

COMP4128 Week 10 Tutorial

Yifan He

`z5173587@unsw.edu.au`

`https://github.com/hharryyf/COMP4128-23T3-tutoring`

Outline

- Division
- The Race
- Problem Set 8 hints by email

Division

Given T ($T \leq 50$) pairs of a and b ($1 \leq a \leq 10^{18}$, $2 \leq b \leq 10^9$), for each pair, find the maximum x such that $x \mid a$ but $b \nmid x$.

- $x \mid y \iff y = k \cdot x$ for some $k \in \mathbb{Z}$

Examples

- $a = 8, b = 4, x = 2$
- $a = 10, b = 4, x = 10$
- $a = 12, b = 4, x = 6$
- $a = 17, b = 5, x = 17$

Division

Analysis

- What is the optimal value of x if $b \nmid a$?

Division

Analysis

- What is the optimal value of x if $b \nmid a$?
- Answer: $x = a$
- Proof Sketch: $x \leq a, a \mid a, b \nmid a \implies x = a$

Division

Analysis

- What is the optimal value of x if $b \nmid a$?
- Answer: $x = a$
- Proof Sketch: $x \leq a, a \mid a, b \nmid a \implies x = a$
- We only need to handle $b \mid a$

Division

Analysis

- Case $b \mid a$
- Consider the prime factorization of a and b
- Let $a = p_1^{c_1} \cdot p_2^{c_2} \dots p_n^{c_n}$ and $b = q_1^{d_1} \cdot q_2^{d_2} \dots q_m^{d_m}$
- What can we tell about $p_1 \dots p_n$ and $q_1 \dots q_m$?

Division

Analysis

- Case $b \mid a$
- Consider the prime factorization of a and b
- Let $a = p_1^{c_1} \cdot p_2^{c_2} \dots p_n^{c_n}$ and $b = q_1^{d_1} \cdot q_2^{d_2} \dots q_m^{d_m}$
- What can we tell about $p_1 \dots p_n$ and $q_1 \dots q_m$?
- $\{q_1, q_2, \dots, q_m\} \in \{p_1, p_2, \dots, p_n\}$
- Rewrite $a = q_1^{c_1} \cdot q_2^{c_2} \dots q_m^{c_m} \cdot M$ such that $q_i \nmid M$ for all $1 \leq i \leq m$
- What can we tell about $c_1 \dots c_m$ and $d_1 \dots d_m$?

Division

Analysis

- Case $b \mid a$
- Consider the prime factorization of a and b
- Let $a = p_1^{c_1} \cdot p_2^{c_2} \dots p_n^{c_n}$ and $b = q_1^{d_1} \cdot q_2^{d_2} \dots q_m^{d_m}$
- What can we tell about $p_1 \dots p_n$ and $q_1 \dots q_m$?
- $\{q_1, q_2, \dots, q_m\} \in \{p_1, p_2, \dots, p_n\}$
- Rewrite $a = q_1^{c_1} \cdot q_2^{c_2} \dots q_m^{c_m} \cdot M$ such that $q_i \nmid M$ for all $1 \leq i \leq m$
- What can we tell about $c_1 \dots c_m$ and $d_1 \dots d_m$?
- $c_1 \geq d_1, c_2 \geq d_2, \dots, c_m \geq d_m$

Division

Solution

- $b = q_1^{d_1} \cdot q_2^{d_2} \dots q_m^{d_m}$
- $a = q_1^{c_1} \cdot q_2^{c_2} \dots q_m^{c_m} \cdot M$ such that $q_i \nmid M$ for all $1 \leq i \leq m$
- $c_1 \geq d_1, c_2 \geq d_2, \dots, c_m \geq d_m$
- How to create the maximum x such that $b \nmid x$ and $x \mid a$?

Division

Solution

- $b = q_1^{d_1} \cdot q_2^{d_2} \dots q_m^{d_m}$
- $a = q_1^{c_1} \cdot q_2^{c_2} \dots q_m^{c_m} \cdot M$ such that $q_i \nmid M$ for all $1 \leq i \leq m$
- $c_1 \geq d_1, c_2 \geq d_2, \dots, c_m \geq d_m$
- How to create the maximum x such that $b \nmid x$ and $x \mid a$?
- x is the maximum of
 - $q_1^{d_1-1} \cdot q_2^{c_2} \dots q_m^{c_m} \cdot M$
 - $q_1^{c_1} \cdot q_2^{d_2-1} \dots q_m^{c_m} \cdot M$
 - \dots
 - $q_1^{c_1} \cdot q_2^{c_2} \dots q_m^{d_m-1} \cdot M$

