COMP4128 Week 03 Tutorial

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https://github.com/hharryyf/COMP4128-23T3-tutoring

Outline

- Ghost Encounters
- Restructuring Company
- Hints
- Additional Problem: Developing Game

Ghost Encounters

There are N ($N \leq 100,000$) ghosts, each is going to appear at position X_i at time T_i seconds. A person starts moving at time S seconds and position 0. For each unit distance, the person needs to use K seconds. (Note that S can be negative). By picking the optimal S, what is the maximum number of ghosts the person can encounter?

Ghost Encounters

Analysis

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- $S_i = K \cdot X_i - T_i$

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- To maximize the total number of ghosts encountered, we need to pick the most frequent number among $K \cdot X_i - T_i$ $(1 \leq i \leq N)$.

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- To maximize the total number of ghosts encountered, we need to pick the most frequent number among $K \cdot X_i - T_i$ $(1 \leq i \leq N)$.
- Time complexity $O(N \cdot log(N))$ with a map.

Restructuring Company

There are N teams ($1 \leq N \leq 2e5$). Design a data structure that supports the following 3 types of queries ($1 \leq Q \leq 5e5$).

- Merge team X and team Y.
- Merge team in a range $[X, Y]$.
- Query if team X and team Y are merged together.

Restructuring Company

Naive approach

- Union-find.
- Type-1. Merge X and Y.
- Type-2. Merge X with $X+1$, $X+1$ with $X+2$, ..., $Y-1$ with Y.
- Type-3. Check if X and Y are in the same CC.

Restructuring Company

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- Type-3. Check if X and Y are in the same CC.
- Type-1 and 3 are $O(1)$ per query, type-2 is $O(N)$ per query.
- Time complexity: $O(N \cdot Q)$. Too slow!

Restructuring Company

Analysis

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- Range tree!

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Solution

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- The parent represents the range $[l, r]$, the left-child contains the range $[l, \frac{l+r}{2}]$, the right-child contains the range $[\frac{l+r}{2} + 1, r]$.

