COMP4128 Week 08 Tutorial

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https://github.com/hharryyf/COMP4128-23T3-tutoring

Outline

- Range tree revision
- Water tree
- Problem Set 6 hints by email

Range tree revision

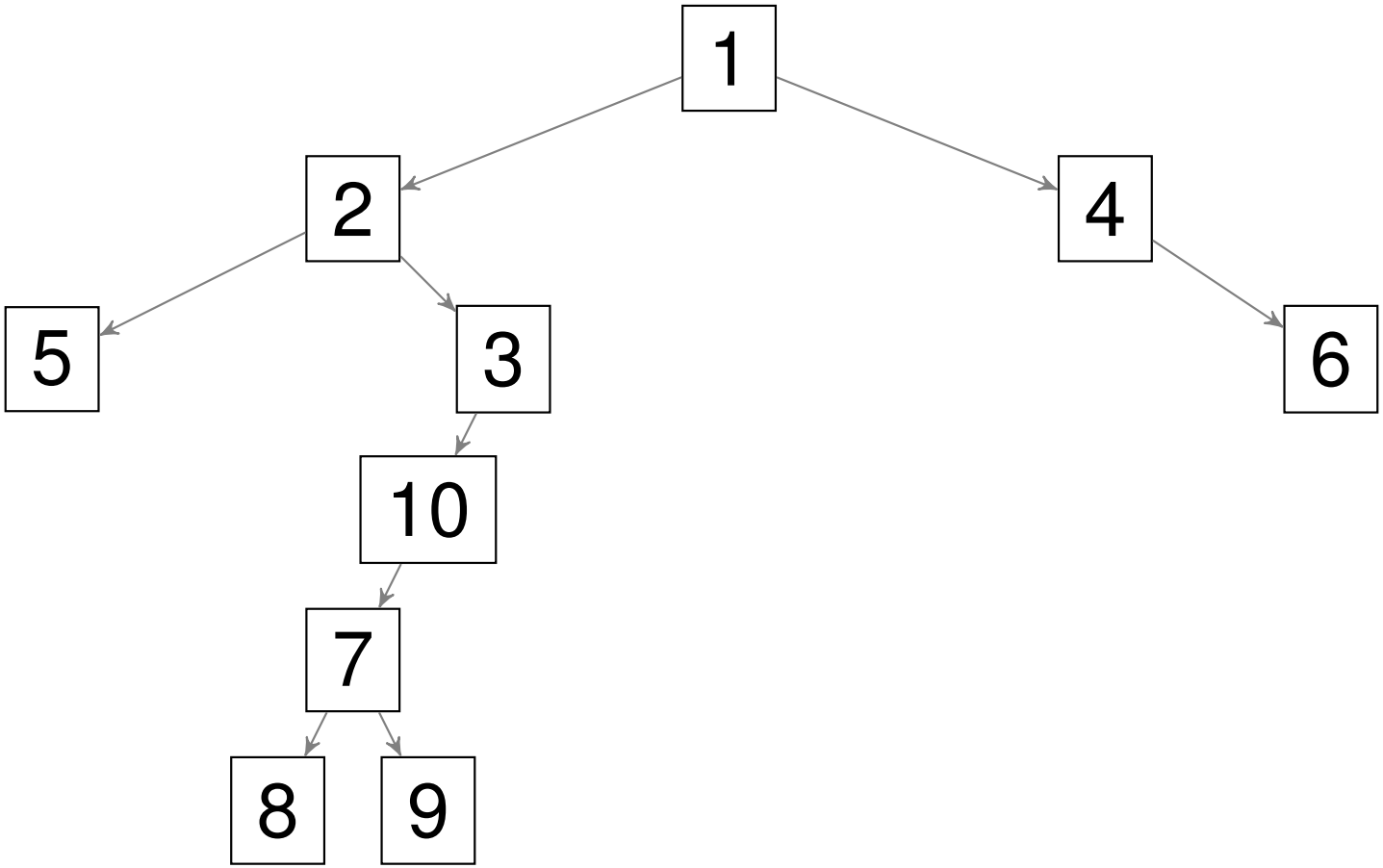
- Range tree is so powerful, it's hard to give a revision in 20 minutes
- Ensure you can solve the following use cases
  - Add v to a range $[l, r]$, query the [max|min|sum] of elements in a range $[l, r]$
  - Set a range $[l, r]$ to v, query the [max|min|sum] of elements in a range $[l, r]$
  - Query the left/rightmost element $[\leq|\geq]$ k in a range
  - Place an interval $[l_i, r_i]$ on the number line, remove a previously placed interval $[l_j, r_j]$ from the number line. Query how many points on the number line are covered by at least 1 interval.
  - Given an array, add v to index i, or query how many numbers are less than v in a range $[l, r]$
  - Target $O(logN)$ or $O(log^2 N)$ per query

