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Simulation of Stress Concentrations in Notches

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Abstract: The fatigue life curves of materials are very sensitive to the magnitude of the stress amplitude. A small change or inaccuracy in the determination of the stress value causes large changes or inaccuracies in the calculated fatigue life estimate. Therefore, the use of computer simulations for fatigue life estimation requires a proper model development methodology. The paper is devoted to the problem of the modeling of components in notches using FEM. The modeling parameters significantly influencing the obtained stress results have been defined. Exact analytical solutions served as a benchmark for comparing the accuracy of the stress values obtained using FEM models. For the selected 2D and 3D notched components, diagrams were created for sensitivity analysis of the influence of the mesh element density at the root of the notch in correlation with the exact analytical solution. The findings from model building were applied to model the stress concentration at the root of a V-weld joint in a gas pipeline.

Keywords: notch; FEM simulation; stress concentration; analytical solution

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1. Introduction

For a reliable assessment of the fatigue life of a structure, it is necessary to define two types of input parameters: load parameters and material parameters. The accuracy and validity with which these parameters are obtained directly affect the results of the fatigue life assessment. For both groups of parameters, it is best to obtain them by direct measurement.

To define a loading of structures in service generally means to define a representative stress in time history at a critical cross-section or point of the structure (the critical cross-section or point of the structure as the cross-section or location with the highest stress variability). The best practice is to directly measure the strain history at the critical point during operation of the structure. Critical locations are most often of a complicated shape because of the stress concentrations, and their overall notching effect determines the fatigue life of that location, and hence, of the entire structure. The possibilities for direct measurement of strain in notches are still limited [1,2]. Measurement with FBG sensors in the notch is not practically feasible. Measurements using optical methods (ESPI, DIC, Photo-Stress, etc.) are applicable on the limited geometry at the measured location and practically excluded in direct operation. Measurement by strain gauges is most commonly used in praxis, but its application in the notch also has limitations.

An acceptable approach for obtaining an accurate and valid stress history at a critical location in a structure may be a combination of direct measurement and computer simulation: direct measurement at the nominal stress (strain) location and accurate simulation of the notching effect at the critical location. Knowledge of the nominal stress (strain) history and the stress concentration factor, as a result of the shape of the notch at the critical location of the structure, allows the nominal stress history to be recalculated in the notch. Several procedures can be found in References [3–5].

Finding the correct value of the notch effect using computer simulation is a challenge that is hard to overcome [6]. However, the model creation procedure offers a number of parameters that have a significant impact on the result obtained. These parameters and their effects will be identified, analyzed and quantified later in the paper in order to objectify the use of computational simulations for fatigue life estimation purposes.

2. FEM Simulation of the Stress Concentration in a Notch

The resulting stress values are affected by the density of the FEM grid, the overall method of mesh formation, boundary conditions, element type and other factors.

To verify the simulation results, as a first step, the following cases of notch effects, for which analytical solutions are known, were chosen:

Two-dimensional problems: a circular hole in the center of a wide sheet in tension, opposite a circular groove in a thin plate loaded in tension and a stepped thin sheet loaded in tension.

Three-dimensional problems: a shaft with a semicircular circumferential groove loaded successively in tension, torsion and bending and a stepped shaft loaded successively in tension and torsion.

The aim of the computational simulations was to find the correct methodology for building the models so that the stress concentration factors obtained by the models were consistent with the analytical solutions [7–10]. In particular, the model creation methodology included the meshing procedure and element selection (with properly set boundary conditions for constraints and loads).

For all modelled cases, an analysis of the influence of boundary conditions, the size of the component under study and the type of element used was performed at the beginning. The bond removal and load application had to be chosen to achieve the theoretical stress distribution in the cross-section (e.g., the tensile stress must have the same value throughout the cross-section outside the notch, so it was necessary to allow for a free displacement in the direction of the applied force both at the bond point and at the point of application of the force). The specimen size was chosen so as not to affect the stress distribution (e.g., the width of the plate with the circular hole must be at least such that the stress increase at the root of the hole drops to the nominal value and the stress distribution is no longer changed when the width of the specimen is further increased).

