whether the following sentence is consistent. Justify your answer. (5 pts.)

$$(y)(\exists x)Rxy \& (x)(y)(z)[(x = y) \vee (x = z) \vee (y = z)] \& \sim (\exists x)(y)Rxy$$

5 Extra Credit (10 points; all or nothing)

Show that the rule RAA of Lemmon's system of propositional logic is redundant; i.e., it can be derived from the remaining 9 primitive rules.