EE2703 - Week 6

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1 Assignment Goals

Assignment 6 requires you to implement gradient descent based optimization.

- Minimum requirement: adapt the code from the presentation to optimize as many of the functions below as possible.
- Write a generic function that will take in 2 other functions as input, and a range of values within which to search, and then implement gradient descent to find the optimum. The basic requirements of gradient descent are already available in the presentation.
- For some assignments, the gradient has not been given. You can either write the function on your own, or suggest other methods that can achieve this purpose.

[1]:
```
import numpy as np
import math
from numpy import cos, sin, pi, exp
import matplotlib.pyplot as plt
```

2 Gradient Descent Algo:

[10]:
```
def test(f, df, vals, left_limit, right_limit, rate=0.1, e=10e-6, iter=10000):

    coords = []

    for i in range(len(vals)):
        sys_break = 0
        cal = vals[i]
        for iterations in range(iter):
            grad = df(cal)
            norm = 0
            for i in grad:
                norm += i**2
            norm = math.sqrt(norm)

            if norm < e:
                break

            for k in range(len(cal)):
                cal[k] -= rate*(grad[k])
```

```
            for i in cal:
                approx_i = round(i, 1)
                if limits_check(approx_i, left_limit, right_limit) == False:
                    sys_break = 1

            if sys_break == 1:
                break

        for i in cal:
            if round(i, 1) == float(round(left_limit, 1)) or round(i, 1) ==␣
↪float(round(right_limit, 1)):
                sys_break = 1

        if sys_break == 0:
            cal = [round(n, 1) for n in cal]
            coords.append(cal)

    unique_pairs = set()

    for pair in coords:
        unique_pairs.add(tuple(pair))

    unique_pairs = list(unique_pairs)

    if len(coords[0]) == 1:
        x = [row[0] for row in unique_pairs]
        y = []
        for i in range(len(x)):
            input_y = [x[i]]
            y.append(f(input_y))

        for i in range(len(unique_pairs)):
            print(f"The coordinates of minima {i+1} are ({x[i]}, {round(y[i],␣
↪1)})")

        return x, y

    if len(coords[0]) == 2:
        x = [row[0] for row in unique_pairs]
        y = [row[1] for row in unique_pairs]
        z = []
        for i in range(len(x)):
            input_z = [x[i], y[i]]
            z.append(f(input_z))

        for i in range(len(unique_pairs)):
```

```
            print(f"The coordinates of minima {i+1} are ({x[i]}, {y[i]},␣
↪{round(z[i], 1)})")

        return x, y, z
```

The `test` function here implements the generalised `gradient descent` algorithm which can be used to find minima's of a multi-variable function. The function can be any variable starting from 1 and going till any number of variables you want.

To solve the problem of multiple minima's is solved using a list of inputs between the limits given so that not a single one would be missed. But this would append the same minima given from multiple inputs, so for this I am adding that particular minima in a set and then converting to a list so that the repeatation will not create an issue.

Another issue that might come is the gradient descent can go to the minimas which doesn't lie in the range provided to us. So for that I am just using limit check function which would check whether the given `x` at a particular instant is between limits or not and if it is not the given input value is discarded.

The above system check for the minima to stand in the range provided in the above function `limit_check` and `sys_break` are used.

These were the major issues which can come while solving for minimas using gradient descent method. So apart from this I have followed the general procedure only to calculate the minima which was discussed in class and hence at the end I am returning the coordinates of minima as well as priniting a statement for all the minimas of the respective function.

[11]:
```
def limits_check(check, left, right):
    if check > right or check < left:
        return False

    return True
```

As I told earlier this is a `limits_check` function which will check whether the input which we are using right now is in the specified limited region given to us or not.

[12]:
```
def get_coords_single_var(function, initial_values):
    x_coords = []
    y_coords = []
    for i in initial_values:
        x, = i
        x_coords.append(x)
        y_coords.append(function(i))

    return x_coords, y_coords
```

I have created this function to give me x and y coordinates of a 1 variable function from the input list which we are giving to the gradient descent function in such a way that it can be user friendly and useful for creating graohs in the latter section of this code

[13]:
```python
def get_coords_doube_var(function, initial_values):
    x_coords = []
    y_coords = []
    z_coords = []
    for i in initial_values:
        x, y = i
        x_coords.append(x)
        y_coords.append(y)
        z_coords.append(function(i))

    return x_coords, y_coords, z_coords
```

