COMP3411 Week 09 Tutorial

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https://github.com/hharryyf/COMP3411-24T1-tutoring

First order logic

- Term: constants, variables, functions applied to terms
- Atomic Formulas: predicates applied to terms
- Logical operators: $\wedge, \vee, \neg, \Longrightarrow, \Longleftrightarrow$
- Quantifiers: $\exists, \forall$
- Example:
  - a constant, x variable, $likes(x, y)$ means x likes y
  - $\exists a \forall x.\ \ likes(x, a)$
  - $\forall x \exists a.\ \ likes(x, a)$
- First order logic is undecidable

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  - Everyone is liked by someone.
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- Example:
  - a constant, x variable, $likes(x, y)$ means x likes y
  - $\exists a \forall x.\ \ likes(x, a)$
  - Everyone is liked by someone.
  - $\forall x \exists a.\ \ likes(x, a)$
  - There is someone liked by everyone.
- First order logic is undecidable

Q1

- $student(x)$: x is a student
- $study(x, c, y)$: x studies the course c in year y.
- $score(x, c, y)$: the score of student x in course c in year y.

1. Some students studied French in 2023.

Q1

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1. Some students studied French in 2023.
   - $\exists x\ student(x) \wedge study(x, French, 2023)$.

Q1

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   - $\exists x\ student(x) \wedge study(x, French, 2023)$.
2. Only one student studied Greek in 2021.
   - At least one student studied Greek in 2021

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   - At least one student studied Greek in 2021
   - $\exists x\ study(x, Greek, 2021)$.

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   - $\exists x\ student(x) \wedge study(x, French, 2023)$.
2. Only one student studied Greek in 2021.
   - At least one student studied Greek in 2021
   - $\exists x\ study(x, Greek, 2021)$.
   - Only one student studied Greek in 2021
   - $\exists x\ study(x, Greek, 2021) \wedge (\forall y study(y, Greek, 2021) \implies x = y)$

Q1

- $student(x)$: x is a student
- $study(x, c, y)$: x studies the course c in year y.
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1. Some students studied French in 2023.
   - $\exists x\ student(x) \wedge study(x, French, 2023)$.
2. Only one student studied Greek in 2021.
   - $\exists x\ study(x, Greek, 2021) \wedge (\forall y study(y, Greek, 2021) \implies x = y)$
3. The highest score in Greek is always higher than the highest score in French.

Q1

- $student(x)$: x is a student
- $study(x, c, y)$: x studies the course c in year y.
- $score(x, c, y)$: the score of student x in course c in year y.

1. Some students studied French in 2023.
   - $\exists x\ student(x) \wedge study(x, French, 2023)$.
2. Only one student studied Greek in 2021.
   - $\exists x\ study(x, Greek, 2021) \wedge$ $(\forall y study(y, Greek, 2021) \implies x = y)$
3. The highest score in Greek is always higher than the highest score in French.
   - $\forall\ y\ \exists x\ \forall\ x'\ score(x, Greek, y) > score(x', French, y)$

Q1

- $person(x)$: x is a person
- $smart(x)$: x is smart
- $policy(p)$: p is a policy
- $buy(x, p)$: person x buys policy p
- $expensive(p)$: policy p is expensive

1. Every person who buys a policy is smart.

Q1

- $person(x)$: x is a person
- $smart(x)$: x is smart
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- $buy(x,p)$: person x buys policy p
- $expensive(p)$: policy p is expensive

1. Every person who buys a policy is smart.
   - $\forall x.\ p.\ (person(x) \wedge policy(p) \wedge buy(x,p)) \implies smart(x)$
2. No person buys an expensive policy.
   - $\forall x.\ p.\ (person(x) \wedge policy(p) \wedge buy(x,p)) \implies \neg expensive(p)$

Q1

- $barber(x)$: x is a barber
- $man(x)$: x is a man
- $shave(x, y)$: x shaves y

1. There is a barber who shaves all men in town who do not shave themselves.
   - $\exists x.barber(x) \wedge (\forall y man(y) \wedge \neg shave(y, y) \implies shave(x, y))$

