Logic and Computer Design Fundamentals

Term Review

Ming Cai cm@zju.edu.cn College of Computer Science and Technology Zhejiang University

Items of Final Exam

- 1. Fill in the blank
- 2. Chose the best Choice
- 3. Verilog
- 4. Kaunaugh Map and Optimization
- 5. Circuit Analysis
 - Combinational circuit
 - Sequential circuit
- 6. Logic Design
 - Combinational circuit
 - Sequential circuit

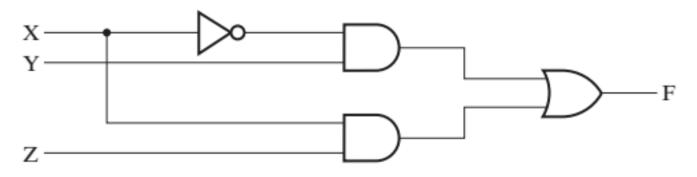
Some Tips for Final Exam

1. Need to understand the terminology!

2. 答题纸内容,一定要字迹整洁、清晰,不然自己吃亏!

Verilog Example

Please use Verilog to describe the following circuit. (No simplification is needed!)



module test(X, Y, Z, F)input X, Y, Z; output F;

assign
$$F = \underline{\qquad \sim X\&Y \mid X\&Z \qquad }$$

endmodule

Course Review

HIGHLIGHTS & PROBLEMS

- Conversion between number systems
 - Binary number, hexadecimal number
 - Eg. $(1101\ 1111)_2 = (DF)_{16}$

Decimal	Binary	Octal	Hexadecimal
369.3125	101110001.0101	561.24	171.5
189.625	10111101.101	275.5	BD.A
214.625	11010110.101	326.5	D6.A
62407.625	1111001111000111.101	171707.5	F3C7.A

- Conversion between binary number and decimal code
 - BCD (binary-coded decimal)
 - Eg. $(1001\ 0101)_{BCD} \longrightarrow (95)_{10} = (0101\ 1111)_2 = (5F)_{16}$
 - Parity Bit
 - $100\ 0001 \rightarrow 0100\ 0001$ (with even parity) $\rightarrow 1100\ 0001$ (with odd parity)
 - How to generate odd parity bit P for any 5-bit binary number $D_4D_3D_2D_1D_0$?
- Gray Code & ASCII Character Code

- Boolean algebra
 - DeMorgan's law: $(\overline{X} + \overline{Y}) = \overline{X} \overline{Y}$ and $(\overline{X}\overline{Y}) = \overline{X} + \overline{Y}$
 - Distributive laws: X+YZ=(X+Y)(X+Z)
 - Dual of an algebraic expression: OR $\leftarrow \rightarrow$ AND, 0's $\leftarrow \rightarrow$ 1's
 - Consensus theorem: XY+XZ+YZ=XY+XZ, (X+Y)(X+Z)(Y+Z)=(X+Y)(X+Z)
- Complement of a function
 - OR $\leftarrow \rightarrow$ AND, 0's $\leftarrow \rightarrow$ 1's, $X \leftarrow \rightarrow \overline{X}$
- Canonical forms, Standard forms
 - Minterms, Maxterms, Canonical forms (SOM, POM)
 - Product terms, sum terms, SOP, POS
 - Relationship between SOM and POM? SOM and SOP?
- Two-level circuit optimization
 - Cost criteria: gate input cost
 - Karnaugh map (K-map), Prime Implicants, Essential Prime Implicants
 - Simplifying in SOP form (with don't care conditions)

- Other gates
 - NAND/NOR, XOR/XNOR, Odd/Even Function, Buffer, 3-state buffer
- Exclusive-OR operator and gates
 - Identities of XOR operation:

$$X \oplus Y = X \oplus Y = (X \oplus Y) \qquad X \oplus (Y \oplus Z) = (X \oplus Y) \oplus Z = (\overline{X \oplus Y \oplus Z})$$

- Odd function and even function
 - Use odd function to generate even parity bit
 - Use even function to generate odd parity bit
 - The even function is obtained by replacing the output gate with an XNOR gate. $P_{odd} = X \oplus Y \oplus Z$, $P_{even} = \overline{P_{odd}} = (X \oplus Y \oplus Z)$
- High-impedance outputs
 - 3-state buffer

(EN)	IN	OUT
0	X	Hi-Z
1	0	0
1	1	1

Problems:

- The dual of an algebraic expression is obtained by 1) <u>interchanging OR and AND operations</u> and 2) 1. replacing 1's by 0's and 0's by 1's.
- Use DeMorgan's Theorem to complement a function: 1) <u>interchange AND and OR operators</u> and 2) 2. complement each constant value and literal
- Four variables odd function has _____ C___ "1" squares in its corresponding K-Map. 3.
 - A. 4
- B. 7

C. 8

- D. 14
- The gate input cost G of function F = AB(C+D) + C(BD+AD) is __A_. 4.
 - A. 15
- B. 14

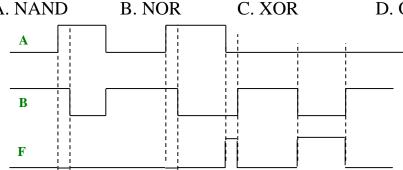
C. 13

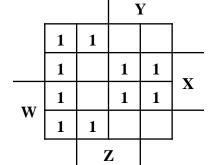
- D. 12
- Which of the following logical gates can be used as a controllable inverter? 5.
 - A. AND gate
- B. XOR gate
- C. Buffer gate
- D. OR gate
- The Essential Prime Implicants in the K-Map given below are ____B___. 6.
 - A. Y'Z', XZ'
- B. X'Y', XY C. XY, XZ'
- D. Y'Z', X'Y'
- Given below are the waveforms of input A, B and output F of a logic device. Then the device is a 7. A gate.





