# HW#4 Plane Stress Problem Using Finite Element Method

**6.6** Evaluate the stiffness matrix for the elements shown in Figure P6–6. The coordinates are given in units of millimeters. Assume plane stress conditions. Let $E =$ 70 GPa, $\nu =$ 0.3 , and $t = 10$ mm.

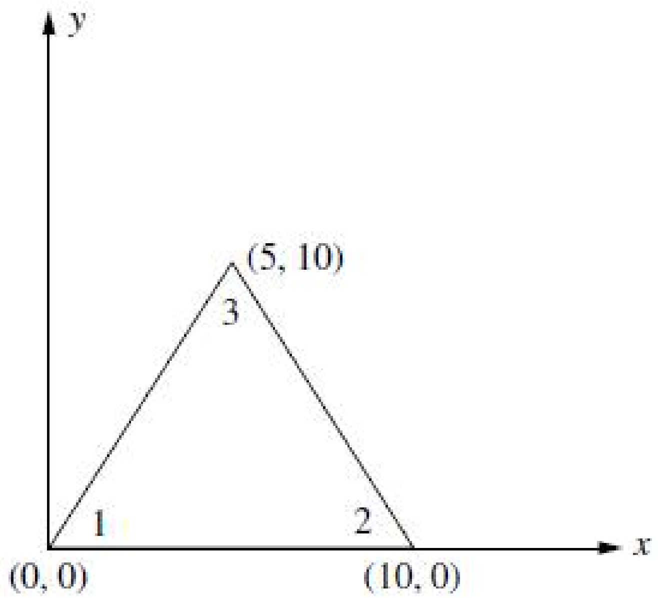
**6.7** For the elements given in Problem 6.6, the nodal displacements are given as

$$u_1 = 2.0 \text{ mm} \qquad v_1 = 1.0 \text{ mm} \qquad u_2 = 0.5 \text{ mm}$$

$$v_2 = 0.0 \text{ mm} \qquad u_3 = 3.0 \text{ mm} \qquad v_3 = 1.0 \text{ mm}$$

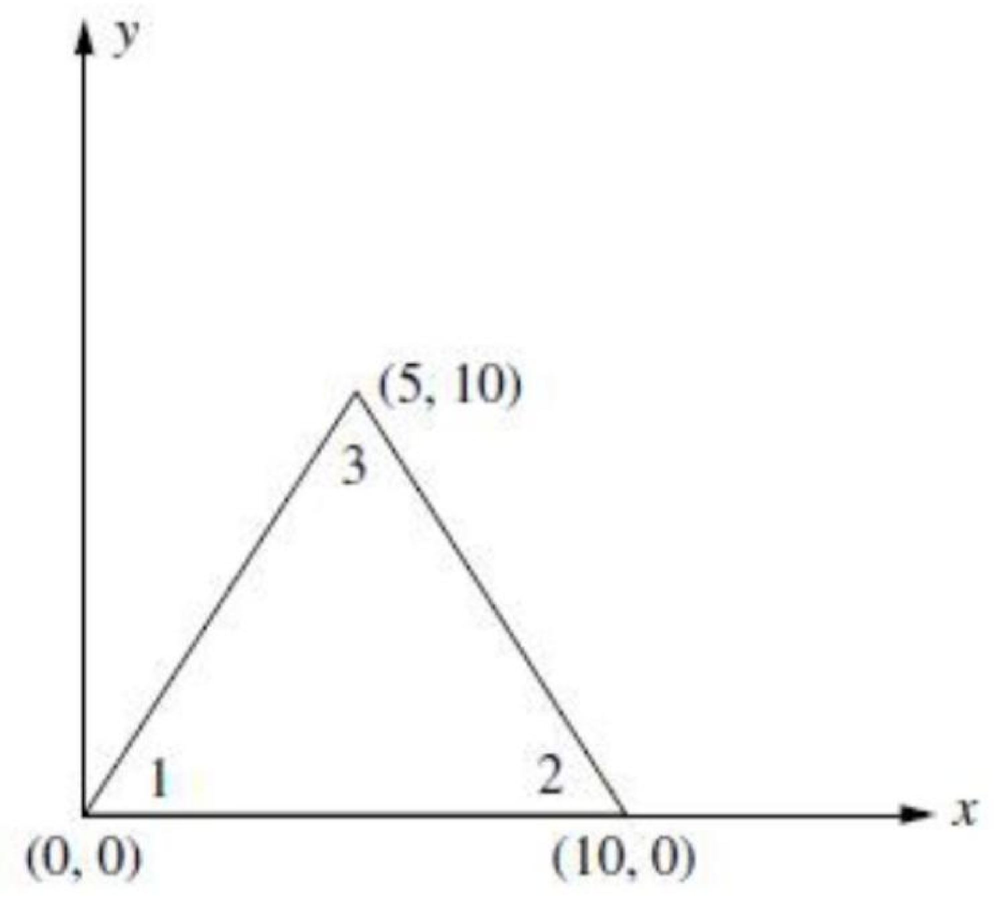
Determine the element stresses $\sigma_x, \sigma_y, \tau_{xy}, \sigma_1,$ and $\sigma_2$.
Use the values of $E, \nu,$ and $t$ given in Problem 6.6.

**6.8** Determine the von Mises stress for problem 6.7




(c)

**6.6** Evaluate the stiffness matrix for the elements shown in Figure P6–6. The coordinates are given in units of millimeters. Assume plane stress conditions. Let $E = 70$ GPa, $\nu = 0.3$, and $t = 10$ mm.



