COMP4128 Week 04 Tutorial

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https://github.com/hharryyf/COMP4128-23T3-tutoring

Outline

- Boredom
- Pebbles

Boredom

Given a sequence a consisting of n integers. The player can make several steps. In a single step he can choose an element of the sequence (let's denote it $a[k]$) and delete it, at that all elements equal to $a[k]-1$ and $a[k]+1$ also must be deleted from the sequence. That step brings $a[k]$ points to the player. What is the maximum number of points the player can get. ($n \leq 1e5$, $a[i] \leq 1e5$).

Example

N = 3, a = [1, 2, 3]
Answer: 4

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- Recursive case: $dp[i] \ = \ max(dp[i-1], dp[i-2] + cnt[i] \cdot i)$
- Final answer: $dp[max(a[i], i = 1..N)]$

Pebbles

You are given an unlimited number of pebbles to distribute across an N ×N game board ($1 \leq N \leq 15$), where each square on the board contains an integer point value between 1 and 99, inclusive. The integers on a given board may not be unique. The player distributes pebbles across the board so that: At most one pebble resides in any given square. No two pebbles are placed on adjacent squares. Two squares are considered adjacent if they are horizontal, vertical, or diagonal neighbors. The goal is to maximize the number of points claimed by your placement of pebbles.

Example

71	24	95	56	54
85	50	74	94	28
92	96	23	71	10
23	61	31	30	46
64	33	32	95	89

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- Not all $m = 0..2^N - 1$ are valid. For example, the binary representation $m = 3$ is 11, invalid!

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- Not all $m = 0..2^N - 1$ are valid. For example, the binary representation $m = 3$ is 11, invalid!
- We should only keep *m* such that its binary representation has no two consecutive 1s.

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- Base case: $dp[0][m] = \sum_{(m>>j)\&1=1} a[0][j]$
- Recursive case: $dp[i][m] = max(dp[i-1][m'] + \sum_{(m>>j)\&1=1} a[i][j],\ m' \ is\ valid)$

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- Even if m' is valid, and m is valid, if m' is the bit representation of row $i-1$, and m is the bit representation of row i, picking m' and m together might not be valid!

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- Cannot put pebbles on $a[i-1][j]$ and $a[i][j]$ for some i, j
- m' and m cannot both have 1 on the j-th bit for some $0 \leq j < N$

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- Cannot put pebbles on $a[i-1][j]$ and $a[i][j+1]$ for some i, j
- Cannot put pebbles on $a[i-1][j]$ and $a[i][j-1]$ for some i, j
- $m \,\&\, (m' << 1) = 0$
- $(m << 1) \,\&\, m' = 0$

Pebbles

Conclusion

- $dp[i][m] = max(dp[i-1][m'] + \sum_{(m>>j)\&1=1} a[i][j],\ m' \ is\ valid)$
- $m \ \& \ m' = 0$
- $m \ \& \ (m' << 1) = 0$
- $(m << 1) \ \& \ m' = 0$

Time complexity

- $O(N \cdot |A|^2),\ A = \{k | k \in [0, 2^N - 1], k \ is\ valid\}$
- $N = 15,\ |A| = 1597$