logic pset4

Resources: Lecture 4 and Chapter 6 of *How Logic Works*.

A. Translation

Represent the form of the following sentences in predicate logic. We've suggested appropriate symbols. (For the sentences about people, you don't need to add an extra predicate for "x is a person.")

1. Only students who do the homework will learn logic. (Sx, Hx, Lx)
2. All students and professors get a discount. (Sx, Px, Dx)
3. Every student respects every professor who respects some student. (Sx, Px, Rxy)
4. There is some student who respects only those professors who respect all students. (Sx, Px, Rxy)

B. Proofs

Prove the following sequents using the propositional logic rules, UE, UI. You may use cut and replacement with any of the "useful sequents" from the back of the textbook.

1. $\forall x(Fx \to \forall y Gy) \vdash \forall x \forall y(Fx \to Gy)$
2. $\forall x \forall y(Fx \to Gy) \vdash \forall x(Fx \to \forall y Gy)$
3. $\vdash \forall x(\forall y Rxy \to Rxx)$

C. Conceptual

It can be proven that $\forall xFx \rightarrow \forall xGx \vdash \forall x(Fx \rightarrow Gx)$. However, the following proof contains an error. Please explain what that error is. A good answer can be as short as one sentence.

1	(1)	$\forall xFx \rightarrow \forall xGx$	A
2	(2)	Fa	A
2	(3)	$\forall xFx$	2 UI
1,2	(4)	$\forall xGx$	1,3 MP
1,2	(5)	Ga	4 UE
1	(6)	$Fa \rightarrow Ga$	2,5 CP
1	(7)	$\forall x(Fx \rightarrow Gx)$	6 UI