

COMP3411 Week 02 Tutorial

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`https://github.com/hharryyf/COMP3411-24T1-tutoring`

Q1

Solve the following Cryptarithmic puzzle. State the heuristics and strategies used. The same character represents the same digit and different characters represent different digits.

$$\begin{array}{r} \text{SEND} \\ +\text{MORE} \\ \hline \text{MONEY} \end{array}$$

Recall CSP variable/value selection heuristics

- Minimum Remaining Values
 - Choose the variable with the fewest legal values
- Degree heuristics (Tie-break MRV)
 - Choose the variable with the most constraints on the remaining variables.
- Least Constraining Value
 - Value selection heuristic: choose the value that rules out the fewest values in the remaining variables

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

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- M = 1 (Minimum Remaining Values)

Q1

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

- M = 1 (Minimum Remaining Values)

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

Q1

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

- M = 1 (Minimum Remaining Values)

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

- O = 0 or 1 (Minimum Remaining Values)

Q1

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

- $M = 1$ (Minimum Remaining Values)

$$\begin{array}{r} \text{SEND} \\ + \text{1ORE} \\ \hline \text{1ONEY} \end{array}$$

- $O = 0$ or 1 (Minimum Remaining Values)
- $O = 0$, since $M \neq O$

$$\begin{array}{r} \text{SEND} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

$$\begin{array}{r} \text{SEND} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- S = 8 or 9 (Minimum Remaining Values)

$$\begin{array}{r} \text{SEND} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- S = 8 or 9 (Minimum Remaining Values)
 - If S = 8

$$\begin{array}{r} 8\text{9ND} \\ +10\text{R9} \\ \hline 10\text{N9Y} \end{array}$$

$$\begin{array}{r} \text{SEND} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- S = 8 or 9 (Minimum Remaining Values)
 - If S = 8

$$\begin{array}{r} 8\textcolor{red}{9}\text{ND} \\ +10\text{R}\textcolor{red}{9} \\ \hline 10\text{N}\textcolor{red}{9}\text{Y} \end{array}$$

- N = 0, conflict!

Q1

$$\begin{array}{r} \text{SEND} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- S = 8 or 9 (Minimum Remaining Values)
 - If S = 8

$$\begin{array}{r} 8\textcolor{red}{9}\text{ND} \\ +10\text{R}\textcolor{red}{9} \\ \hline 10\text{N}\textcolor{red}{9}\text{Y} \end{array}$$

- N = 0, conflict!
- S = 9

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$
 - $N + R = 10 + E$ or $N + R + 1 = 10 + E$

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$
 - $N + R = 10 + E$ or $N + R + 1 = 10 + E$
 - If $N + R = 10 + E$, $R = 9$, conflict!

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$
 - $N + R = 10 + E$ or $N + R + 1 = 10 + E$
 - If $N + R = 10 + E$, $R = 9$, conflict!
 - $N + R + 1 = 10 + E$ is the only possibility, $R = 8$

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$
 - $N + R = 10 + E$ or $N + R + 1 = 10 + E$
 - If $N + R = 10 + E$, $R = 9$, conflict!
 - $N + R + 1 = 10 + E$ is the only possibility, $R = 8$

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$
 - $N + R = 10 + E$ or $N + R + 1 = 10 + E$
 - If $N + R = 10 + E$, $R = 9$, conflict!
 - $N + R + 1 = 10 + E$ is the only possibility, $R = 8$

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$

Q1

$$\begin{array}{r} 9\text{END} \\ +10\text{RE} \\ \hline 10\text{NEY} \end{array}$$

- $E + 1 = N$
 - $N + R = 10 + E$ or $N + R + 1 = 10 + E$
 - If $N + R = 10 + E$, $R = 9$, conflict!
 - $N + R + 1 = 10 + E$ is the only possibility, $R = 8$

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$
- remaining values 2, 3, 5, 6, 7

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y, E + 1 = N$
- remaining values 2, 3, 5, 6, 7

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y, E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y, E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)
 - If $Y = 3, E = 6$ or 7

