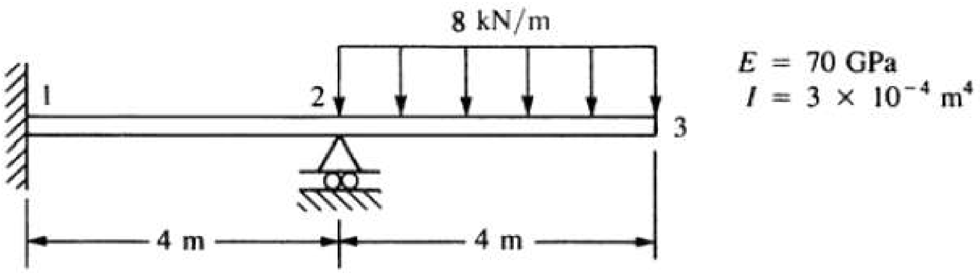
HW#3 Beam Problem Using Finite Element Method



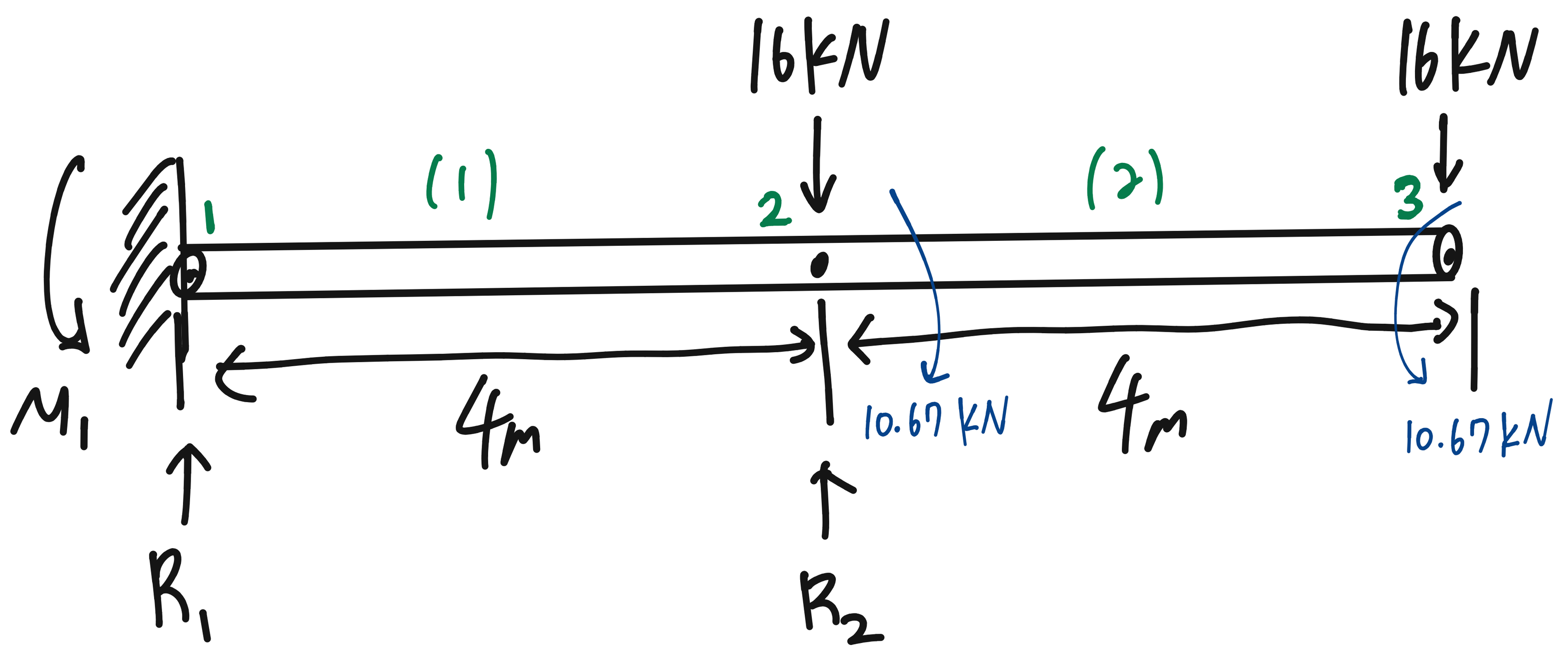
위 빔에서,

(1) 3절점 2요소 유한요소법으로 풀어, 각 절점의 변위, 반력힘과 반력굽힘모멘트를 구하라

(2) 요소 1의 절점 1, 2의 빔 단면에서 최대 수직응력을 각각 구하라.(빔은 사각단면이며 높이는 250mm, 폭은 230.4mm)

(3) 이론해와 MATLAB코드의 Assembled 강성행렬, 변위해, 반력해와 비교하라.