Q1

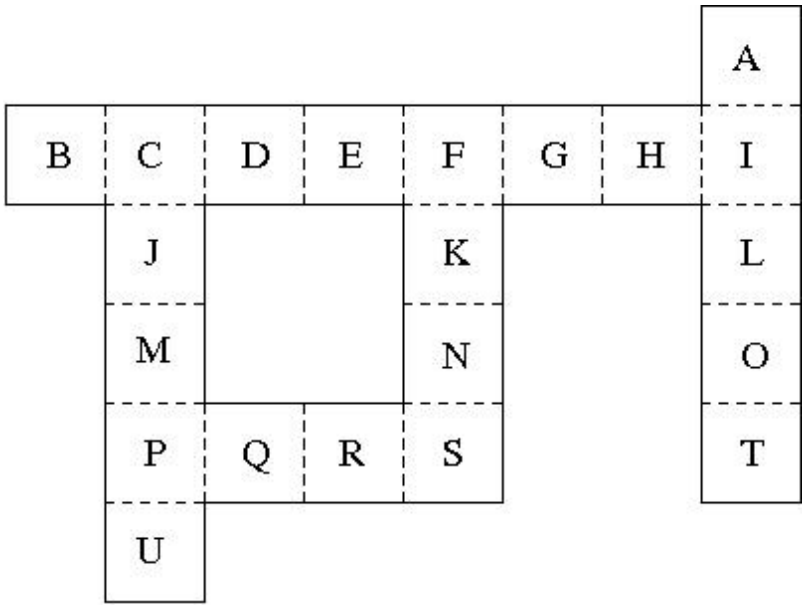
- $politician(p)$: p is a politician
- $fool(p, x, t)$: p fools the person x at time t

1. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.
   - $\forall p.(politician(p) \implies ((\exists x \forall t.fool(p, x, t)) \wedge (\forall x \exists t.fool(p, x, t)) \wedge (\exists x\ t.\neg fool(p, x, t))))$

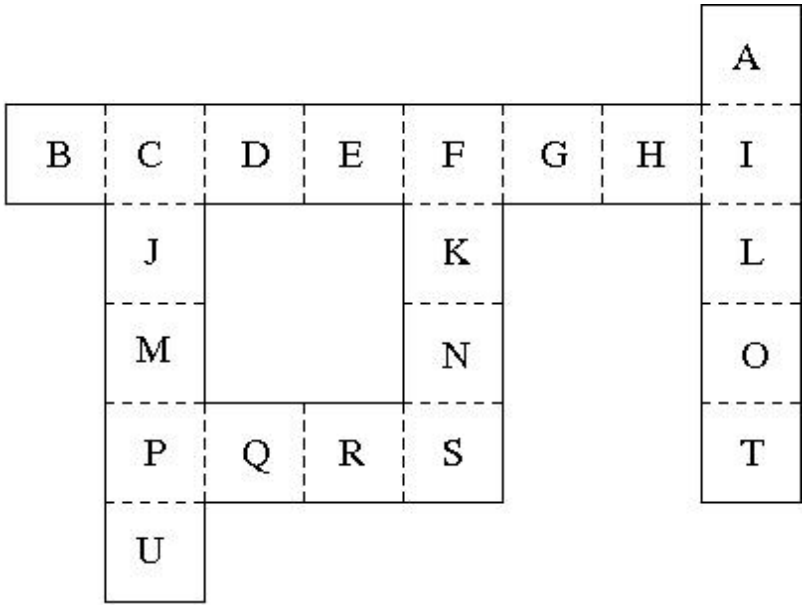
Q2

Robot navigation

- A robot with imperfect information
- Only knows if the left/right/front/back is a wall
- Given a walking trajectory, can the robot eventually know its location