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)
 - If $Y = 3$, $E = 6$ or 7
 - $E = 6$ means $N = 7$ and $D = 7$, conflict

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)
 - If $Y = 3$, $E = 6$ or 7
 - $E = 6$ means $N = 7$ and $D = 7$, conflict
 - $E = 7$ means $N = 8$, conflict

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)
 - If $Y = 3$, $E = 6$ or 7
 - $E = 6$ means $N = 7$ and $D = 7$, conflict
 - $E = 7$ means $N = 8$, conflict
 - $Y = 2$, and $E = 5$ or 7

Q1

$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)
 - If $Y = 3$, $E = 6$ or 7
 - $E = 6$ means $N = 7$ and $D = 7$, conflict
 - $E = 7$ means $N = 8$, conflict
 - $Y = 2$, and $E = 5$ or 7
 - $E = 5$ and $D = 7$ and $N = 6$ is the only solution

Q1

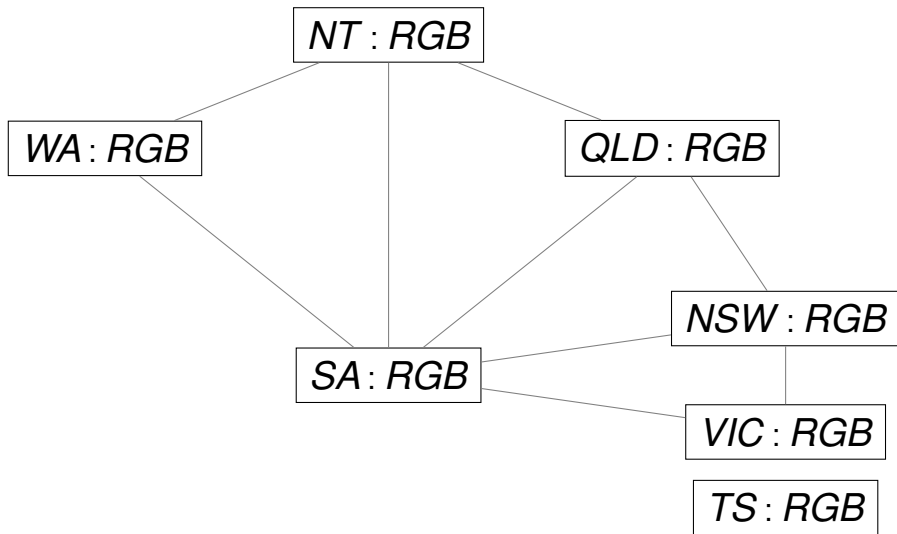
$$\begin{array}{r} 9\text{END} \\ +108\text{E} \\ \hline 10\text{NEY} \end{array}$$

- $D + E = 10 + Y$, $E + 1 = N$
- remaining values 2, 3, 5, 6, 7
- $Y = 2$ or 3 (Minimum Remaining Values)
 - If $Y = 3$, $E = 6$ or 7
 - $E = 6$ means $N = 7$ and $D = 7$, conflict
 - $E = 7$ means $N = 8$, conflict
 - $Y = 2$, and $E = 5$ or 7
 - $E = 5$ and $D = 7$ and $N = 6$ is the only solution

