

Logic precept 3

Proofs

Review of \vee -elimination

1. $(P \rightarrow Q) \vee (P \rightarrow R) \vdash P \rightarrow (Q \vee R)$
2. $\neg P \vee \neg Q \vdash \neg(P \wedge Q)$
3. $\neg P \vee Q \vdash P \rightarrow Q$

Reductio ad Absurdum

1. $P \rightarrow Q \vdash \neg(P \wedge \neg Q)$
2. $\neg(P \rightarrow Q) \vdash \neg Q$
3. $\neg(P \rightarrow Q) \vdash Q \rightarrow R$
4. $\neg(P \vee Q) \vdash \neg P$
5. $P \rightarrow Q \vdash \neg P \vee Q$
6. $P \rightarrow (Q \vee R) \vdash (P \rightarrow Q) \vee R$

Challenge problem: Pierce's law

$$\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$$

Truth tables

Key Concepts

- arguments: valid, invalid
- counterexample
- truth-value
- main connective
- sentences (syntactic): atomic, conjunction, negation, disjunction, conditional, biconditional
- sentences (semantic): tautology, inconsistency, contingency
- two sentences: equivalent, inconsistent, independent

For arguments

Determine whether the following arguments are valid or not. Explain your answer by showing the existence of a row of a truth table, or by pointing to a full truth table, or something of the sort. Your answer should be articulated in English prose so that it can convince anyone else who is familiar with truth tables.

1. $P \rightarrow (Q \vee R) \vdash (P \rightarrow Q) \vee R$
2. $\vdash (P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (Q \leftrightarrow R)$
3. $P \rightarrow (Q \rightarrow R) \vdash (P \wedge Q) \rightarrow R$
4. $P \rightarrow R \vdash (P \vee Q) \rightarrow R$
5. $(P \leftrightarrow Q) \leftrightarrow R \vdash P \vee R$
6. $\vdash (P \rightarrow Q) \vee (Q \rightarrow R)$