COMP3411 Week 08 Tutorial

Yifan He

z5173587@unsw.edu.au
https://github.com/hharryyf/COMP3411-24T1-tutoring

Logic revision

Propositional Logic

- $\phi = p \mid \phi \wedge \phi \mid \phi \vee \phi \mid \neg\phi \mid \phi \implies \phi \mid \phi \iff \phi$
- Semantic definition:
  - $\phi = p$ is true iff p is true
  - $\phi = \neg\, \phi'$ is true iff ϕ' is false
  - $\phi = \phi_1 \;\wedge\; \phi_2$ is true iff both ϕ_1 and ϕ_2 are true
  - $\phi = \phi_1 \;\vee\; \phi_2$ is true iff either ϕ_1 or ϕ_2 is true
  - $\phi_1 \implies \phi_2$ can be written as $\neg\phi_1 \vee \phi_2$
  - $\phi_1 \iff \phi_2$ is the same as $(\phi_1 \implies \phi_2) \wedge (\phi_2 \implies \phi_1)$

Logic revision

Conjunctive Normal Form

- A literal is a variable p or its negation $\neg p$
- A clause is a disjunction ($\vee$) of literals
- A propositional formula is CNF iff it is a conjunction ($\wedge$) of clauses

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Satisfiability ($KB \models \alpha$)

- Model: a satisfiability assignment for KB
- Valid: true in all models
- Satisfiable: true in some models
- Unsatisfiable: true in no models

Logic revision

Propositional logic remark

- ϕ is satisfiable iff there is some assignment that satisfies ϕ
- SAT is *NP*-complete

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Resolution

- A method of checking satisfiability
- Sound and complete
  - Convert ϕ to CNF (can be done in linear time)
  - $C_1, C_2 \in \phi$ such that $l \in C_1$ and $\neg l \in C_2$
  - $\phi = \phi \wedge ((C_1 \setminus l) \vee (C_2 \setminus \neg l))$
  - Example: $(x_1 \vee x_2) \wedge (x_3 \vee \neg x_2)$
    - $(x_1 \vee x_2) \wedge (x_3 \vee \neg x_2) \wedge (x_1 \vee x_3)$
  - ϕ is unsatisfiable iff empty clause is derivable

Q3

- Valid, satisfiable, or neither using truth tables or inference rules. For those that are satisfiable, list all the models that satisfy them.
- $(Smoke \implies Fire) \implies ((Smoke \wedge Heat) \implies Fire)$

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- $(Smoke \implies Fire) \implies ((Smoke \land Heat) \implies Fire)$
  - $(\neg Smoke \lor Fire) \implies ((Smoke \land Heat) \implies Fire)$

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  - $(Smoke \wedge \neg Fire) \vee ((Smoke \wedge Heat) \implies Fire)$
  - $(Smoke \wedge \neg Fire) \vee ((\neg(Smoke \wedge Heat)) \vee Fire)$

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    - $(Smoke \wedge \neg Fire) \vee (\neg Smoke \vee \neg Heat \vee Fire)$
    - $(Smoke \vee \neg Smoke \vee \neg Heat \vee Fire) \wedge (\neg Fire \vee \neg Smoke \vee \neg Heat \vee Fire)$
    - Valid

Q3

- $(Big \wedge Dumb) \vee \neg Dumb$

Q3

- $(Big \wedge Dumb) \vee \neg Dumb$
    - $(Big \vee \neg Dumb) \wedge (Dumb \vee \neg Dumb)$

Q3

- $(Big \land Dumb) \lor \neg Dumb$
  - $(Big \lor \neg Dumb) \land (Dumb \lor \neg Dumb)$
  - $Big \lor \neg Dumb$
  - Satisfiable

Q3

- $(Big \wedge Dumb) \vee \neg Dumb$
  - $(Big \vee \neg Dumb) \wedge (Dumb \vee \neg Dumb)$
  - $Big \vee \neg Dumb$
  - Satisfiable
  - $\{Big\}$ $\{Big, Dumb\}$ $\{\}$

