HW#1

2.1 **a.** Obtain the global stiffness matrix $\underline{K}$ of the assemblage shown in Figure P2–1 by superimposing the stiffness matrices of the individual springs. Here k_1, k_2, and k_3 are the stiffnesses of the springs as shown.

b. If nodes 1 and 2 are fixed and a force P acts on node 4 in the positive x direction, find an expression for the displacements of nodes 3 and 4.

c. Determine the reaction forces at nodes 1 and 2.

(*Hint:* Do this problem by writing the nodal equilibrium equations and then making use of the force/displacement relationships for each element as done in the first part of Section 2.4. Then solve the problem by the direct stiffness method.)

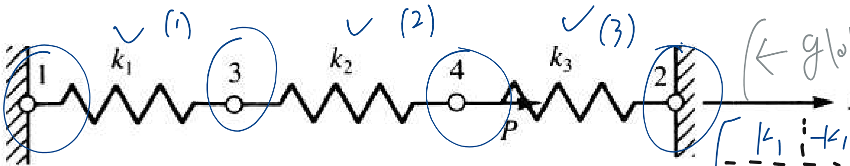


Figure P2–1

(← global stiffness matrix →)

$$\begin{bmatrix} k_1 & -k_1 & & \\ -k_1 & k_1+k_2 & -k_2 & \\ & -k_2 & k_2+k_3 & -k_3 \\ & & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{3x} \\ d_{4x} \\ d_{2x} \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ 0 \\ 0 \\ F_{2x} \end{Bmatrix}$$

$d_{1x} = 0$, $d_{2x} = 0$, P

3.12 Solve for the axial displacement and stress in the tapered bar shown in Figure P3–12 using one and then two constant-area elements. Evaluate the area at the center of each element length. Use that area for each element. Let $A_0 = 2\text{ in}^2$, $L = 20$ in., $E = 10 \times 10^6$ psi, and $P = 1000$ lb. Compare your finite element solutions with the exact solution.

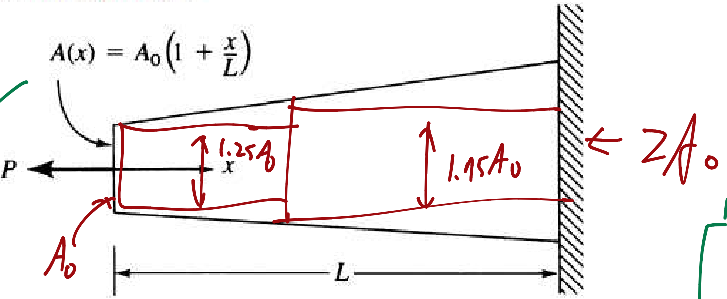


Figure P3–12

(one element)

$P \leftarrow$ 1 — 2 $\rightarrow F_{2x}$

$1.5A_0$

$$\begin{bmatrix} \frac{1.5EA_0}{L} & -\frac{1.5EA_0}{L} \\ -\frac{1.5EA_0}{L} & \frac{1.5EA_0}{L} \end{bmatrix} \begin{pmatrix} u_{1x} \\ u_{2x} \end{pmatrix} = \begin{pmatrix} F_{1x} \\ F_{2x} \end{pmatrix}$$