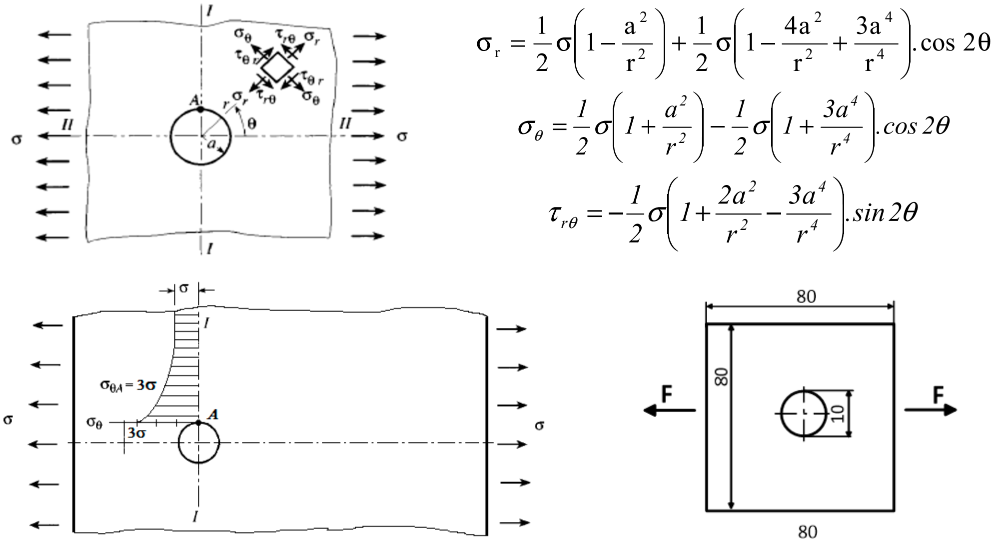
In the simulation results, the following occurred:

- The values of the stresses at the nodal points and the stresses in the elements were compared.
- The uses of linear and quadratic elements were compared.
- The principal stresses were selected for comparison with the analytical solution (non-equivalent von Mises stresses).

2.1. Simulations of Notch Effects in Plane Problems

An analytical solution of stress concentration that can be considered fundamental is the solution of a circular hole in the center of a wide sheet under tensile loading (Timoshenko, Goodier in 1970 [11]), as shown in (Figure 1).

The magnitude of the loading force of 10,000 N was chosen so that stresses are generated only in the linear region of the material curve (Young's modulus 2.1 $\times$ 10^5 MPa, Poisson ratio 0.3). Elements in the models were generated using concentric circles in the notch region with sizes of 0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 3.0, 4.0 and 5.0 mm. Dividing by the element length of 0.05 mm in the notch, the number of elements on the perimeter of the hole was 691, and the total number of elements in the model was 2.53 million. Using the smallest element length of 0.01 mm, the number of elements exceeded 10 million. The thickness of the elements was always the same as the thickness of the sheet. The method of formation of the elements around the hole, and the results documenting the strong influence of their size on the resulting peak stress values at the root of the notch, are shown in Figure 2.



$$\sigma_r = \frac{1}{2}\sigma\left(1-\frac{a^2}{r^2}\right)+\frac{1}{2}\sigma\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right).\cos 2\theta$$

$$\sigma_\theta = \frac{1}{2}\sigma\left(1+\frac{a^2}{r^2}\right)-\frac{1}{2}\sigma\left(1+\frac{3a^4}{r^4}\right).\cos 2\theta$$

$$\tau_{r\theta} = -\frac{1}{2}\sigma\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right).\sin 2\theta$$

Figure 1. Analytical solution and stress diagram for circular hole in the center of a wide sheet loaded in tension (K_σ = 2.625, σ_{max} = 375 MPa [10,11]).