Q4

If the unicorn in mythical, then it is immortal, but if it is not mythical, then it is mortal and a mammal. If the unicorn in either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

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Variables: *mythical*, *mortal*, *mamal*, *magical*, *horned*

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Q4: Convert to CNF

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   - $(mythical \vee mortal) \wedge (mythical \vee mammal)$

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   - $mythical \vee (mortal \wedge mammal)$
   - $(mythical \vee mortal) \wedge (mythical \vee mammal)$
3. $(\neg mortal \vee mammal) \implies horned$

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   - $mythical \vee (mortal \wedge mammal)$
   - $(mythical \vee mortal) \wedge (mythical \vee mammal)$
3. $(\neg mortal \vee mammal) \implies horned$
   - $(\neg(\neg mortal \vee mammal)) \vee horned$
   - $(mortal \wedge \neg mammal) \vee horned$
   - $(mortal \vee horned) \wedge (\neg mammal \vee horned)$

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   - $(mythical \vee mortal) \wedge (mythical \vee mammal)$
3. $(\neg mortal \vee mammal) \implies horned$
   - $(\neg(\neg mortal \vee mammal)) \vee horned$
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4. $horned \implies magical$

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   - $(mythical \vee mortal) \wedge (mythical \vee mammal)$
3. $(\neg mortal \vee mammal) \implies horned$
   - $(\neg(\neg mortal \vee mammal)) \vee horned$
   - $(mortal \wedge \neg mammal) \vee horned$
   - $(mortal \vee horned) \wedge (\neg mammal \vee horned)$
4. $horned \implies magical$
   - $\neg horned \vee magical$

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   - $(\neg(\neg mortal \vee mammal)) \vee horned$
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   - $(mortal \vee horned) \wedge (\neg mammal \vee horned)$
4. $horned \implies magical$
   - $\neg horned \vee magical$
5. $(\neg mythical \vee \neg mortal) \wedge (mythical \vee mortal) \wedge (mythical \vee mammal) \wedge (mortal \vee horned) \wedge (\neg mammal \vee horned) \wedge (\neg horned \vee magical)$

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- $(\neg mythical \vee \neg mortal) \wedge (mythical \vee mortal) \wedge (mythical \vee mammal) \wedge (mortal \vee horned) \wedge (\neg mammal \vee horned) \wedge (\neg horned \vee magical) \wedge \neg horned$ is UNSAT

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- $(\neg mythical \vee \neg mortal) \wedge (mythical \vee mortal) \wedge (mythical \vee mammal) \wedge (mortal \vee horned) \wedge (\neg mammal \vee horned) \wedge (\neg horned \vee magical) \;\wedge$ $\neg horned \wedge \neg mammal$

Q4

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- $(\neg mythical \vee \neg mortal) \wedge (mythical \vee mortal) \wedge (mythical \vee mammal) \wedge (mortal \vee horned) \wedge (\neg mammal \vee horned) \wedge (\neg horned \vee magical) \wedge \neg horned \wedge \neg mammal \wedge mythical$

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Q4

- $(\neg mythical \vee \neg mortal) \wedge (mythical \vee mortal) \wedge (mythical \vee mammal) \wedge (mortal \vee horned) \wedge (\neg mammal \vee horned) \wedge (\neg horned \vee magical) \wedge \neg horned \wedge \neg mammal \wedge mythical \wedge mortal \wedge \neg mythical$

Q4

- $(\neg mythical \lor \neg mortal) \land (mythical \lor mortal) \land (mythical \lor mammal) \land (mortal \lor horned) \land (\neg mammal \lor horned) \land (\neg horned \lor magical) \land \neg horned \land \neg mammal \land mythical \land mortal \land \neg mythical$
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Q4

Encoding (we know *horned* is true)

1. $mythical \implies \neg mortal$
2. $\neg mythical \implies (mortal \wedge mammal)$
3. $(\neg mortal \vee mammal) \implies horned$
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- Write out all models, is magical/mythical always true?

