A. Appendices

In the Appendices, we provide a proof (sketch) of Proposition 1.

Proposition 1. *Suppose that G is a valid GDL description and φ is a GCL formula over G, then $G \models_t \varphi$ if and only if the following QASP is satisfiable.*

$$\mathbf{Q}_d \;\; S_0^{true}(0) \cup Enc(\varphi, 0) \cup \{tdom(0).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\varphi, 0)\} \tag{E1}$$

Proof. We prove a more general statement. Suppose that S is a state reachable from S_0 with some valid play sequence, Pos_φ is the set of all positions of all subformulas of φ, and $\mathscr{V}_\varphi$ is a valid position naming function. Suppose that $pos(\psi) = \pi$, we will show that $G, S \models_t \psi$ where ψ is a subformula of φ if and only if the following QASP holds.

$$\mathbf{Q}_d \;\; S^{true}(\mathscr{V}_\varphi(\pi)) \cup Enc(\psi, \mathscr{V}_\varphi(\pi)) \cup \{tdom(\mathscr{V}_\varphi(\pi)).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi, \mathscr{V}_\varphi(\pi))\} \tag{E2}$$

where $\mathbf{Q}_d$ in (E2) is defined the same way as in Definitions 11 and 12. It is trivial that if the general statement holds, the original statement must also hold. The proof is by induction on the structure of ψ.

$\underline{\psi = p(\vec{t})}$: From definition 9, we know that $\eta(p(\vec{t}), \mathscr{V}_\varphi(\pi)))$ can be justified if and only if $p(\vec{t}, (\mathscr{V}_\varphi(\pi)), 0))$ can be justified. Since $p(\vec{t}, (\mathscr{V}_\varphi(\pi)), 0))$ only appears in $P_{ext}(G) \cup S^{true}(\mathscr{V}_\varphi(\pi))$. From Theorem 2, we know that $P_{ext}(G) \cup S^{true}(\mathscr{V}_\varphi(\pi)) \models p(\vec{t}, (\mathscr{V}_\varphi(\pi)), 0))$ if and only if $G, S^{true} \models p(\vec{t})$. Hence, the base case when $\psi = p(\vec{t})$ holds.

$\underline{\psi = \neg p(\vec{t})}$: The proof of this case is analogous to the proof of the previous case.

$\underline{\psi = \psi_1 \wedge \psi_2}$: From definition 9, we know that $\eta(\psi, \mathscr{V}_\varphi(\pi)))$ can be justified if and only if both $\eta(\psi_1, \mathscr{V}_\varphi(\pi\wedge_1)))$ and $\eta(\psi_2, \mathscr{V}_\varphi(\pi\wedge_2)))$ can be justified. Note that due to position naming, $\mathscr{V}_\varphi(\pi) = \mathscr{V}_\varphi(\pi\wedge_1) = \mathscr{V}_\varphi(\pi\wedge_2)$, and as a result, $S^{true}(\mathscr{V}_\varphi(\pi)) = S^{true}(\mathscr{V}_\varphi(\pi\wedge_1)) = S^{true}(\mathscr{V}_\varphi(\pi\wedge_2))$. Let $\mathscr{V}_\varphi(\pi) = \mathscr{V}_\varphi(\pi\wedge_1) = \mathscr{V}_\varphi(\pi\wedge_2) = i$. The inductive hypothesis says that $\mathbf{Q}_d \;\; S^{true}(i) \cup Enc(\psi_1, i) \cup \{tdom(i).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi_1, i)\}$ is satisfiable if and only if $G, S^{true}(i) \models_t \psi_1$; and $\mathbf{Q}_d \;\; S^{true}(i) \cup Enc(\psi_2, i) \cup \{tdom(i).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi_2, i)\}$ is satisfiable if and only if $G, S^{true}(i) \models_t \psi_2$. Due to our quantification method, we can ensure that $\eta(\psi_1 \wedge \psi_2, i)$ is quantified no earlier than $\eta(\psi_1, i)$ and $\eta(\psi_2, i)$. Hence, it is trivial that $\mathbf{Q}_d \;\; S^{true}(i) \cup Enc(\psi_1, i) \cup Enc(\psi_2, i) \cup \{tdom(i).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi_1, i)\} \cup \{ :\text{-}\; not\; \eta(\psi_2, i)\}$ is satisfiable if and only if both $\mathbf{Q}_d \;\; S^{true}(i) \cup Enc(\psi_1, i) \cup \{tdom(i).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi_1, i)\}$ and $\mathbf{Q}_d \;\; S^{true}(i) \cup Enc(\psi_2, i) \cup \{tdom(i).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi_2, i)\}$ are satisfiable, which is equivalent to $\mathbf{Q}_d \;\; S^{true}(i) \cup Enc(\psi, i) \cup \{tdom(i).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\psi_1 \wedge \psi_2, i)\}$ is satisfiable, which is also equivalent to both $G, S^{true}(i) \models_t \psi_1$ and $G, S^{true}(i) \models_t \psi_2$ holds. Hence, the case when $\psi = \psi_1 \wedge \psi_2$ holds.

$\underline{\psi = \psi_1 \vee \psi_2}$: The proof of this is analogous to the proof of the previous case.

