Logic and Computer Design Fundamentals Chapter 3 – Combinational Logic Design

Part 2 – Combinational Functional Blocks

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Overview

- Part 2 Combinational Logic
 - Functions and functional blocks
 - Rudimentary logic functions
 - Decoding using Decoders
 - Implementing Combinational Functions with Decoders
 - Encoding using Encoders
 - Selecting using Multiplexers
 - Implementing Combinational Functions with Multiplexers

Functions and Functional Blocks

- The functions considered are those found to be very useful in design.
- Corresponding to each of the functions is a combinational circuit implementation called a functional block.
- In the past, functional blocks were packaged as small-scale-integrated (SSI), medium-scale integrated (MSI), and large-scale-integrated (LSI) circuits.
- Today, they are often simply implemented within a very-large-scale-integrated (VLSI) circuit.

Rudimentary Logic Functions

- Four elementary combinational logic functions
 - Value-Fixing: F = 0 or F = 1, no Boolean operator
 - Transferring: F = X, no Boolean operator
 - Inverting: $F = \overline{X}$, involves one logic gate
 - Enabling: F = X EN or F = X + EN, involves one or two logic gates
 - The first three are functions of a single variable X

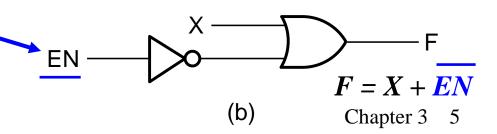
 \underline{V}_{CC} or \underline{V}_{DD} Table Functions of one variable $F \equiv X$ $E \equiv 1$ $F \equiv 1$ (c) $F = X \quad F = X \quad F = 1$ F = 0 $\underline{F} = \underline{0}$ (b) (a) (d)

Enabling Function

- *Enabling* permits an input signal to pass through to an output.
- Disabling blocks an input signal from passing through to an output with a fixed value.
- When it is disabled, the output can be 0 and 1 in comparison to Hi-Z by three-state buffers.

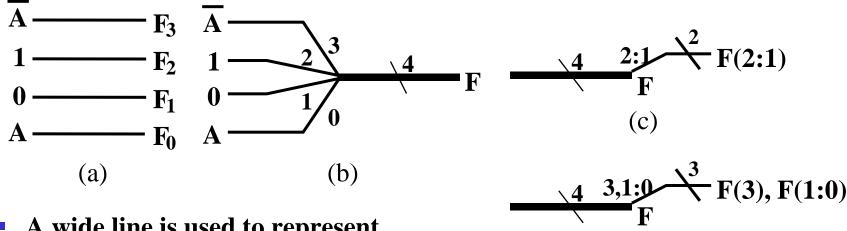
• When disabled, 0 output X = F(a) F = X EN

When disabled, 1 output



Multiple-bit Rudimentary Functions

Multi-bit Examples:

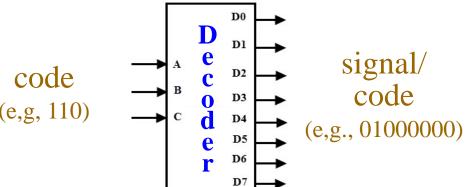


- A wide line is used to represent a bus which is a vector signal
- In (b) of the example, $F = (F_3, F_2, F_1, F_0)$ is a bus.
- The bus can be split into <u>individual bits</u> as shown in (b)
- **Sets of bits** can be split from the bus as shown in (c) for bits 2 and 1 of F.
- The sets of bits need not be continuous as shown in (d) for bits 3, 1, and 0 of F.

(d)

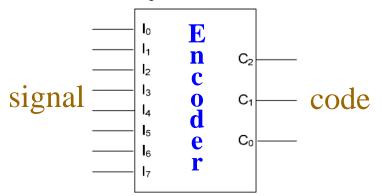
Big Picture of Four Functional Blocks

- Decoder
 - Minterm Detector
 - Code Translation Decoder

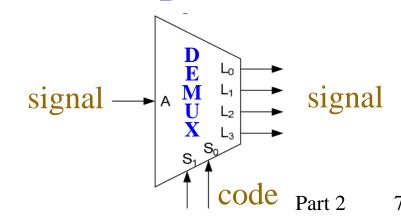


- Multiplexer
 - $\begin{array}{c|c}
 \hline
 & D_0 \\
 \hline
 & D_1 \\
 & U \\
 \hline
 & D_2 \\
 \hline
 & D_3 \\
 \hline
 & Code
 \end{array}$ signal

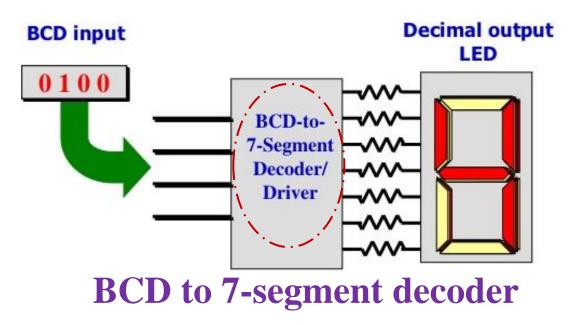
- Encoder
 - Decimal-to-BCD Encoder
 - Priority Encoder



Demultiplexer



Decoding



- Decoding the conversion of an n-bit input code to an *m*-bit output code with $n \le m \le 2^n$ such that each valid code word produces a unique output code
- Circuits that perform decoding are called decoders

Types of decoder

Types of decoder

- Variable Decoder (minterm detector)
- Display Decoder
- Code Translation Decoder

minterm A B Decoder D3 D4 D5 D6 D7

3-to-8 binary decoder

Commonly used decoders:

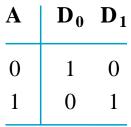
- decoder with 2 input and 4 output, 74LS139 (2-to-4-Line Decoder)
- decoder with 3 input and 8 output, 74LS138 (3-to-8-Line Decoder)
- decoder with 4 input and 16 output, MC14514(4-to-16-Line Decoder)

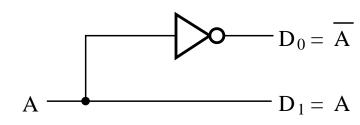
Variable Decoder (Minterm Detector)

A	В	C	$\mathbf{Y_0}$	\mathbf{Y}_1	\mathbf{Y}_{2}	\mathbf{Y}_3	Y_4	\mathbf{Y}_{5}	\mathbf{Y}_{6}	Y_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Variable Decoder Examples

1-to-2-Line Decoder A





2-to-4-Line Decoder

\mathbf{A}_1	\mathbf{A}_0	\mathbf{D}_0	\mathbf{D}_1	\mathbf{D}_2	\mathbf{D}_3					
0	0	1	0	0	0					
0	1	0	1	0	0					
1	0	0	0	1	0					
1	1	0	0	0	1					
	(a)									