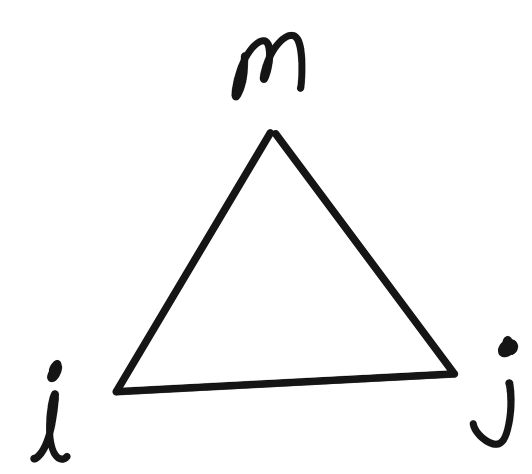
(c)

$$[k] = tA[B]^T[D][B]$$

평면응력 : $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

$$\rightarrow [D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} \beta_i & 0 & \beta_j & 0 & \beta_m & 0 \\ 0 & \gamma_i & 0 & \gamma_j & 0 & \gamma_m \\ \gamma_i & \beta_i & \gamma_j & \beta_j & \gamma_m & \beta_m \end{bmatrix}$$




$\alpha_i = x_j y_m - y_j x_m$ $\quad$ $\alpha_j = y_i x_m - x_i y_m$ $\quad$ $\alpha_m = x_i y_j - y_i x_j$

$\beta_i = y_j - y_m$ $\quad$ $\beta_j = y_m - y_i$ $\quad$ $\beta_m = y_i - y_j$

$\gamma_i = x_m - x_j$ $\quad$ $\gamma_j = x_i - x_m$ $\quad$ $\gamma_m = x_j - x_i$

$$[D] = \frac{70\times10^9}{1-0.3^2}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & \frac{1-0.3}{2} \end{bmatrix} = 7.7\times10^{10}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$$

$2A = 100\ mm^2$ $\quad$ $\beta_i = -10$, $\beta_j = 10$, $\beta_m = 0$

$= 100\times10^{-6}\ m^2$, $\quad$ $\gamma_i = -5$, $\gamma_j = -5$, $\gamma_m = 10$

$$[k] = 0.01\times50\times10^{-6}\times\frac{10^{-3}}{100\times10^{-6}}\begin{bmatrix} -10 & 0 & -5 \\ 0 & -5 & -10 \\ 10 & 0 & -5 \\ 0 & -5 & 10 \\ 0 & 0 & 10 \\ 0 & 10 & 0 \end{bmatrix} 7.7\times10^{10}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}\frac{10^{-3}}{100\times10^{-6}}\begin{bmatrix} -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & -5 & 0 & -5 & 0 & 10 \\ -5 & -10 & -5 & 10 & 10 & 0 \end{bmatrix}$$

$$= 3.85\times10^6\begin{bmatrix} 108.75 & 32.5 & -91.25 & -2.5 & -17.5 & -30 \\ 32.5 & 60 & 2.5 & -10 & -35 & -50 \\ -91.25 & 2.5 & 108.75 & -32.5 & -17.5 & 30 \\ -2.5 & -10 & -32.5 & 60 & 35 & -50 \\ -17.5 & -35 & -17.5 & 35 & 35 & 0 \\ -30 & -50 & 30 & -50 & 0 & 100 \end{bmatrix}$$

**6.7** For the elements given in Problem 6.6, the nodal displacements are given as

$$u_1 = 2.0 \text{ mm} \qquad v_1 = 1.0 \text{ mm} \qquad u_2 = 0.5 \text{ mm}$$

$$v_2 = 0.0 \text{ mm} \qquad u_3 = 3.0 \text{ mm} \qquad v_3 = 1.0 \text{ mm}$$

Determine the element stresses $\sigma_x, \sigma_y, \tau_{xy}, \sigma_1$, and $\sigma_2$.
Use the values of $E, \nu$, and $t$ given in Problem 6.6.

$E = 70GPa$ , $\nu = 0.3$ , $t = 10mm$

$$\{\sigma\} = [D]\{\varepsilon\} = [D][B]\{d\}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \frac{1}{2A}\begin{bmatrix} \beta_i & 0 & \beta_j & 0 & \beta_m & 0 \\ 0 & \gamma_i & 0 & \gamma_j & 0 & \gamma_m \\ \gamma_i & \beta_i & \gamma_j & \beta_j & \gamma_m & \beta_m \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{Bmatrix}$$

$$= \frac{70\times10^9}{1-0.3^2}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix} \frac{10^{-3}}{100\times10^{-6}}\begin{bmatrix} -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & -5 & 0 & -5 & 0 & 10 \\ -5 & -10 & -5 & 10 & 10 & 0 \end{bmatrix} 10^{-3}\begin{Bmatrix} 2 \\ 1 \\ 0.5 \\ 0 \\ 3 \\ 1 \end{Bmatrix}$$

$$\boxed{\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{pmatrix} -1.0385\times10^{10} \\ 385\times10^{6} \\ 202\times10^{7} \end{pmatrix} N/m^2}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\begin{aligned} \sigma_1 &= 751.4\times10^{6} \ N/m^2 \\ \sigma_2 &= -1.075\times10^{10} \ N/m^2 \end{aligned}}$$

**6.8** Determine the von Mises stress for problem 6.7

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2}$$

$$= \sqrt{(-1.0385 \times 10^{10})^2 + (385 \times 10^6)^2 - (-1.0385 \times 10^{10})(385 \times 10^6) + 3 \times (202 \times 10^7)^2}$$

$$= \underline{1.115 \times 10^{10} \ N/m^2}$$