Restructuring Company

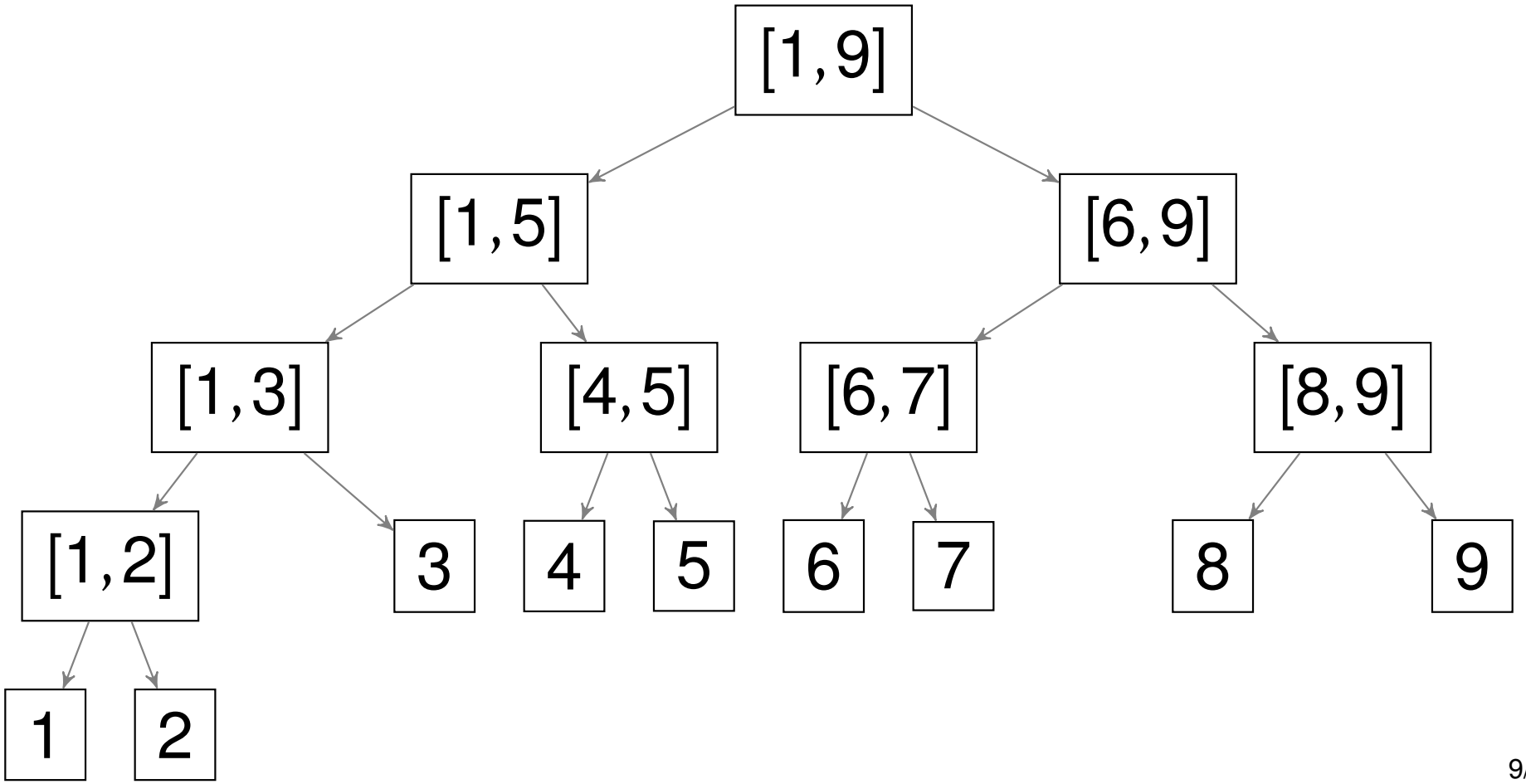
Solution

- Recall that in a range tree, the root represents the range $[1, N]$.
- The parent represents the range $[l, r]$, the left-child contains the range $[l, \frac{l+r}{2}]$, the right-child contains the range $[\frac{l+r}{2} + 1, r]$.
- Merge $[X, Y]$ means merging X with all the "top-level" nodes representing the range $[X + 1, Y]$.

Restructuring Company

Example

Merge range [5, 7] <==> Merge 5 with range [6, 7].