 A_{0} A_{0} A_{1} A_{2} A_{1} A_{2} A_{1} A_{2} A_{1} A_{2} A_{1} A_{2} A_{2} A_{1} A_{3} A_{1} A_{2} A_{1} A_{2} A_{1} A_{2} A_{2} A_{3} A_{4} A_{1} A_{2} A_{3} A_{4} A_{2} A_{3} A_{4} A_{4} A_{5} A_{5} A_{1} A_{2} A_{3} A_{4} A_{5} A_{1} A_{2} A_{3} A_{4} A_{5} A_{5} A_{1} A_{2} A_{3} A_{4} A_{5} A_{5} A_{5} A_{7} A_{1} A_{2} A_{3} A_{4} A_{5} A_{5} A_{5} A_{7} A_{7

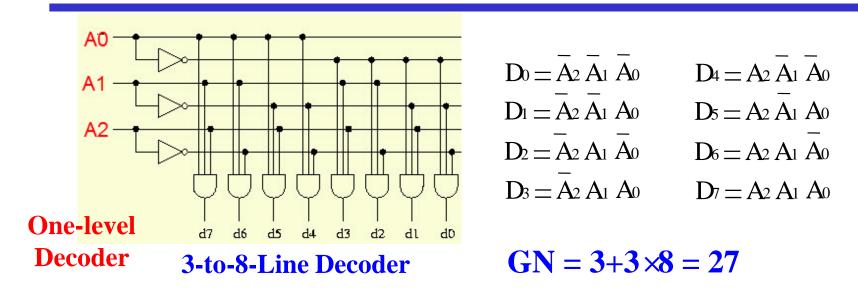
(b)

Note that the 2-4-line made up of 2 1-to-2-

line decoders and 4 AND gates.

 $-D_3 = A_1 A_0$

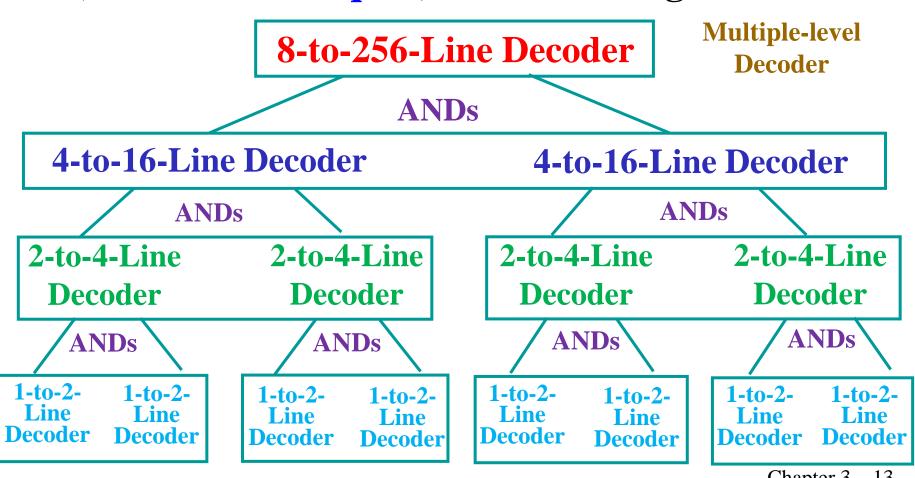
Variable Decoder Expansion



- Large decoders can be constructed by implementing each minterm function using a single AND gate with more inputs.
- Unfortunately, as decoders become larger, this approach gives a high fan-in and gate-input cost.
- We give a procedure that uses hierarchical design and collections of AND gates to construct any decoder with a lower fan-in and gate-input cost by shared terms.

Variable Decoder Expansion (Cont.)

This procedure applies hierarchical design (divide-and-conquer) to build a large decoder.



Variable Decoder Expansion (Cont.)

- This procedure builds a multi-level decoder backward from the outputs:
 - 1. The output AND gates are driven by two decoders with their numbers of inputs either equal or differing by 1.
 - 2. These decoders are then designed using the same procedure until 2-to-1-line decoders are reached.
- The procedure can be modified to apply to decoders with the number of outputs $\neq 2^n$

Decoder Expansion - Example 1

- 3-to-8-Line Decoder
 - Number of output ANDs = 8
 - Hierarchically, divide the input signals equally
 - 2-to-4-Line decoder
 1-to-2-Line decoder
- 2-to-4-Line Decoder
 - Number of output ANDs = 4
 - Divide the input signals equally
 - Two 1-to-2-Line decoder

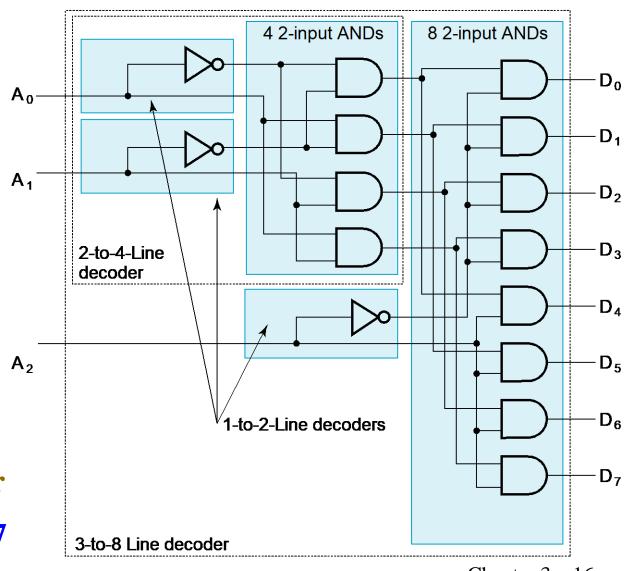
Decoder Expansion - Example 1

Result shared terms $D_0 = A_2 (A_1 A_0)$ $D_1 = A_2 (A_1 A_0)$ $D_2 = A_2 (A_1 A_0)$ $D_3 = A_2 (A_1 A_0)$ $D_4 = A_2 (A_1 A_0)$ $D_5 = A_2 (A_1 A_0)$ $D_6 = A_2 (A_1 A_0)$



