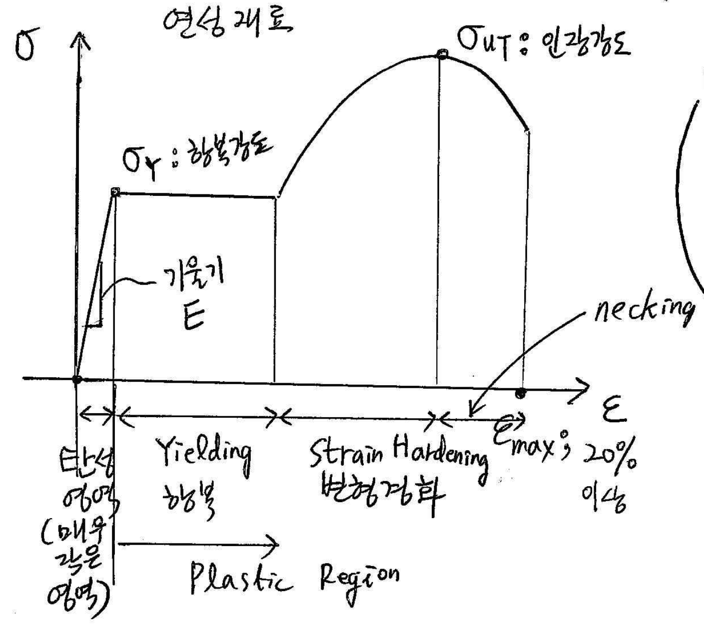
2. Fatigue and Buckling

2-1. Dog Bone Test Specimen

① Fatigue

Stress – Strain Relationship

1. Ductile Mat'l (Steel, Al)
연성재료



; $\sigma = E\varepsilon$ or $\tau = G\gamma$ (선형 관계)

이값은 책끝의 부록 참조할것

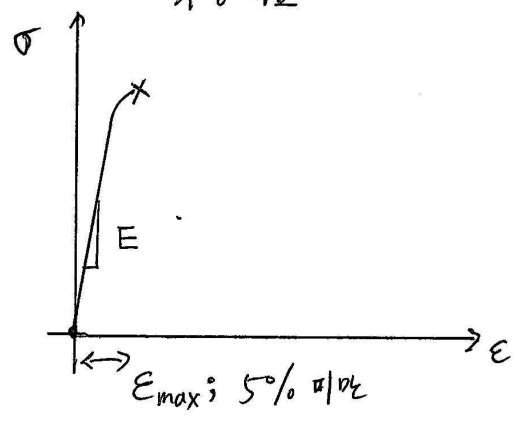
E ; Elastic Modulus (Young's Modulus)

σ_Y : Yield strength

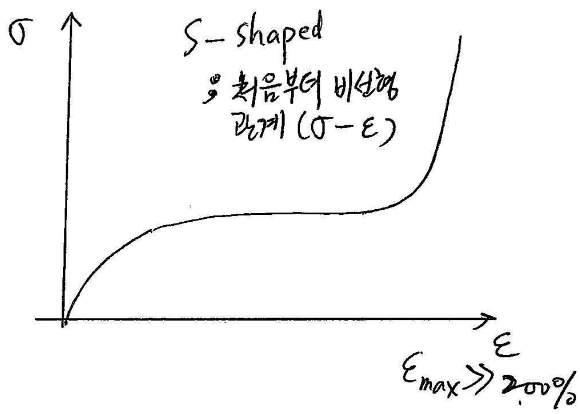
σ_{UT}, σ_{UC} ; Ultimate Tensile Strength
" Compressive "

Standard Specimen (KS 규격)
; $d = 12.7$ mm, $l_0 = 50$ mm
(www.standard.go.kr)

2. Brittle Mat'l (Iron, Glass)
취성재료



3. Hyperelastic Mat'l (Rubber)



강도비교

- SS400 ; $S_Y = 250$ MPa, $S_{UT} = 400$ MPa, $\varepsilon_{max} \approx 30\%$
- Al6061-T6 ; $S_Y = 220$ MPa, $S_{UT} = 290$ MPa, $\varepsilon_{max} \approx 15\%$
- GC200 ; $S_{UT} = 200$ MPa, $S_{uc} = 700$ MPa, ε_{max} small
- 플라스틱 ; $S_{UT} \approx 20 \sim 40$ MPa