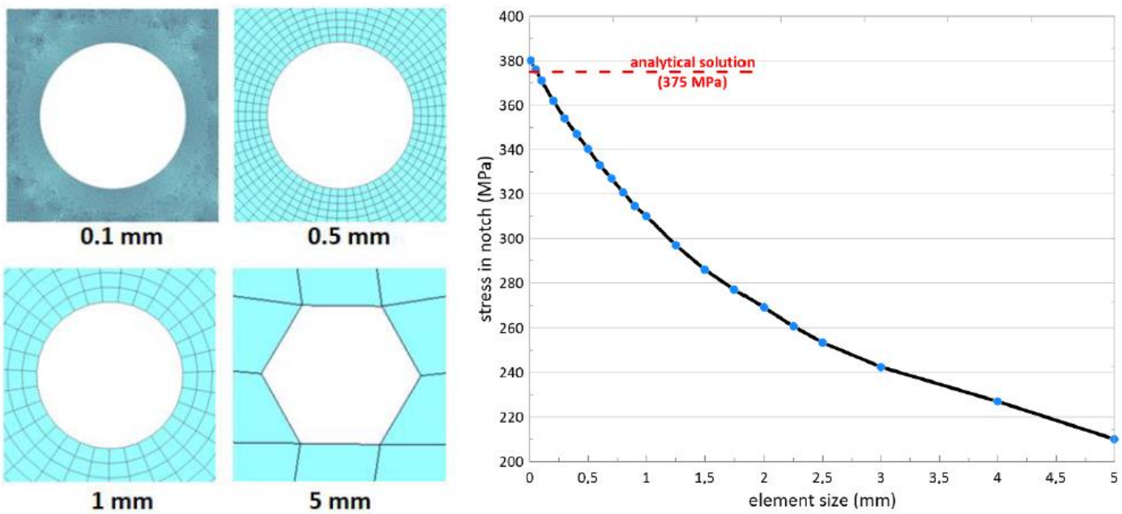


Figure 2. Mesh with an element sizes 0.1, 0.5, 1 and 5 (mm), and diagram of the peak stress in the root of the notch relative to element size diagram.

The same analysis procedure as that carried out for the first notch was performed for the tensile loaded plate with opposite semicircular grooves and for the stepped thin sheet loaded in tension (Figure 3). Simulations were performed for element sizes 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0 mm. The element mesh was constructed to minimize the use of triangular elements. The selected cases and the modelling method can be seen in Figure 3.

It is practical to compare notch root stresses for different element densities in a diagram where the vertical axis is the ratio of the stress from the simulation to the analytical solution, and the horizontal axis is the ratio of the element size to the radius of curvature at the notch root.

A comparison of the results in Figure 4 shows a practically insignificant difference in the results when using linear elements compared with using quadratic elements, and significantly more accurate results when using nodal point stresses versus element stresses.

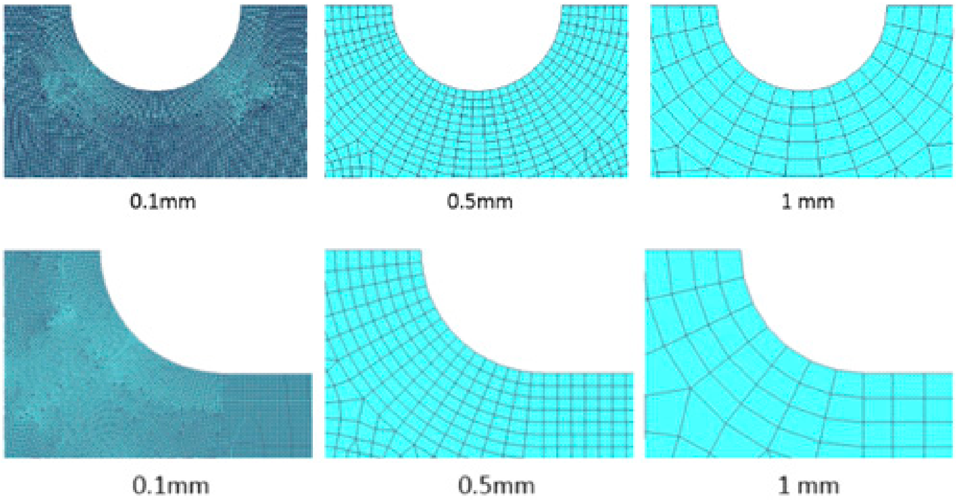


Figure 3. Element network on a plate (**top**) with a opposite semicircular groove under tensile load (K_{σ} = 2.64, σ_{max} = 377.7 MPa [10,12]) and on a stepped plate (**bottom**) under tensile load (K_{σ} = 1.9, σ_{max} = 380 MPa [10,13]).