 $D_7 = A_2 (A_1 A_0)$

$$GN = 3+2 \times 8+2 \times 4 = 27$$



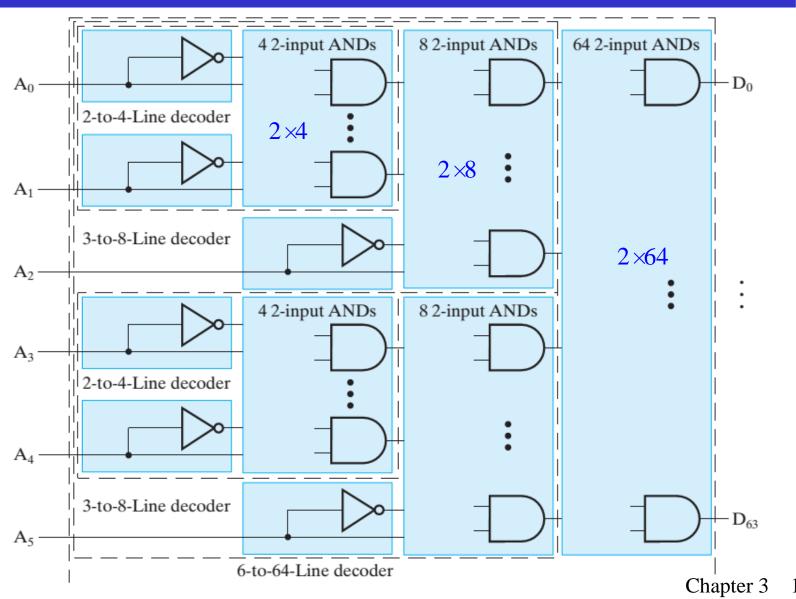
Decoder Expansion - Example 2

- 6-to-64-line decoder
 - Number of inputs to decoders: 6
 - Number of output ANDs = 64
 - Closest possible split to equal
 - 3-to-8-line decoder
 - 3-to-8-line decoder
 - Complete using known 3-to-8 line and 2-to-4 line decoders
- For gate input cost
 - $GN = 6+6\times64 = 390$
 - $GN = 6+2\times64+2\times2\times8+2\times2\times4 = 182$

One-level Decoder

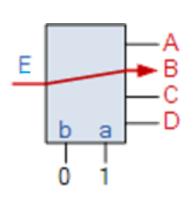
Three-level Decoder

6-to-64-line Decoder

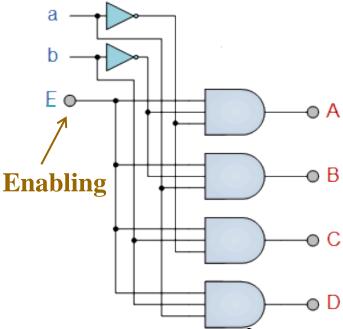


Decoder with Enable

- In general, attach *m*-enabling circuits to the outputs
- See truth table below for function
 - Note use of X's to denote both 0 and 1
 - Combination containing two X's represent four binary combinations
- Alternatively, can be viewed as distributing value of signal EN to 1 of 4 outputs (redefinition)
- In this case, called a demultiplexer

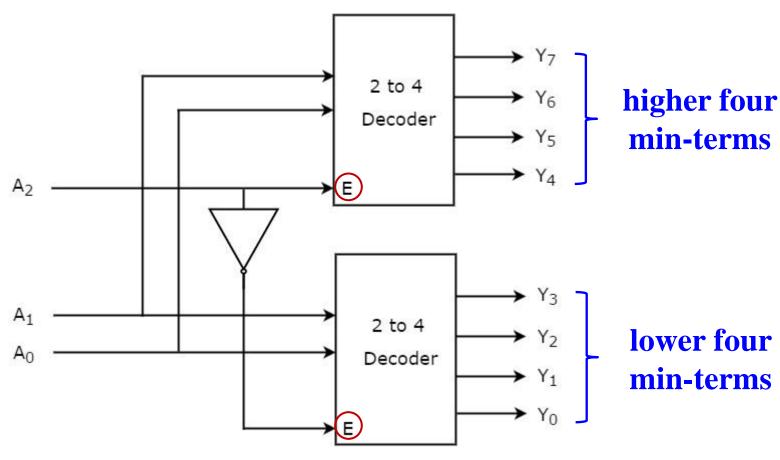


Е	b	a	A	В	C	D
0	X	X	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1



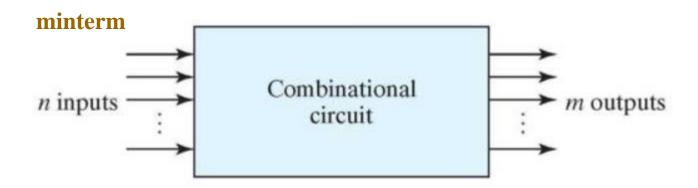
Decoder Expansion-using Lower-Order Decoders with Enable

 Using two 2-to-4-Line Decoders for implementing one 3-to-8-Line Decoder



Combinational Logic Implementation

- Decoder and OR Gates



- Implement m functions of n variables with:
 - Sum-of-minterms expressions
 - One *n*-to-2ⁿ-line decoder
 - m OR gates, one for each output
- Approach
 - Find the truth table for the functions
 - Find the minterms for each output function
 - OR the minterms together

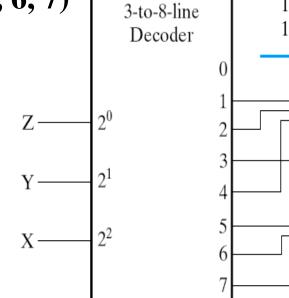
Decoder and OR Gates Example

- Implement a binary Adder
- Finding sum of minterms expressions

$$S(X, Y, Z) = \Sigma_{m}(1, 2, 4, 7)$$

 $C(X, Y, Z) = \Sigma_{m}(3, 5, 6, 7)$

Find circuit



X	Υ	Z	С	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Truth Table

Decoder and OR Gates Example

 Implement the following set of odd parity functions of (W, X, Y, Z)

$$P_1 = W \oplus Y \oplus Z$$

$$P_2 = W \oplus X \oplus Z$$

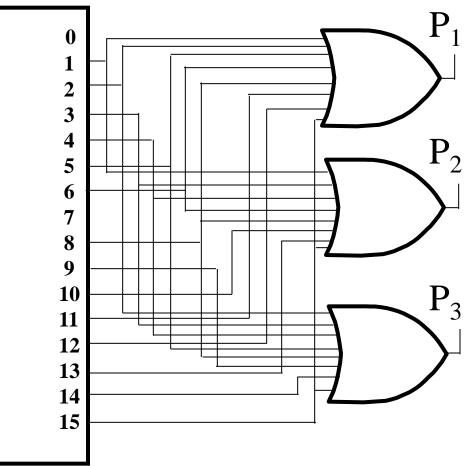
$$P_3 = W \oplus X \oplus Z$$

$$\mathbf{P}_3 = \mathbf{W} \oplus \mathbf{X} \oplus \mathbf{Y} \qquad \mathbf{X}$$

 Finding sum of minterms expressions

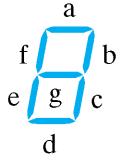
$$\begin{aligned} \mathbf{P}_1 &= \Sigma_{\mathrm{m}}(1,2,5,6,8,11,12,15) \\ \mathbf{P}_2 &= \Sigma_{\mathrm{m}}(1,3,4,6,8,10,13,15) \\ \mathbf{P}_3 &= \Sigma_{\mathrm{m}}(2,3,4,5,8,9,14,15) \end{aligned}$$

- Find circuit
- Is this a good idea?