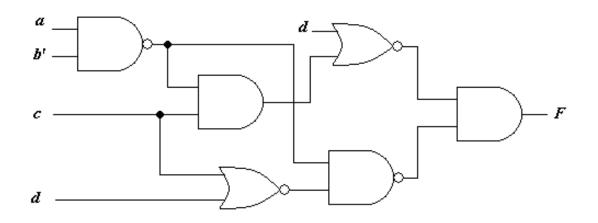






Problems:

According to the following logic circuit diagram, write down the corresponding Boolean function and optimize it to the form of SOP

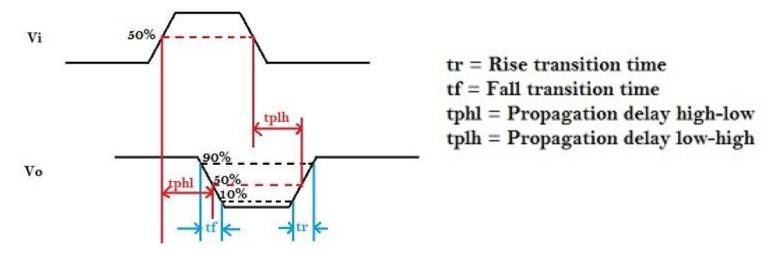


2 Gvena Boolean function

$$F(W,X,Y,Z) = \sum m(4,6,7,8,12,15) + \sum d(2,3,5,10,11)$$

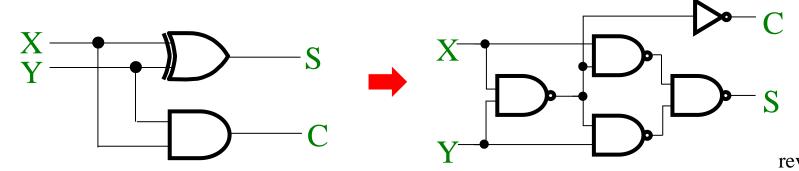
Optimize Ftogether with the don't-care conditions dusing a K-map

- Technology parameters
 - fan-in, fan-out, noise margin, cost, transition time, propagation delay, power dissipation



- Delay Model: transport delay, inertial delay, rejection time
- How to calculate gate delay based on fan-out?

- Methods of Describing Logic Events
 - Truth Table, Timing Diagram, Boolean Function, Karnaugh Maps, Logic Circuit
- Design procedure: specification, formulation, optimization, technology mapping, verification
 - Hierarchical Design
- Seven-segment display
 - How to design a BCD-to-Seven-Segment decoder? example 3-2
- Technology mapping
 - How to implement a Boolean function with NAND gates?



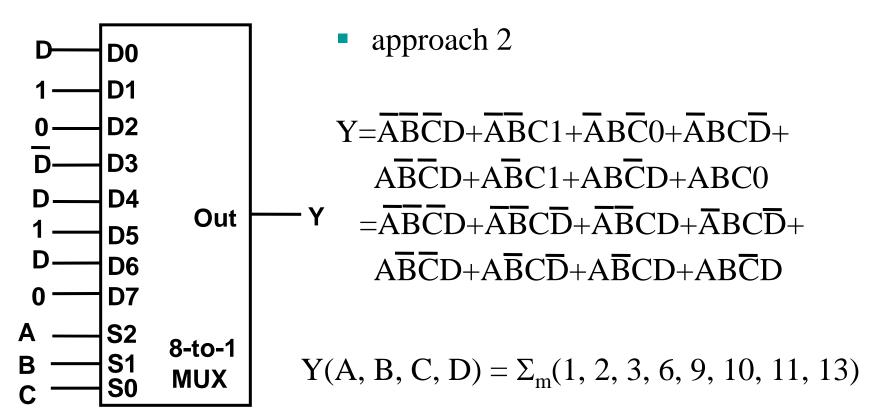
- n-to-m-Line Decoder
 - n inputs and m outputs with $n \le m \le 2^n$
- m-to-n-Line Encoder
 - m inputs and n outputs with $n \le m \le 2^n$
- Multiplexer
 - n control inputs (selection inputs), m inputs and one output with $m < 2^n$
- Demultiplexer: Decoder with Enable
- Combinational Function Implementation
 - Decoders and OR gates
 - Multiplexers
 - ROMs
 - PALs: doesn't provide full decoding of the variables, so it doesn't generate all the minterms
 - PLAs: similar to the PALs
 - Lookup Tables

• Problem: Give the canonical sum of product expression for the function which is implemented using the following circuit.

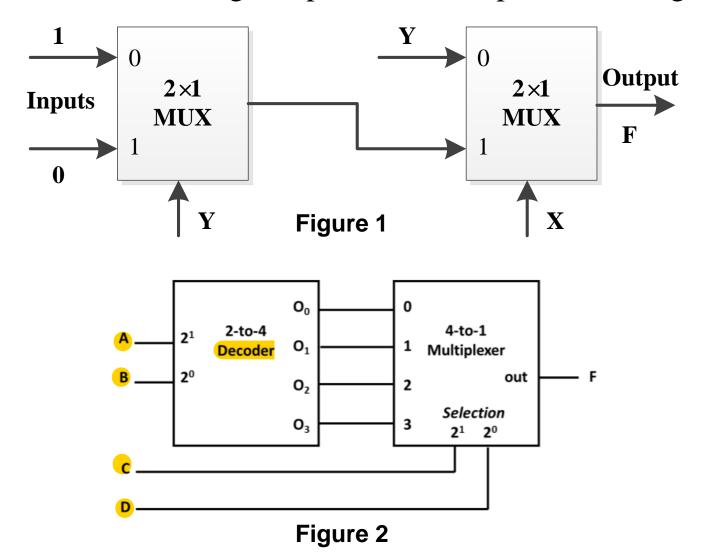
A	В	С	D	Y		¬
0	0	0	0	0	D D0	
0	0	0	1	1		
0	0	1	0	1	1 — D1	
0	0	1	1	1	∫' 0——D2	
0	1	0	0	0	\overline{D} D3	
0	1	0	1	0		
0	1	1	0	1	\Box D—D4	v
0	1	1	1	0	\overline{D} 1 — D5 Out	"
1	0	0	0	0	$D \longrightarrow D6$	
1	0	0	1	1	_	
1	0	1	0	1	0 — D7	
1	0	1	1	1	Δ \$2	
1	1	0	0	0	D 61 8-t0-1	
1	1	0	1	1		
1	1	1	0	0		_
1	1	1	1	0	0	

 $Y(A, B, C, D) = \Sigma_m(1, 2, 3, 6, 9, 10, 11, 13)$

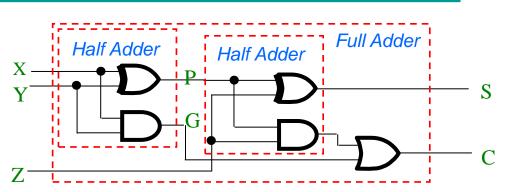
• Problem: Give the canonical sum of product expression for the function which is implemented using the following circuit.