E값비교

- Steel ; 200 GPa
- Al ; 70 GPa
- Mg ; 45 GPa
- Glass ; 70 GPa
- 플라스틱 ; 1~3 GPa

★ 구조용강철 SM45C (0.45%C)
$\sigma_Y = 343$ MPa
$\sigma_{UT} = 568$ MPa

★ 합금강 SCM440 (0.40%C)
$\sigma_Y = 850$ MPa
$\sigma_{UT} = 1000$ MPa

★ 스텐강 SUS304 ; 니켈, 크롬으로 부식방지

- www.matweb.com ; 재료의 기계적 성질
- standard.go.kr ; 재료기호

Failure Theory (파손이론)

(1) 취성재료 (Brittle) ; $\underset{\text{최대인장응력}}{\sigma_1} \le \dfrac{S_{ut}}{N}$

S_{ut} — 재료의 인장강도
N — 안전계수

(2) 연성재료 (Ductile) ; ① 최대전단응력이론

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \le \frac{\tau_Y}{N}$$

$\tau_Y \approx \dfrac{S_Y}{2}$ (재료의 항복강도)
N — 안전계수

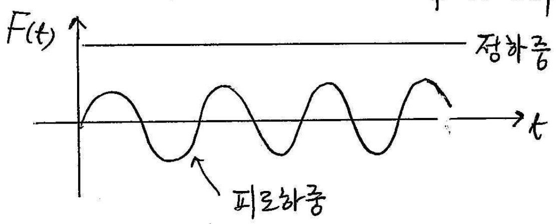
② 최대비틀림에너지이론 (Von-Mises 등가응력)

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \le \frac{S_Y}{N}$$

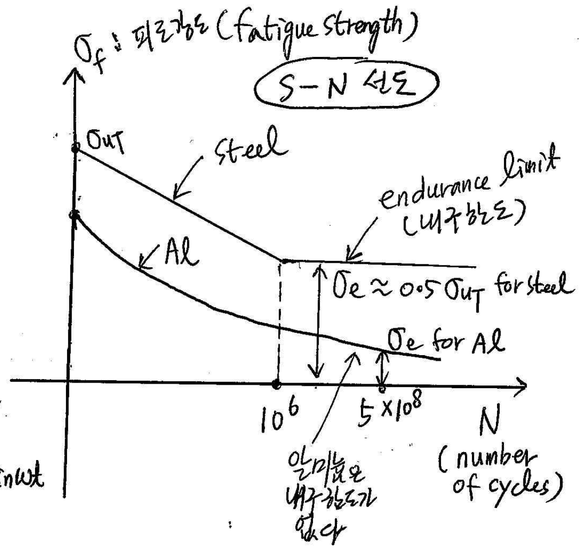
3D — or $\sigma_{VM} = \sqrt{\dfrac{(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_x-\sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}}$

Fatigue (피로)

Under repeated loading & unloading
→ Failure occurs at $\sigma_f \ll \sigma_{UT}$



$F(t) = F_0 \sin\omega t \rightarrow \sigma(t) = \sigma_0 \sin\omega t$



피로파손이론 (Fatigue Failure Theory)

무어의 피로실험 → S-N 곡선, σ_e' (내구한도) 도출 ← 실험실 환경
(Fully reversed fatigue stress)

(강철 at 10^6 cycles ← $\sigma_e' \approx 0.5\sigma_{UT}$
알루미늄 at 5×10^8 cycles) ← $\sigma_e' = 0.4\sigma_{UT}$

수정피로한도

$$; \sigma_e = C_f\, C_r\, C_s\, C_t \frac{1}{K_f} \sigma_e'$$

① C_f : 표면처리계수 → 표면 처리 상태, 단단할수록 불리

② C_r : 신뢰도 계수 → 신뢰도를 높일수록 낮아진다

③ C_s : 크기계수 → 지름 7.62 mm 보다 크면 불리 (원형단면이 아니면 등가지름 활용)

④ C_t : 온도계수 → 고온에서 불리

⑤ K_f : 피로응력집중계수

$$K_f = 1 + (K_t - 1)\,q$$

K_t : 기하학적 응력집중계수

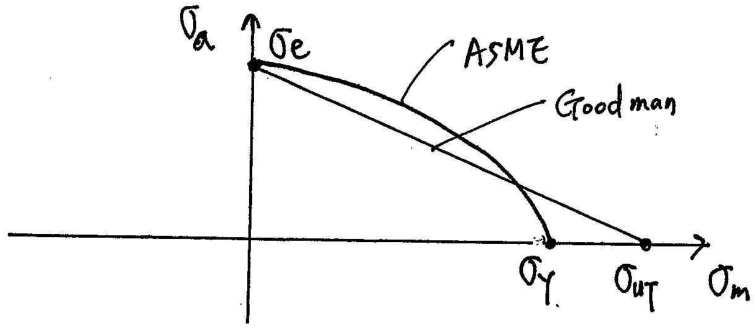
q : 노치민감도 ($0 \le q \le 1$), 강도가 높을수록 1에 가깝다 (불리) $K_f \approx K_t$