Water Tree

Given a tree with N vertices, with the root at vertex 1. For each vertex, the reservoirs of its children are located below the reservoir of this vertex, and the vertex is connected with each of the children by a pipe through which water can flow downwards. Mike wants to do the following operations with the tree:

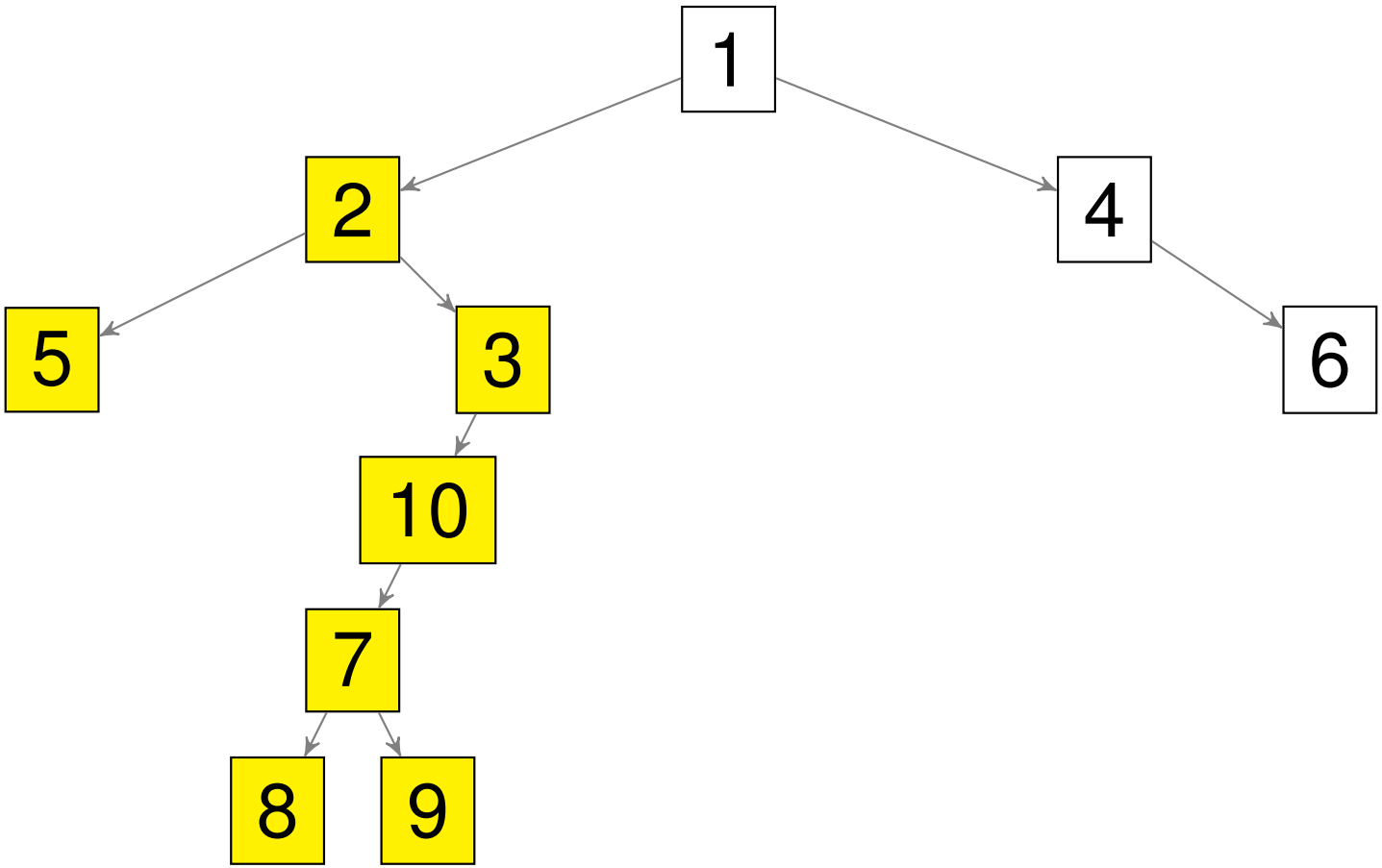
1. Fill vertex v with water. Then v and all its children are filled with water.
2. Empty vertex v. Then v and all its ancestors are emptied.
3. Determine whether vertex v is filled with water at the moment.