$$\begin{array}{r} 9567 \\ +1085 \\ \hline 10652 \end{array}$$

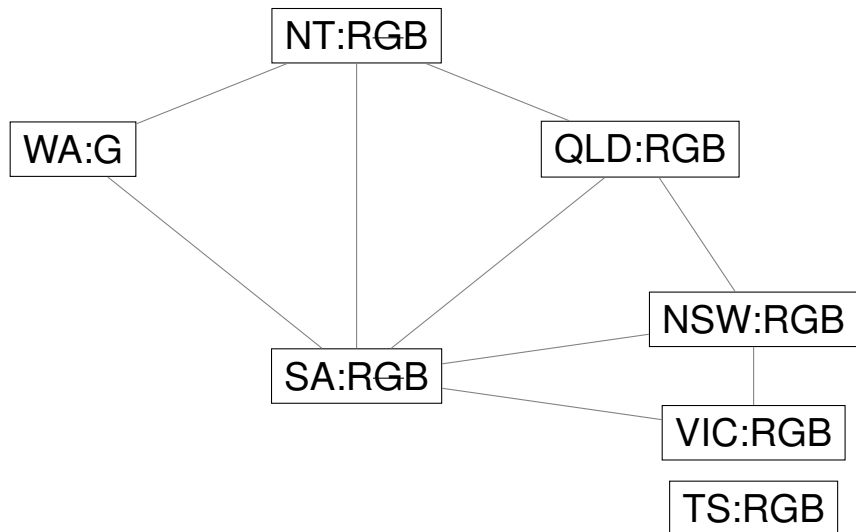
- Forward Checking (FC)
- Maintain Arc consistency (MAC)