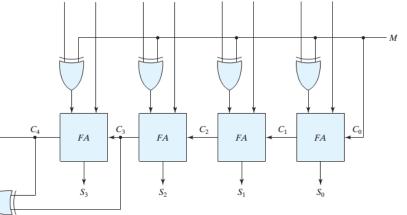


Problem: Find the logic expression of output F in the figure.



- Half Adder
- Full Adder
 - carry generate: X Y
 - *carry propagate:* X⊕Y
- Binary Ripple Carry Adder
- Carry Lookahead Adder
- Binary subtraction
 - Unsigned 2's Complement subtraction
 - Signed 2's Complement subtraction
 - **1**100 0011 = 1001
 - 0011 1100 = 0111
- Signed-Magnitude Arithmetic _c_
- Signed-Complement Arithmetic

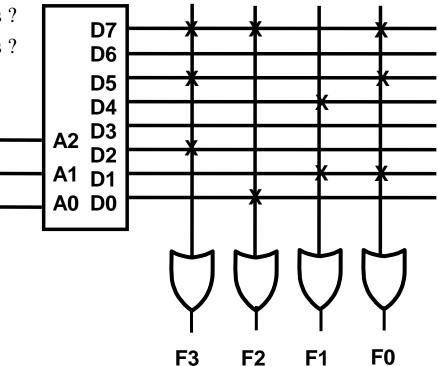




- Programmable implementation technologies
 - PROM, PAL, PLA, FPGA

AND	OR	DEVICE
Fixed	Fixed	Not Programmable
Fixed	Programmable	PROM
Programmable	Fixed	PAL
Programmable	Programmable	PLA

- Programmable implementation technologies
 - PROM
 - Can any combinational circuit with *n* inputs and *m* outputs be implemented with
 - a PROM with *n* inputs and *m* outputs?
 - a PLA with *n* inputs and *m* outputs ?
 - a PAL with *n* inputs and *m* outputs?



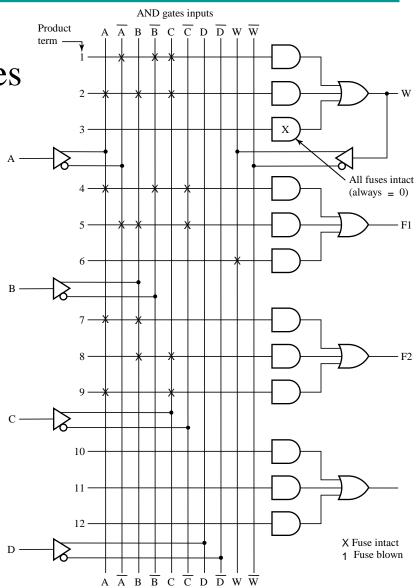
Programmable implementation technologies

PAL

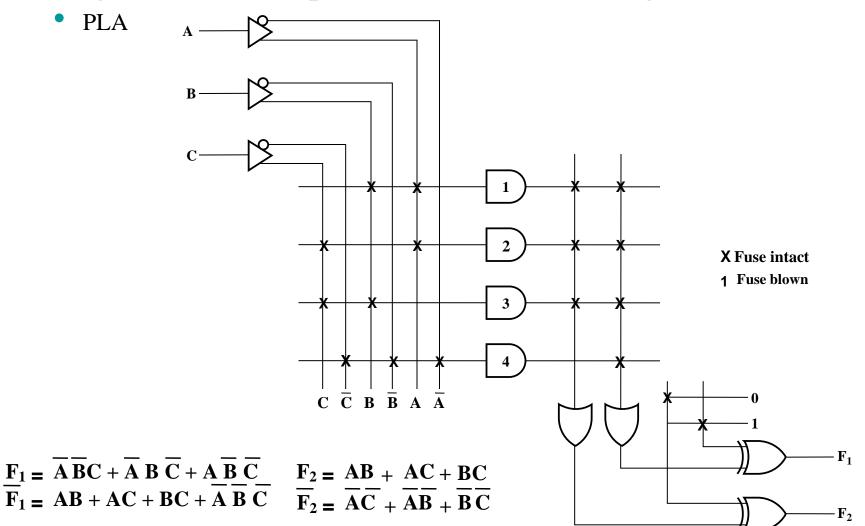
W=ABC+AB(

F1=X= ABC+ABC+W

F2=Y= **AB**-B C-A (



- Programmable implementation technologies
 - PLA



- Latches and flip-flops
- Master-slave flip-flop and edge-triggered flip-flop
 - SR master-slave flip-flop
 - edge-triggered D flip-flop
- Direct inputs: preset (direct set), clear (direct reset)
- Flip-flop timing
 - Setup time, hold time, propagation delay time
 - Timing Equations: $t_p = (t_{pd,FF} + t_{pd,COMB} + t_s) + t_{slack}$ for t_{slack} greater than or equal to zero, $t_p \ge \max (t_{pd,FF} + t_{pd,COMB} + t_s)$ for all paths from flip-flop output to flip-flop input
 - How to calculate the maximum frequency of operation of circuit?

