LAB2: Resonance of Multi-level Building

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1. PreLAB

Find the dynamic formula for the system below, derive the response of the system displacement x(t) for external force F(t) = F0sin(wt), and explain the resonance (assuming the system is underdamped)

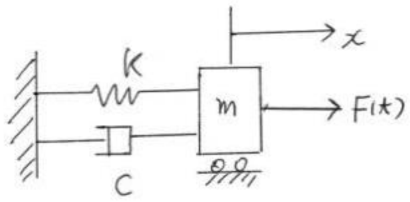


Fig 1. DOF system

Fig 1 is a single degree of freedom system. The equation of motion for a single degree of freedom system is expressed as follows.

$$m\ddot{x} + c\dot{x} + kx = f(t), \qquad f(t) = F_0 \cdot \sin(\omega t)$$

m is the mass, c is the damping coefficient, k is the spring constant, x is the displacement, F_0 is the amplitude of the external force, and ω is the frequency of the external force. The above expression can be written as follows.

$$w_n = \sqrt{\frac{k}{m}}, \qquad \xi = \frac{c}{2\sqrt{mk}}, \qquad \ddot{x} + 2\xi w_n \dot{x} + w_n^2 x = \frac{F_0 \cdot \sin(\omega t)}{m}$$

Analyzing the harmonic motion ($F(t) = F_0 \sin(wt)$), the x(t) value can be obtained as follows.

$$x(t) = x_h(t) + x_p(t)$$

The system is underdamped, so the condition is $\xi > 1$. To find $x_h(t)$, we use a characteristic equation. Considering this condition, $x_h(t)$ can be calculated as follows.

$$x(t) = e^{st} \rightarrow \; ms^2 + cs + k = 0$$

$$x_h(t) = C_1 e^{\left[-\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right]t} + C_2 e^{\left[-\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right]t}$$

We can set the value as follows and we can write x(t) as follows.

$$w_n = \sqrt{\frac{k}{m}}, \qquad \xi = \frac{c}{2\sqrt{km}}$$

$$\rightarrow \ x_h(t) = C_1 e^{\left(-\xi+\sqrt{\xi^2-1}\right)w_n t} + C_2 e^{\left(-\xi-\sqrt{\xi^2-1}\right)w_n t}$$

Applying the following initial conditions to the above equation results in the following form.

$$I.C \rightarrow t = 0, \qquad x(t) = x_0, \qquad \dot{x}(0) = \dot{x_0}$$

$$X_0 = \frac{\sqrt{x_0^2 w_n^2 + x_0^2 + 2x_0\dot{x}_0\xi w_n}}{\sqrt{1-\xi^2}\cdot w_n}, \qquad \phi_0 = \tan^{-1}\left(\frac{x_0 + \dot{\xi} w_n x_0}{x_0 w_n\sqrt{1-\xi^2}}\right)$$

$$\rightarrow \ x_h(t) = \frac{\sqrt{x_0^2 w_n^2 + x_0^2 + 2x_0\dot{x}_0\xi w_n}}{\sqrt{1-\xi^2}\cdot w_n}\cdot e^{-\xi w_n t}\cdot \sin\left(\sqrt{1-\xi^2}\cdot w_n t - \tan^{-1}\left(\frac{x_0 + \dot{\xi} w_n x_0}{x_0 w_n\sqrt{1-\xi^2}}\right)\right)$$

If $x_p(t) = X\cdot\sin(wt-\phi)$, we can write as follows.

$$X = \frac{F_0}{\sqrt{(k-mw^2)^2+(cw)^2}} = \frac{\delta_{st}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}, \qquad \phi = \tan^{-1}\left(\frac{cw}{k-mw^2}\right) = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$$r = \frac{w}{w_n}, \quad \delta_{st} = \frac{F_0}{k}$$

$$\rightarrow \ x_p(t) = \frac{\delta_{st}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}\cdot\sin\left(wt - \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)\right)$$

The displacement x(t) of the system is as follows.

$$x(t) = x_h(t) + x_p(t) = X_0 e^{-\xi w_n t}\cdot\sin\left(\sqrt{1-\xi^2}\cdot w_n t - \phi_0\right) + X sin(wt-\phi)$$

In $X = \frac{\delta_{st}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$ expression, the amplitude ratio is $\frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$. Therefore, the graph of r for amplitude ratio M is as follows.

