Chapter 7: Kant, Incongruent Counterparts, and Absolute Space

1. Test for substantivalism: Imagine a possible world with a single glove. Would there be a fact of the matter about whether that glove is left or right handed?

   Relationalism: "...contrary to Kant, for a hand standing alone there aren't two different actions of crative cause for God to choose between." (p 138)

2. Recall that substantivalists supposedly think that shifts create new possibilities.

   Would the substantivalist count the reflected triangle as a different possible world?

3. "Broad imagines that the set of space points occupied by B and the set of points occupied by B' might differ in some geometrical properties that are not manifested in the relations among the occupying particles of matter ..." (p 140)

   HH: But space and its structures provide a common standard — in the same way that space and its structures provide a standard for travelling on a straight line.

4. "Nerlich is incorrect in holding that Kant is right if we interpret him as saying that the enantiomorphism of a hand depends upon the relation between it and the absolute container space considered as a unity." (p 142)

5. Fact: The group of symmetries of Euclidean space has three subgroups: translations, rotations, and reflections. The translations and rotations are continuously connected to the identity I, and their determinant is 1. The reflections are *not* continuously connected to the identity I, and their determinant is -1.

6. Fact: Symmetries preserve properties of, and relations between, shapes.

7. Fact: The relation of being incongruent counterparts is definable in terms of inner products of vectors. i.e. Euclidean geometry defines a

relation “same handed”, although it doesn’t define predicates for “left” and “right”.

The bases $\{u_1, \ldots, u_n\}$ and $\{v_1, \ldots, v_n\}$ are co-oriented if and only if:

$$\sum_{i_1, i_2, \ldots, i_n = 1}^{n} \varepsilon_{i_1 i_2 \ldots i_n} \langle u_1, v_{i_1} \rangle \langle u_2, v_{i_2} \rangle \cdots \langle u_n, v_{i_n} \rangle > 0,$$

where $\varepsilon_{i_1 i_2 \ldots i_n}$ is the Levi-Civita symbol. (The equation on the left is just the determinant of the linear operator that maps $u_1, \ldots, u_n$ to $v_1, \ldots, v_n$.)

So a triangle alone in space “has an orientation”, it’s just neither left nor right!

This fact doesn’t contradict what we said above about reflections preserving all properties and relations. If X and Y are same handed, then $M(X)$ and $M(Y)$ are same handed.

8. Fact: The binary relation of same-handedness cannot be analyzed into a conjunction of unary predicates.

   Recall that Leibniz believes that all relational properties are ultimately grounded in monadic properties.