Restructuring Company

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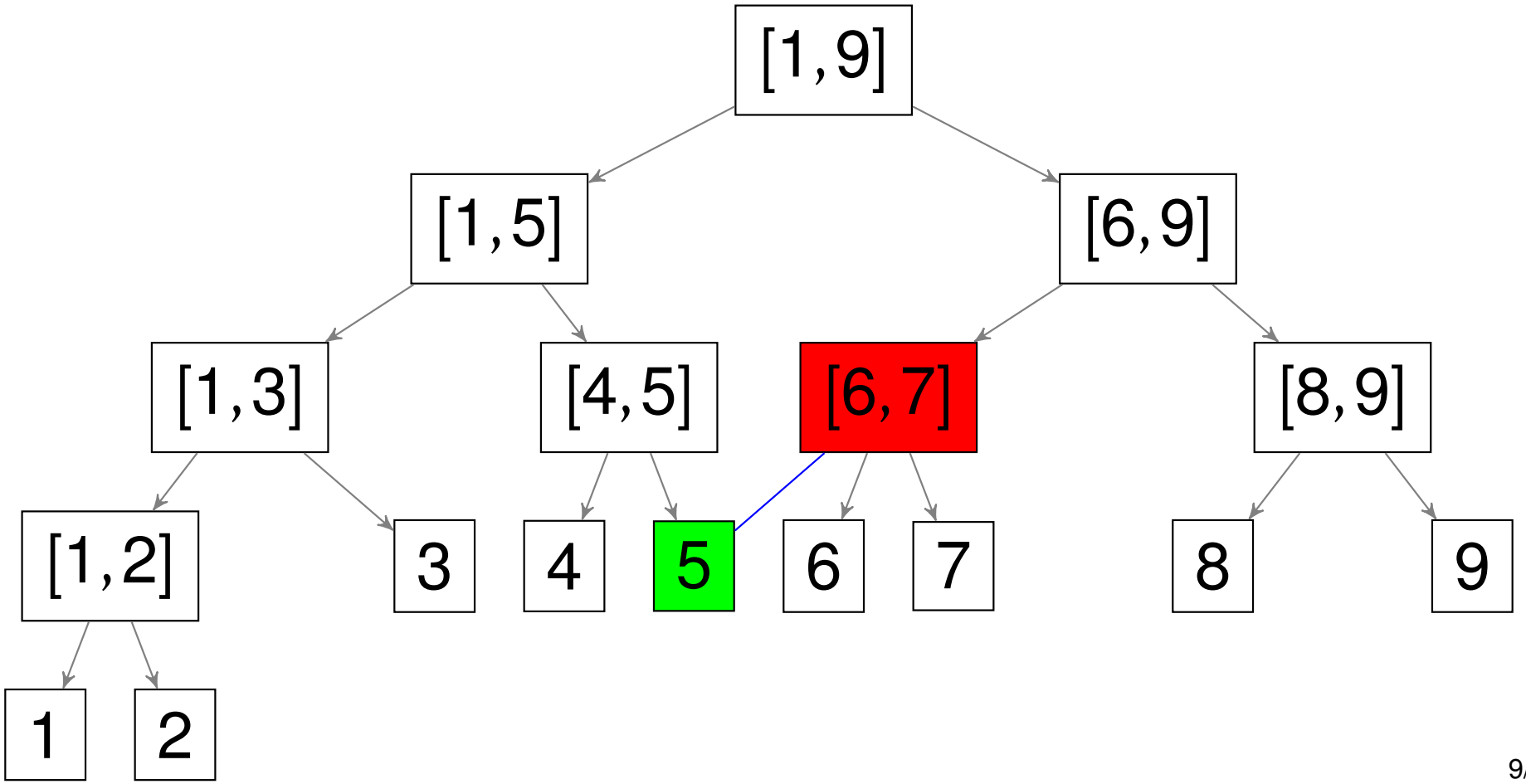
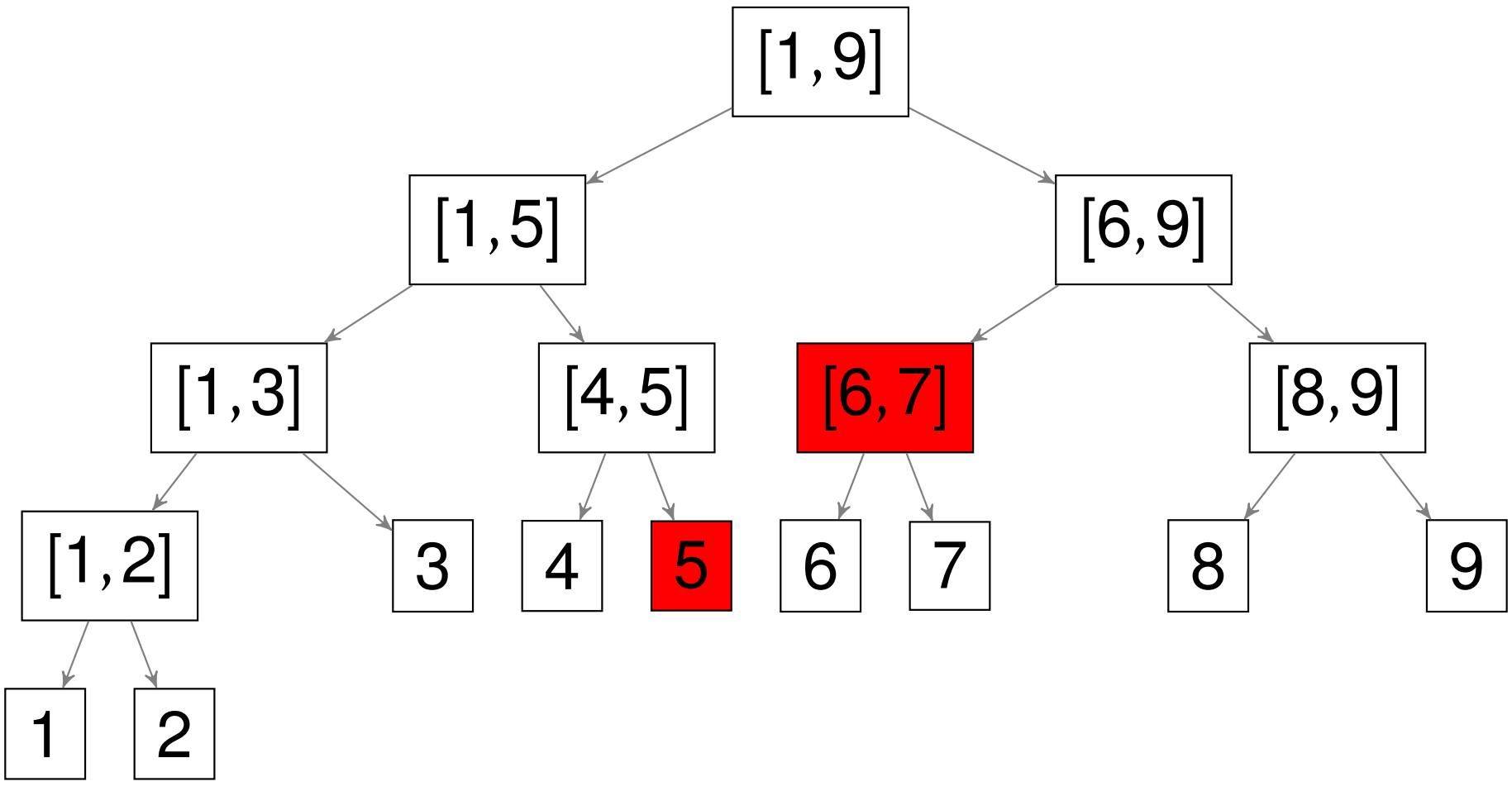