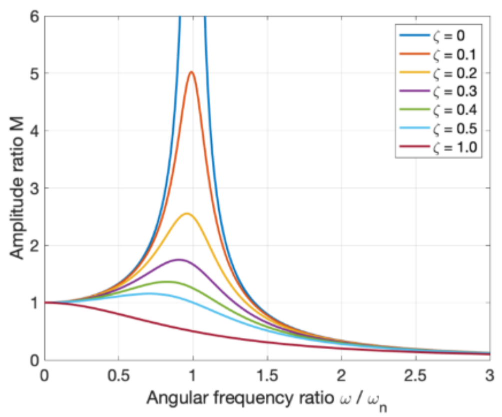


Fig 2. Amplitude ratio graph

In Figure 2, when $w = w_n$, resonance occurs. Just looking at the graph, you can see that when the two values are the same, the vibration is amplified.

2. Results

2-1 Experiment

Analyze the data of the spectrum analyzer to find the two resonant frequencies and mode sizes (amplitude).

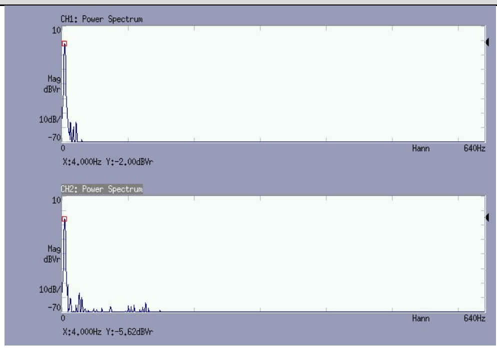
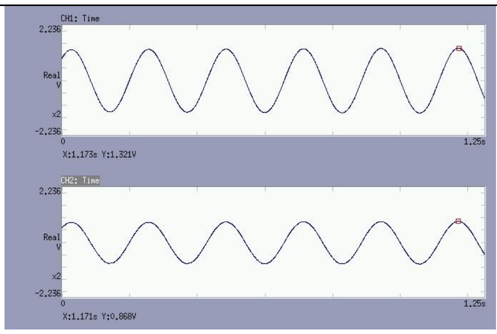
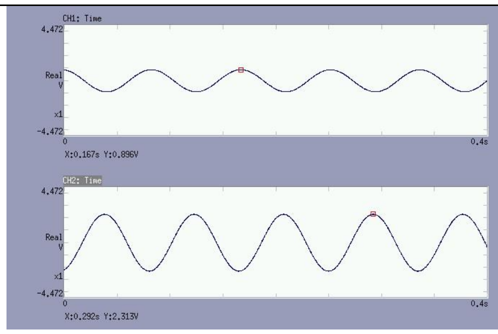
	1^{st}	2^{nd}
Resonance frequency	 $f_n = 4\ Hz$	 $f_n = 11.25\ Hz$
Mode shape	 $\frac{X_2^{(1)}}{X_1^{(1)}} = \frac{CH2\_Y}{CH1\_Y} = 0.657$	 $\frac{X_2^{(2)}}{X_1^{(2)}} = \frac{CH2\_Y}{CH1\_Y} = -2.581$

Table 1. Result of experiment

In Table 1, it can be seen that the resistance frequency in mode 1 is 4 Hz and 11.25 Hz in mode 2. In the case of Mode shape, since CH2/CH1 must be performed, it can be seen that 0.657 was found in Mode 1 and -2.581 was found in Mode 2. In mode 1, the phase of the waveform of the two channels is the same, but in mode 2, the phase is reversed, so the mode shape has a negative value.

2-2 Theory

Modeled as a two-degree mass-spring to find the resonant frequency and mode shape for two lateral bends.

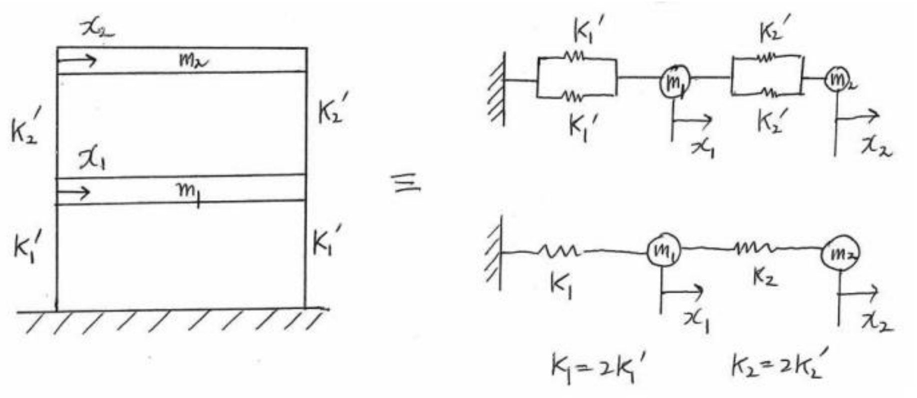


Fig 3. Two-Freedom Mass-Spring Modeling

Equation of motion can be calculated as follows through the figure in Figure 1.