Sequential circuit analysis

- Input equation
- Next state equation
- Output equations: Mealy model circuits, Moore model circuits
- State table: present state, input, next state, output
- State diagram: don't-care condition

Sequential circuit design

- State diagram: state minimization, state assignment, don't-care condition
- State table
- Excitation equation
- Input equations
- Output equations
- Optimization
- Technology Mapping
- Verification

Difference between Latches and Flip-Flops

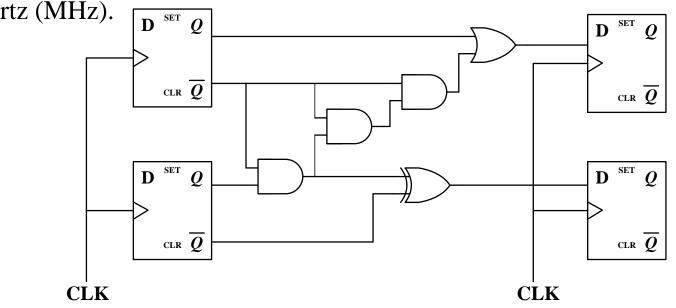
Latch	Flip-Flop
Latches are level sensitive devices	Flip-flops are edge sensitive devices
Latches are sensitive to glitches	Flip-flops are immune to glitches
Latches take less gates and power	Flip-flops take more gates and power
Latches are faster	Flip-flops are slower

- Latches are transparent.
- Master-slave flip-flops use alternating clocks to break the path from input to output.
- S-R/J-K master-slave flip-flops have "1s catching" behavior.
- Edge-triggered flip-flops respond to the input at a well-defined moment (at the clock-transition).

- Four basic descriptors for flip-flops
 - Characteristic (truth) tables
 - Characteristic equations
 - Excitation tables
 - Excitation equations
- Four different types of flip-flops
 - SR (S-Set, R-Reset) flip-flop
 - JK (J-Set, K-Reset) flip-flop
 - D (Data or Delay) flip-flop
 - T (Toggle) flip-flop
- Transformation among flip-flops
 - Mealy model circuits and Moore model circuits
 - Flip-Flop Conversion

Problem: Sequential circuit analysis

The timing parameter in the sequential circuit are as follows: AND Gate: $t_{pd} = 7.0$ ns; OR Gate: $t_{pd} = 8.0$ ns; XOR Gate: $t_{pd} = 11.0$ ns; Flip-flop: $t_{pd} = 7.0$ ns, $t_{s} = 4.0$ ns, $t_{h} = 2.0$ ns. Determine the maximum frequency of operation of the circuit in megahertz (MHz).



1) find the longest path delay from clock edge to clock:

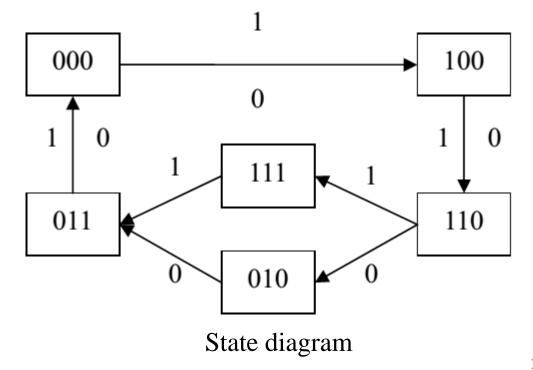
 $t_{delay\text{-clock edge to clock edg}} = FF t_{pd} + 3*AND t_{pd} + OR t_{pd} + FF t_{s} = 7 + 3*7 + 8 + 4 = 40 \text{ ns}$

2) determine the maximum frequency: $f_{max} = 1/t = 25 \text{ MHz}$

Problem: Sequence generation

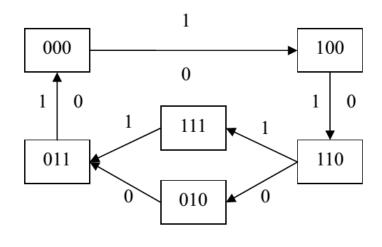
Design a controllable sequence counter. When control input C = 1, counter sequence is $000 \rightarrow 100 \rightarrow 110 \rightarrow 111 \rightarrow 011 \rightarrow 000$; When C = 0, sequence is $000 \rightarrow 100 \rightarrow 110 \rightarrow 011 \rightarrow 000$. Please derive the state diagram, state table, next state functions and output functions, and draw the circuit diagram.

Answer:



Problem: Sequence generation

Answer:



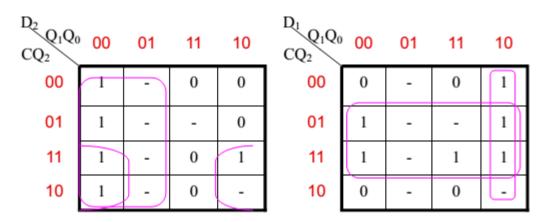
State diagram

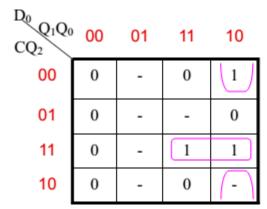
Current	Next State				
State	C = 0	C = 1			
Q2Q1Q0	Q2'Q1'Q0'	Q2'Q1'Q0'			
000	100	100			
001	XXX	XXX			
010	011	XXX			
011	000	000			
100	110	110			
101	XXX	XXX			
110	010	111			
111	XXX	011			

State table

Problem: Sequence generation

Answer:





Input equations

$$D_2 = \overline{Q}_1 + C\overline{Q}_0$$

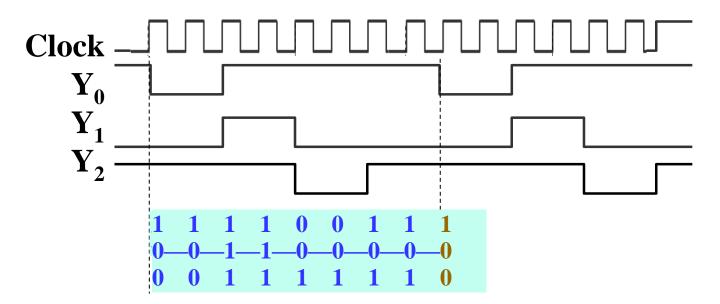
$$D_1 = Q_2 + Q_1\overline{Q}_0$$

$$D_0 = CQ_2Q_1 + \overline{Q}_2Q_1\overline{Q}_0$$

Problem: Sequence generation

Design a waveform generator using D flip-flops and NOR gates. The waveforms of output Y0~Y2 are shown below. Requirement:

- Draw the Moore state diagram for the circuit.
- Find the state table and make a state assignment.
- Derive the next state functions and output functions.
- Design the circuit using D flip-flops and NOR gates, draw the circuit diagram.