– 평균응력 (σ_m)과 교번응력 (σ_a)이 공존하는 경우

; 피로파손 선도를 그려서 판단함

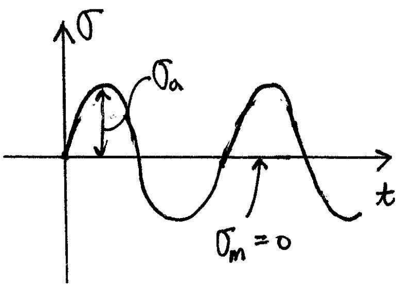
Goodman 선도 ; $\dfrac{\sigma_a}{\sigma_e} + \dfrac{\sigma_m}{\sigma_{UT}} = 1$ 일반 기계 부품

ASME 선도 ; $\left(\dfrac{\sigma_a}{\sigma_e}\right)^2 + \left(\dfrac{\sigma_m}{\sigma_Y}\right)^2 = 1$ 회전축

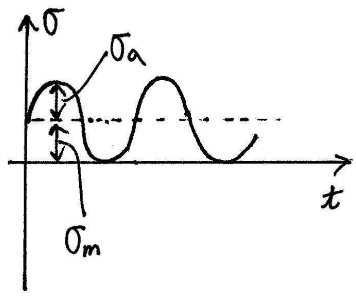


- Fatigue types

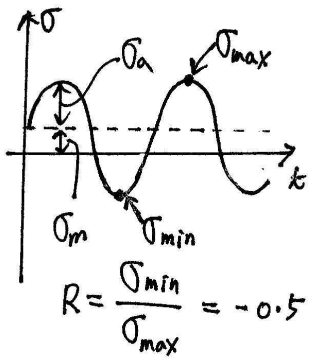
Fully Reversed

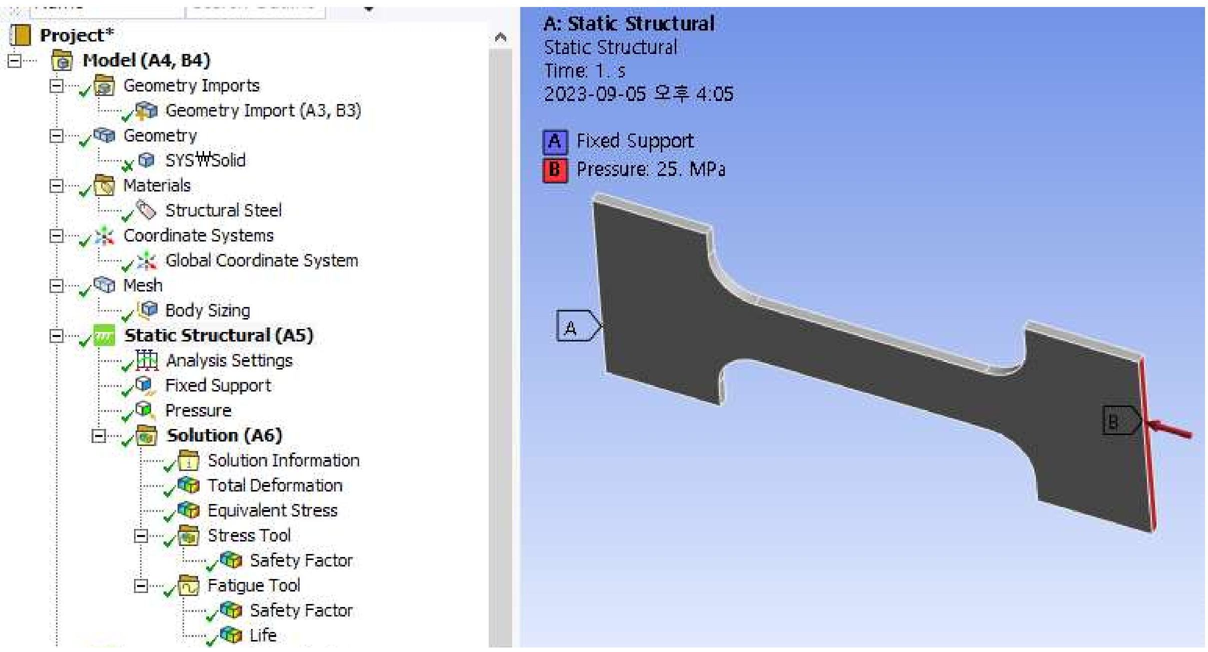


Zero-Based



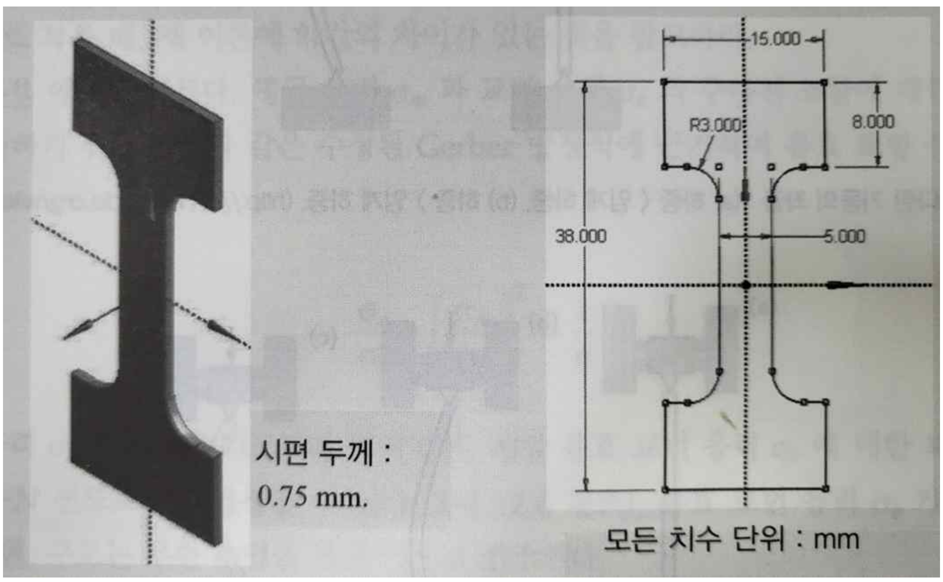
Ratio-Based





(1) Geometry

- Dog bone part (width=5.0, thickness=0.75mm)