Example



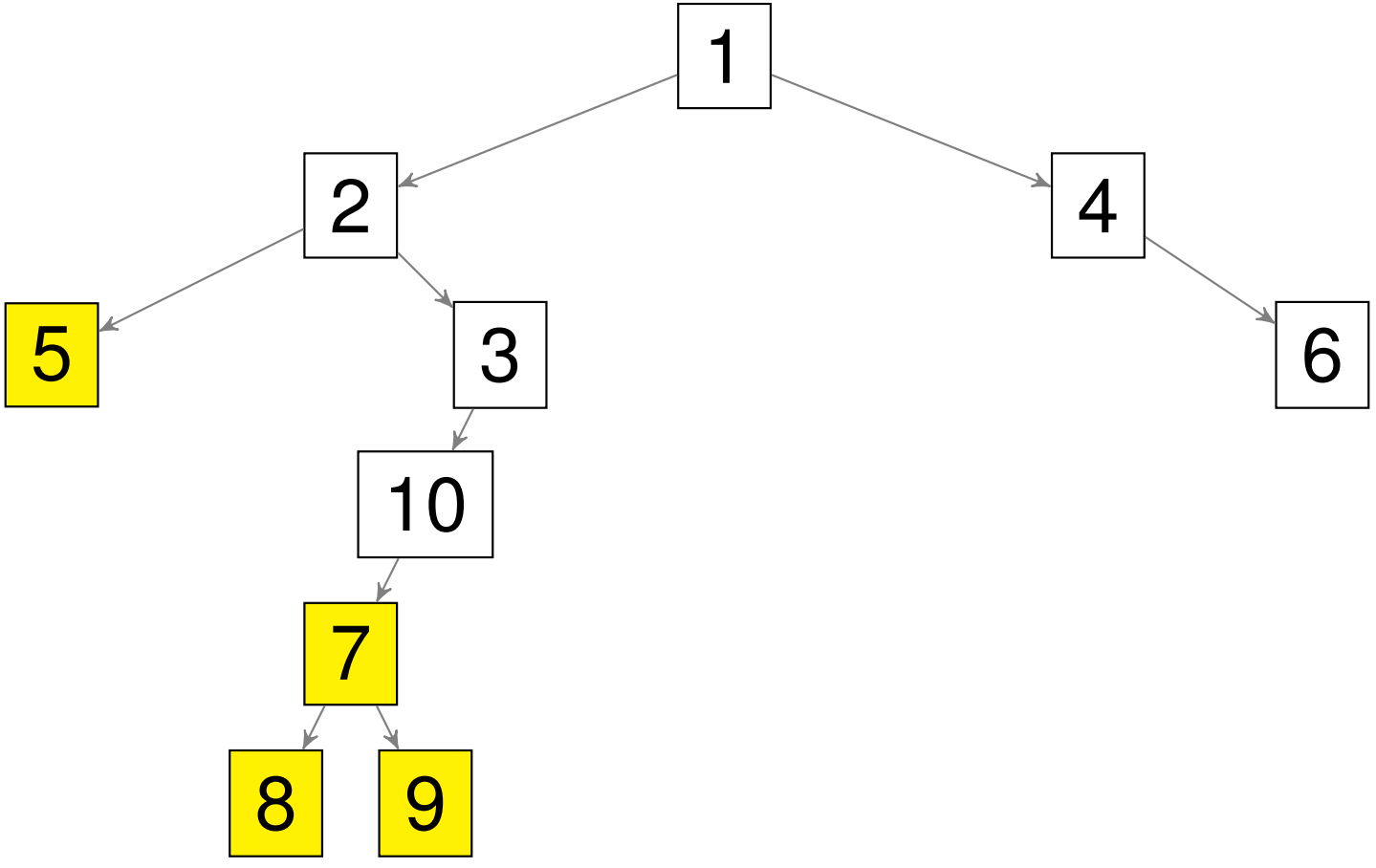
Example

- Fill 2 with water
- Query if 3 has water? Yes.



Example

- Empty vertex 10
- Query if 3 has water? No.