BCD-to-Segment Decoder (Display Decoder)

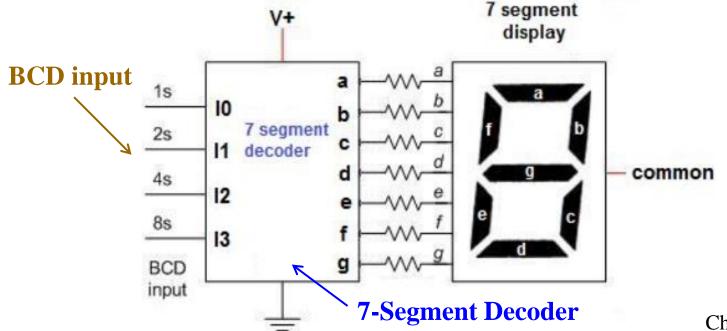
Seven-Segment Displayer



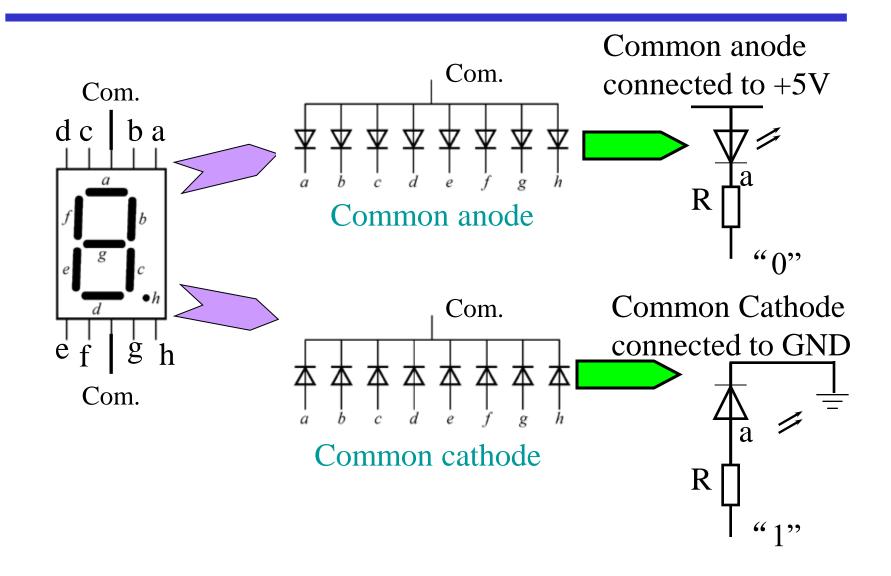


(a) Segment designation

(b) Numeric designation for display



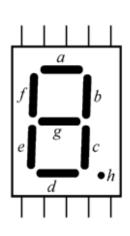
Seven-Segment Decoder



Seven-Segment Decoder (Cont.)

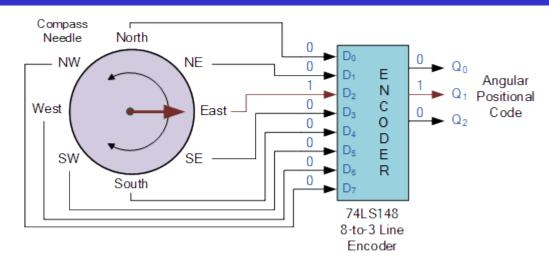
Truth Table for BCD-to-Seven-Segment Decoder

Common cathode



BCD Input			t	Seven-Segment Decoder								
Α	В	С	D		а	b	С	d	е	f	g	
0	0	0	0		1	1	1	1	1	1	0	
0	0	0	1		0	1	1	0	O	O	O	
0	0	1	0		1	1	0	1	1	0	1	
0	0	1	1		1	1	1	1	0	0	1	
0	1	0	0		0	1	1	0	0	1	1	
0	1	0	1		1	0	1	1	0	1	1	
0	1	1	0		1	0	1	1	1	1	1	
0	1	1	1		1	1	1	0	0	0	0	
1	0	0	0		1	1	1	1	1	1	1	
1	0	0	1		1	1	1	1	0	1	1	
All other inputs			outs		0	0	0	0	0	0	0	

Encoding

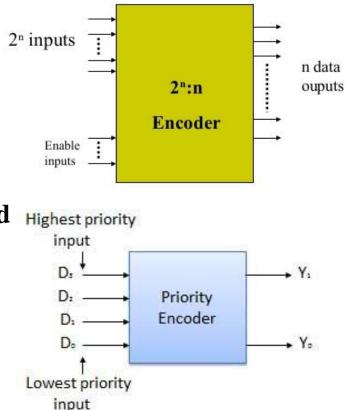


Encoding

- the opposite of decoding the conversion of an m-bit input code to a n-bit output code with $n \le m \le 2^n$ such that each valid code word produces a unique output code
- Circuits that perform encoding are called encoders
- An encoder has 2^n (or fewer) input lines and n output lines which generate the binary code corresponding to the input values

Types of Encoder

- Typically, an encoder converts a code containing exactly one bit that is 1 to a binary code corresponding to the position in which the 1 appears.
- Types of encoder:
 - Decimal-to-BCD Encoder
 - Instruction Encoder
 - Priority Encoder (widely used in computer priority interrupt system and keyboard coding system)
 - Cypher Encoder



Encoder Example

- A decimal-to-BCD encoder
 - Inputs: 10 bits corresponding to decimal digits 0 through 9 (D_0 , ..., D_9) ——one-hot encodings
 - Outputs: 4 bits with BCD codes
 - Function: If input bit D_i is a 1, then the output (A_3, A_2, A_1, A_0) is the BCD code for i,
- The truth table could be formed, but alternatively, the equations for each of the four outputs can be obtained directly.

Encoder Example (continued)

• Input D_i is a term in equation A_j if bit A_j is 1 in the binary value for i.

Input D_i is a term in equation D_i if bit D_i is 1 in the binary value for i.

Equations:

$$A_3 = D_8 + D_9$$

$$A_2 = D_4 + D_5 + D_6 + D_7$$

$$A_1 = D_2 + D_3 + D_6 + D_7$$

$$A_0 = D_1 + D_3 + D_5 + D_7 + D_9$$

one-hot → binary

• $F_1 = D_6 + D_7$ can be extracted from A_2 and A_1 Is there any cost saving?

Priority Encoder

- If more than one input value is 1 (e.g., binary), then the encoder just designed does not work.
- One encoder that can accept all possible combinations of input values and produce a meaningful result is a *priority encoder*.
- Among the 1s that appear, it selects the most significant input position (or the least significant input position) containing a 1 and responds with the corresponding binary code for that position.

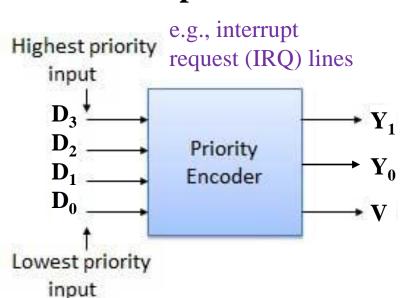
Priority Encoder Example 1

• There are four input D_3 , D_2 , D_1 , D_0 and two output Y_1 , Y_0 . D_3 has the highest priority and D_0 has the lowest priority.

• If $D_3 = 1$ then $Y_1Y_0 = 11$ irrespective of the other inputs.

• If $D_3 = 0$ and $D_2 = 1$ then $Y_1Y_0 = 10$ irrespective of the

other inputs.