$\underline{\psi = \langle\!\langle C \rangle\!\rangle \mathbf{X} \phi}$: From definition 9, we know that

- $$\begin{aligned} Enc(\langle\!\langle C \rangle\!\rangle \mathbf{X}\phi, \mathscr{V}_\varphi(\pi)) = \{&\eta(\langle\!\langle C \rangle\!\rangle \mathbf{X}\phi, \mathscr{V}_\varphi(\pi)) \;:\text{-}\; \eta(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}))\;, not\; terminal((\mathscr{V}_\varphi(\pi), 0)).\} \cup \\ &P_{gen}(G, C, \mathscr{V}_\varphi(\pi), \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})) \cup Enc(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})) \cup \\ &\{succ(\mathscr{V}_\varphi(\pi), \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup \{tdom(\mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup \\ &\{\_exists(r, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}))). \mid r \in C\} \end{aligned}$$

Note that $G, S \not\models_t \psi$ if S is a terminal state, in this case, the QASP is also unsatisfiable because of the consequence of Theorem 2 and the fact that $\eta(\langle\!\langle C \rangle\!\rangle \mathbf{X}\phi, \mathscr{V}_\varphi(\pi))$ can never be justified if S is a terminal state. We now focus on the case when S is a non-terminal state. From the inductive hypothesis, we also know that for all S' that are reachable from S_0, we have that $G, S' \models_t \phi$ if and only if $\mathbf{Q}_d \;\; S'^{true}(\mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})) \cup Enc(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})) \cup \{tdom(\mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}))\}$ is satisfiable. Due to the consequence of Theorem 2, we can thus ensure that if A is a legal joint action at state S and $S \xrightarrow{A} S'$, then, $\mathbf{Q}_d \;\; S^{true}(\mathscr{V}_\varphi(\pi)) \cup A^{does}(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}) \cup Enc(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})) \cup \{succ(\mathscr{V}_\varphi(\pi), \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup \{tdom(\mathscr{V}_\varphi(\pi)).\} \cup \{tdom(\mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup P_{ext}(G) \cup \{ :\text{-}\; not\; \eta(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}))\}$ is satisfiable if and only if $G, S' \models_t \phi$, which is equivalent to $\mathbf{Q}_d \;\; S^{true}(\mathscr{V}_\varphi(\pi)) \cup A^{does}(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}) \cup Enc(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})) \cup \{succ(\mathscr{V}_\varphi(\pi), \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup \{tdom(\mathscr{V}_\varphi(\pi)).\} \cup \{tdom(\mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X})).\} \cup P_{ext}(G) \cup \{\eta(\psi, \mathscr{V}_\varphi(\pi)) \;:\text{-}\; \eta(\phi, \mathscr{V}_\varphi(\pi\langle\!\langle C \rangle\!\rangle \mathbf{X}))\;, not\; terminal((\mathscr{V}_\varphi(\pi), 0)).\} \cup \{ :\text{-}\; not\; \eta(\psi, \mathscr{V}_\varphi(\pi))\}$ is satisfiable.

Rules (3) and (4) in P_{gen} (cf. Definition 10) ensure that if we apply the action generator at a non-terminal state S, exactly 1 legal action of each player can be generated. Similarly, all possible legal joint actions of players in S can be generated. In other words, due to the consequence of Theorem 2, suppose that A is a legal joint action at state S and $S \xrightarrow{A} S'$. We denote the ASP program $S^{true}(\mathscr{V}_\varphi(\pi))) \cup P_{gen}(G, C, \mathscr{V}_\varphi(\pi)), \mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})))$ as P. Then, we know that P must admit an answer set $\mathbf{M}$ such that the set of all positive instances of **does** in $\mathbf{M}$ is equal to $A^{does}(\mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})))$ and the set of all positive instances of **true** in $\mathbf{M}$ is equal to $S^{true}(\mathscr{V}_\varphi(\pi))) \cup S'^{true}(\mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})))$. Conversely, we also know that for any stable model $\mathbf{M}$ of P. If $A^{does}(\mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})))$ is the set of all positive instances of **does** in $\mathbf{M}$, then A must be a legal joint actions of the players at state S.

As a result, due to our quantification methods and the effect of the logarithmic encoding of the actions of the players in $R \setminus C$, we can ensure that the actions of the players in C should be fixed before the actions of the players in $R \setminus C$, and $\mathbf{Q}_d\ \ S^{true}(\mathscr{V}_\varphi(\pi)) \cup P_{gen}(G, C, \mathscr{V}_\varphi(\pi), \mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})) \cup Enc(\phi, \mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})) \cup \{succ(\mathscr{V}_\varphi(\pi), \mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})).\} \cup \{tdom(\mathscr{V}_\varphi(\pi)).\} \cup \{tdom(\mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X})).\} \cup P_{ext}(G) \cup \{\eta(\psi, \mathscr{V}_\varphi(\pi))\ \texttt{:-}\ \eta(\phi, \mathscr{V}_\varphi(\pi\langle\langle C\rangle\rangle\,\mathbf{X}))\ , not\ terminal((\mathscr{V}_\varphi(\pi), 0)).\} \cup \{\texttt{:-}\ not\ \eta(\psi, \mathscr{V}_\varphi(\pi))\}$ is satisfiable if and only if there exists some legal joint actions of players in C such that for all legal joint actions of players in $R \setminus C$, ϕ holds in the next state, which implies that $G, S \vDash_t \psi$, which is equivalent to $\mathbf{Q}_d\ \ S^{true}(\mathscr{V}_\varphi(\pi)) \cup Enc(\psi, \mathscr{V}_\varphi(\pi)) \cup \{tdom(\mathscr{V}_\varphi(\pi)).\} \cup P_{ext}(G) \cup \{\texttt{:-}\ not\ \eta(\psi, \mathscr{V}_\varphi(\pi))\}$ is satisfiable. Hence, the case when $\psi = \langle\langle C\rangle\rangle\,\mathbf{X}\phi$ holds.

$\underline{\psi = [[C]]\,\widetilde{\mathbf{X}}\phi}$: The proof of this case is similar to the previous case. $\square$