Water Tree

Simplified Problem

Only need to support 2 types of query

1. Fill a vertex with water
2. Query if a vertex has water

- Type 1: set the subtree of vertex v to 1
- Type 2: query if a vertex has value 1

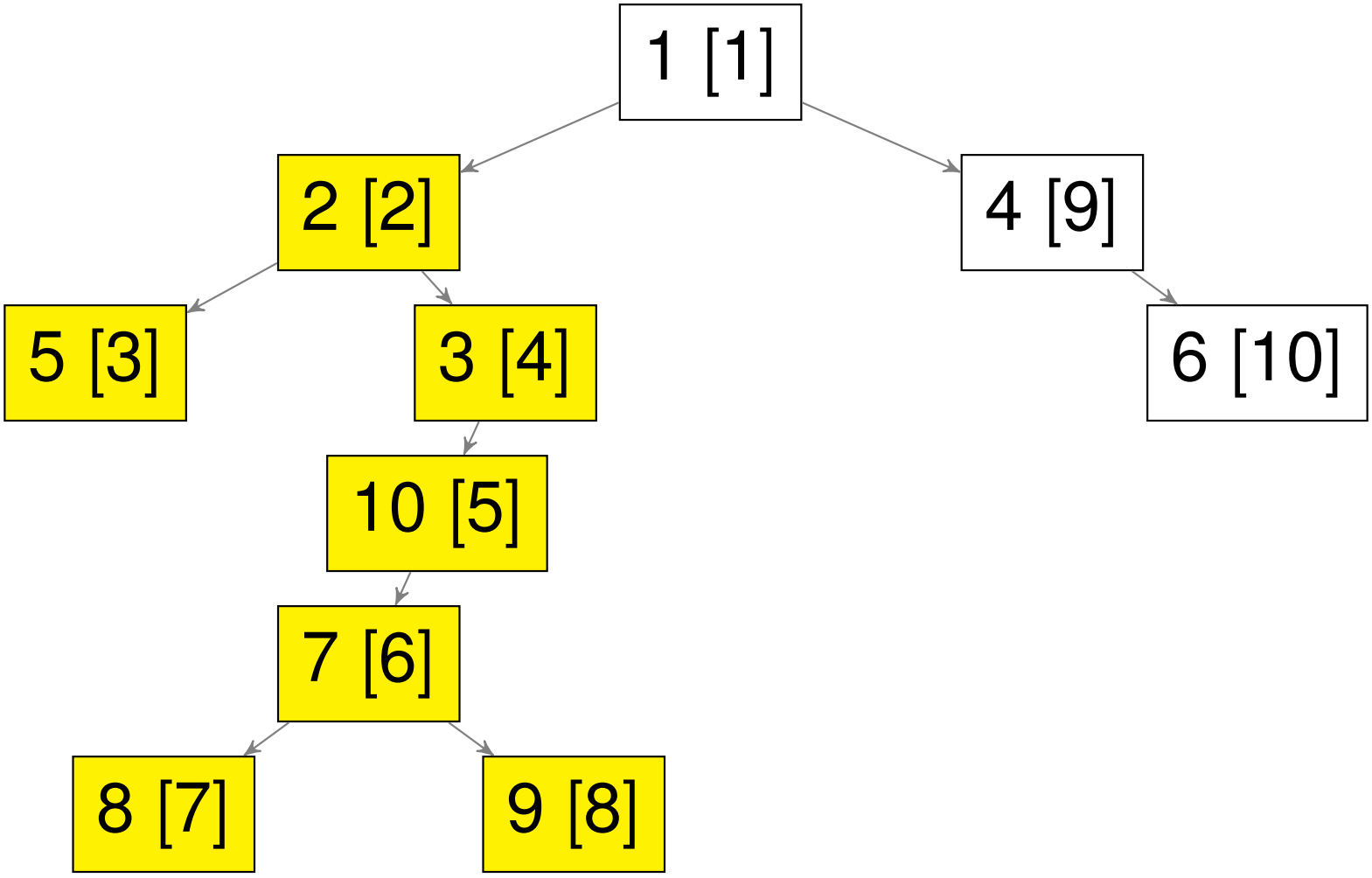
- Standard technique: label the nodes with their dfs order
- Transform the problem to:
  - Set the range to u
  - Query if the value of a point is greater than 0

Water Tree

- For each node v, suppose it has dfs order $id[v]$, and the subtree of v has size $sz[v]$
- Set the subtree of node v to u: set the range $[id[v], sz[v] + id[v] - 1]$ to u
- Query the value of v: query the value of $id[v]$

Water Tree

- Fill water to vertex 2 ($id[2] = 2, sz[2] = 7$)
- Set the nodes with dfs order in range $[2, 8]$ to 1.