$$m_1\ddot{x_1} + (k_1 + k_2)x_1 - k_2x_2 = 0$$

$$m_2\ddot{x_2} - k_2x_1 + k_2x_2 = 0$$

And harmonic motion can be written in the following form

$$\begin{bmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The first matrix must be zero for the above expression to be established. In addition, since the resonance frequency is w, the equation for obtaining w can be calculated as follows.

$$\begin{bmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{bmatrix} = 0$$

$$w^4 m_1 m_2 - w^2(m_2k_1 + m_2k_2 + m_1k_2) + k_1k_2 = 0$$

$$w^2 = \frac{(m_2k_1 + m_2k_2 + m_1k_2) \pm \sqrt{(m_2k_1 + m_2k_2 + m_1k_2)^2 - 4m_1m_2k_1k_2}}{2m_1m_2}$$

The k value to obtain the resonance frequency can be written by the boundary condition as follows.

$$k = \frac{24EI}{l^3}$$

The values presented in the question are as follows.

$$m_1 = 1.12kg, \qquad m_2 = 1.37kg, \qquad k_1 = k_2 = k$$

$$E = 70\ GPa, \qquad I = \frac{bh^3}{12} = \frac{0.05 \times 0.0015^3}{12}, \qquad l = 0.225$$

Therefore, if the resonance frequency is obtained with the above values, it is as follows.

$$w_1^2 = \frac{(m_1 + 2m_2)k - \sqrt{\left((m_1 + 2m_2)k\right)^2 - 4m_1m_2k^2}}{2m_1m_2} = \frac{3.86k - 2.96k}{3.07} = 0.293k$$

$$w_1 = 0.541 \times \sqrt{\frac{24EI}{l^3}} = 26.38\ rad/s = 4.2\ Hz$$

$$w_2^2 = \frac{(m_1 + 2m_2)k + \sqrt{\left((m_1 + 2m_2)k\right)^2 - 4m_1m_2k^2}}{2m_1m_2} = \frac{3.86k + 2.96k}{3.07} = 2.221k$$

$$w_2 = 1.49 \times \sqrt{\frac{24EI}{l^3}} = 72.64\ rad/s = 11.56\ Hz$$

The mode shape can be obtained as follows using the harmonic motion equation.

$$r_1 = \frac{x_1^{(1)}}{x_2^{(1)}} = \frac{-w_1^2 m_1 + k_1 + k_2}{k_2} = -0.293 \cdot m_1 + 2 = 1.67$$

$$\rightarrow\ \overrightarrow{x^{(1)}} = \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 1.67 \end{Bmatrix} x_1^{(1)}$$

$$\rightarrow\ 1_{st}\ mode\ shape = \frac{1.0}{1.67} = 0.6$$

$$r_2 = \frac{x_1^{(2)}}{x_2^{(2)}} = \frac{-w_2^2 m_1 + k_1 + k_2}{k_2} = -2.221 \cdot m_1 + 2 = -0.488$$

$$\rightarrow\ \overrightarrow{x^{(2)}} = \begin{Bmatrix} x_1^{(2)} \\ x_2^{(2)} \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ -0.488 \end{Bmatrix} x_1^{(2)}$$

$$\rightarrow\ 2_{nd}\ mode\ shape = \frac{1.0}{-0.488} = -2.05$$

In summary, the theory value is as follows.

$$1_{st}\ \ f_n = 4.2\ Hz,\ \ 2_{nd}\ \ f_n = 11.53\ Hz,\ \ 1_{st}\ mode\ shape = 0.6,\ \ 2_{nd}\ mode\ shape = -2.05$$

2-3 Simulation

Find 5 resonance frequencies and mode shapes from ANSYS simulation

In the ANSYS simulation process, the material of the vertical beams is aluminum alloy and the material of the horizontal beams is structural steel.

Five resonant frequencies and mode shapes were obtained through ANSYS simulation. The resonance frequency of each mode can be confirmed through Figure 2.