Q2



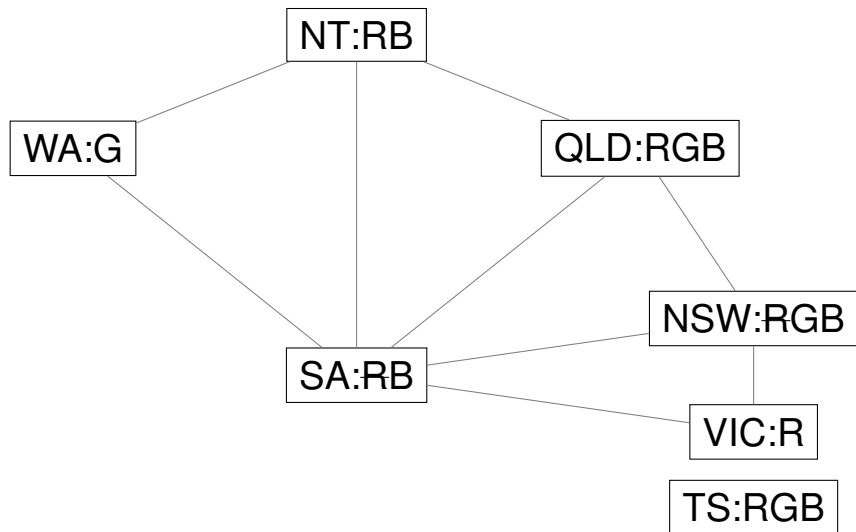
Q2: FC only

- Assign WA: G



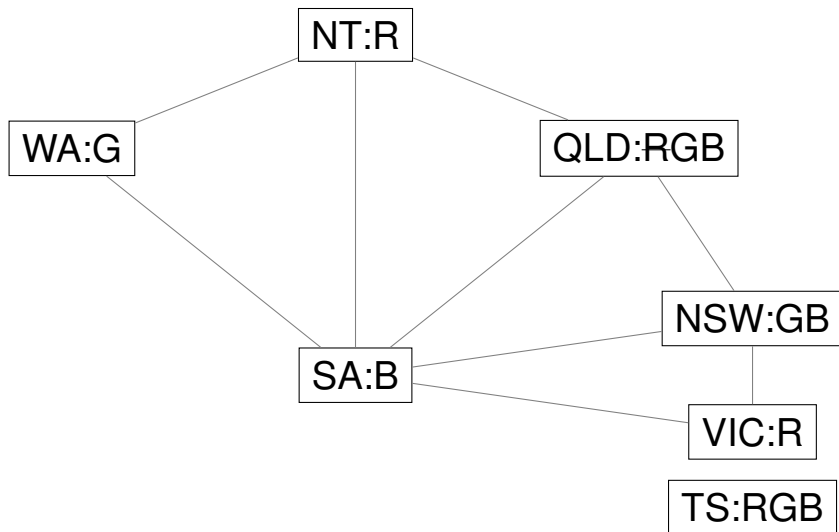
Q2: FC only

- Assign WA: G, VIC: R



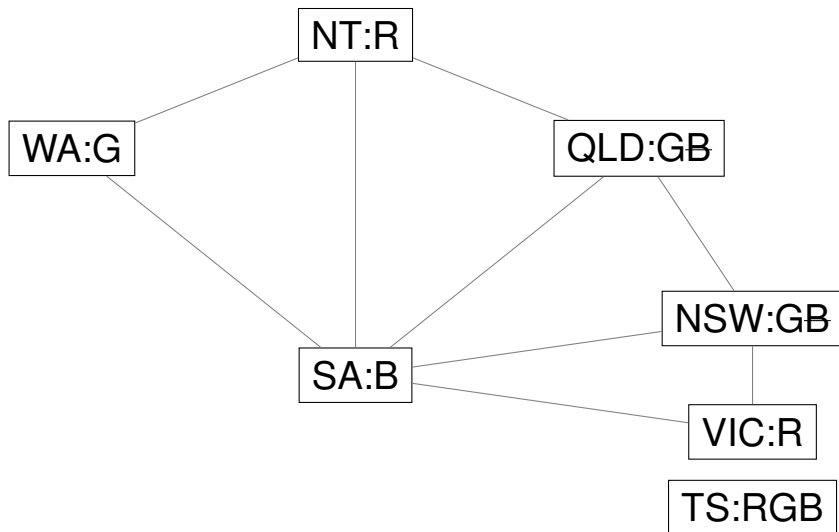
Q2: FC only

- Assign WA: G, VIC: R, NT: R



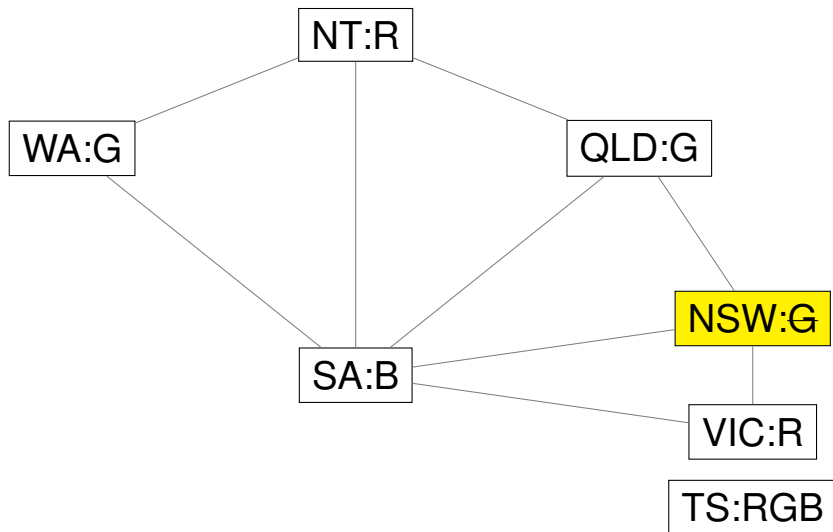
Q2: FC only

- Assign WA: G, VIC: R, NT: R, SA can only be B

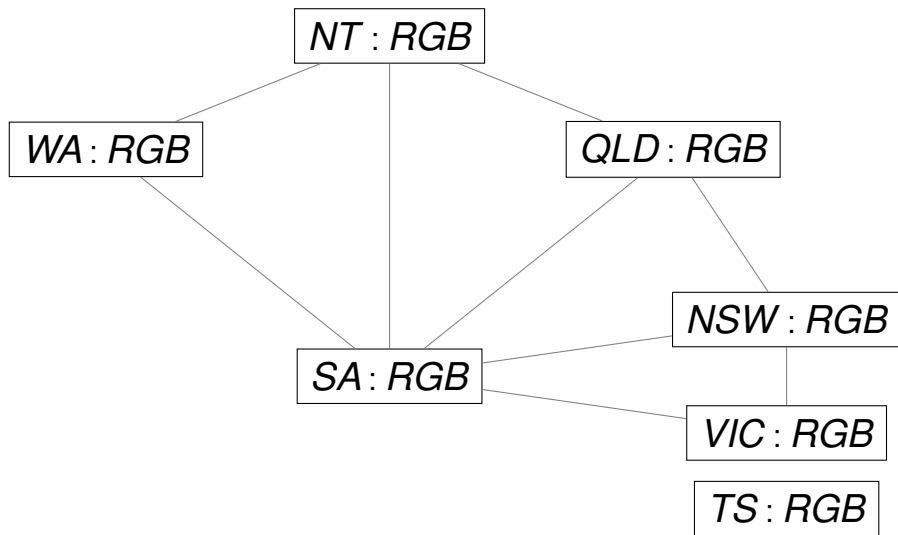


Q2: FC only

- Assign WA: G, VIC: R, NT: R, SA: B, QLD: G

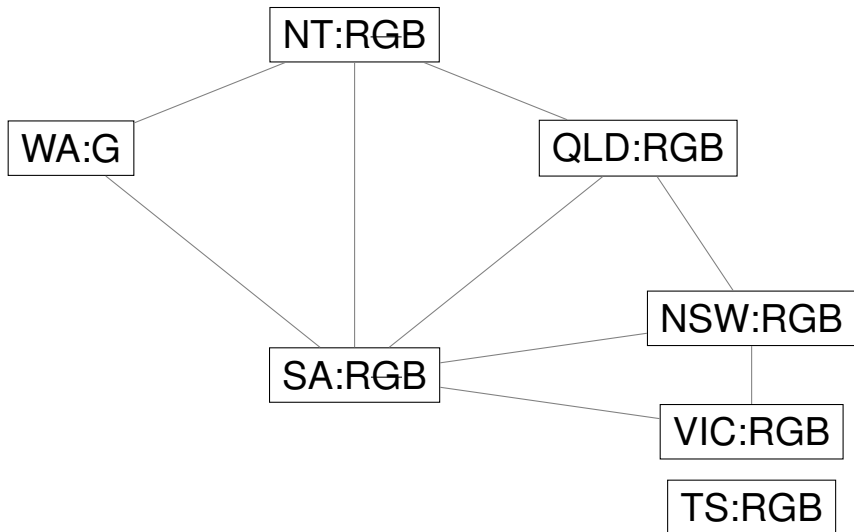


Q2



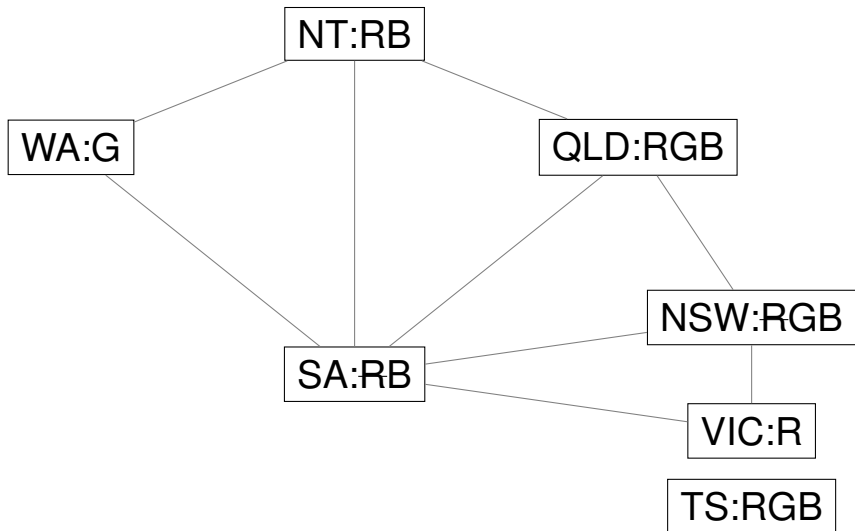
Q2: FC + MAC

- Assign WA: G



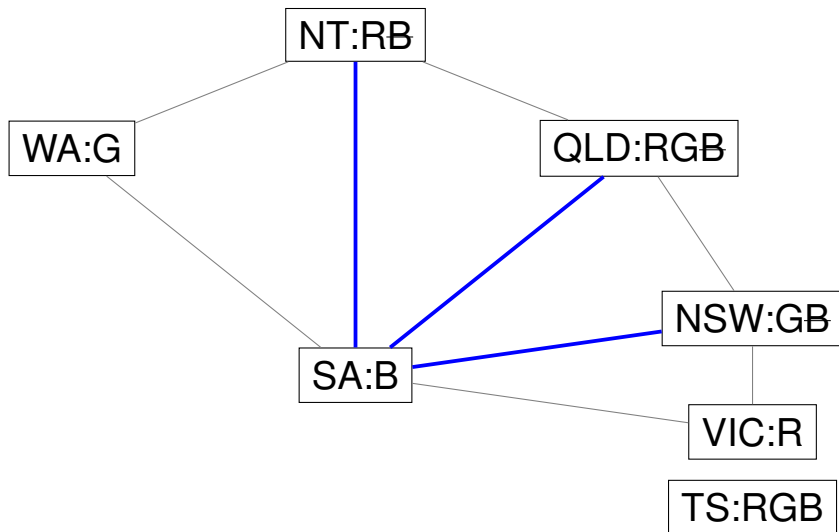
Q2: FC + MAC

- Assign WA: G, VIC: R



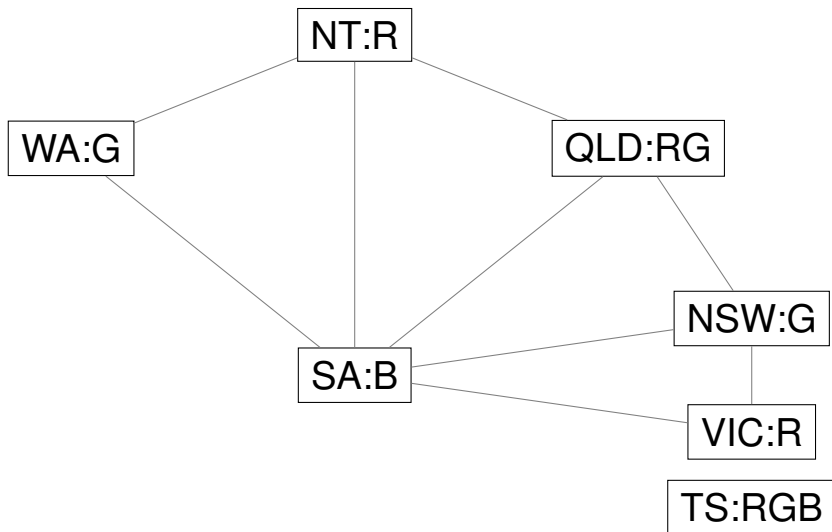
Q2: FC + MAC

- Assign WA: G, VIC: R