Water Tree

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- How to deal with empty a node?
- This is difficult based on what is covered in the lecturer

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- We only need to know if a node is filled with water, not how many times it is filled with water
- Claim: node v has no water if and only if there is some nodes in the subtree of v without water

Water Tree

Full Problem

- How to deal with empty a node?
- This is difficult based on what is covered in the lecturer
- We only need to know if a node is filled with water, not how many times it is filled with water
- Claim: node v has no water if and only if there is some nodes in the subtree of v without water
- v has water $\equiv$ the total number of nodes with water in the subtree of v is $sz[v] \equiv$ the range sum of $[id[v], id[v] + sz[v] - 1]$ is $sz[v]$

Water Tree

Solution

- Check if v has water: check if the range sum of $[id[v], sz[v] + id[v] - 1]$ is $sz[v]$
- Empty a node v: set $id[v]$ to 0
- Fill a node v with water:
  - $flag \leftarrow$ If v has water
  - Set $[id[v], sz[v] + id[v] - 1]$ to 1
  - If $flag$ is true, set $id[parent[v]]$ to 0

Water Tree

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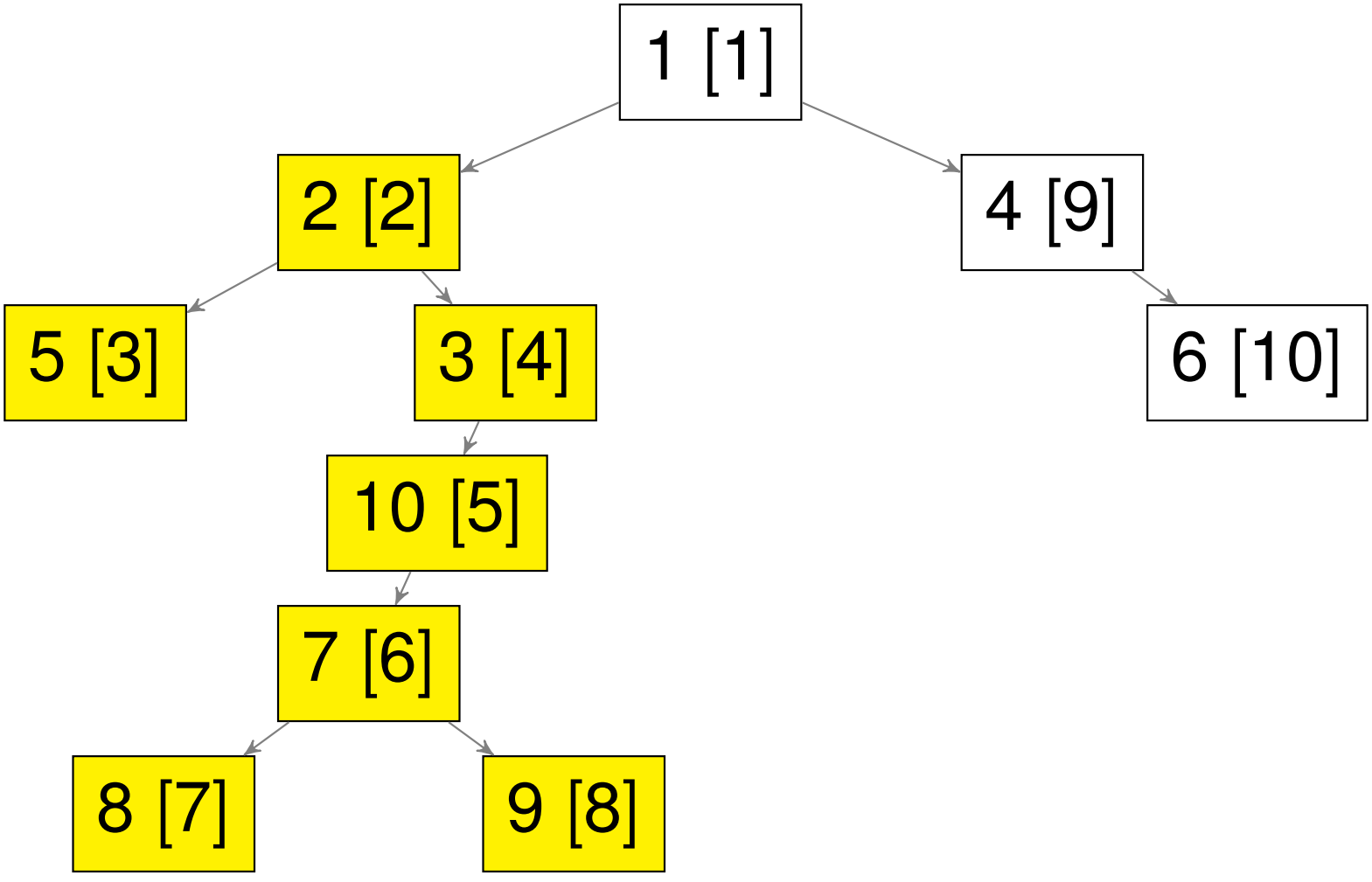
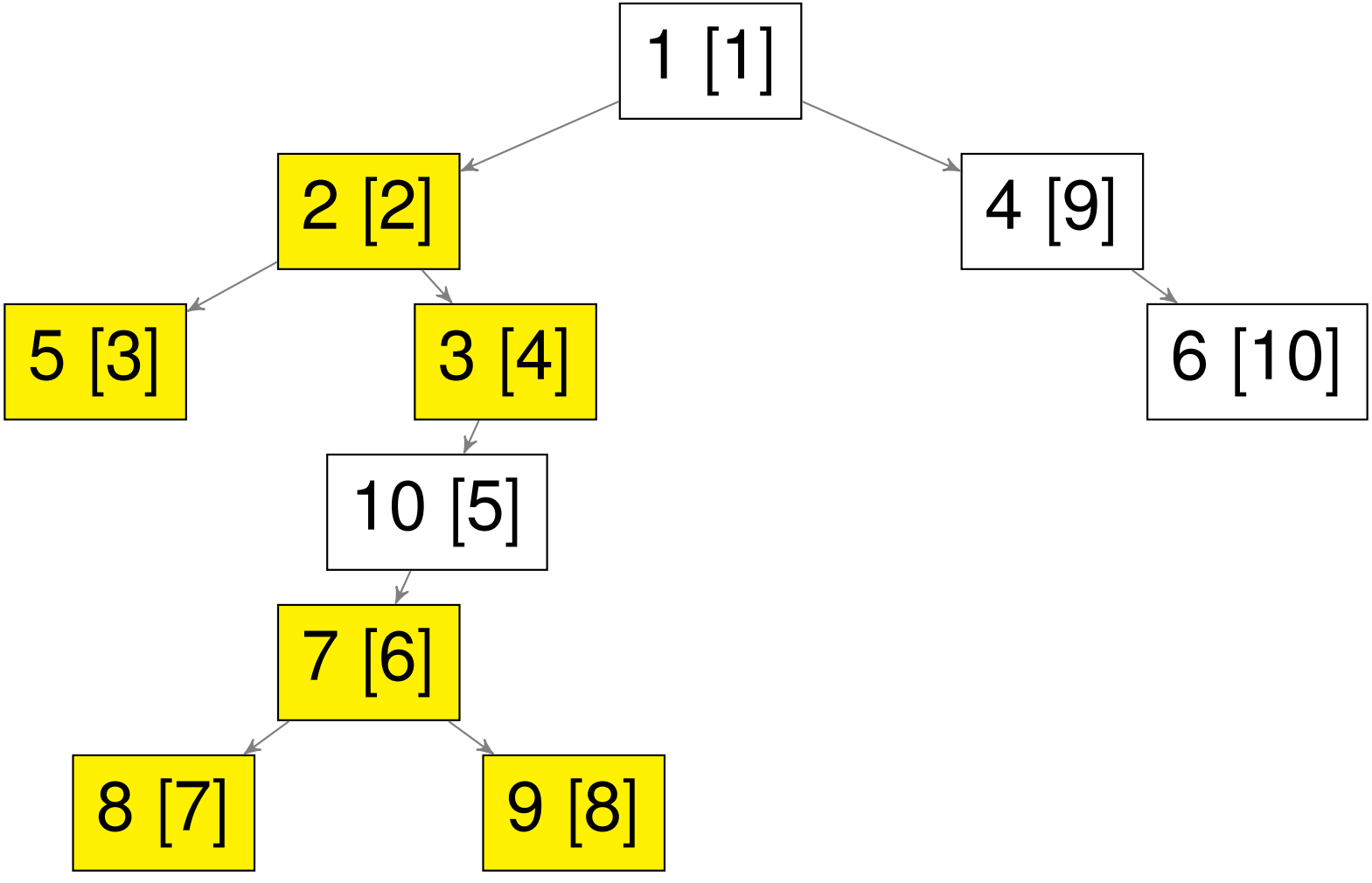