Restructuring Company

Example

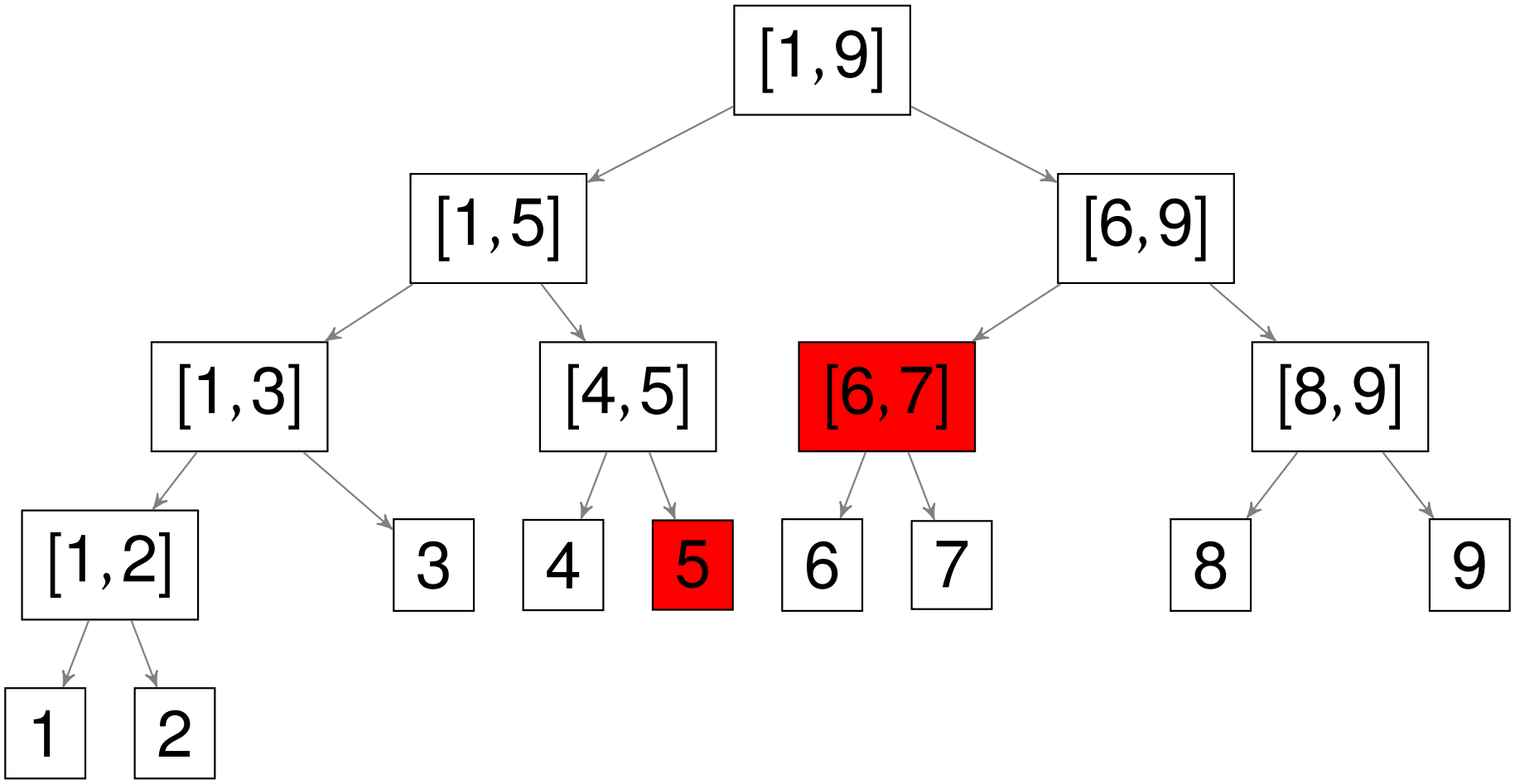
Is that all? No!