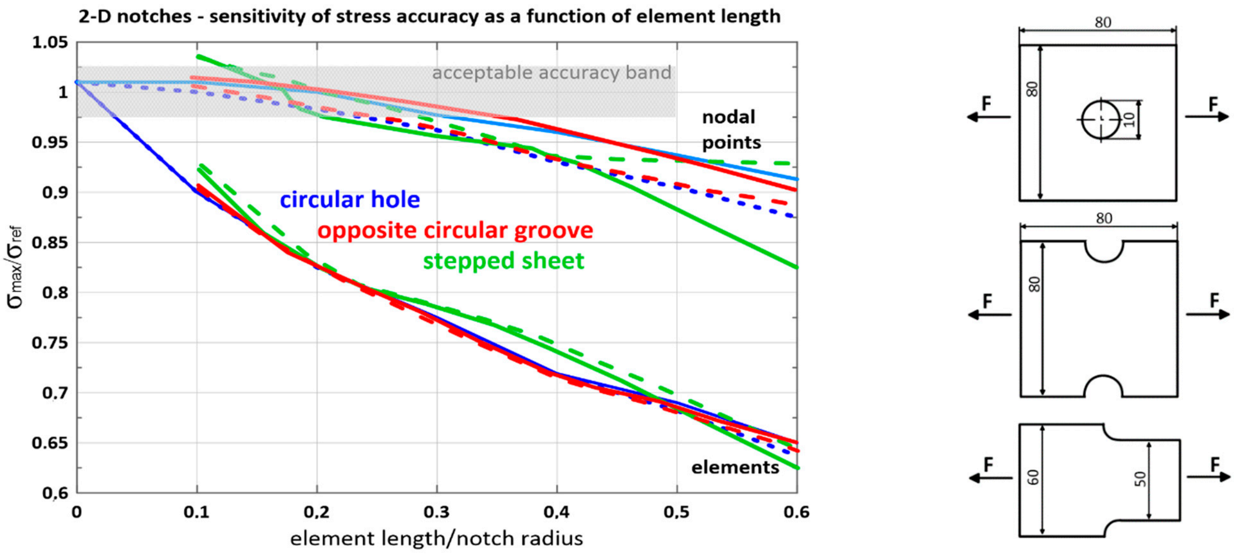


Figure 4. Analysis of the accuracy of notch root stress obtained by FEM simulation as a function of element mesh density for 3 selected cases of 2D notched components (dimensions are in millimeters): solid lines—linear elements; dashed lines—quadratic elements.

2.2. Simulations of Notch Effects in 3D Tasks

For the simulation of spatial notched solids, the chosen case was that of a shaft with a circumferential groove loaded in bending and a stepped shaft loaded in tension and torsion, respectively (Figure 5).

These simulations were significantly more demanding on computing power. In the finest mesh cases, the total number of models elements reached tens of millions. The aim of the correct choice of loading conditions, anchorage and model dimensions was to achieve theoretical stress distributions in the cross-sections of the modelled shafts for a linear material behavior [10,13]. The output of each simulation was a comparison of the principal stresses at nodal points or elements in the root of the notch at the chosen mesh density with the analytical solution. The results of the effect of element density on the accuracy of the results of the stresses in the notches are shown in Figure 6. The results for the case of a stepped shaft loaded in tension and torsion are shown with solid (linear elements) and empty circles (quadratic elements) for a ratio of 0.2 (element length/notch radius) only.