- Register Transfer Operations (RTL)
 - Microoperations
 - Let R1 = 10101010 and R2 = 11110000, then after the operation, R0 becomes:
 - Conditional Transfer
- Register Transfer Structures
 - Multiplexer-Based Transfers
 - Bus-Based Transfers
 - Three-State Bus
 - Other Transfer Structures
- Shift Registers
 - Parallel Load Shift Registers
 - Bidirectional Shift Register

R0	Operation
10101010	R0 ← R1
11111010	$R0 \leftarrow R1 \lor R2$
10100000	R 0 ← R 1 ∧ R 2
01011010	R 0 ← R 1 ⊕ R 2
01010100	R 0 ← sl R 1

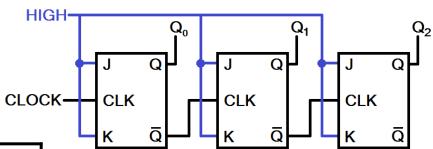
Mode Control		Register
$\mathbf{S_1}$	S_0	Operation
0	0	No change
0	1	Shift down
1	0	Shift up
1	1	Parallel load

Counters

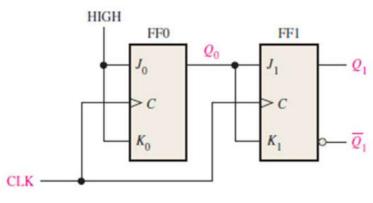
- Asynchronous/Ripple Counter
- Synchronous Counter
 - Counter with Parallel Load

Load	Count	Action	
0	0	Hold Stored Value	
0	1	Count Up Stored Value	
1	X	Load D	

- Divide-by-n (Modulo n) Counter
 - A synchronous 4-bit binary counter with a synchronous load and an asynchronous clear is used to make a Modulo 7 counter
 - How to design a Modulo-17 counter?



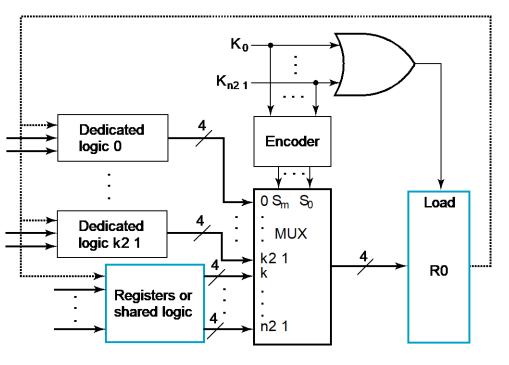
Asynchronous/Ripple Counter

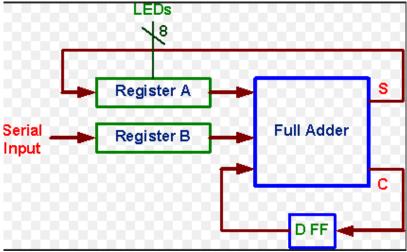


Synchronous Counter

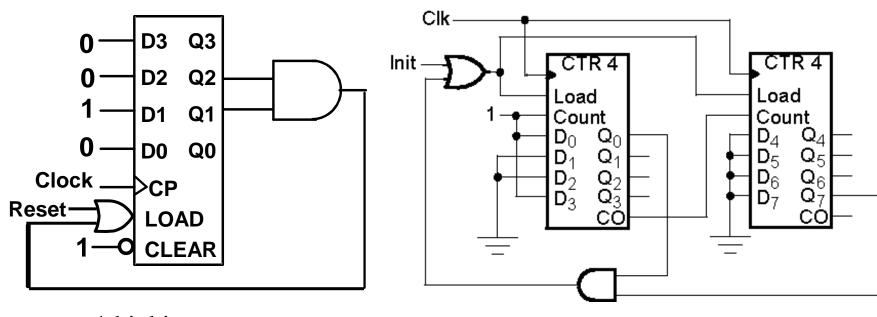
- Register Cell Design
 - Sequential Circuit Design Approach
 - Multiplexer Approach

Serial microoperations





Problem: Circuit Analysis



a 4-bit binary counter

0010-0110

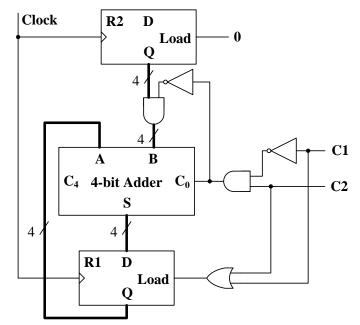
an 8-bit binary counter

00001001-10000001

Problem: Circuit Analysis

Analyze the following register transfer circuit, finish the function table, and write down the corresponding register transfer operation statements in RTL forms.

C1	C2	Input Load of R1	C_0	Next state of R1	Function
0	0	0	0	R1	No change
0	1				
1	0				
1	1				

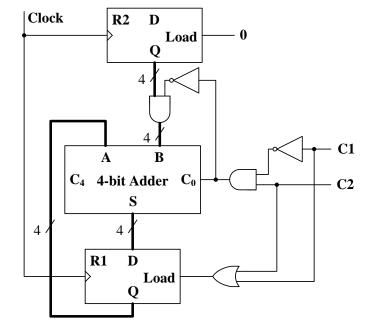


Problem: Circuit Analysis

Analyze the following register transfer circuit, finish the function table, and write down the corresponding register transfer operation statements in

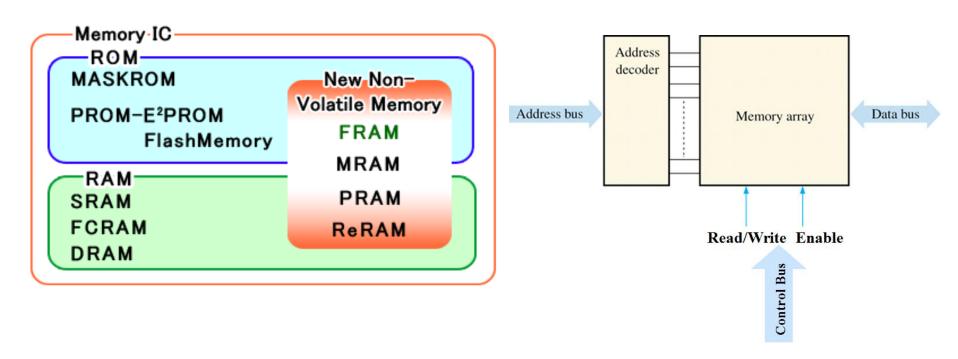
RTL forms.