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  - By rule (4), we know *magical* must be true

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- Write out all models, is magical/mythical always true?
  - By rule (4), we know *magical* must be true
  - We only need to consider the truth value of *mythical*, *mortal*, and *mammal*
  - We just use a truth table

Q4

Encoding (we know *horned* and *magical* are true)

1. $mythical \implies \neg mortal$
2. $\neg mythical \implies (mortal \wedge mammal)$

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Encoding (we know *horned* and *magical* are true)

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mythical	mortal	mammal	(1)	(2)
T	T	T	**F**	-
T	T	F	**F**	-
T	F	T	**T**	**T**
T	F	F	**T**	**T**
F	T	T	**T**	**T**
F	T	F	**T**	**F**
F	F	T	**T**	**F**
F	F	F	**T**	**F**

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F	T	T	**T**	**T**
F	T	F	**T**	**F**
F	F	T	**T**	**F**
F	F	F	**T**	**F**

- $\{horned, magic, myth, mammal\}$
- $\{horned, magic, myth\}$
- $\{horned, magic, mortal, mammal\}$ no *mythical*!

Hint on assignment 3

9-board tic-tac-toe

- You are playing tic-tac-toe on a 9x9 “sudoku” board, it has 3x3 subboard of size 3x3.
- The first step is made by the *x* player randomly. Then, two players take turns to put pieces on the board.
- If a player plays at the *i*-th cell on the *j*-th subboard. The next player must make a move on the *i*-th subboard.
- Game ends if one player has a 3 in a row/column/diagonal.
- Game ends in a draw if someone cannot make a move.

Hint on assignment 3

Algorithms

- Two player zero-sum turn-taking game
- Monte Carlo Tree Search (MCTS) + UCT
  - Works very well in games that a heuristic evaluation function is hard to come up with
  - I recommend you try this approach if you know MCTS
- Alpha-beta pruning with heuristic function

Hint on assignment 3

Important components of alpha-beta search

- Efficient data structure for the board
    - Update a move, undo a move
    - Knows the termination of a game
    - Knows who wins
    - Evaluate how “good” a position is
    - Sometimes this is critical
    - You can increase the searching depth with good DS
- Gradually increase the searching depth as the game progress
- A well designed heuristic function for middle game positions
- Health warning with Python! Too slow!

Alpha-beta pruning algorithm Review

- $alphabeta(state, \alpha = -\infty, \beta = \infty, depth)$
  1. If *state* is terminal or $depth = 0$, return: heuristic score of the *max* player
  2. If it is the turn of *max*
     - For each *next* state of *state*
     - $\alpha = max(\alpha, alphabeta(next, \alpha, \beta, depth - 1))$
     - If $\alpha \geq \beta$ return α
  3. return α
  4. If it is the turn of *min*
     - For each *next* state of *state*
     - $\beta = min(\beta, alphabeta(next, \alpha, \beta, depth - 1))$
     - If $\alpha \geq \beta$ return β
  5. return β

A heuristic function to start with

- $Score = \sum_{i=1}^{9} \; eval(subboard(i))$
  - $eval(subboard(i)) = 1000000$ if x has 3 in a line
  - $eval(subboard(i)) = -1000000$ if o has 3 in a line
  - $eval(subboard(i)) = 0$ if the subboard is full
  - $eval(subboard(i)) + = 1000$ if x has one 2 in a line
  - $eval(subboard(i)) - = 1000$ if o has one 2 in a line
  - $eval(subboard(i)) + = 100$ if x has one 1 in a line
  - $eval(subboard(i)) - = 100$ if o has one 1 in a line

		x
	o	
x	o	

  - $Score = -1000 + 3 * 100 - 2 * 100 = -900$