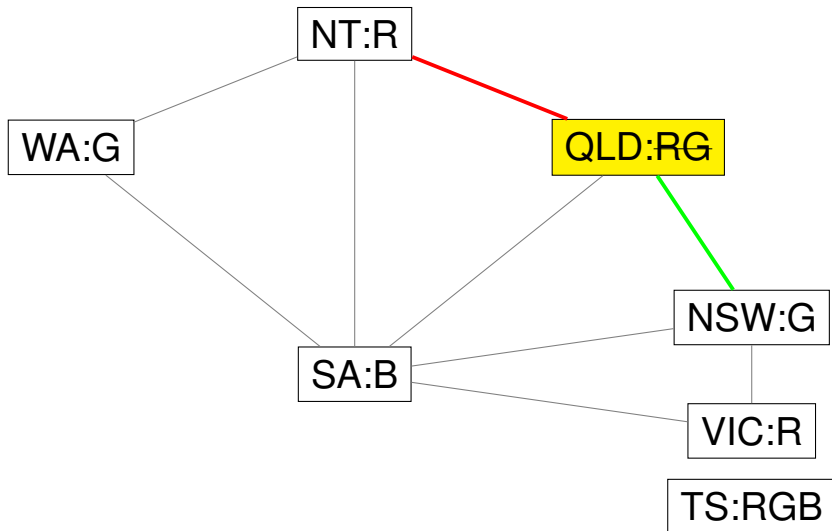
Q2: FC + MAC

- Assign WA: G, VIC: R

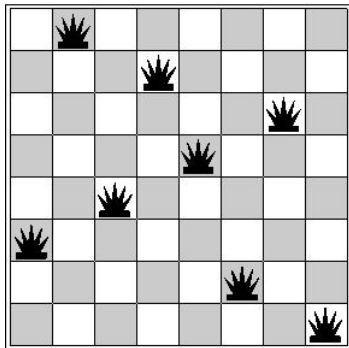


Q2: FC + MAC

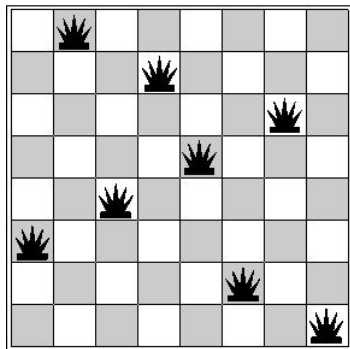
- Assign WA: G, VIC: R



Q3

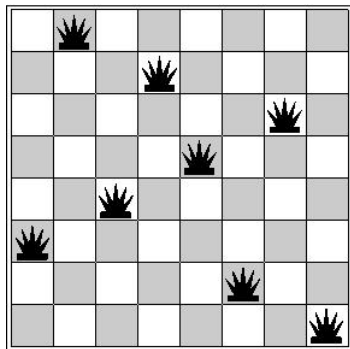


Q3



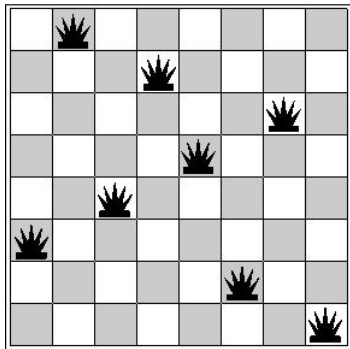
- The queens in columns 2 and 5 are attacking each other ($h = 1$)

Q3



- The queens in columns 2 and 5 are attacking each other ($h = 1$)
- Easy to verify that it is impossible to reduce h by **hill-climbing** alone

Q3



- The queens in columns 2 and 5 are attacking each other ($h = 1$)
- Easy to verify that it is impossible to reduce h by hill-climbing alone
 - We need to move either the queen in column 2 or 5, but that would create at least a "row" conflict.