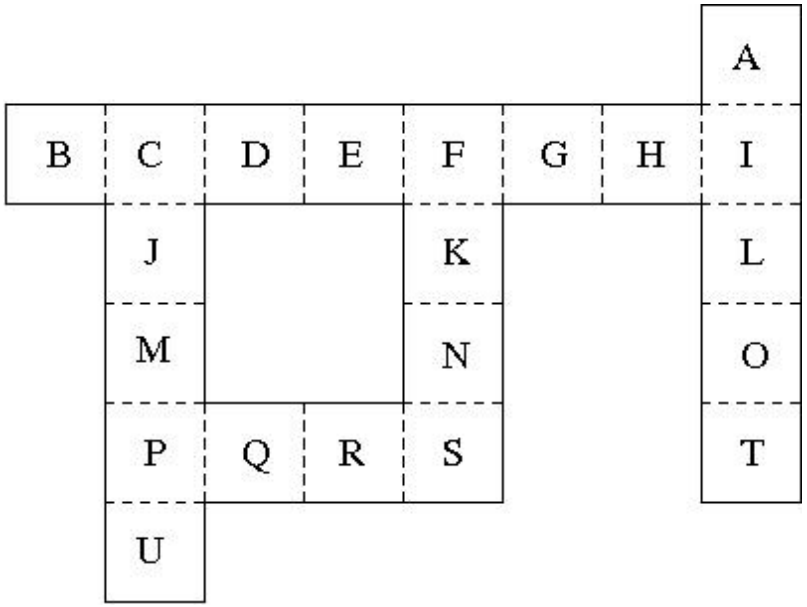


Q2



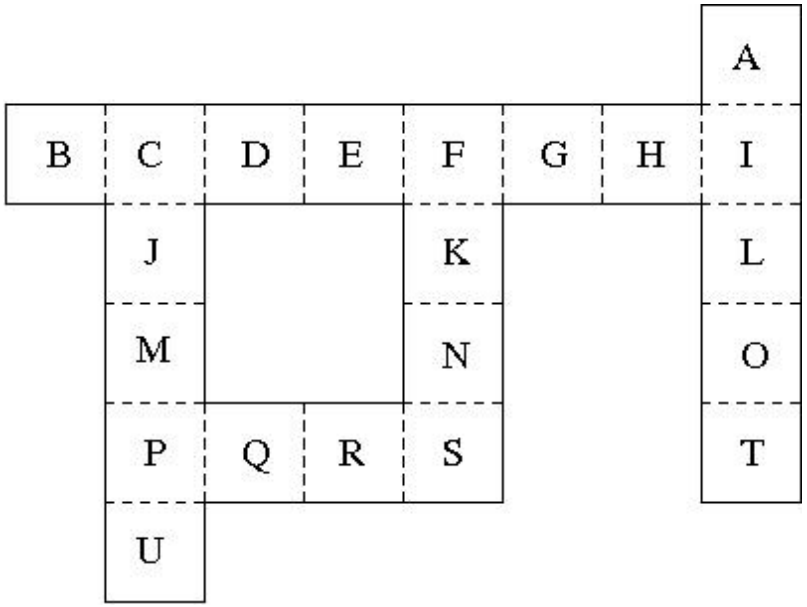
- Initially the robot is at *N* moving upward
- Which locations and orientations are indistinguishable for the robot?

Q2



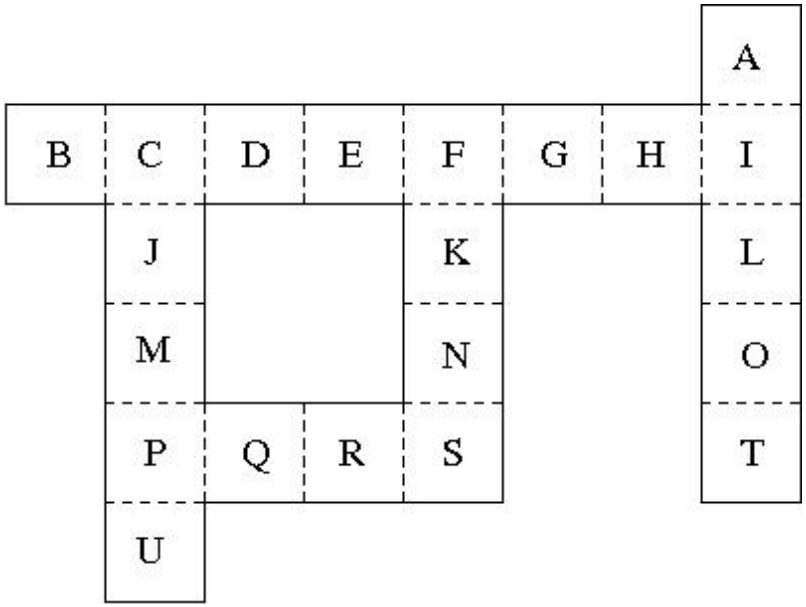
- Initially the robot is at *N* moving upward
- Which locations and orientations are indistinguishable for the robot?
  - Left/right are walls, front back are not walls