Restructuring Company

Example

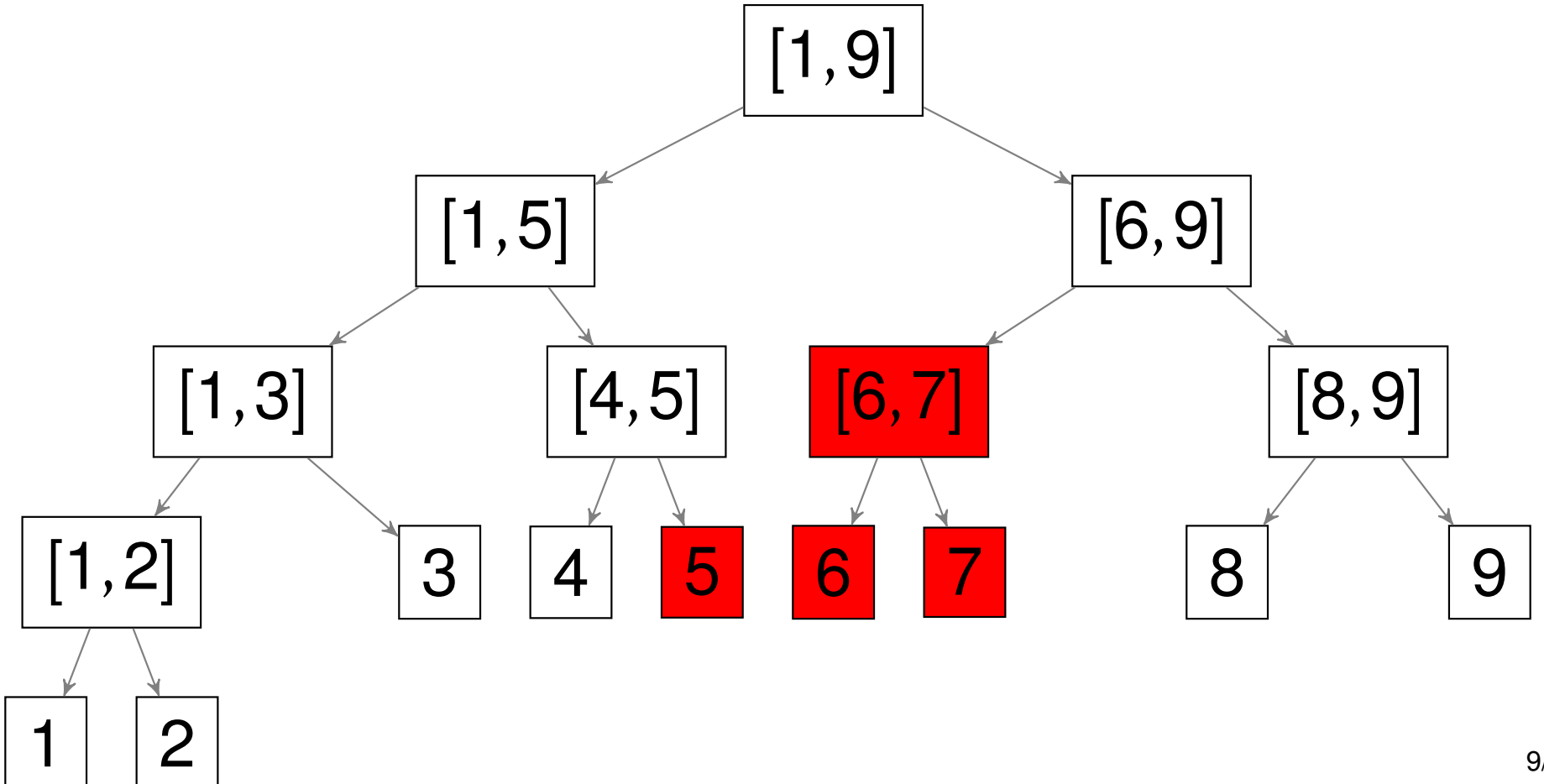
For example, 5 and 6 are not really merged.