C1	C2	Input Load of R1	C_0	Next state of R1	Function
0	0	0	0	R1	No change
0	1	1	1	R1 + 1	Increment
1	0	1	0	R1 + R2	Addition
1	1	1	0	R1 + R2	Addition



C1: R1←R1 + R2

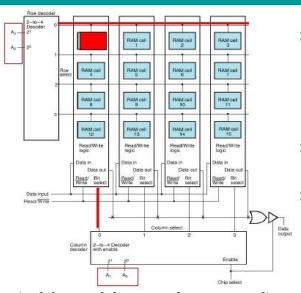
C1 C2: R1←R1 + 1

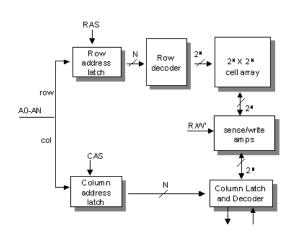


Computer memory

Block Diagram of RAM

- Memory Operation Timing
 - Read timing
 - Write timing
- Coincident Decoding
 - Row select
 - Column select
- Memory Expansion
 - Word-Capacity Expansion: Word extension (address bits are increased)
 - Word-Length Expansion: Bit extension
- DRAM
 - Address multiplexing
 - Refresh policy
 - Burst read
 - How to calculate memory bandwidth?





38

Problem:

The memory units that follow are specified by the number of words times the number of bits per word. How many address lines and data lines are needed in each case?

- (a) 8K * 16 (b) 2G * 8

- (c) 16M * 32 (d) 256K * 64

Answer:

(a)
$$8 K * 16 = 2^{13} * 16$$
 $A = 13, D = 16$

$$A = 13, D = 16$$

(b)
$$2 G * 8 = 2^{31} * 8$$

$$A = 31, D = 8$$

(c)
$$16 \text{ M} * 32 = 2^{24} * 32$$
 $A = 24, D = 32$

$$A = 24, D = 32$$

(d)
$$256 \text{ K} * 64 = 2^{18} * 64$$
 $A = 18, D = 64$

$$A = 18, D = 64$$

Problem:

A 16K * 4 memory uses coincident decoding by splitting the internal decoder into X-selection and Y-selection.

- (a) What is the size of each decoder, and how many AND gates are required for decoding the address?
- (b) Determine the X and Y selection lines that are enabled when the input address is the binary equivalent of 6,000.

Answer:

- (a) 16 K = 2^{14} = $2^7 * 2^7$ = 128 * 128 Each decoder is a 7-to-128-line decoder Decoders require 256 AND gates, each with 7 inputs
- (b) $(6,000)_{10} = (0101110 \ 1110000)_2$ X = 46, Y = 112

Problem:

A DRAM chip uses two-dimensional address multiplexing. It has 13 common address pins, with the row address having one bit more than the column address. What is the capacity of the memory?

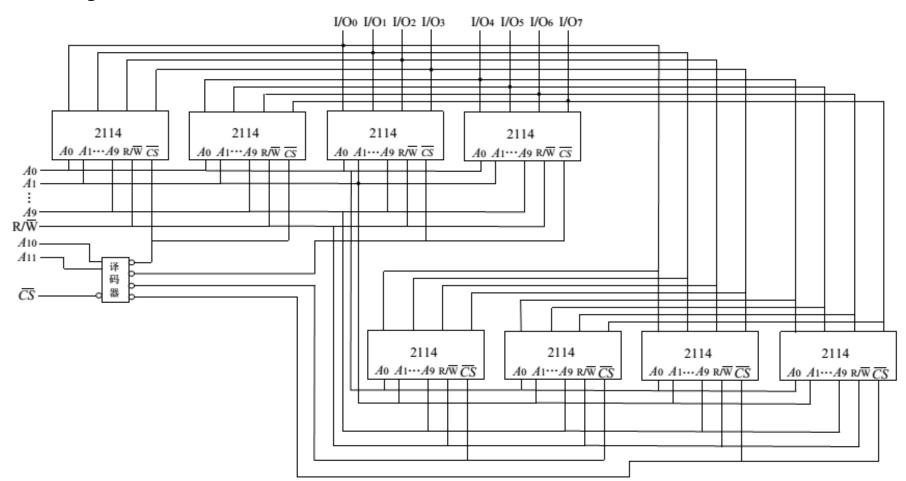
Answer:

13 + 12 = 25 address lines.

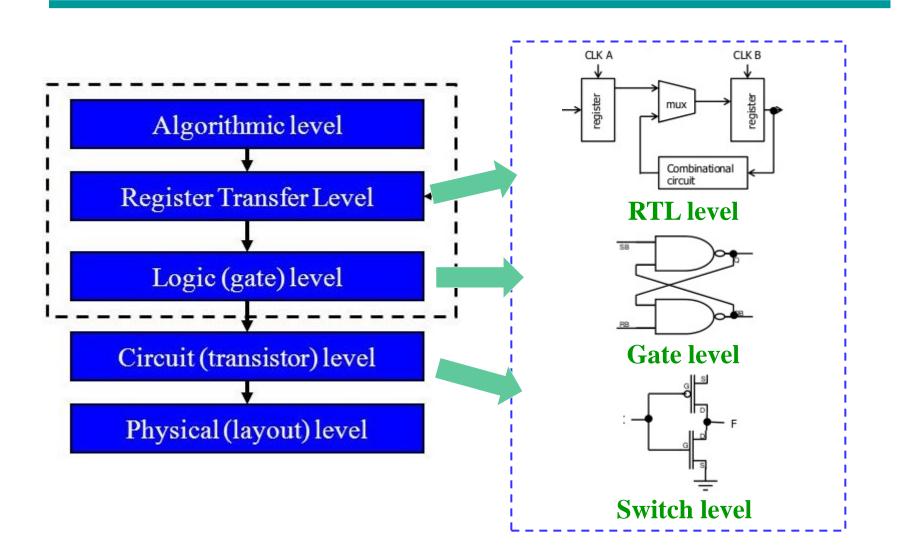
Memory capacity = 2^{25} words.