Q2



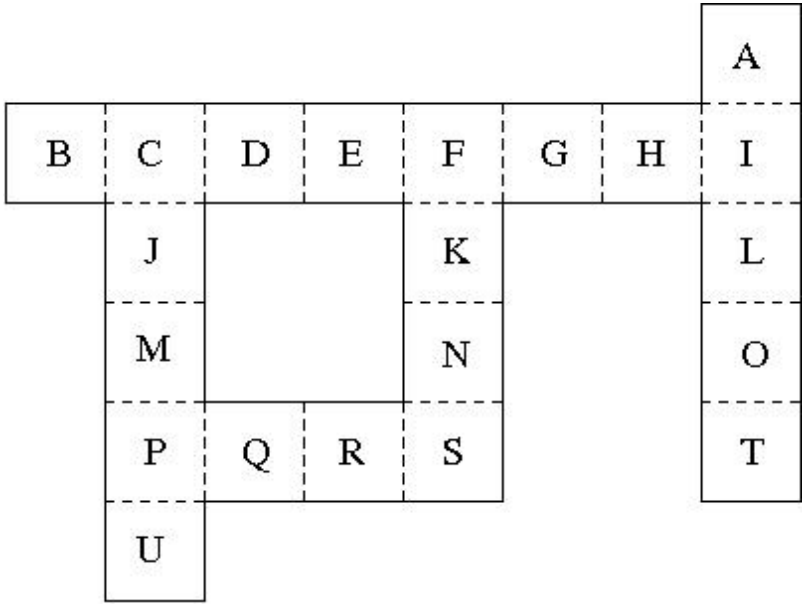
- Initially the robot is at *N* moving upward
- Which locations and orientations are indistinguishable for the robot?
  - Left/right are walls, front back are not walls
  - $J^{ud}, M^{ud}, D^{lr}, E^{lr}, K^{ud}, N^{ud}, Q^{lr}, R^{lr}, G^{lr}, H^{lr}, L^{ud}, O^{ud}$

Q2



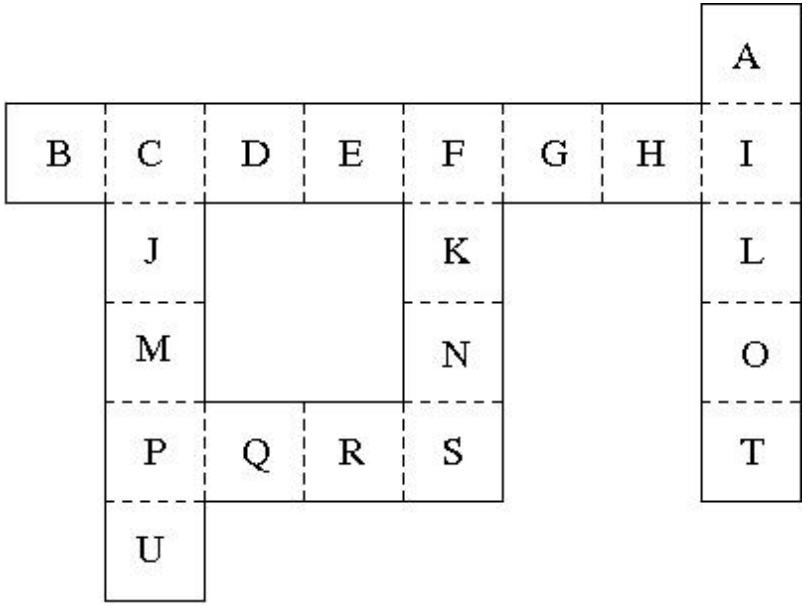
- Initially the robot is at *N* moving upward
- Which locations and orientations are indistinguishable for the robot?
  - Left/right are walls, front back are not walls
  - $J^{ud}, M^{ud}, D^{lr}, E^{lr}, K^{ud}, N^{ud}, Q^{lr}, R^{lr}, G^{lr}, H^{lr}, L^{ud}, O^{ud}$
  - In total 24 possibilities

Q2



- If the robot moves in the order N, K, F. Which locations and orientations are indistinguishable for the robot at each step?