Priority Encoder Example 2

Priority encoder with 5 inputs $(D_4, D_3, D_2, D_1, D_0)$ - highest priority to most significant 1 present - Code outputs A2, A1, A0 and V where V indicates at least one 1 present.

No. of Min-]	Input	S					
terms/Row	D4	D3	D2	D1	D 0	A2	A1	A0	V
0	0	0	0	0	0	X	X	X	0
1	0	0	0	0	1	0	0	0	1
2	0	0	0	1	X	0	0	1	1
4	0	0	1	X	X	0	1	0	1
8	0	1	X	X	X	0	1	1	1
16	1	X	X	X	X	1	0	0	1

Xs in input part of table represent 0 or 1; thus table entries correspond to product terms instead of minterms. The column on the left shows that all 32 minterms are present in the product terms in the table.

Priority Encoder Example 2 (continued)

Could use a K-map to get equations, but can be read directly from table and manually optimized if careful:

$$A_2 = D_4$$

$$A_1 = \overline{D}_4 D_3 + \overline{D}_4 \overline{D}_3 D_2 = \overline{D}_4 (D_3 + D_2) = \overline{D}_4 F_1$$

$$A_0 = \overline{D}_4 D_3 + \overline{D}_4 \overline{D}_3 \overline{D}_2 D_1 = \overline{D}_4 (D_3 + \overline{D}_2 D_1)$$

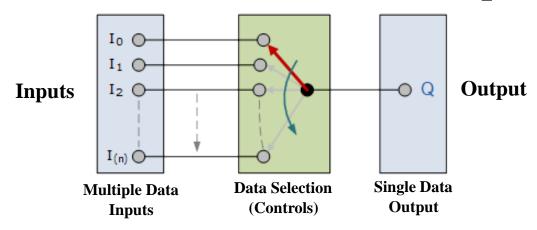
$$V = D_4 + D_3 + D_2 + D_1 + D_0 = D_4 + F_1 + D_1 + D_0$$

Selecting

- Selecting of data or information is a critical function in digital systems and computers
- Circuits that perform selecting have:
 - A set of information inputs from which the selection is made
 - A set of control lines for making the selection
 - A single output
- Logic circuits that perform selecting are called multiplexers
- Selecting can also be done by decoder plus **AND-OR** gates or three-state buffers

Multiplexers

A multiplexer selects information from an input line and directs the information to an output line



- A typical multiplexer has n control inputs $(S_{n-1}, ...$ S_0) called selection inputs, 2^n information inputs (I_2^n) $_{1}, \dots I_{0}$), and one output Y
- A multiplexer can be designed to have m information inputs with $m < 2^n$ as well as n selection inputs

2-to-1-Line Multiplexer

- Since $2 = 2^1$, n = 1
- The single selection variable S has two values:
 - S = 0 selects input I_0
 - S = 1 selects input I_1
- The equation:

$$\mathbf{Y} = \overline{\mathbf{S}}\mathbf{I}_0 + \mathbf{S}\mathbf{I}_1$$

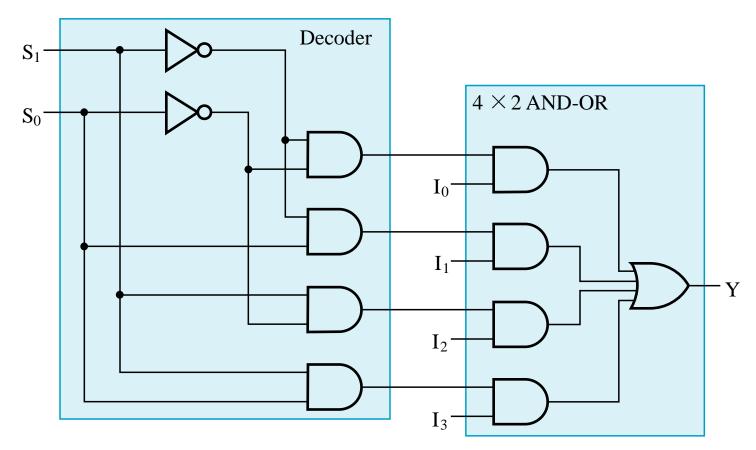
2-to-1-Line Multiplexer (continued)

- Note the regions of the multiplexer circuit shown:
 - 1-to-2-line Decoder
 - 2 Enabling circuits
 - 2-input OR gate
- To obtain a basis for multiplexer expansion, we combine the Enabling circuits and OR gate into a 2 × 2 AND-OR circuit:
 - 1-to-2-line decoder
 - 2×2 AND-OR
- In general, for an 2^n -to-1-line multiplexer:
 - n-to- 2^n -line decoder
 - $2^n \times 2$ AND-OR

Example: 4-to-1-line Multiplexer

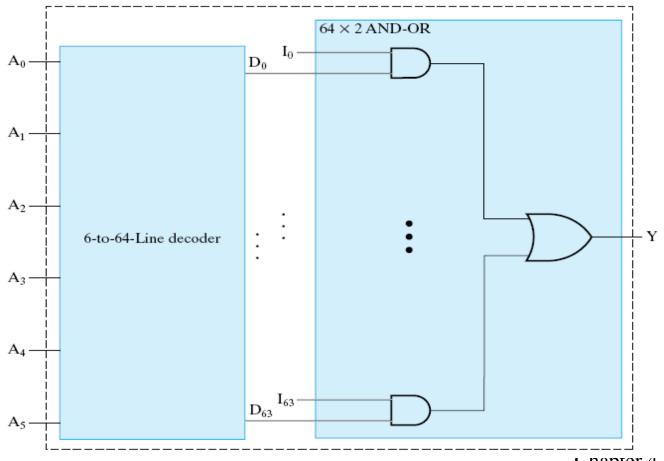
- 2-to-2²-line decoder
- $^{\circ}$ 2² × 2 AND-OR