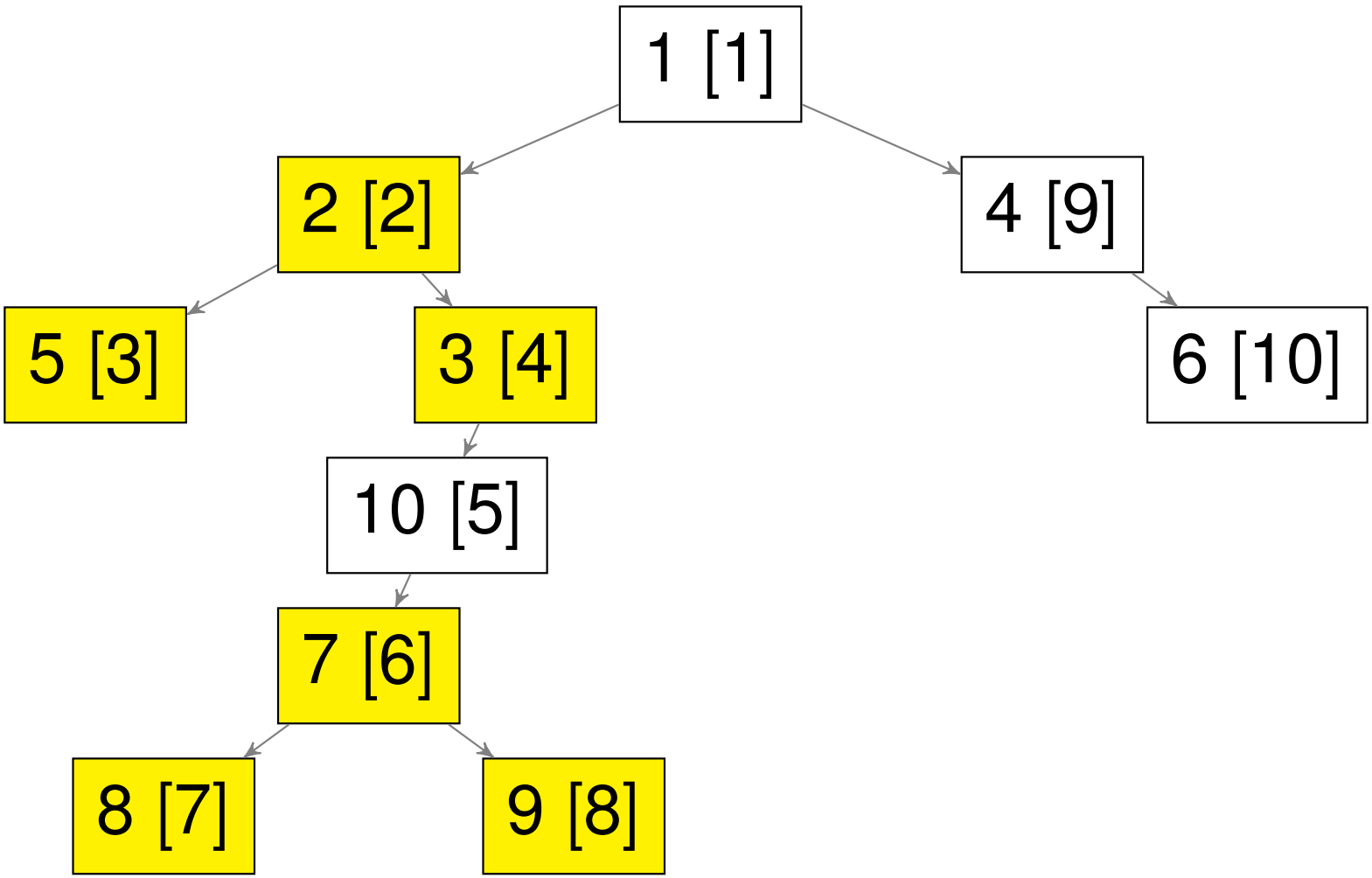
Water Tree

- Empty vertex 10 ($id[10] = 5, sz[10] = 4$)
- Set the node 10 to 0.