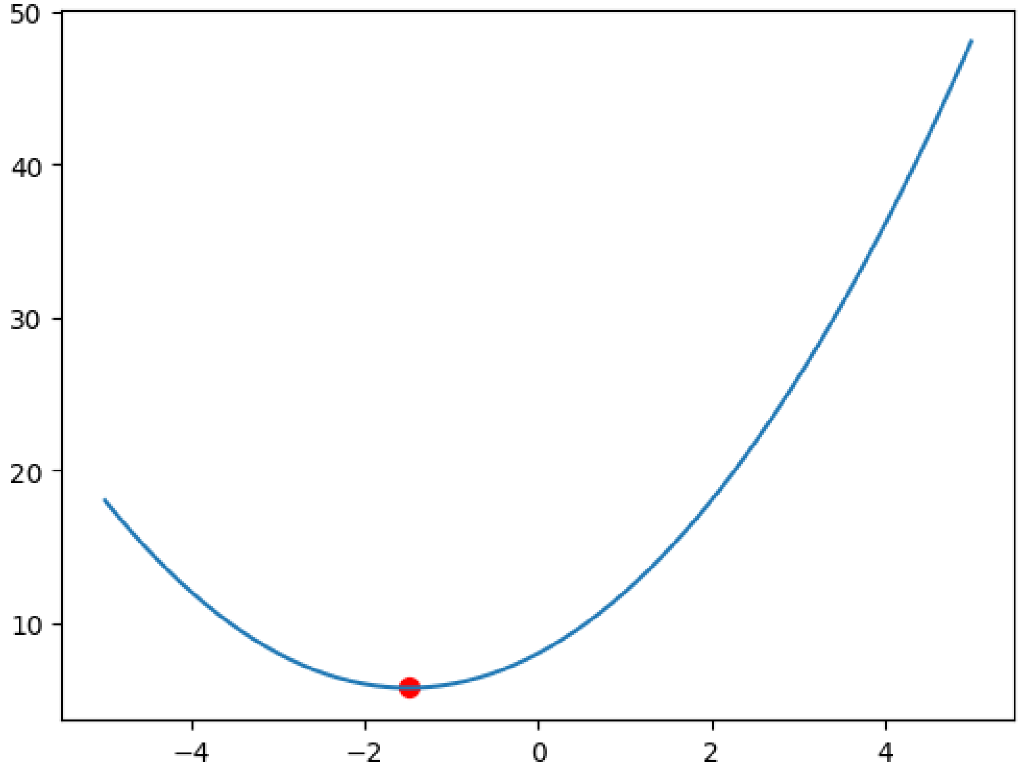
Similarly like above this piece of code give us x, y and z coordinates of a 2 varialbe function.

2.1 Problem 1 - 1-D simple polynomial

The gradient is not specified. You can write the function for gradient on your own. The range within which to search for minimum is [-5, 5].

[14]:
```python
x = np.linspace(-5, 5, 100)
coords = []
for i in range(len(x)):
    coords.append([x[i]])

def f1(val):
    x, = val
    return x**2 + 3*x + 8

def f1d(val):
    x, = val
    return np.array([2*x + 3])

x1, y1 = get_coords_single_var(f1, coords)
plt.plot(x1, y1)

a, b = test(f1, f1d, coords, -5, 5)

plt.scatter(a, b, s=50, c='red', marker='o')
plt.show()
```

```
The coordinates of minima 1 are (-1.5, 5.8)
```