$$GN = 4+8+8+2 = 22$$



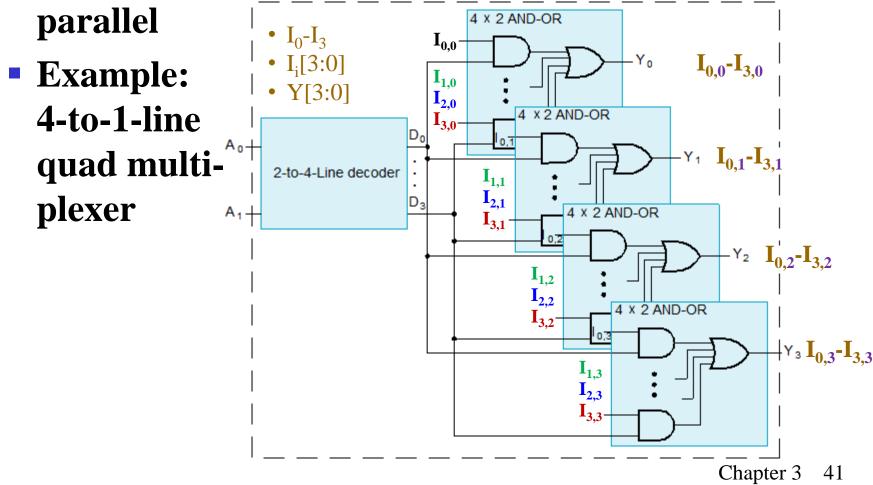
Example: 64-to-1-line Multiplexer

- 6-to-2⁶-line decoder
- $^{\circ}$ 2⁶ × 2 AND-OR



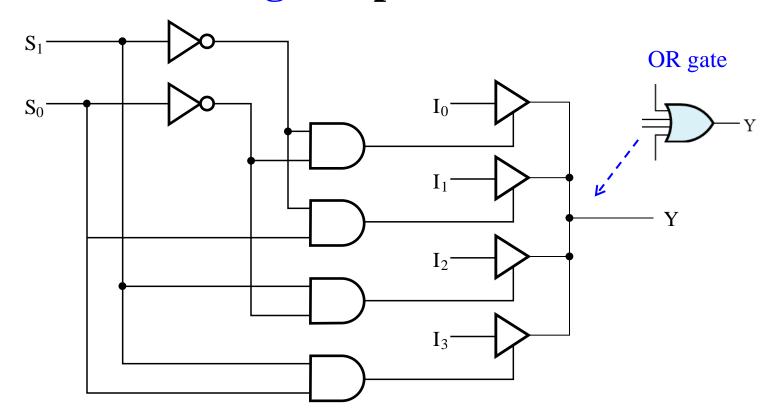
Multiplexer Width Expansion

- Select "vectors of bits" instead of "bits"
- Use multiple copies of $2^n \times 2$ AND-OR in



Other Selection Implementations

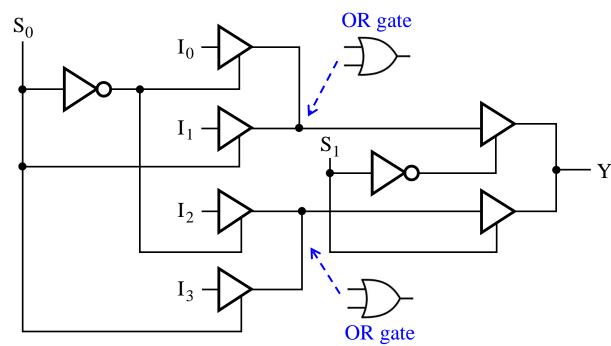
Three-state logic in place of AND-OR



- Gate input cost with NOTs:18 (8+8+2)
- Gate input cost with NOTs of AND-OR gates: 22

Other Selection Implementations

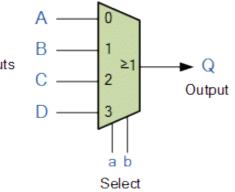
Distributing the decoding across the three-state
 drivers so



- Gate input cost with NOTs of AND-OR gates: 22
- Gate input cost with NOTs of 3-state drivers: 18
- Gate input cost with NOTs: 14 (4+8+2)

Combinational Logic Implementation

- Multiplexer Approach 1
- Implement m functions of n variables with:
 - Sum-of-minterms expressions
 - An m-wide 2^n -to-1-line multiplexer
- Design:
 - Find the truth table for the functions.
 - In the order they appear in the truth table:
 - Apply the function input variables to the multiplexer selection inputs $S_{n-1}, ..., S_0$
 - Label the outputs of the multiplexer with the output variables
 - Value-fix the information inputs to the multiplexer using the values from the truth table (for don't cares, apply either 0 or 1)



Example: Gray to Binary Code

- Design a circuit to convert a 3-bit Gray code to a binary code
- The formulation gives the truth table on the right
- It is obvious from this table that X = C and the
 Y and Z are more complex

Gray	Binary	
ABC	хуz	
000	000	
100	0 0 1	
110	010	
010	0 1 1	
011	100	
111	101	
101	110	
001	1 1 1	

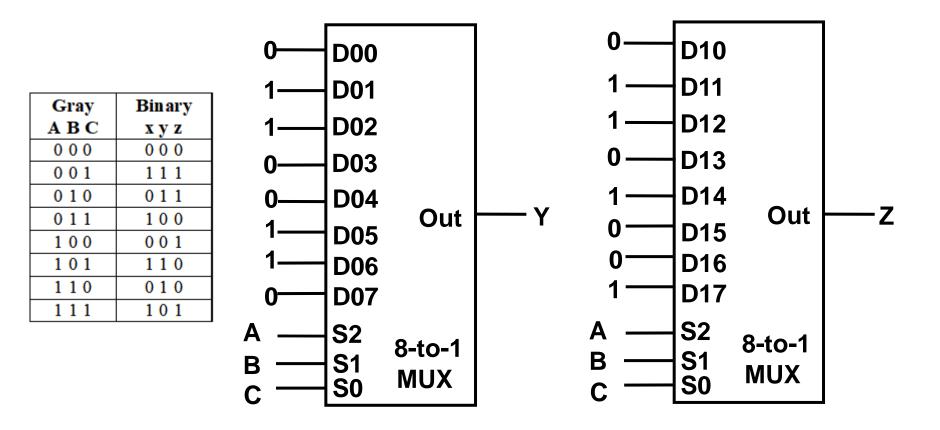
Gray to Binary (continued)

- Rearrange the table so that the input combinations are in counting order
- Functions y and z can be implemented using a dual 8-to-1-line multiplexer by:

Gray	Binary
A B C	хуz
$0 \ 0 \ 0$	0 0 0
0 0 1	1 1 1
010	0 1 1
0 1 1	100
100	0 0 1
101	1 1 0
110	0 1 0
1 1 1	101

- connecting A, B, and C to the multiplexer select inputs
- placing y and z on the two multiplexer outputs
- connecting their respective truth table values to the inputs

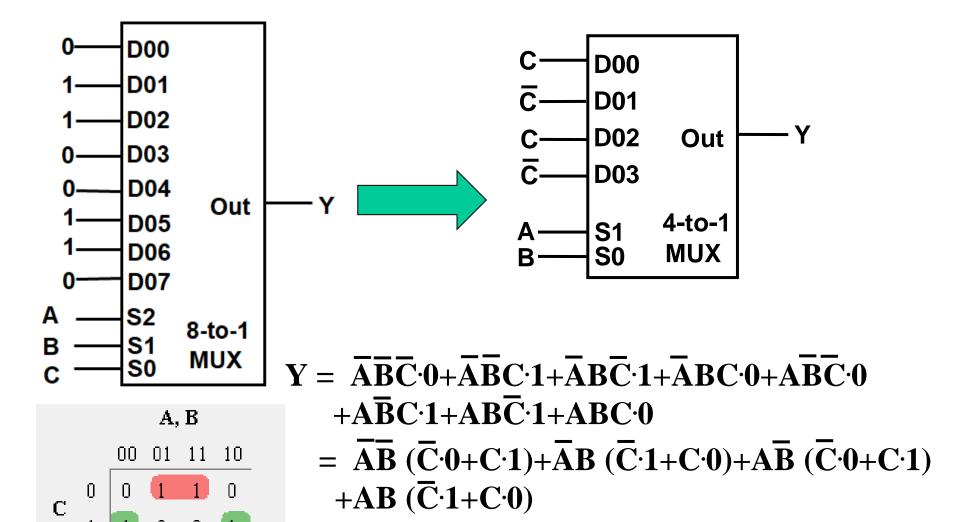
Gray to Binary (continued)



Note that the multiplexer with fixed inputs is identical to a ROM with 3-bit addresses and 2-bit data!