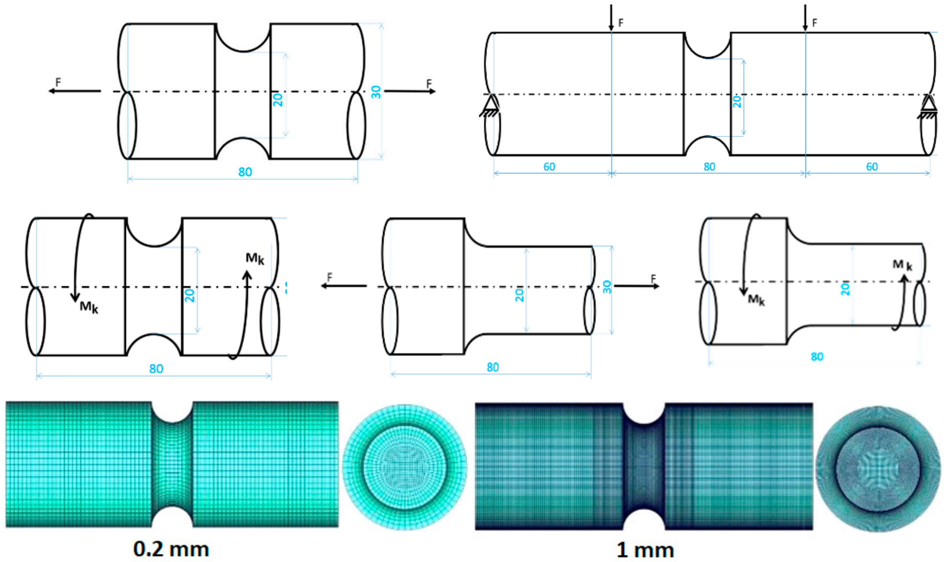


Figure 5. Cases of notched 3D components (dimensions are in millimeters) and the formation of a mesh of elements for a shaft with a semicircular circumferential groove (element length 1.0 mm and 0.2 mm).

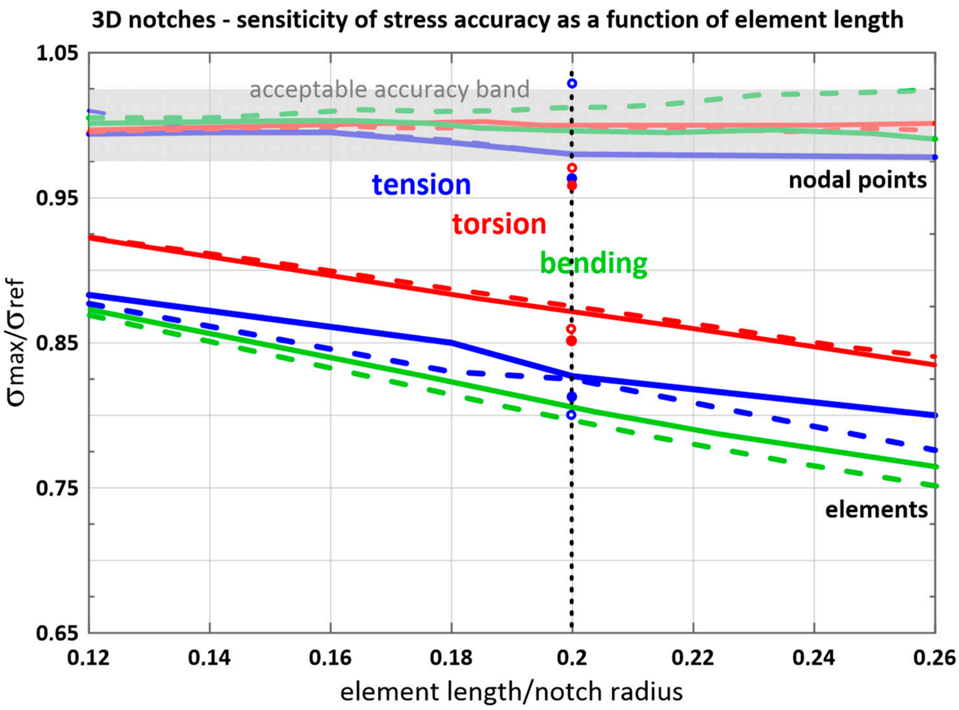


Figure 6. Analysis of the accuracy of notch root stress obtained by FEM simulation as a function of element mesh density for 3 selected cases of 3D notched components: solid lines—linear elements; dashed lines—quadratic elements.

As with the comparison of the results in the 2D simulations, it is also true for the 3D simulations that more accurate stress values were obtained by subtracting the stresses at the nodal points, and the effect of the quadratic element compared with the linear element is insignificant.