Q2



- If the robot moves in the order N, K, F. Which locations and orientations are indistinguishable for the robot at each step?
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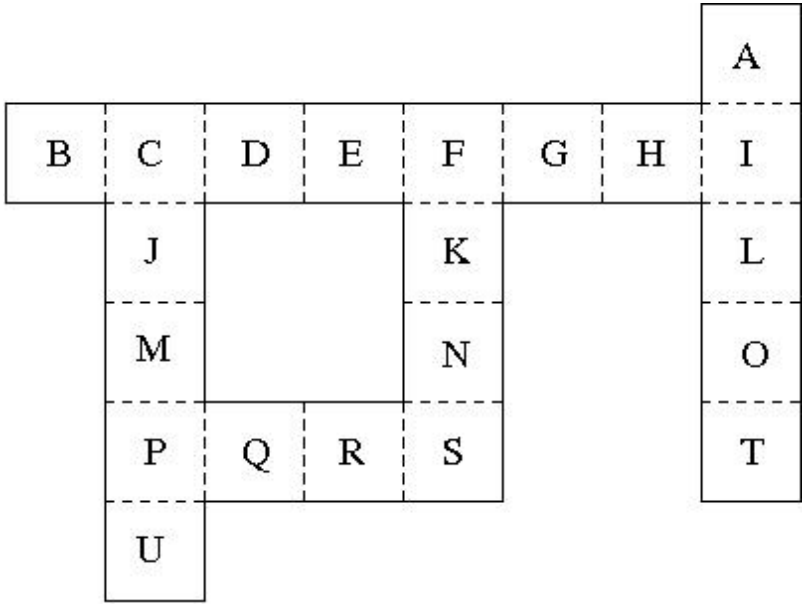
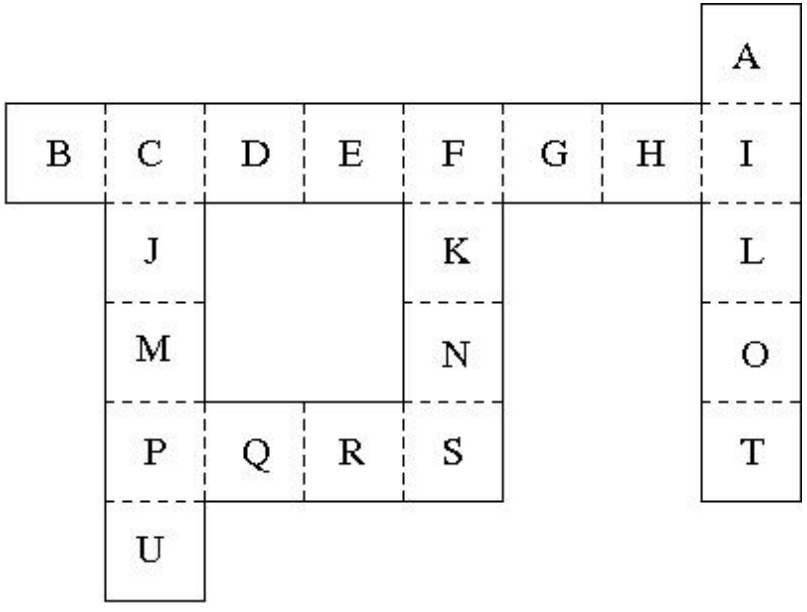