Combinational Logic Implementation

- Multiplexer Approach 2



 $= \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} = \sum m(1,2,5,6)$

Combinational Logic Implementation

- Multiplexer Approach 2
- Implement any m functions of n variables by using:
 - An m-wide 2^{n-1} -to-1-line multiplexer
 - A single inverter
- Design:
 - The first n 1 variables are applied to the selection inputs.
 - For each combination of the selection variables, the output is a function of the last variable $(0, 1, X, \overline{X})$.
 - These values are then applied to the appropriate data inputs.

Example: Gray to Binary Code

- Design a circuit to convert a 3-bit Gray code to a binary code
- The formulation gives the truth table on the right
- It is obvious from this
 table that X = C and the
 Y and Z are more complex

Gray A B C	Binary
	хух
0 0 0	0 0 0
100	0 0 1
110	010
010	0 1 1
011	100
111	101
101	110
001	1 1 1

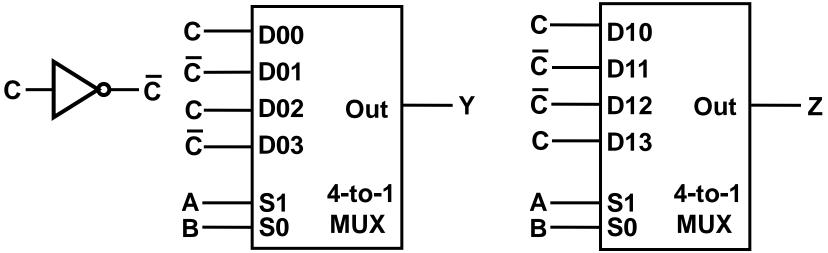
Gray to Binary (continued)

Rearrange the table so that the input combinations are in counting order, pair rows, and find rudimentary functions

Gray A B C	Binary x y z	Rudimentary Functions of C for y	Rudimentary Functions of C for z
000	000	$\mathbf{F} = \mathbf{C}$	$\mathbf{F} = \mathbf{C}$
0 1 0 0 1 1	0 1 1 1 0 0	$\mathbf{F} = \overline{\mathbf{C}}$	$\mathbf{F} = \overline{\mathbf{C}}$
100	001	$\mathbf{F} = \mathbf{C}$	$\mathbf{F} = \overline{\mathbf{C}}$
110 111	0 1 0 1 0 1	$\mathbf{F} = \overline{\mathbf{C}}$	$\mathbf{F} = \mathbf{C}$

Gray to Binary (continued)

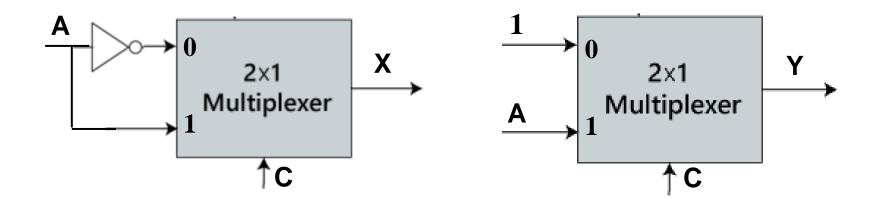
Assign the variables and functions to the multiplexer inputs:



- Note that this approach (Approach 2) reduces the cost by almost half compared to Approach 1.
- This result is no longer ROM-like.
- **Extending, a function of more than** *n* **variables is decomposed into several** <u>sub-functions</u> defined on a subset of the variables. The multiplexer then selects among these sub-functions.

Find Boolean Function

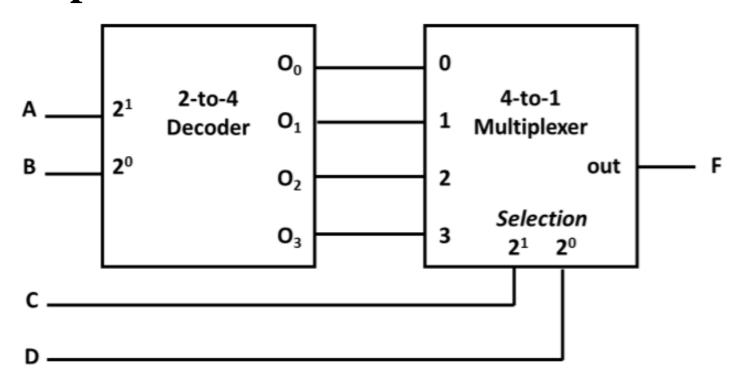
 Problem 1: Please find the logic expressions of output X and Y.



- X = ____
- Y = ____

Find Boolean Function (continued)

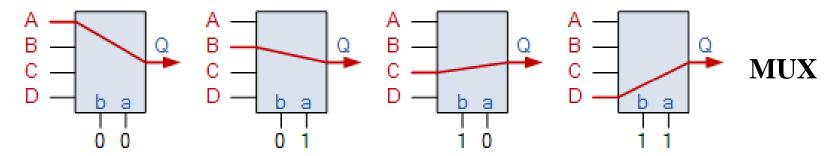
 Problem 2: Please find the logic expression of output F.



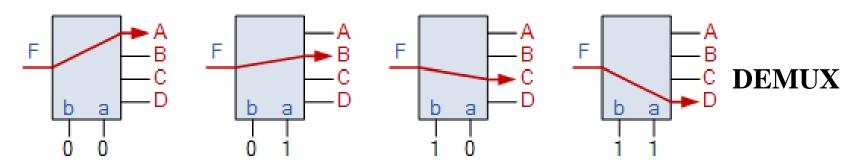
• F = _____

Multiplexer and Demultiplexer

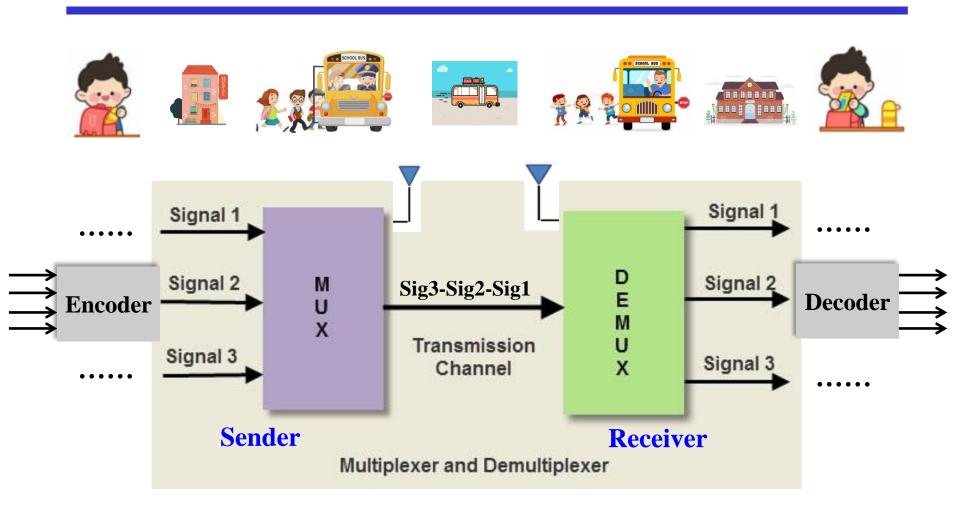
 Multiplexer means many into one, which is used to select one of the several input signals to a signal output.