(2) Material

- Structural steel
- Check S-N curve

Table of Properties Row 20: S-N Curve

	A
1	Mean Stress (Pa)
2	0
*	

	B	C
1	Cycles	Alternating Stress (Pa)
2	10	3.999E+09
3	20	2.827E+09
4	50	1.896E+09
5	100	1.413E+09
6	200	1.069E+09
7	2000	4.41E+08
8	10000	2.62E+08
9	20000	2.14E+08
10	1E+05	1.38E+08
11	2E+05	1.14E+08
12	1E+06	8.62E+07
*		

(3) Mesh

- Body sizing 0.5mm

(4) Boundary Condition and Load Condition

- One face fixed
- Opposite face 25MPa pressure

(5) Results

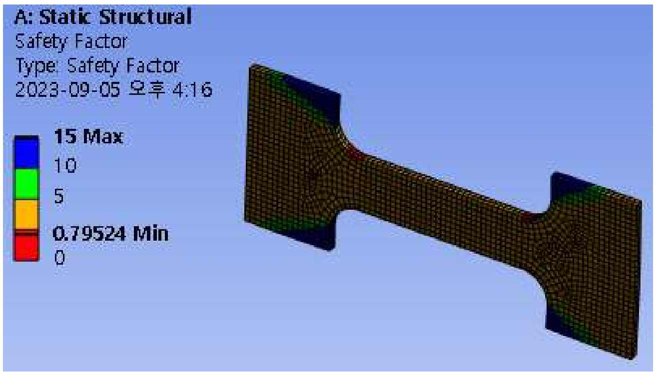
- Stress: Von-Mises stress
- Stress Tool: Safety factor