From the plots in Figures 4 and 6, a key conclusion can also be drawn regarding the influence of the element mesh density on the accuracy of the stresses in the notch, determined by FEM simulations. The required accuracy of determining the value of the stress in the notch is established from the parameters of the S–N curves. In Figure 7, Basquin curves [14] for low carbon steel and high strength alloy steel are given as an example to represent the parameter ranges (σ_f', b) of structural steels. If we choose a maximum permissible error in fatigue life of ±25% (blue dashed lines in Figure 7), then for the so-called high-cycle region, the permissible inaccuracy in the determination of the stress amplitude is $\pm(1.8 \div 3)\%$.

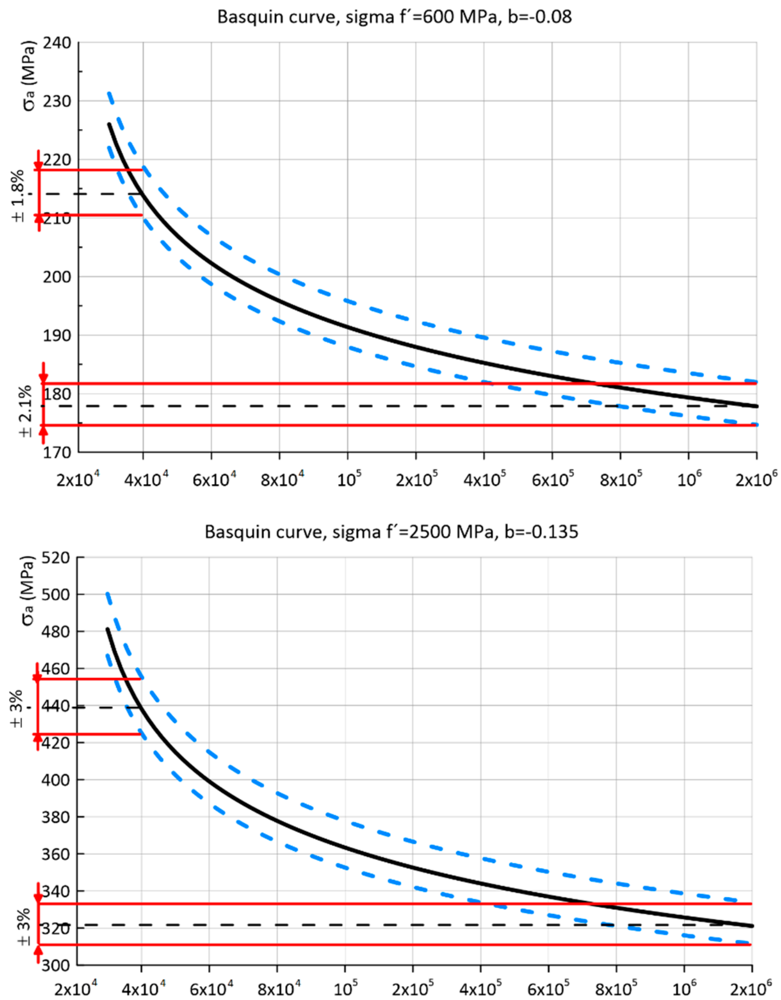


Figure 7. Determination of the permissible error for the calculation of the stress amplitude.

If we apply this error to the results of the above simulations, as indicated by the approximately 2% band in Figures 4 and 6, we read the ratio of the element size to the radius of curvature of the notch on the horizontal axis at 0.2–0.25. This means that 4–5 elements are required along the length of the circumference with a given radius of curvature of the notch in the FEM model for 25% accuracy in determining fatigue life. If we choose a different value for the required accuracy in the fatigue life calculation, these ratios change. Due to the high values of the variances of the cyclic material properties, the use of cumulative rules and other steps in fatigue life estimation procedures, this value is acceptable for engineering practice.

3. Practical Applications

The above procedure for modelling the stresses at the root of the notch was used to determine the stress concentration factor at the root of a weld. The objective of the FEM simulation was to determine the value of stress concentration in the weld root penetration or top cap weld of the pipeline. The weld preheat geometry of the V-welds of the gas transmission pipelines was analyzed directly on the pipeline sections and confronted with the IIW recommendations (Figure 8) [15–20]. At the root of the top and bottom weld elevation, five elements were applied to the radius of the notch curvature (in agreement with previous findings).