- If the robot moves in the order N, K, F. Which locations and orientations are indistinguishable for the robot at each step?
  - N Left/right are walls, front back are not walls
  - K Left/right are walls, front back are not walls
  - $M \rightarrow J$, $J \rightarrow M$, $D \rightarrow E$, $E \rightarrow D$, $K \rightarrow N$, $N \rightarrow K$
  - $G \rightarrow H$, $H \rightarrow G$, $L \rightarrow O$, $O \rightarrow L$, $R \rightarrow Q$, $Q \rightarrow R$
  - 12 possibilities

Q2



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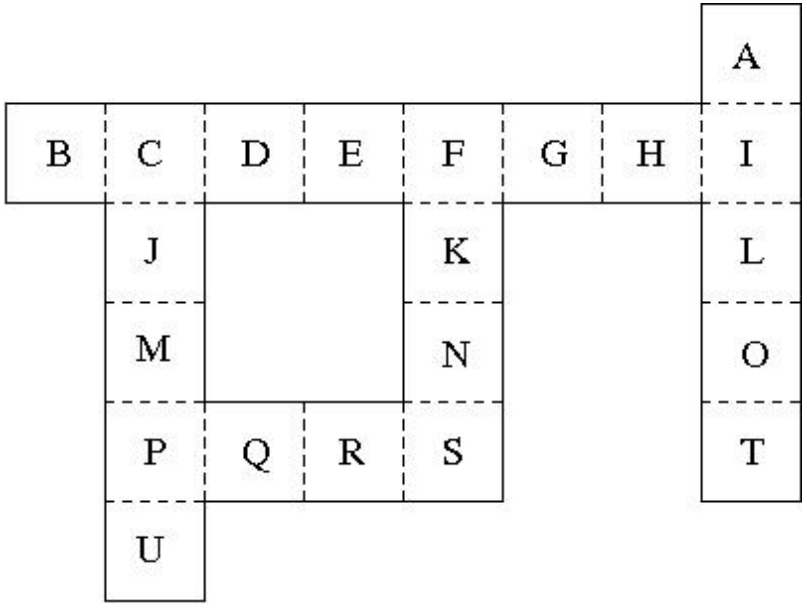

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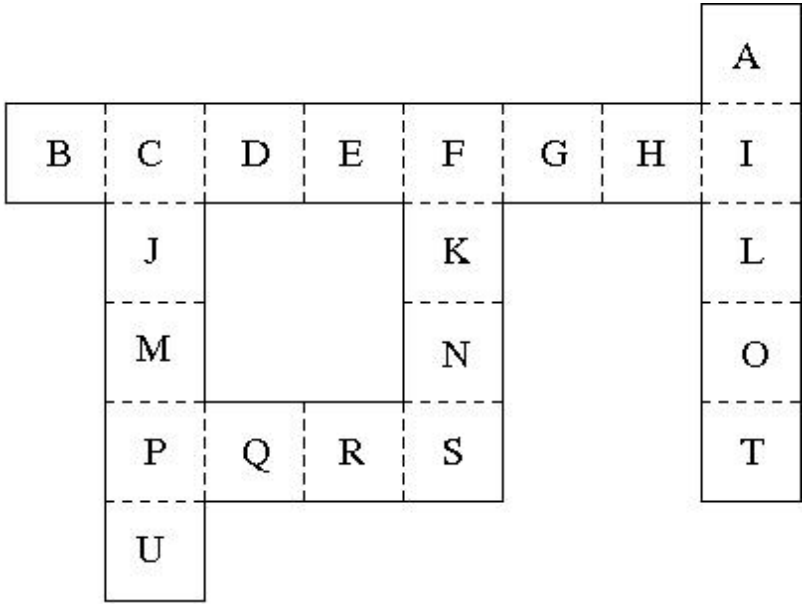
  - F left/right/back are not walls, front is a wall

Q2



- If the robot moves in the order N, K, F. Which locations and orientations are indistinguishable for the robot at each step?
  - N Left/right are walls, front back are not walls
  - K Left/right are walls, front back are not walls
  - F left/right/back are not walls, front is a wall
  - $M \rightarrow J \rightarrow C$, $N \rightarrow K \rightarrow F$, $G \rightarrow H \rightarrow I$, $R \rightarrow Q \rightarrow P$
  - 4 possibilities