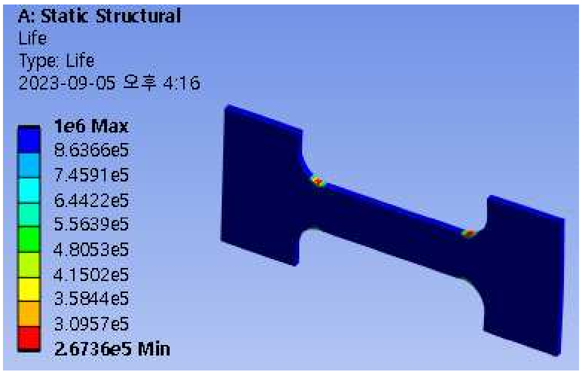
(6) Fatigue Tool

- Fatigue Strength Factor K_f=1.0
- Loading Scale Factor: 1.0
- Analysis Type: Stress Life
- Loading Type: Fully Reversed, Zero-based
- Mean Stress Theory: Goodman, Equivalent Von-Mises

(7) Results

- Static: Von-Mises Stress, Safety Factor
- Fatigue: Safety Factor(Design Life 10^6 cycles), Life: cycles

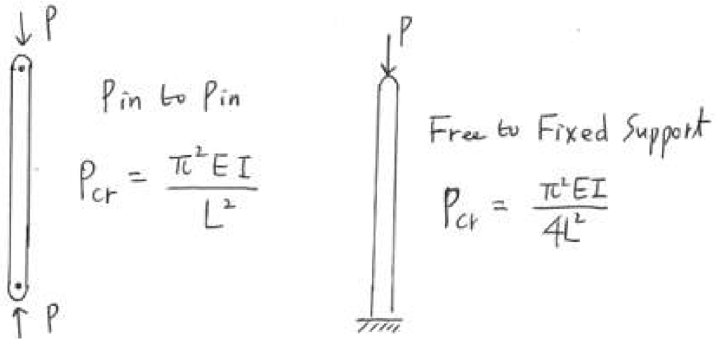




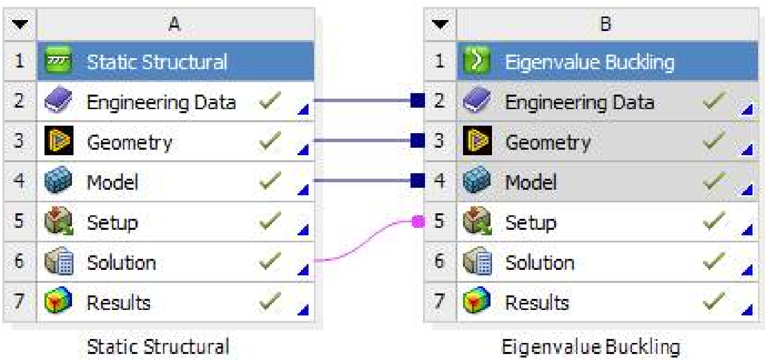
② Buckling

- Critical compressive force exists for slender members
  : 단면적에 비해 긴 부재가 압축력을 받는 경우, 임계하중($P=P_{cr}$)에서 갑작스럽게 구부러지는 현상으로 P_y(Yield Load) 다 작은 P_{cr}에서 파손 된다.
- Theory: $P_{cr} = C\frac{\pi^2 EI}{L^2}$

  C depends on boundary conditions
  C=1/4 for free to fix, C=1 for pin to pin



- "Static Structural" analysis is used to generate "Eigenvalue Buckling" analysis
- RMB on solution tab --> Transfer data to "Eigenvalue Buckling



