Problem 1:

- Derive the matrix formula du/dt=Au for solving the heat equation with fixed-fixed boundary condition.

- Derive the Matrix formula for three methods of time stepping scheme discussed in class:

a) Forward explicit in time.
b) Backward implicit in time.
c) The Crank-Nicolson scheme (centered in time).

Problem 2:

1. Let's rescale the Schrodinger equation to get into dimensionless form shown in class by a change of variables for length, time, and potential for appropriately chosen scale factors, which have units of distance, time, and energy respectively.

2. Derive the matrix formula for solving the Schrodinger equation with the Crank-Nicolson (centered) time stepping scheme.

Problem 3: Centered Difference

Show that the center difference formula is exact for x and x^2.

Problem 4: Boundary value problem.

a) Consider the 1D Poisson equation -u''=f with constant source (f=1) and fixed boundary conditions (u=1 at both boundaries). Calculate the analytic solution.

b) Set up the linear algebra problem: Au=f. Write a code to solve the equation.

c) Make a graph to check if the numerical solution is exact to the analytic solution.