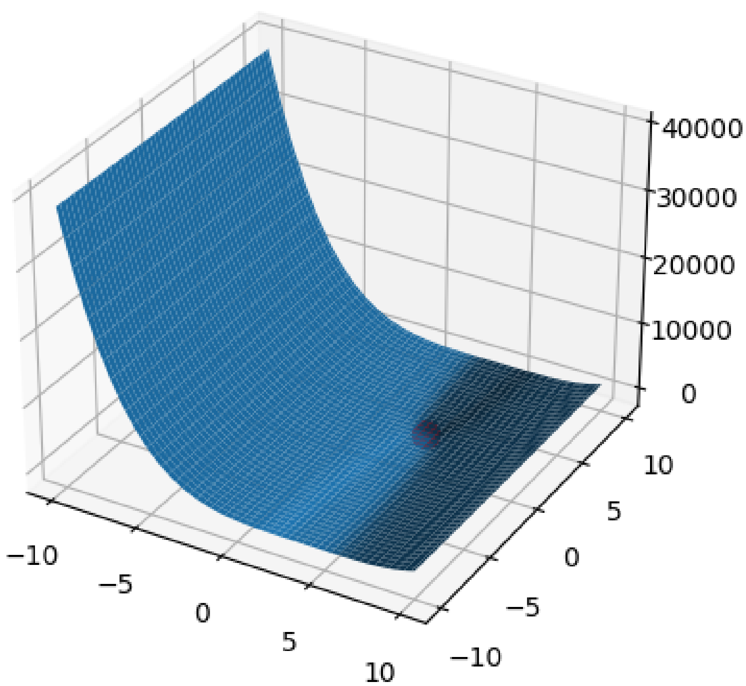
2.2 Problem 2 - 2-D polynomial

Functions for derivatives, as well as the range of values within which to search for the minimum, are given.

[15]:
```
xlim3 =  [-10, 10]
ylim3 =  [-10, 10]

x = np.linspace(-10, 10, 100)
y = np.linspace(-10, 10, 100)

coords = []
for i in range(len(x)):
    coords.append([x[i], y[i]])

def f3(values):
    x, y = values
    return x**4 - 16*x**3 + 96*x**2 - 256*x + y**2 - 4*y + 262

def f3_d(values):
    x, y = values
    return np.array([4*x**3 - 48*x**2 + 192*x - 256, 2*y - 4])
```

```
x_coords = np.outer(np.linspace(-10, 10, 100), np.ones(100))
y_coords = x_coords.copy().T
z_coords = x_coords**4 - 16*x_coords**3 + 96*x_coords**2 - 256*x_coords +
  ↪y_coords**2 - 4*y_coords + 262

ax = plt.axes(projection ='3d')
ax.plot_surface(x_coords, y_coords, z_coords)

a, b, c = test(f3, f3_d, coords, -10, 10)
ax.scatter3D(a, b, c, s=100, c='r', marker='o')
plt.show()
```

```
The coordinates of minima 1 are (4.0, 2.0, 2.0)
```



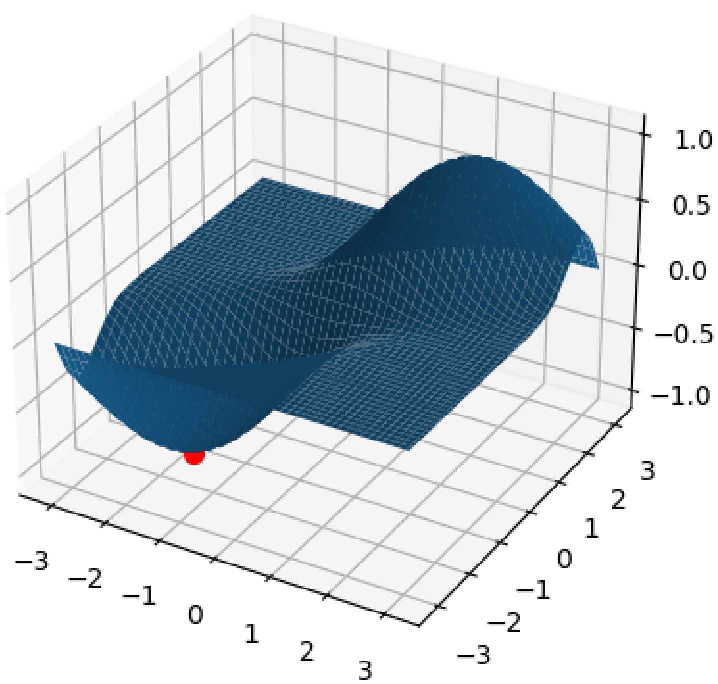
2.3 Problem 3 - 2-D function

Derivatives and limits given.