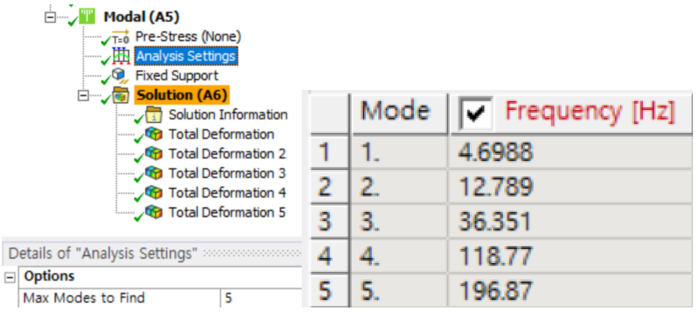


Fig 4. Five resonance frequency

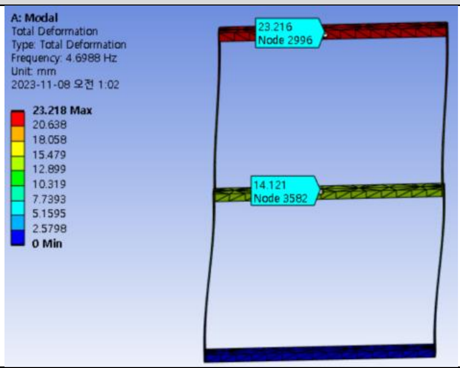
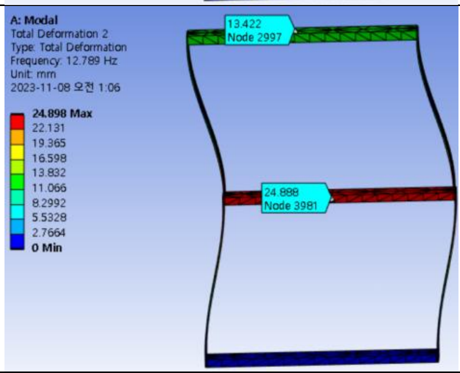
	Simulation of modal	Mode shape
Mode 1		$\frac{14.121}{23.216} = 0.61$
Mode 2		$-\frac{24.888}{13.422} = -1.85$

Table 2. Result of Simulation

As shown in Table 2, the mode shape was calculated through simulation.

3. Discussions

(1) Make a comparison table as shown in Table 1 and discuss the differences between experiments, theories, and simulations.

	1^{st} $\mathrm{f_n}$	2^{nd} $\mathrm{f_n}$	1^{st} mode shape $X_2^{(1)}/X_1^{(1)}$	2^{nd} mode shape $X_2^{(2)}/X_1^{(2)}$
Experiment	4.0 Hz	11.25 Hz	0.657	-2.581
Theory	4.2 Hz	11.56 Hz	0.6	-2.05
Simulation	4.7 Hz	12.79 Hz	0.61	-1.85

Table 3 Comparison of natural frequencies and mode shapes

The values of experiments, theories, and simulations all came out differently. The resonance frequency of the experimental values was smaller than that of theory and simulation, and the mode shape was larger than that of theory and simulation. In particular, there was a big difference in the second mode shape. The material properties can be considered for reasons that have different values. The physical properties of materials used in theories and simulations may not be completely the same as those of materials used in experiments. In addition, there is a possibility that the inaccuracy of the sensors used in the experiment or the surrounding environment may be affected. When measuring the resonance frequency and mode shape in the experiment, it was confirmed with the eye that the building structure was shaking a lot and measured by the waves at that time, so it cannot be said that the exact values were measured. In the case of simulation, a difference may occur because the mesh size is set to default. It is necessary to adjust the mesh size for the correct value.

(2) Discuss the vibration characteristics of multi-degree-of-freedom structures compared to one-degree-of-freedom structures.

One degree of freedom structure is a simple structural system that can be moved or transformed independently. Multi degree of freedom structures are more complex structural systems than one degree of freedom structures and should be analyzed using matrix and differential equations. In addition, interactions between structures are important and have a lot of freedom.

Similar to what was mentioned above, one degree of freedom structure has a single vibration mode. It means that the system moves only in a single vibration direction. It is easy to interpret because its vibration characteristics are simple, and it is analyzed using the displacement or angle of vibration. On the other hand, in the case of a multi degree of freedom structure, it has several vibration modes. It means that the system moves in different vibration directions. Due to various vibration modes, vibration characteristics are more complex and difficult to interpret than one degree of freedom. Therefore, various mathematical calculation methods are required. Multi-degree of freedom structure is complex because it deals with more diverse forms of vibration, but it is more accurate than one degree of freedom structure.

(3) Consider ways to prevent resonance when a building's roof has a large cooling fan attached to it and causes resonance in the building.

The building has a natural vibration frequency. However, resonance occurs when the natural frequency of the cooling fan on the roof of the building is the same or has little difference. Therefore, the structure can be strengthened to prevent resonance to control the building's natural vibration frequency, but this method is expensive, difficult and complex. In addition to this method, vibration generated by the cooling fan may be subjected to silent treatment. We can use a vibrating noise spring or a vibrating noise device. There is also a method of mitigating vibration by installing a vibration insulating device between the cooling fan and the building.