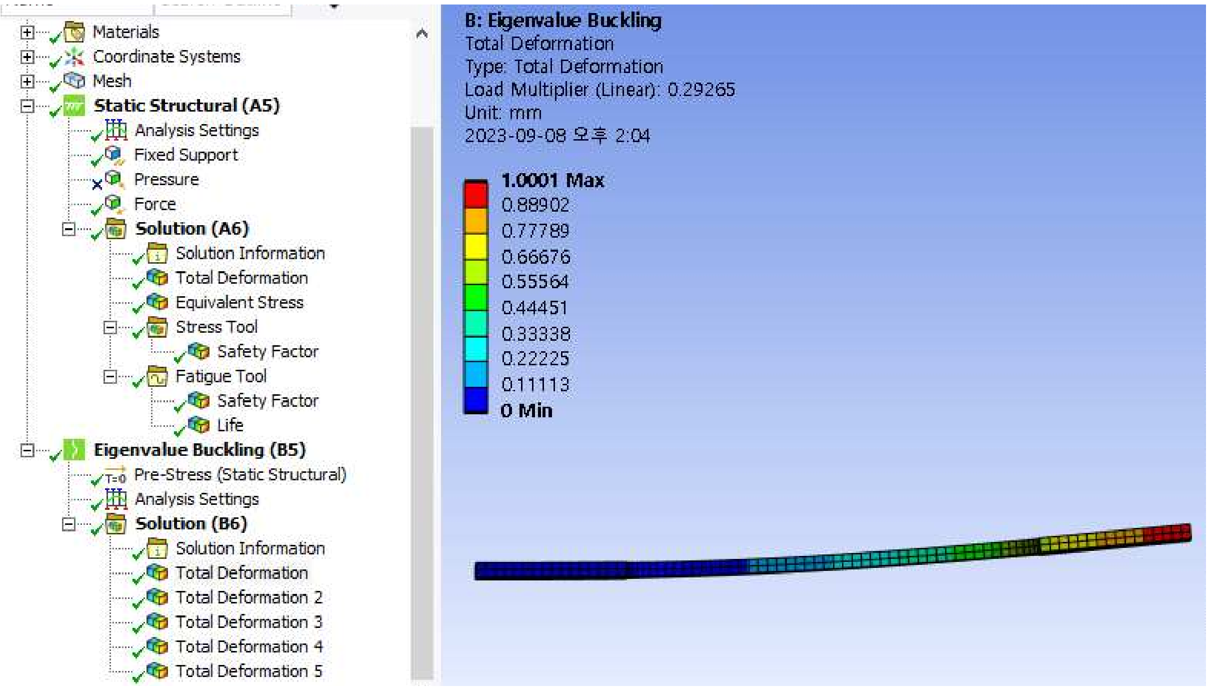
- If you want to get the buckling safety factor, you have to use the force(281.25N) instead of the pressure(25MPa) as the boundary condition. Therefore, remove the pressure load and apply compressive force P on the same face.(figure on next page)
  --> P = p * A = 25 x (15 x 0.75) = 281.25 N

- Eigenvalue Bucking --> Analysis Settings, set "Max. Modes to find" to 5 and solve

- Solution tab --> tubular data, select the load multipliers and RMB "create mode shapes"
  The positive load multipliers and mode shapes represent safety factors for buckling

Tabular Data

	Mode	Load Multiplier
1	1.	0.29265
2	2.	2.8086
3	3.	7.9671
4	4.	15.379
5	5.	15.466

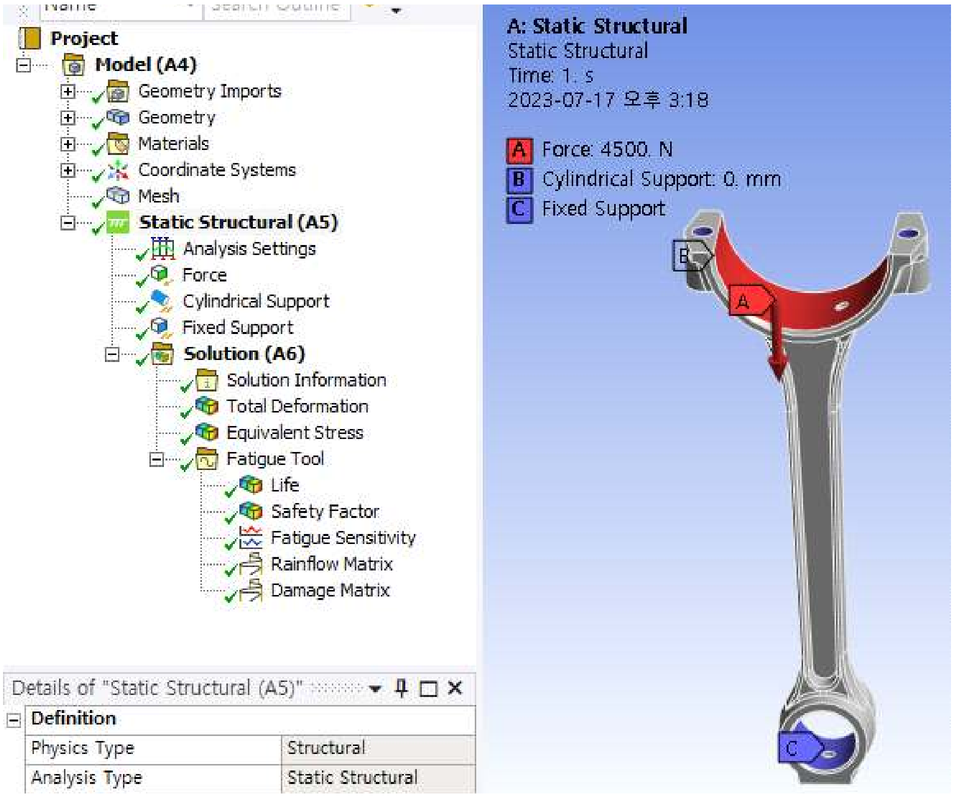


- Calculate an approximate theoretical value for the buckling load multiplier and compare it with the simulation results

$$P_{cr} = C\frac{\pi^2 EI}{L^2} = 0.25\frac{\pi^2 \cdot 200 \cdot 10^9 \cdot \frac{0.005 \cdot 0.00075^3}{12}}{0.038^2} = 60.0\,N$$

$$\text{Load Multiplier (Safety Factor for Buckling)} = \frac{P_{cr}}{P} = \frac{60.0}{281.25} = 0.213$$