Problem: Given a 256 \times 8 ROM chip with Enable input, show the external connections 256 x 8 D_{0-7} ROM necessary to construct a $2K \times 8$ ROM with eight chips and a decoder. Е 3x8 256 x 8 D_{0-7} Decodei ROM A10 -Е 256 x 8 D₀ - 7 ROM Е

Problem: Given a $1K \times 4$ ROM chip with Enable input, show the external connections necessary to construct a $4K \times 8$ ROM with eight chips and a decoder.

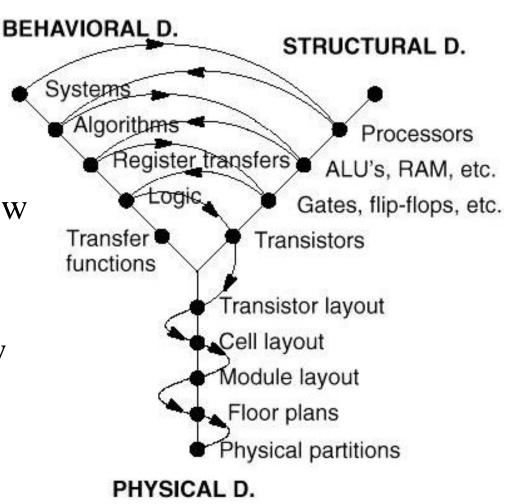


Levels of Logic Design



Levels of Logic Design (continued)

- Behavioral: specifies what a specific system does
- Structural: specifies how entities are connected together
- Physical: specifies how to build a structure



From Structural Design Perspective

- Gates: AND, OR, NOT, NAND, NOR, 3-state buffer (Hi-Z), XOR, XNOR
- Gate-Level Technology Mapping
- Rudimentary functions: decoder, encoder, multiplexer, demultiplexer
- Arithmetic functions: half/full adder, ripple carry/carry lookahead adder, adder/subtractor, multiplication, shifter, ALU
- Programmable devices: PROM, PAL, PLA, FPGA

From Structural Design Perspective (continued)

- Latches: SR, JK, D, T
- Flip-flops: Master-Slave/Edge-triggered, SR/JK/D/T
- Registers: counter, shifter
- Register Transfer Structures: MUX-based, MUX bus, 3-state bus
- Memories: ROM, SRAM, DRAM, SDRAM

From Behavioral Design Perspective

- Methods of designing combinational circuit
 - ➤ Designing theory: time-independent logic
 - Describing methods: truth table (canonical form),
 Karnaugh maps, timing diagram, boolean function,
 logic circuit
 - ➤ Optimization: two-level circuit optimization (Boolean algebra), iterative array, contraction
 - Timing and performance: gate input cost, fanin/fan-out, delay model

From Behavioral Design Perspective (continued)

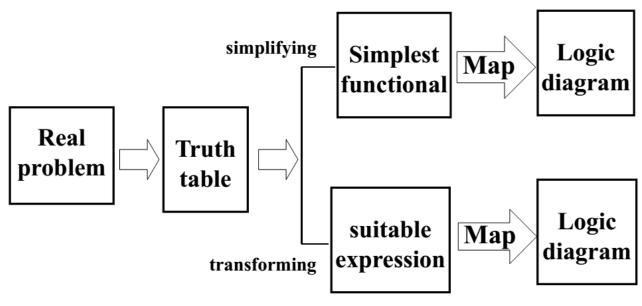
- Methods of designing sequential circuit
 - Designing theory: finite state machine (Mealy model and Moore model)
 - Describing methods: state table, state diagram, next state equation(characteristic equation), input equation, excitation equation, output equation
 - ➤ Optimization: state minimization, state assignment
 - Encoding: Counting order, Gray and One-hot code
 - Timing and performance: glitch, 1's catching, t_s, t_h, t_w, t_{px}

From Behavioral Design Perspective (continued)

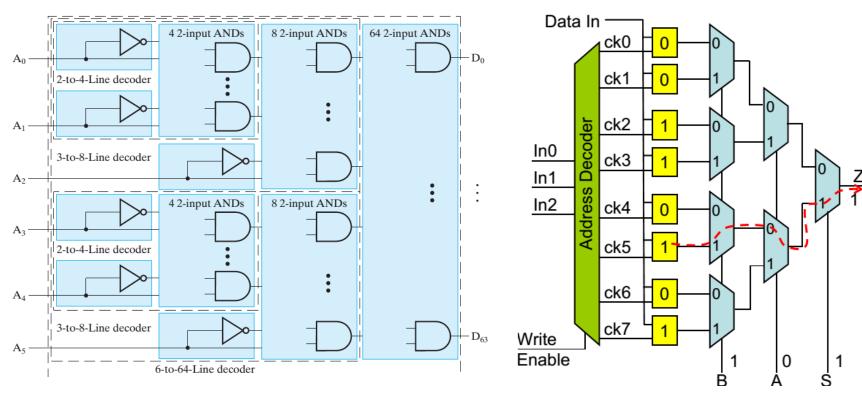
- Methods of designing digital system
 - ➤ Designing theory: state-machine diagram, register design model, datapath+control unit
 - ➤ Describing method: Register Transfer Language(RTL)
 - Timing and performance: system timing equation $t_p = t_{pd,FF} + t_{pd,COMB} + t_s + t_{slack}$, maximum frequency
- Methods of designing memory
 - Describing methods: address, data, operation
 - ➤ Optimization: coincident decoding, address multiplexing, burst read
 - ➤ Timing and performance: read timing, write timing, memory bandwidth

Method of Logic Design

- Methods of designing digital circuits
 - Design by truth table/state table
 - Design by bisection
 - Design in a hierarchical structure
 - Design by iteration
 - Design by contraction/expansion