The demultiplexer means one into many, which takes one single input data line and then switches it to any one of a number of individual output lines.



Multiplexer and Demultiplexer



An example of a communication system

Assignments

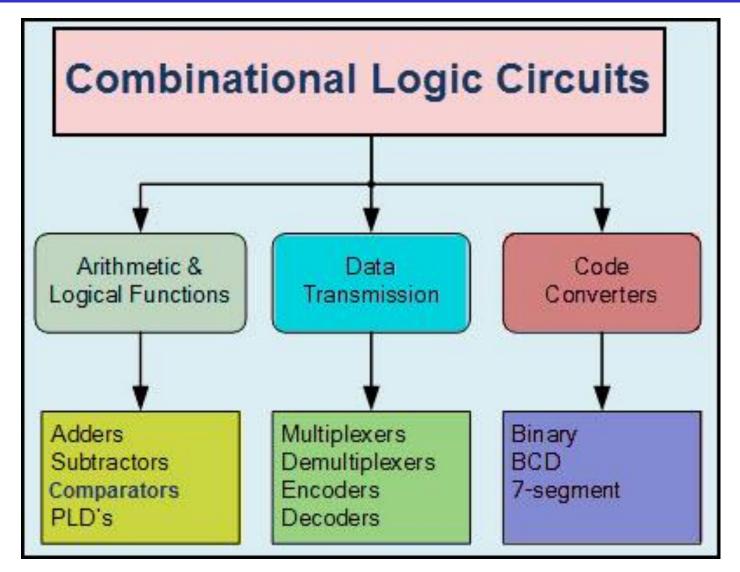
Reading

■ 3.4 – 3.7

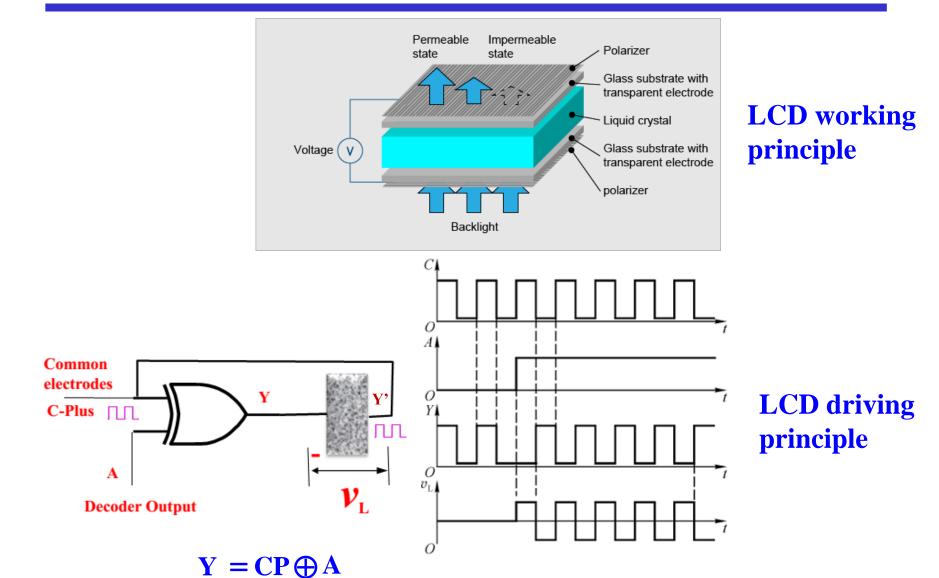
Problem assignment

3-28, 3-29, 3-37, 3-44, 3-47

Appendix A: Classification of Combinational Logic

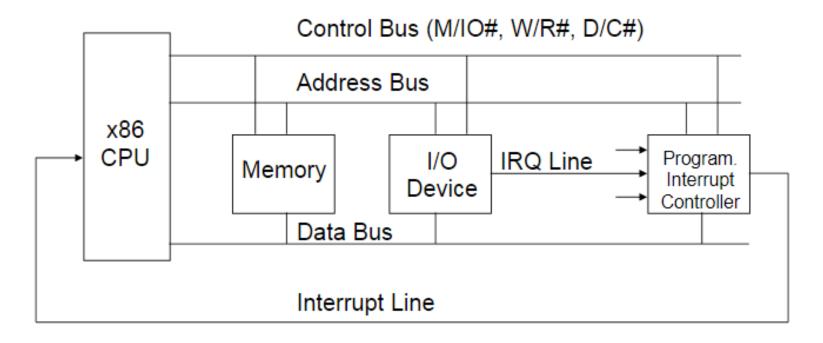


Appendix B: LCD Driving Principle



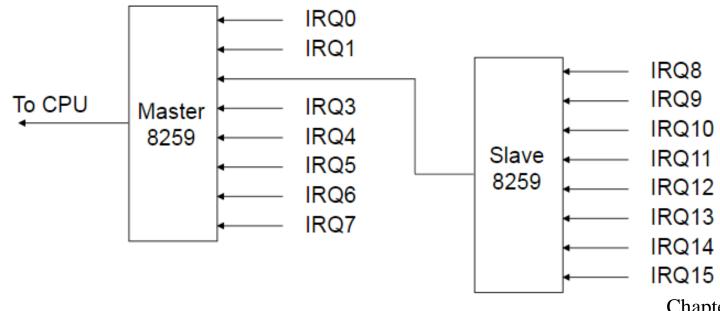
Appendix C: Priority Encoder Example 1

- Adding interrupt to the hardware
 - IRQ line from I/O device to Programmable Interrupt Controller (PIC)
 - Interrupt line from PIC to CPU



Priority Encoder Example 1 (continued)

- Programmable Interrupt Controller (8259A chip)
 - Support eight interrupt request (IRQ) lines
 - Two chips used in PC, called "master" and "slave"
 - Priority: highest to lowest order is IRQ(0-1, 8-15, 3-7)
 - Asserts INTR to CPU, responds to resulting INTA with interrupt type code



Appendix D: Transmission Gate Multiplexer

Transmission Gate Multiplexer

Gate input

cost = 8

compared

to 14 for

3-state logic

and 18 or 22

for gate logic 12-