Restructuring Company

Example

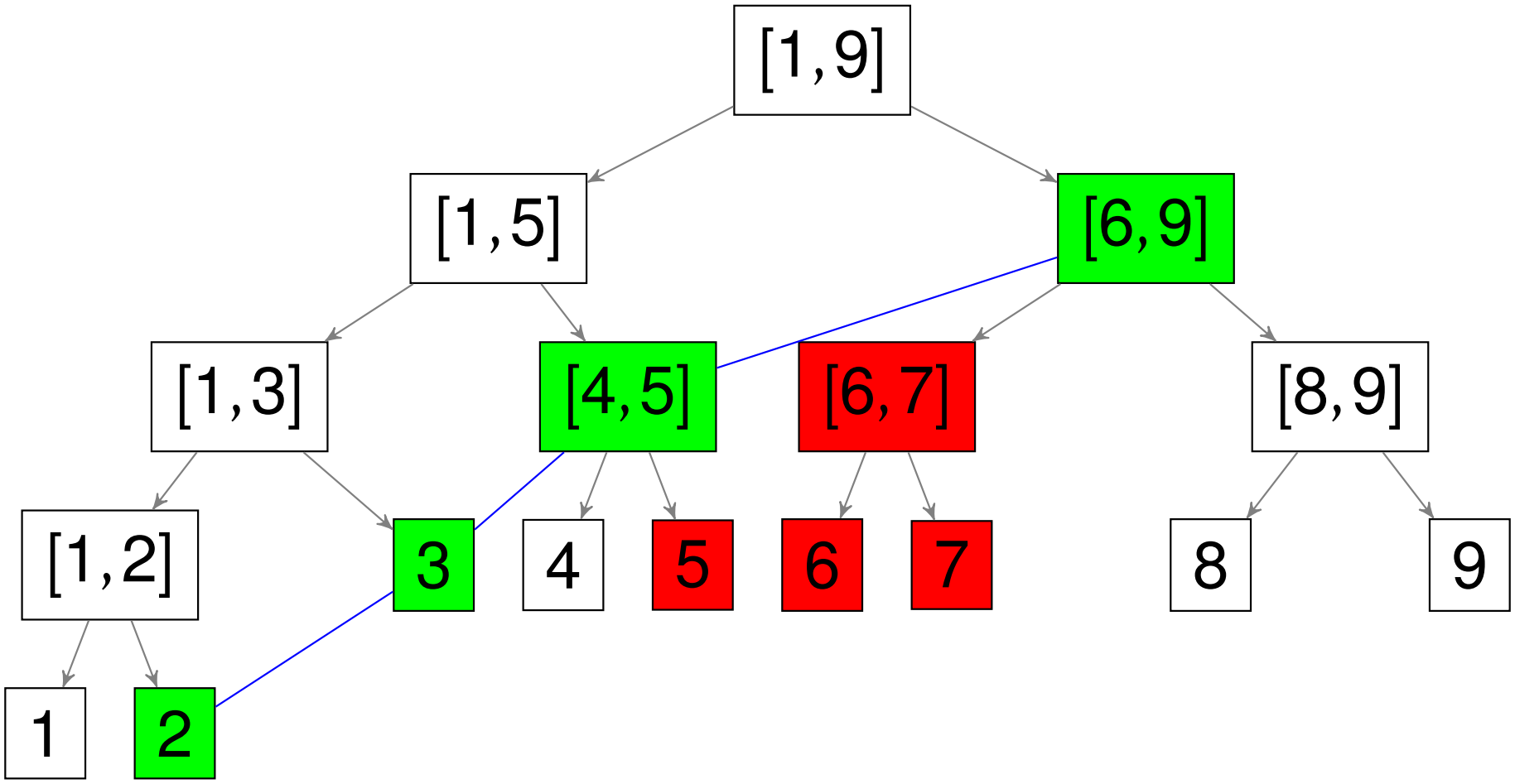
We need to propagate the range to the leaf, or until we meet a merged range.