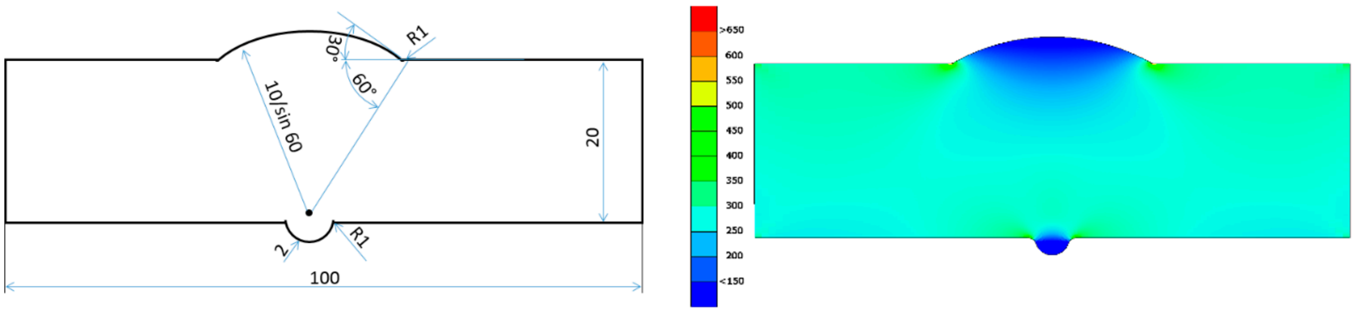


Figure 8. V-weld geometry (dimensions are in millimeters) and stress distribution around the weld obtained by FEM simulation (stresses are in MPa).

Based on the scatter of the actual V-weld geometry of the pipe faces, models were developed for different combinations of weld geometry parameters. Stress concentration factors from the most favorable geometry to the most unfavorable geometry of weld parameters of DN500 × 20 and DN700 × 20 pipes were in the interval K = 1.38 ÷ 1.88 in the simulation results. These results are in good correlation with the experimental results presented in [21–25]. Knowledge of the magnitude of the stress concentration caused by the weld geometry provides a basis for discussing the extension of the service life of a given spot by its additional grinding [24].

4. Conclusions

The stress value at the critical location of the structure, where the stress concentration is usually present, is one of the key input values determining the accuracy of the computational estimation of its fatigue life.

In this paper, the influence of the FEM model development methodology on the accuracy of the calculation of the stresses in the root of the notch was analyzed. The benchmark for determining the accuracy of the simulation results was the known analytical solutions of selected cases of 2D and 3D notched components. In all cases, the conditions for the application of the component loads and constraints were consistently chosen so that the resulting stress distributions in the FEM simulations respected the analytical solutions of the stress distributions in the cross-sections. The main findings from the simulation results (more than 200 simulations, models with the finest element mesh contained tens of millions of elements) for the methodology of creating notched component models are as follows:

- To assess the stress state at the root of the notch, it is necessary to know the components of the principal stresses; in addition, this is also a necessary input in the case of the multiaxial approach to fatigue life and strength estimation.
- The significant difference in the evaluation of the principal stress values lies in the elements and at the nodal points.
- Significantly better agreement of stresses can be achieved at nodal points compared with stresses in elements at the same element network density.
- Non-significant difference was found between stress values in quadratic and linear elements in both 2D and 3D models.

- For objectively defined accuracy of fatigue life calculation (±25%), it is necessary to choose the mesh density at the critical location, such that the ratio of element length to notch root radius is 1/5 to 1/4.
- The proposed methodology, applied to simulate the stress concentration at the weld root of a gas pipe, is in good agreement with published experimental results, and may help to determine possible fatigue life extension by deburring the weld geometry with a grinder.

The proposed methodology can also be used in crack modelling for crack propagation analysis. [26]. In cases where it is not possible to apply the proposed modelling methodology to the structure as a whole, the sub-modelling method can be used to achieve the required accuracy of the results by applying the proposed procedures.

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Abbreviations

Abbreviations

FEM	finite element method
FBG	fiber Bragg gratting
ESPI	electronic speckle pattern interferometry
DIC	digital image correlation method
IIW	International Institute of Welding

Nomenclature

K_σ	stress concentration factor
σ_{max}	maximum stress in the notch
σ_{ref}	the value of the maximum stress in the notch obtained by the analytical solution

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