Division

Examples

- $a = 17, b = 5, x = 17$ ($b \nmid a$)
- $a = 64, b = 4, x = 2$
 - $a = 2^6$
 - $b = 2^2$
 - $x = 2^1 = 2$
- $a = 180, b = 12, x = 90$
 - $a = 2^2 \cdot 3^2 \cdot 5^1$
 - $b = 2^2 \cdot 3^1$
 - $x = \max(2^1 \cdot 3^2 \cdot 5^1, 2^2 \cdot 3^0 \cdot 5^1) = \max(90, 20) = 90$

Division

Demo

The Race

Consider a turn-taking game. In player 0's turn, he would toss a coin, and get 1 point if the coin shows up head (0 points otherwise). In player 1's turn, he would choose a number T and toss a coin for T times. If the coin shows up head in all T times, he will get 2^{T-1} points (0 points otherwise). Player 0 goes first, and the first person that gets 100 points wins. Find the probability of player 1 wins.

Answer: 0.83648556

The Race

- Player 1 chooses the number T before each turn, is T bounded?

The Race

- Player 1 chooses the number T before each turn, is T bounded?
- Yes, $1 \leq T \leq 8$
- If $T \geq 8$, when the coin shows up head for T times, player 1 would win in the current turn

The Race

- Player 1 chooses the number T before each turn, is T bounded?
- Yes, $1 \leq T \leq 8$
- If $T \geq 8$, when the coin shows up head for T times, player 1 would win in the current turn
- Two-player zero-sum turn-taking probabilistic (fully observable) game
- Solvable with Dynamic Programming
 - If you know some AI, Max^n or Minimax
- At the current game state what is the maximum probability of the current player winning the game

The Race

- How to capture the game state?

The Race

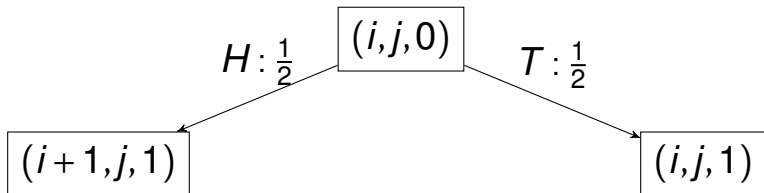
- How to capture the game state?
- (player 0's points, player 1's points, control)

The Race

- How to capture the game state?
- (player 0's points, player 1's points, control)
- Subproblem: Let $dp[i][j][k]$ be the maximum probability of the player k winning the game when player 0 has i points and player 1 has j points
- Base case:
 - 1 $i \geq 100$ $dp[i][j][k] = 1 - k$
 - 2 $j \geq 100$ $dp[i][j][k] = k$
- Answer: $1 - dp[0][0][0]$

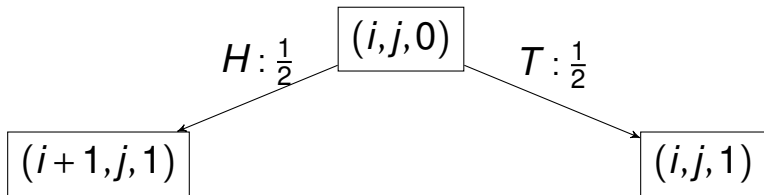
The Race

Player 0 taking control



The Race

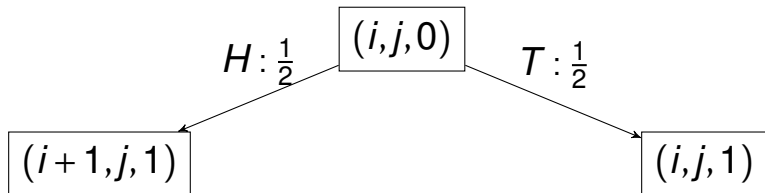
Player 0 taking control



- $$dp[i][j][0] = \frac{1}{2} \cdot dp[i][j][1] + \frac{1}{2} \cdot dp[i+1][j][1]$$

The Race

Player 0 taking control



- $dp[i][j][0] = \frac{1}{2} \cdot dp[i][j][1] + \frac{1}{2} \cdot dp[i+1][j][1]$ **x**
- Probability of player 0 winning at $(i, j, 1)$ is $1 - dp[i][j][1]$
- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$

The Race

Player 0 taking control

- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$

The Race

Player 0 taking control

- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$

Player 1 taking control

- Choose the T that maximize $dp[i][j][1]$
- Let $score = dp[i][j + 2^{T-1}][0]$ and $p = \frac{1}{2^T}$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$

The Race

Player 0 taking control

- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$

Player 1 taking control

- Choose the T that maximize $dp[i][j][1]$
- Let $score = dp[i][j + 2^{T-1}][0]$ and $p = \frac{1}{2^T}$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$
- Questions?

