Precept exercises: Week 9

1. What does it mean for a predicate logic sentence to be *consistent*?
2. Suppose that we had an algorithm that determined whether sentences are consistent. Explain how we could use this algorithm to determine if arguments are valid.
3. Consider the following English sentences, along with the two possible translations into predicate logic. Are the two PL sentences logically equivalent? Does one imply the other? How does this information bear on your judgment about which is the best translation?

   (a) Only students who do the homework will learn logic. (Sx, Hx, Lx)

   $$\forall x(Lx \to (Sx \wedge Hx))$$

   $$\forall x((Sx \wedge Lx) \to Hx)$$

   (b) There is some student who respects only those professors who respect all students. (Sx, Px, Rxy)

   $$\exists x(Sx \wedge \forall y(Rxy \to (Py \wedge \forall z(Sz \to Ryz))))$$

   $$\exists x(Sx \wedge \forall y((Py \wedge Rxy) \to \forall z(Sz \to Ryz)))$$

4. Explain why the sentence $\exists x(Mx \to Dx)$ is *not* a good translation of "There is a melancholy Dane."
5. Provide models to show that the following sequents are invalid:

   (a) $\forall x(Fx \vee Gx) \vdash \forall xFx \vee \forall xGx$

   (b) $\forall xFx \to \forall xGx \vdash \forall x(Fx \to Gx)$

   (c) $\exists x(Fx \to P) \vdash \exists xFx \to P$

6. The EE rule requires that the arbitrary name that is used in the instance of the existential formula does *not* appear in (a) the existential formula, (b) the auxiliary assumptions used to derive the conclusion, and (c) the conclusion itself. Explain why dropping any one of these three restrictions would lead to an unsound rule.

7. Which of the following sentences are true in which of the diagrams below.

(a) $\forall x \forall y (Rxy \to Ryy)$

(b) $\forall x \exists y (Rxy \land Ryx)$

(c) $\exists x \forall y (Rxy \to \exists z Ryz)$

(d) $\forall x \exists y (Rxy \land \forall z (Ryz \to Rxz))$

(e) $\exists x \exists y (Rxy \land \neg Ryx)$