2-2. Connecting Rod



(1) Geometry

- Connecting rod

(2) Material

- Generate a new material SS400

Properties of Outline Row 3: SS400

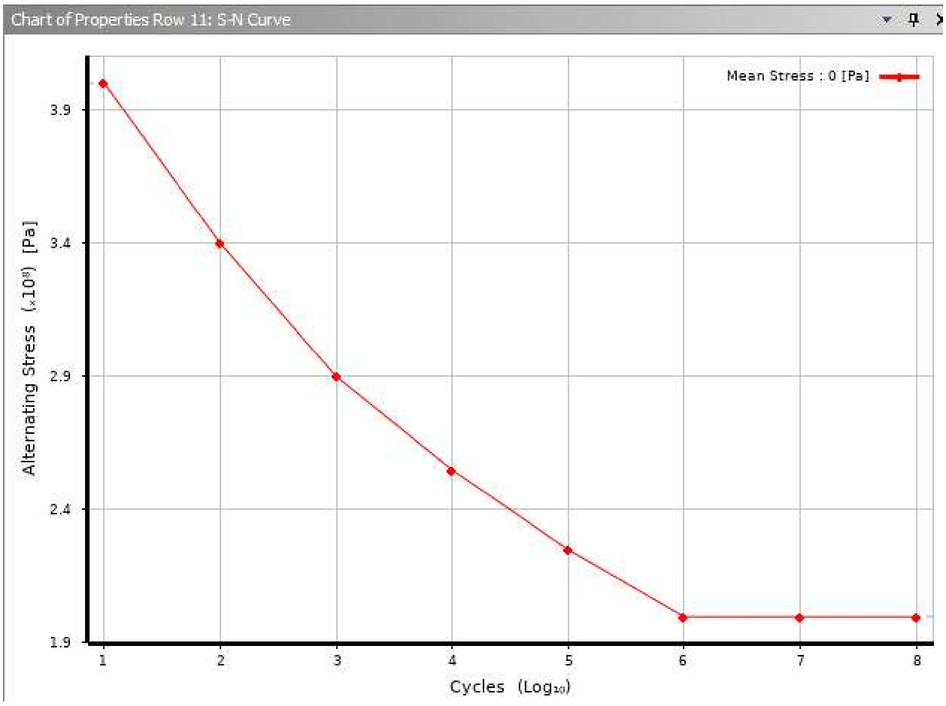
	A	B	C
1	Property	Value	Unit
2	Material Field Variables	Table	
3	Density	7850	kg m^-3
4	Isotropic Elasticity		
5	Derive from	Young's Modulus and Poisson's Ratio	
6	Young's Modulus	200	GPa
7	Poisson's Ratio	0.3	
8	Bulk Modulus	1.6667E+11	Pa
9	Shear Modulus	7.6923E+10	Pa
10	S-N Curve	Tabular	
11	Interpolation	Semi-Log	
12	Scale	1	
13	Offset	0	Pa
14	Tensile Yield Strength	250	MPa
15	Compressive Yield Strength	250	MPa
16	Tensile Ultimate Strength	400	MPa
17	Compressive Ultimate Strength	400	MPa

- Generate a S-N curve manually

Table of Properties Row 10: S-N Curve

	A
1	Mean Stress (Pa)
2	0
*	

	B	C
1	Cycles	Alternating Stress (MPa)
2	10	400
3	100	340
4	1000	290
5	10000	255
6	1E+05	225
7	1E+06	200
8	1E+07	200
9	1E+08	200
*		



(3) Mesh

- Body sizing 5mm

(4) Boundary Condition

- Fixed support at small end
- Cylindrical support on two bolt holes at large end

    --> Set the radial direction to “fixed”, set axial and tangential to “free”