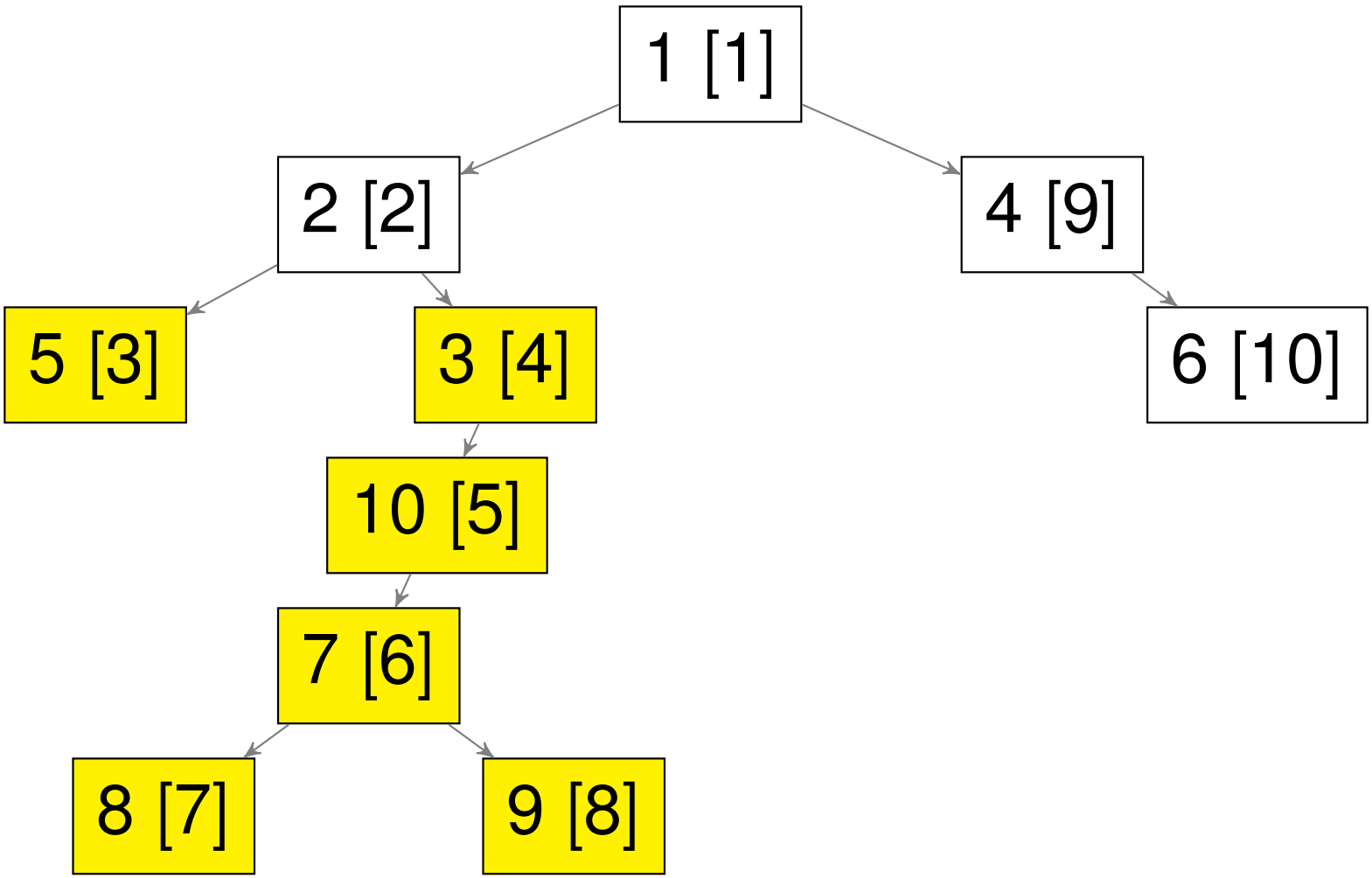
Water Tree

- If node 3 has water ($id[3] = 4, sz[3] = 5$)?
- Query if node with dfs order $[4, 8]$ has range sum 5. No!



Water Tree

- Fill node 3 with water ($id[3] = 4, sz[3] = 5$).
- 3 has no water, empty node $parent[3] = 2$
- Fill 3 with water



Water Tree

Demo