[16]:
```
xlim4 = [-pi, pi]
x = np.linspace(-np.pi, np.pi, 100)
y = np.linspace(-np.pi, np.pi, 100)
```

```
coords = []
for i in range(len(x)):
    coords.append([x[i], y[i]])

def f4(value):
    x, y = value
    return exp(-(x - y)**2)*sin(y)

def f4d(value):
    x, y = value
    return np.array([-2*exp(-(x - y)**2)*sin(y)*(x - y), exp(-(x - y)**2)*cos(y)␣
↪+ 2*exp(-(x - y)**2)*sin(y)*(x - y)])

x_coords = np.outer(np.linspace(-np.pi, np.pi, 100), np.ones(100))
y_coords = x_coords.copy().T
z_coords = exp(-(x_coords - y_coords)**2)*sin(y_coords)

ax = plt.axes(projection ='3d')
ax.plot_surface(x_coords, y_coords, z_coords)

x, y, z = test(f4, f4d, coords, -np.pi, np.pi)
ax.scatter(x, y, z, s=50, c='r', marker='o')
plt.show()
```

```
The coordinates of minima 1 are (-1.6, -1.6, -1.0)
```



2.4 Problem 4 - 1-D trigonometric

Derivative not given. Optimization range [0, 2*pi]

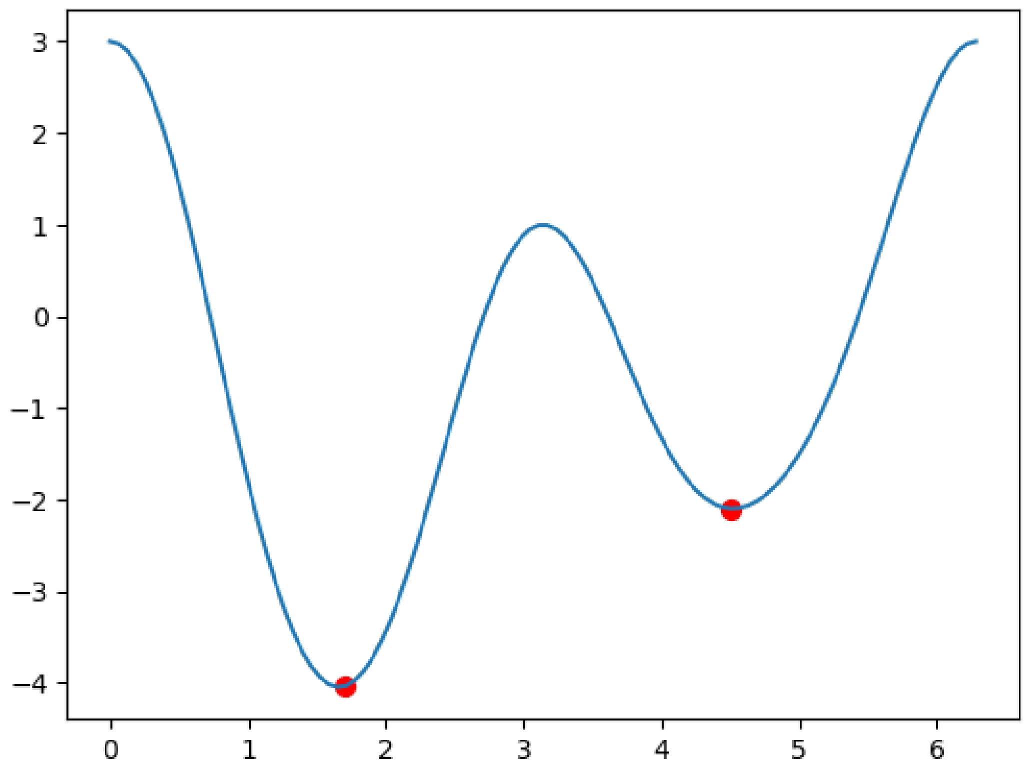
[17]:

```
x = np.linspace(0, 2*(np.pi), 100)

coords = []
for i in range(len(x)):
    coords.append([x[i]])

def f5(value):
    x, = value
    return cos(x)**4 - sin(x)**3 - 4*sin(x)**2 + cos(x) + 1

def f5d(value):
    x, = value
    term1 = -4*(cos(x)**3)*(sin(x))
    term2 = -3*(sin(x)**2)*(cos(x))
    term3 = -8*sin(x)*cos(x)
    term4 = -sin(x)
    term = term1 + term2 + term3 + term4
    return np.array([term])

x4, y4 = get_coords_single_var(f5, coords)
plt.plot(x4, y4)

a, b = test(f5, f5d, coords, 0, 2*(np.pi))
plt.scatter(a, b, s=50, c='red', marker='o')
```

```
The coordinates of minima 1 are (1.7, -4.0)
The coordinates of minima 2 are (4.5, -2.1)
```

[17]:

```
<matplotlib.collections.PathCollection at 0x7fa8b57c8dc0>
```


3
2
1
0
−1
−2
−3
−4
0
1
2
3
4
5
6