Restructuring Company

Example

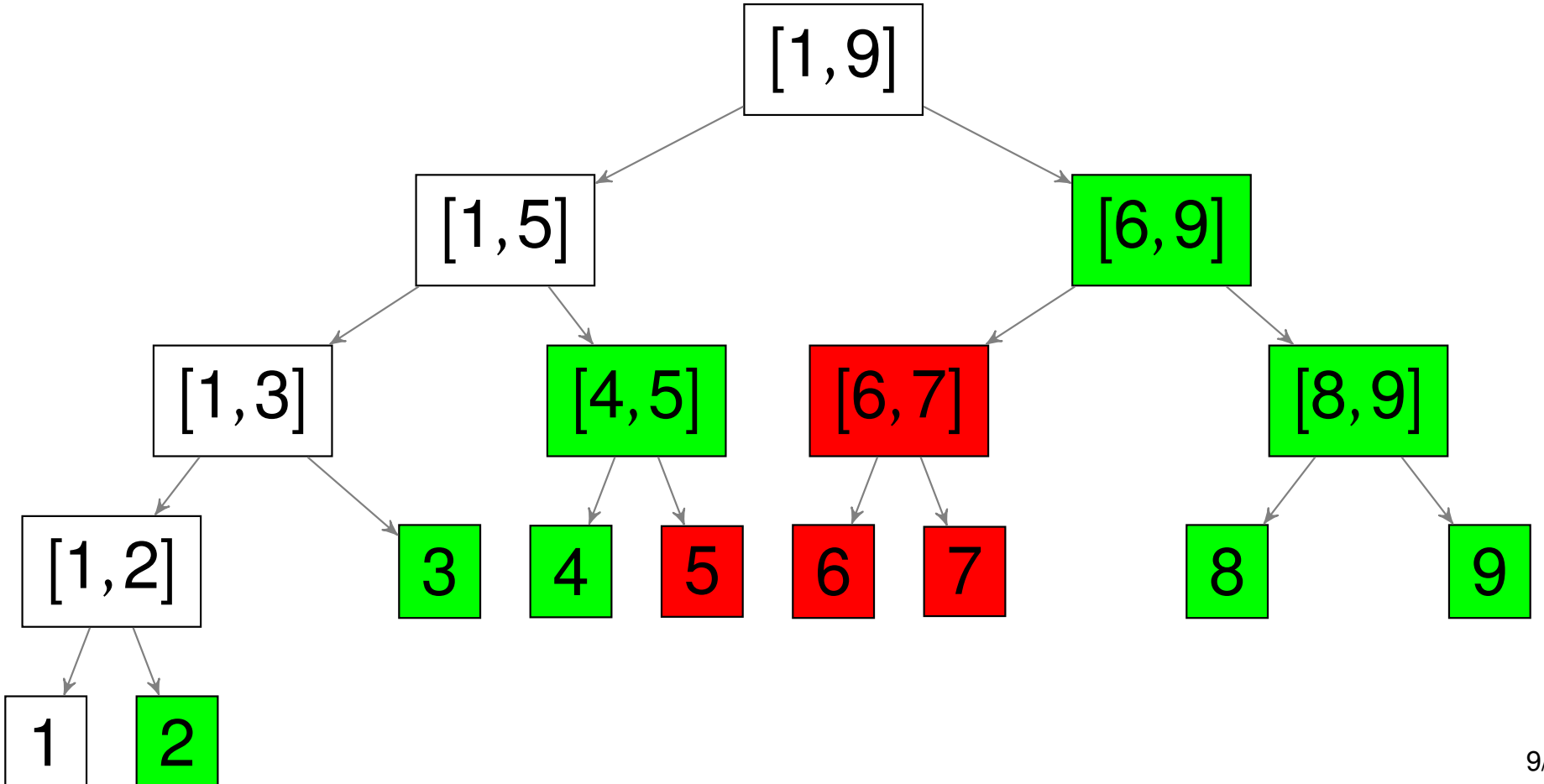
Merge 2 with [3, 9].