(1) 3절점 2요소 유한요소법으로 풀어, 각 절점의 변위, 반력힘과 반력굽힘모멘트를 구하라



힘 : $\frac{wL}{2}$

굽힘모멘트 : $\frac{wL^2}{2}$

$$\begin{pmatrix} R_1 \\ M_1 \\ R_2-16k \\ -10.67k \\ -16k \\ 10.67k \end{pmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{pmatrix} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \\ d_{3y} \\ \phi_3 \end{pmatrix}$$

$$\underset{\substack{\text{유효} \\ \text{절점력}}}{F_e} = \underset{\substack{\text{외력} \\ \text{or} \\ \text{반력}}}{F} + \underset{\text{등가하중}}{F_0} = \hat{k} \cdot \hat{d}$$

$d_{1y} = \phi_1 = d_{2y} = 0 \rightarrow$ Reduced Stiffness Matrix

$$\begin{pmatrix} -10.67k \\ -16k \\ 10.67k \end{pmatrix} = \frac{EI}{L^3} \begin{bmatrix} 8L^2 & -6L & 2L^2 \\ -6L & 12 & -6L \\ 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{pmatrix} \phi_2 \\ d_{3y} \\ \phi_3 \end{pmatrix}$$

$$= \frac{70\times10^9 \times 3\times10^{-4}}{4^3} \begin{bmatrix} 128 & -24 & 32 \\ -24 & 12 & -24 \\ 32 & -24 & 64 \end{bmatrix} \begin{pmatrix} \phi_2 \\ d_{3y} \\ \phi_3 \end{pmatrix}$$

$\Rightarrow$ $d_{1y} = 0$, $\phi_1 = 0$

$d_{2y} = 0$, $\phi_2 = -3.05 \times 10^{-3}$ (rad)

$d_{3y} = -24.4\times10^{-3}$ m , $\phi_3 = -7.11 \times 10^{-3}$ (rad)

$$\begin{pmatrix} R_1 \\ M_1 \\ R_2-16k \\ -10.67k \\ -16k \\ 10.67k \end{pmatrix} = 328125 \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.05\times10^{-3} \\ -24.4\times10^{-3} \\ -7.11\times10^{-3} \end{pmatrix}$$

$$R_1 = 328125 \times 6L \times (-3.05\times10^{-3}) = -24019N = \underline{24kN}$$

$$M_1 = 328125 \times 2L^2 \times (-3.05\times10^{-3}) = -32025N = \underline{32kN}$$

$$R_2 - 16k = 328125 \times \{-12 \times (-24.4\times10^{-3}) + 6L \times (-7.11\times10^{-3})\} = 40084N = 40kN$$

$$\Rightarrow R_2 = \underline{56kN}$$

(2) 요소 1의 절점 1, 2의 빔 단면에서 최대 수직응력을 각각 구하라.(빔은 사각단면이며 높이는 250mm, 폭은 230.4mm)

$$\sigma_x = E\cdot\varepsilon_x = -E\hat{y}\cdot\hat{v}''(\hat{x})$$

$$\hat{v}(x) = \left[\frac{1}{L^3}(2\hat{x}^3 - 3\hat{x}^2L + L^3),\ \frac{1}{L^3}(\hat{x}^3L - 2\hat{x}^2L^2 + \hat{x}L^3),\ \frac{1}{L^3}(-2\hat{x}^3 + 3\hat{x}^2L),\ \frac{1}{L^3}(\hat{x}^3L - \hat{x}^2L^2)\right] \begin{pmatrix} \hat{d}_{1y} \\ \hat{\phi}_1 \\ \hat{d}_{2y} \\ \hat{\phi}_2 \end{pmatrix}$$