- You may press F1 key on the cylindrical support menu to get on line help or watch the appropriate web site such as “Determining which Support to Use” (20 minutes)

https://courses.ansys.com/index.php/courses/structural-boundary-conditions/lessons/determining-which-support-to-use-lesson-1/

(5) Load Condition

- Compressive force 4500N at large end

(6) Results

- Von-Mises stress, Static safety factor

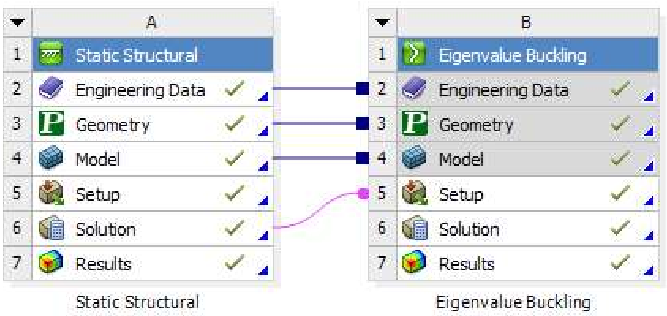
(7) Fatigue Tool
- Fatigue Strength Factor K_f=0.7 (Reflect a surface factor in service condition)
- Analysis Type: Stress Life
- Mean Stress Theory: Goodman, Signed Von-Mises
- Loading Type: Compare 4 cases
    - a. Fully Reversed: Loading Scale Factor=1.0
    - b. Zero_Based: Loading Scale Factor=1.0
    - c. R=-0.5: Loading Scale Factor=1.0
    - c. History Data
        - Import SAEBracketHistory.dat
        - Loading Scale Factor: 0.005 (200 gages used to estimate the load)

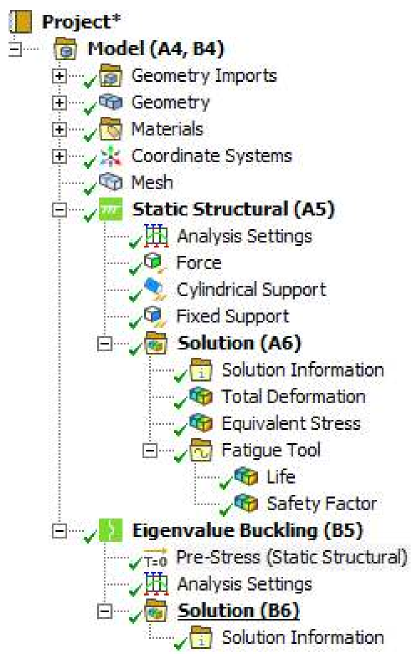
$$\left(\frac{1\,\text{FEM load}}{1000\,lbs}\right)\times\left(\frac{1000\,lbs}{200\,\text{strain gauge}}\right)=\left(\frac{1\,\text{FEM load}}{200\,\text{strain gauge}}\right)=.005$$

        - Bin Size: 32
        - Infinite Life: 10^9 blocks

(8) Fatigue Results
- Fully Reversed, Zero-Based, Ratio R=-0.5
    - Safety Factor: Design Life for 10^6 cycles
    - Life: Cycles
- History Data
    - Safety Factor: Design Life for 100 blocks
    - Life: Blocks

(9) Further Study
    - Check for buckling load multipliers





2-3. Investigation on Fatigue Types

For the dog bone fatigue, find the safety factor and the number of repetitive loads before fatigue failure (Life) for the following four cases. For cases 1, 2, and 3, compare the analytical results with the Goodman theory.
In this case, a new SS400 material is created and used for the analysis (E=200GPa, v=0.3, tensile strength 400MPa, yield strength 250MPa, endurance limit at laboratory condition S_e=200MPa at 10^6 cycles) Apply the fatigue strength factor = 0.50 considering $C_f \cdot C_r \cdot C_s \cdot C_t$ values at the service condition.

① Fully Reversed
Apply 10^6 cycles for infinite life
(1) Fatigue safety factor for 10^6 cycles
(2) Life: Cycles
(3) Compare results with Goodman's theory

② Zero-based
Apply 10^6 cycles for infinite life
(1) Fatigue safety factor for 10^6 cycles
(2) Life: Cycles
(3) Compare results with Goodman's theory

③ R=-0.5
Apply 10^6 cycles for infinite life
(1) Fatigue safety factor for 10^6 cycles
(2) Life: Cycles
(3) Comparison with Goodman's theory

④ History data: Apply SAE Bracket History
Apply 10^9 blocks for infinite life
(1) Fatigue safety factor for 100 blocks load history
(2) Life: Blocks