The Race

Player 0 taking control

- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$

Player 1 taking control

- Choose the T that maximize $dp[i][j][1]$
- Let $score = dp[i][j + 2^{T-1}][0]$ and $p = \frac{1}{2^T}$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$
- Questions?
- This is incorrect, because $dp[i][j][0]$ and $dp[i][j][1]$ forms circular dependency

The Race

Remove circular dependency

- A classic technique used in MDP (Markov)
- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$
- Replace $dp[i][j][0]$ in the second equation with the RHS of the first equation

The Race

Remove circular dependency

- A classic technique used in MDP (Markov)
- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$
- Replace $dp[i][j][0]$ in the second equation with the RHS of the first equation
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - \frac{1}{2} \cdot (1 - dp[i][j][1]) - \frac{1}{2} \cdot (1 - dp[i+1][j][1]))$

The Race

Remove circular dependency

- A classic technique used in MDP (Markov)
- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$
- Replace $dp[i][j][0]$ in the second equation with the RHS of the first equation
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - \frac{1}{2} \cdot (1 - dp[i][j][1]) - \frac{1}{2} \cdot (1 - dp[i+1][j][1]))$
- $= p \cdot (1 - score) + \frac{1-p}{2} \cdot dp[i][j][1] + \frac{1-p}{2} \cdot dp[i+1][j][1]$

The Race

Remove circular dependency

- A classic technique used in MDP (Markov)
- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - dp[i][j][0])$
- Replace $dp[i][j][0]$ in the second equation with the RHS of the first equation
- $dp[i][j][1] = p \cdot (1 - score) + (1 - p) \cdot (1 - \frac{1}{2} \cdot (1 - dp[i][j][1]) - \frac{1}{2} \cdot (1 - dp[i+1][j][1]))$
- $= p \cdot (1 - score) + \frac{1-p}{2} \cdot dp[i][j][1] + \frac{1-p}{2} \cdot dp[i+1][j][1]$
- $dp[i][j][1] = \frac{2 \cdot p}{1+p} \cdot (1 - score) + \frac{1-p}{1+p} \cdot dp[i+1][j][1]$

The Race

Summary

- $dp[i][j][0] = \frac{1}{2} \cdot (1 - dp[i][j][1]) + \frac{1}{2} \cdot (1 - dp[i+1][j][1])$
- Choose the T that maximize $dp[i][j][1]$
- Let $score = dp[i][j + 2^{T-1}][0]$ and $p = \frac{1}{2^T}$
- $dp[i][j][1] = \frac{2 \cdot p}{1+p} \cdot (1 - score) + \frac{1-p}{1+p} \cdot dp[i+1][j][1]$

The Race

Demo

Final Slide

- COMP4128 is a very challenging subject
- Hope you find the tutorials helpful

Final Slide

- COMP4128 is a very challenging subject
- Hope you find the tutorials helpful
- Hope your interview skills have improved a lot

Final Slide

- COMP4128 is a very challenging subject
- Hope you find the tutorials helpful
- Hope your interview skills have improved a lot
- If you are interested in ICPC, this is a good starting point
 - It took me only 1 year after COMP4128 to lead my first team to get the regional 2nd place
 - ~~First team because my second team carried me in 2021~~

Final Slide

- COMP4128 is a very challenging subject
- Hope you find the tutorials helpful
- Hope your interview skills have improved a lot
- If you are interested in ICPC, this is a good starting point
 - It took me only 1 year after COMP4128 to lead my first team to get the regional 2nd place
 - ~~First team because my second team carried me in 2021~~
 - Quantitatively: 1200+ problems away

Final Slide

- COMP4128 is a very challenging subject
- Hope you find the tutorials helpful
- Hope your interview skills have improved a lot
- If you are interested in ICPC, this is a good starting point
 - It took me only 1 year after COMP4128 to lead my first team to get the regional 2nd place
 - ~~First team because my second team carried me in 2021~~
 - Quantitatively: 1200+ problems away
- Contest 3, this weekend
- Final exam, December (more challenging)
- Glad to reply to any email related to revision tips