$$\hat{v}''(x) = \left[\frac{1}{L^3}(12\hat{x} - 6L),\ \frac{1}{L^3}(6\hat{x}L - 4L^2),\ \frac{1}{L^3}(-12\hat{x} + 6L),\ \frac{1}{L^3}(6\hat{x}L - 2L^2)\right] \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.05\times10^{-3} \end{pmatrix}$$

$$\hat{v}''(0) = \frac{1}{L^3}\begin{bmatrix} -6L & -4L^2 & 6L & -2L^2 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.05\times10^{-3} \end{pmatrix} = -\frac{2}{L}\times(-3.05\times10^{-3})$$

$$= 1.53\times10^{-3}$$

$$v''(4) = \frac{1}{L^3}\left[48 - 6L,\ 24L - 4L^2,\ -48 + 6L,\ 24L - 2L^2\right] \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.05\times10^{-3} \end{pmatrix}$$

$$= \left(\frac{24}{L^2} - \frac{2}{L}\right)\times(-3.05\times10^{-3}) = -3.05\times10^{-3}$$

$x = 0 \rightarrow \sigma_x = -E \cdot \hat{y} \cdot \hat{v}''(0) = 70\times10^{9} \times \frac{0.25}{2} \times 1.525 \times10^{-3} = \underline{13.4\,MPa}$

$x = 4 \rightarrow \sigma_x = -E \cdot \hat{y} \cdot \hat{v}''(4) = 70\times10^{9} \times \frac{0.25}{2} \times (-3.05 \times10^{-3}) = \underline{26.7\,MPa}$

$x=4$ 에서의 응력이 더 크다.

(3) 이론해와 MATLAB코드의 Assembled 강성행렬, 변위해, 반력해와 비교하라.

Theory Solution

$$328125 \begin{bmatrix} 12 & 24 & -12 & 24 & 0 & 0 \\ 24 & 64 & -24 & 32 & 0 & 0 \\ -12 & -24 & 24 & 0 & -12 & 24 \\ 24 & 32 & 0 & 128 & -24 & 32 \\ 0 & 0 & -12 & -24 & 12 & -24 \\ 0 & 0 & 24 & 32 & -24 & 64 \end{bmatrix}$$

$$= \begin{bmatrix} 3937500 & 7875000 & -3937500 & 7875000 & 0 & 0 \\ 7875000 & 21000000 & -7875000 & 10500000 & 0 & 0 \\ -3937500 & -7875000 & 7875000 & 0 & -3937500 & 7875000 \\ 7875000 & 10500000 & 0 & 42000000 & -3937500 & 7875000 \\ 0 & 0 & -3937500 & -7875000 & 3937500 & -7875000 \\ 0 & 0 & 7875000 & 10500000 & -3937500 & 21000000 \end{bmatrix}$$

MATLAB

```
  GK: Assembled Global Stiffness Matrix
ans =

   1.0e+07 *

    0.3937    0.7875   -0.3937    0.7875         0         0
    0.7875    2.1000   -0.7875    1.0500         0         0
   -0.3937   -0.7875    0.7875         0   -0.3937    0.7875
    0.7875    1.0500         0    4.2000   -0.7875    1.0500
         0         0   -0.3937   -0.7875    0.3937   -0.7875
         0         0    0.7875    1.0500   -0.7875    2.1000
```

이론해와 MATLAB 코드의 Assembled 강성행렬은 같다.

Theory Solution

$$\begin{pmatrix} d_{1y} \\ \phi_1 \\ d_{2y} \\ \phi_2 \\ d_{3y} \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3.05\times10^{-3} \\ -24.4\times10^{-3} \\ -7.11\times10^{-3} \end{pmatrix}$$

MATLAB

```
GU: Displacement
ans =

         0
         0
   -0.0000
   -0.0030
   -0.0244
   -0.0071
```

이론해와 MATLAB 코드의 변위해는 같다.

Theory Solution

$$\begin{pmatrix} R_1 \\ M_1 \\ R_2 - 16k \\ -10.67k \\ -16k \\ 10.67k \end{pmatrix} = \begin{pmatrix} 24000 \\ 32000 \\ 40000 \\ -10670 \\ -16000 \\ 10670 \end{pmatrix}$$

MATLAB

```
GFF: Reaction force
ans =

   1.0e+04 *

   -2.4000
   -3.2000
    4.0000
   -1.0670
   -1.6000
    1.0670
```

이론해와 MATLAB 코드의 반력해는 같다.