- Why Simulated Annealing may Succeed eventually

Q3

- Why Simulated Annealing may Succeed eventually
- Recall that SA is a local search algorithm
 - 1 Start from a random \mathbf{x} configuration
 - 2 Draw a random configuration in the neighborhood
 - 3 If the new configuration \mathbf{x}' is better, let $x = x'$
 - 4 Otherwise, draw a random number $p \in [0, 1]$, let $x = x'$ with probability p , return to step 1.
- p is drawn from a probability distribution similar to the Boltzmann distribution in Physics

Q3

- Why Simulated Annealing may Succeed eventually
- Recall that SA is a local search algorithm
 - 1 Start from a random \mathbf{x} configuration
 - 2 Draw a random configuration in the neighborhood
 - 3 If the new configuration \mathbf{x}' is better, let $x = x'$
 - 4 Otherwise, draw a random number $p \in [0, 1]$, let $x = x'$ with probability p , return to step 1.
- p is drawn from a probability distribution similar to the Boltzmann distribution in Physics
- Step 1-3 of the algorithm is just hill-climbing, step 4 might help us to escape the local minimum

- If you want to read more about local search, the paper "General local search methods" is highly recommended ¹
- It contains 3 very popular local search methods:
 - Simulated Annealing
 - Tabu Search
 - Genetic Algorithm

¹<https://www.sciencedirect.com/science/article/pii/S0377221796000070>

- We'll skip the problem, you can solve it in your free time
- If you know Prolog or ASP, it is a good exercise