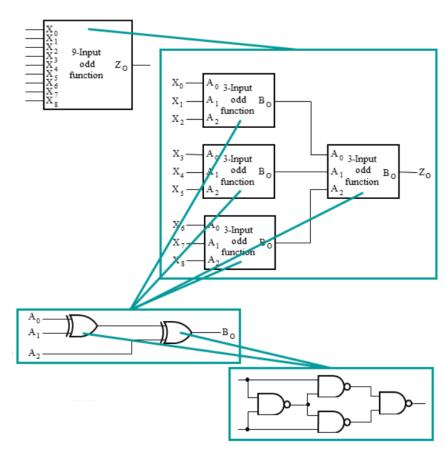
Design by bisection



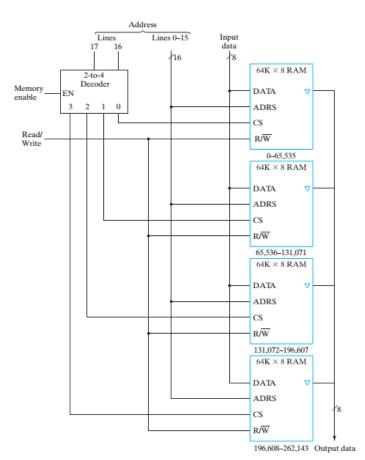
Decoder expansion

Lookup tables

Design in a hierarchical structure

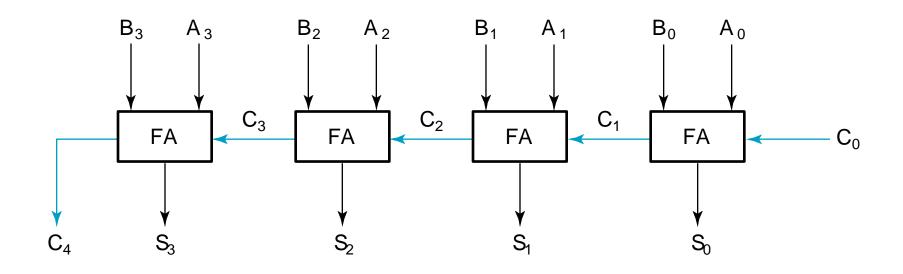


9-input parity tree



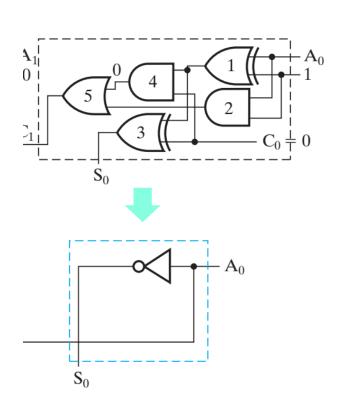
256K * 8 RAM with four 64K * 8 RAM

Design by iteration

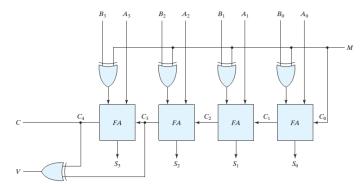


A Four-bit Ripple Carry Adder

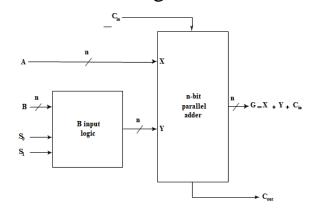
Design by contraction/expansion



Contraction of Adder to Incrementer



Expansion of inputs for constructing substractor



Expansion of inputs for constructing ALU

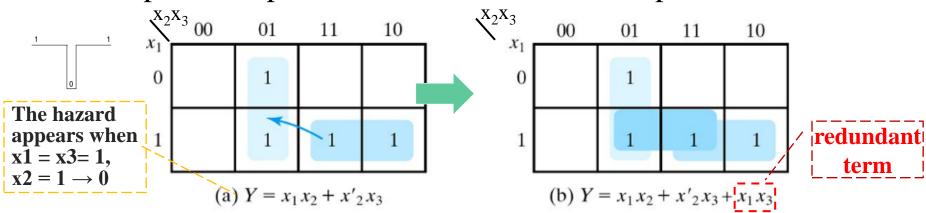
Design Tradeoffs in Logic Circuits

- Performance-Cost tradeoff
 - Two-level circuit vs. Multiple-level circuit
 - Carry lookahead adder vs. Ripple carry
 - Parallel adder vs. Serial adder
 - >Synchronous counter vs. Asynchronous counter
 - >SRAM vs. DRAM



Design Tradeoffs in Logic Circuits (continued)

- Performance-Reliability tradeoff
 - Latch vs. Flip-flop
 - ➤ Mealy model vs. Moore model
 - ➤ Not encoded (One-hot Code) vs. Encoded (Gray Code)
- Cost-Reliability tradeoff
 - ➤ No self-correcting vs. Self-correcting
 - > Optimal implementation vs. redundant implementation



End

Final Examination Time:

Jan. 11, 2024 10:30~12:30

Good luck!



Problems:

- The dual of an algebraic expression is obtained by 1) <u>interchanging OR and AND operations</u> and 2) replacing 1's by 0's and 0's by 1's.
- Use DeMorgan's Theorem to complement a function: 1) <u>interchange AND and OR operators</u> and 2) 2. complement each constant value and literal
- Four variables odd function has _____ C___ "1" squares in its corresponding K-Map. 3.
 - A. 4
- B. 7

C. 8

- D. 14
- The gate input cost G of function F = AB(C+D) + C(BD+AD) is __A_. 4.
 - A. 15
- B. 14

C. 13

- D. 12
- Which of the following logical gates can be used as a controllable inverter? _____B___. 5.
 - A. AND gate
- B. XOR gate
- C. Buffer gate
- D. OR gate
- The Essential Prime Implicants in the K-Map given below are _____B___. 6.
 - A. Y'Z', XZ'

- B. X'Y', XY C. XY, XZ' D. Y'Z', X'Y'
- Given below are the waveforms of input A, B and output F of a logic device. Then the device is a 7. <u>C</u> gate.