$F_{1x} = -P$, $u_{2x} = 0$

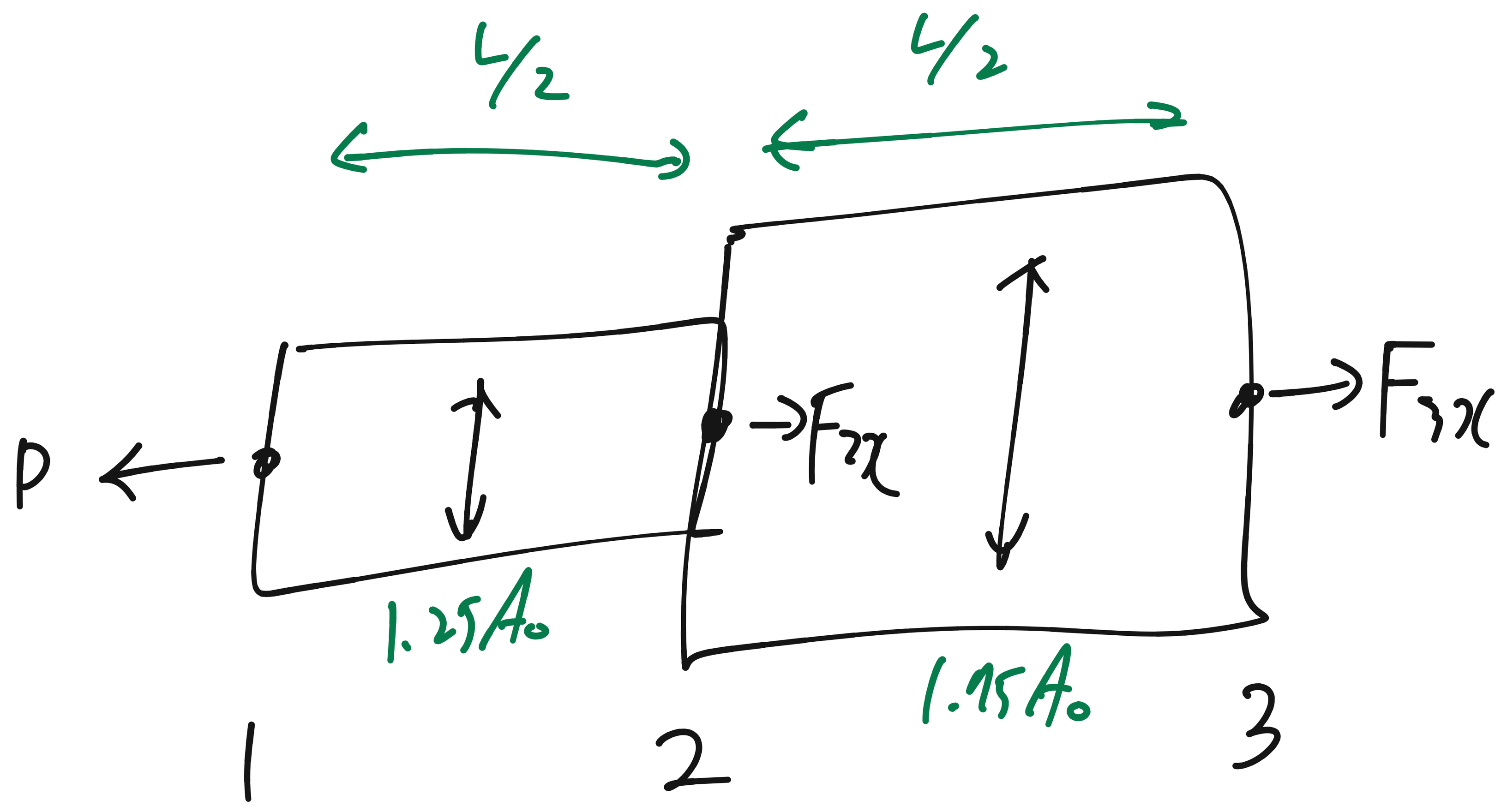
$$\sigma^{(1)} = \frac{P}{A} = \frac{P}{1.5A_0} = 333\text{ Psi}$$

$$\delta = \frac{F \cdot L}{EA} \quad : \quad \sigma = E \cdot \varepsilon, \quad \sigma = \frac{F}{A}, \quad \varepsilon = \frac{\delta}{L}$$

$$\delta = \int_0^L \frac{F}{EA} \cdot dx = \int_0^L \frac{P\,dx}{EA_0\left(1+\frac{x}{L}\right)} = \frac{PL}{EA_0} \ln\left(1+\frac{x}{L}\right)\Big|_0^L = \bigcirc$$

P = 1000 lb, E = 10^6 Psi, A_0 = 2 in², L = 20 in

$$\sigma = \frac{F}{A} = \frac{P}{A_0\left(1+\frac{x}{L}\right)}$$



$$k = \frac{EA}{L}$$

$$\begin{bmatrix} \frac{1.25EA_0}{L/2} & \frac{-1.25EA_0}{L/2} & 0 \\ \frac{-1.25EA}{L/2} & \boxed{\frac{1.25EA_0}{L/2} + \frac{1.75EA_0}{L/2}} & -\frac{1.75EA_0}{L/2} \\ 0 & \frac{-1.75EA_0}{L/2} & \frac{1.75EA_0}{L/2} \end{bmatrix} \begin{pmatrix} U_{1x} \\ U_{2x} \\ U_{3x} \end{pmatrix} = \begin{pmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{pmatrix}$$

$U_{3x} = 0$, $F_{1x} = -P$, $F_{2x} = 0$

$$\sigma^{(1)} = \frac{P}{A} \leftarrow 1.25A_0$$

$$\sigma^{(2)} = \frac{P}{A} \leftarrow 1.75A_0$$