Restructuring Company

Example

Since [6, 7] are merged, don't need to merge all the way to the leaf!



Hints for Problem Set 2

- Problem A: (1) each position, greedily consider what's the minimum possible character. (2) use stack or set.
- Problem B: this problem is equivalent to given a set of intervals, find the maximum number of intervals such that no points on the line is covered by more than K intervals.
- Problem C: range tree, point update & query.
- Problem D: given an arbitrary K, can you decide whether the bugs can be fixed within K days?
- Problem E: union-find cannot deal with deletion. Solve the queries in a backward direction.
- Problem F: very difficult for beginners, classic line sweep + range tree problem.

Developing Game [1]

There are N ($1 \leq N \leq 1e5$) people, each person has an ability v_i, and wants to work with people with ability in range $[l_i, r_i]$ ($1 \leq l_i, r_i \leq 3e5$ and $l_i \leq v_i \leq r_i$). What is the maximum number of people you can pick such that they all want to work with each other?

[1] https://codeforces.com/contest/377/problem/D

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- It is difficult to design a polynomial algorithm
- *A* and *B* can work together, *B* and *C* can work together, doesn't mean *A* and *C* can work together.

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- For a group of people, there must exist an acceptance bound L and R such that $l_p \leq L$ and $R \leq r_p$ and $L \leq v_p$ and $v_p \leq R$ for all p in the group.

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- For any bound $[L, R]$, the i-th people can be included in the answer if $l_i \leq L \leq v_i \leq R \leq r_i$.

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- Find the $[L, R]$ such that the maximum number of people can be included in the answer!

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Analysis

- Naively this works in $O(N \cdot MAXR^2)$, can be done in $O(MAXR \cdot log(MAXR))$ or even $O(N \cdot log(N))$.

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Analysis

- Do you know how to solve given a set of intervals, find a point that is covered by the most amount of intervals?
- This problem is the same problem in the 2-d space!

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Analysis

- View $[L, R]$ as the point (L, R) and each person as a rectangle with the lower left corner (l_i, v_i) and top-right corner (v_i, r_i).

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Analysis

- View $[L, R]$ as the point (L, R) and each person as a rectangle with the lower left corner (l_i, v_i) and top-right corner (v_i, r_i).
- Find the point (L, R) on the 2-d plane that is covered by the maximum number of rectangles!

Developing Game

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Solution

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- use range tree + line sweep
- for each rectangle, we create 2 events, an add event $[l_i, v_i]$ at y-coordinate v_i, and a delete event $[l_i, v_i]$ at y-coordinate r_i.
- we sweep across all y coordinates, for each coordinate, we range update all the add events, then find the maximum value in the range, and finally update all the delete events.

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- This problem is very difficult, it's perfectly normal if you don't understand the first time

[2] https://leetcode.com/problems/rectangle-area-ii/

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- However, problems like finding the point on the plane that is covered by the maximum number of rectangles are standard.

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- Given a set of rectangles with sides parallel to the axis, calculate the area of the union of the rectangles [2] (Bound can be $1e5$).
- Given a set of points on a 2-d plane, you are also given Q queries like: given a rectangle with sides parallel to the axis, how many points are contained in this rectangle?

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- Given a set of points on a 2-d plane, you are also given Q queries like: given a rectangle with sides parallel to the axis, how many points are contained in this rectangle?
- All can be done in $O(N \cdot log(N))$

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