Algorithms and Data Structures.

Homework 1

Asymptotic Analysis

Problem 1

a) $f(x) = 3x \quad g(x) = x^3,$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{3x}{x^3}$$

$$= \lim_{x\to\infty} \frac{3}{x^2} \to 0$$

$\therefore$ since limit as x approaches ∞ is 0

$f(x) = o(g(x))$ non tight upper bound.

conversely,

$$\lim_{x\to\infty} \frac{g(x)}{f(x)} \to \lim_{x\to\infty} \frac{x^3}{3x} \to \lim_{x\to\infty} \frac{x^2}{3} = \infty$$

hence

$g(x) = \omega(f(x))$ non tight lower bound.

with $n_0 = \sqrt{3}$

b. $f(n) = 7n^{0.7} + 2n^{0.2} + 13\log n \quad g(n) = \sqrt{n}$

$$\lim_{n\to\infty} \frac{7n^{0.7} + 2n^{0.2} + 13\log n}{\sqrt{n}} = \infty$$

$\therefore$ $f(n) = \omega(g(n))$ non tight

and conversely.

$g(n) = o(f(n))$ non tight

with $n_0 = 0.5$

c) $f(n) = \frac{n^2}{\log n}$ $g(n) = n\log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{n^2}{\log n}}{n\log n} = \infty$$

$f(n) = \Omega(g(n))$

and conversely,

$g(n) = O(f(n))$

~~with $n_0 = 1.5$~~

d) $f(n) = (\log(3n))^3$ $g(n) = 9\log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \rightarrow \lim_{n\to\infty} \frac{(\log(3n))^3}{9\log n} = \infty$$

Thus.

$f(n) = \Omega(g(n))$

and conversely,

$g(n) = O(f(n))$.

with $n_0 = 4$.

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