# Lecture 7: First Steps in Predicate Logic

Hans Halvorson

Princeton University

September 29, 2025

# Overview

1. Propositional logic is inadequate
2. Enlarging the grammar
    - Names and predicates
    - Variables and quantifiers
3. Proofs
    - Universal elimination
    - Universal introduction

# Propositional logic is inadequate

# Propositional logic is inadequate

All people are mortal.
Socrates is a person.
Therefore, Socrates is mortal.

All whales are mammals.
All mammals have lungs.
All whales have lungs.

# Enlarging the grammar

# Predicate,Name $\mapsto$ Proposition

Alice is French. Bernard is French.

Alice is German.

# Predicate,Quantifier $\mapsto$ Proposition

All Finns are gregarious.

Some Finns are gregarious.

No Finns are gregarious.

Some Finns are not gregarious.

All happy Finns are gregarious.

All Finns and Germans are happy.

No dogs or cats are permitted in the restaurant.

$\forall x(Fx \to P)$ $\forall x Fx \to P$

# Translating relational statements

Maren is taller than Niels.

Maren is taller than someone.

Someone is taller than Niels.

Everyone is taller than someone.

Someone is taller than everyone.

There is a student who likes every subject.

There is a subject that every student likes.

Every student likes some subject.

# Proofs with quantifiers

# Universal elimination

$$\forall x(Fx \to Gx), Fa \vdash Ga$$

$\forall x \forall y(Fx \wedge Gy) \vdash Fa \wedge Gb$

$$\forall x \forall y (Fx \land Gy) \vdash Fa \land Ga$$

$$P \to \forall x F x \vdash P \to Fa$$

$\neg Fa \vdash \neg\forall x Fx$

# Warnings

1. Only apply UE when the sentence on the line is universally quantified.
   $\forall x(Fx \rightarrow P)$
   $\forall x(Fx \rightarrow \forall yGy)$
   $\forall xFx \rightarrow Ga$
   $\forall x \forall yRxy$
2. When applying UE, replace all instances of the relevant variable with the same name.

$$\forall x(Fx \rightarrow \forall y Rxy)$$

# Universal introduction

$\forall x(Fx \to Gx), \forall x Fx \vdash \forall x Gx$

$$\vdash \forall x(Fx \to (Fx \lor Gx))$$

$$\forall x(P \to Fx) \vdash P \to \forall x Fx$$

Arguing to a universal sentence via an instance

$P \to \forall x Fx \vdash \forall x(P \to Fx)$

| | | | |
|---|---|---|---|
| 1 | (1) | $\forall x \forall y Rxy$ | A |
| 1 | (2) | $\forall y Ray$ | 1 UE |
| 1 | (3) | $Raa$ | 2 UE |
| 1 | (4) | $\forall x Rxx$ | 3 UI |

| | | | |
|---|---|---|---|
| 1 | (1) | $\forall x \forall y Rxy$ | A |
| 1 | (2) | $\forall y Ray$ | 1 UE |
| 1 | (3) | $Rab$ | 2 UE |
| 1 | (4) | $\forall x Rxb$ | 3 UI |
| 1 | (5) | $\forall y \forall x Rxy$ | 4 UI |

# Universal introduction

Restriction: To apply UE to a line $\Gamma \vdash \phi(a)$, the name $a$ must not appear in $\Gamma$ or in $\phi$.

| | | | |
|---|---|---|---|
| 1 | (1) | $\forall x Rxx$ | A |
| 1 | (2) | $Raa$ | 1 UE |
| 1 | (3) | $\forall x Rxa$ | 2 UI |
| 1 | (4) | $\forall y \forall y Rxy$ | |