# COMP4128 Week 01 Workshop

#### Yifan He

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## **Tutorial schedule**

Week	Tutor		
1-5	Yifan He		
7-10	Adam Stucci		

### About me

- Year 2 PhD student, studying the field of logic & general game playing
  - Advertisement: Prof Michael Thielscher is great
  - If you enjoy SAT/QBF/Logic-related AI, do honours with him
- My competition history (in chronological order)
  - In the 2019 divisional contest (14th place ©)
  - Did COMP4128 in 19T3, full mark + course prize
  - In the 2020 competition, got regional final 2nd place
  - In the 2021 competition, got regional final 2nd place

## Reminder

- Contest 1 (this weekend)
- Problem Set 1

### **Outline**

- What to expect in COMP4128
- Structure of the tutorial sessions
- Problem solving

## More information about COMP4128

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#### What you might take away from COMP4128

- Be smarter?
- Smash interview?
- More advanced DS & algorithms
- Practice problem solving in general

# About COMP4128 (cont.)

#### How to do well

- Don't practice leetcode hard problems
- Solve every problem set problems individually
- Solve similar problems on Codeforces
- You can email me for problem recommendations

#### Warning

- Don't copy solutions directly
  - Especially, don't spend effort renaming variables and trying to bypass the plagiarism detection system!
- Use contest 1 to "predict" your success rate

#### **Tutorial Structure**

1. How many online tutorials (Thursday 12:00-13:30) would you expect to come in the term, there are 9 in total?



2. What's your preferred style of the online tutorial?

A peer discussion workshop (we give you discussion questions and provide feedback)

22%

A standard tutorial (lecture revision + tutorial question and solutions)

78%

- For week 1-5:
  - Will do some revision
  - Solve tutorial question
  - Hints for problem sets
  - A new problem (if we have time)

# Adoption (2021 Final exam)

You have N dogs, N cats, and 2N people ( $N \le 1e5$ ). Each person would select exactly one pet. The happiness of person i getting a dog is d[i], and a cat is c[i]. Calculate the maximum total happiness of the 2N people.

#### Example (N=2)

<b>c[i]</b>	2	c[i]	<b>d[i]</b> 2	<b>c[i]</b> 1	d[i]
2	1 3 -1	2	1	2	1
0	3	0		0	3
2	-1	2	5	-1	-2

- We can formulate it as an ILP problem
- We introduce 2N variables x[1..2N],  $x[i] \in \{0,1\}$ .
- x[i] = 1 (resp. x[i] = 0) means person i selects cat (resp. dog).
- Constraint:  $\sum_{i=1}^{2N} x[i] = N$ .
- Objective: Max  $\sum_{i=1}^{2N} (c[i] \cdot x[i] + d[i] \cdot (1 x[i]))$

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- How can we maximize the expression?
- Just select the N largest c[i] d[i]!

#### **Full solution**

- Calculate the sum of d[1..2N] denote it as X
- Calculate a[i] = c[i] d[i] and sort array a in descending order
- Calculate the sum of a[1..N] denote it as Y
- Answer is X + Y

## Demo

Give a string of length N ( $N \le 50,000$ ), the string is nice if there exists a substring of length 26 where each letter of 'A' to 'Z' appears exactly once. You are given a string where some of its letters are missing (represented with '?'). Is it possible to fill in the missing letters with 'A' to 'Z' so that the resulting string is nice? If yes, find an example.

#### Example

input: ABC??FGHIJK???OPQR?TUVWXY? output: ABCDEFGHIJKLMNOPQRSTUVWXYZ

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- Maintain a window of size 26, and dynamically maintain (#distinct letters, #letters), and check they are equal. Time complexity O(N).

How to complete the string to be nice?

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- Detect the missing letters
- Replace the '?' with the missing letters
- Don't forget about '?' in other substrings.
- Just fill it with 'A'.

## Demo

You are given a set of points  $x_1, x_2, ..., x_n$  on the number line. Two points i and j can be matched with each other if the following conditions hold:

- neither i nor j is matched with any other point;
- $\bullet |X_i X_j| \geq Z.$

What is the maximum number of pairs of points you can match with each other?

•  $2 \le n \le 200,000$  and  $1 \le z \le 10^9$ 

<sup>1</sup>https://codeforces.com/contest/1156/problem/C

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#### Example

$$n = 4, z = 2$$
  
  $x = [1, 3, 3, 7]$ 

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#### Can we do greedy matching?

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- Iterate from the largest  $x_n$ , match it with the largest  $x_i$  such that  $|x_n x_i| \ge z$ ?
- Wrong answer!
- N = 6, Z = 9
- 1 2 3 10 13 19
- But a better solution is 1 2 3 10 13 19

#### A Modified Problem

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- Can be proved by the exchange argument
- If there is a match of size K, can we find a match of size K-1?
- Binary search to find the maximum K!

## Demo