

Problem 2.2 Recurrences.

a) $T(n) = 36T(n/6) + 2n.$

\therefore Master method since of the form $T(n) = aT(n/b) + f(n).$

$$\Rightarrow n^{\log_b(a)} = n^{\log_6 36} \Rightarrow n^2$$

$$f(n) = 2n.$$

$$\therefore 2n < n^2$$

Since the function $f(n)$ is polynomially smaller than $n^{\log_b a}$

$$T(n) = \Theta(n^2)$$

b) $T(n) = 5T(n/3) + 17n^{1.2}$

Master method.

$$f(n) = 17n^{1.2}$$

$$n^{\log_b a} = n^{1.465}$$

$$\therefore f(n) < n^{1.465}$$

$$= T(n) = \Theta(n^{1.465})$$

c) $T(n) = 12T(n/2) + n^2 \lg n$

$$n^{\log_b a} = n^{3.585}$$

$$f(n) = n^2 \lg n$$

since $n^{\log_b a} > f(n)$

$$T(n) = \Theta(n^{3.585})$$

d) $T(n) = 3T(n/5) + T(n/2) + 2^n$
using the master method.

$$3T(n/5) + T(n/2) \prec 11T(n/10)$$

$$\therefore 11T(n/10) + 2^n$$

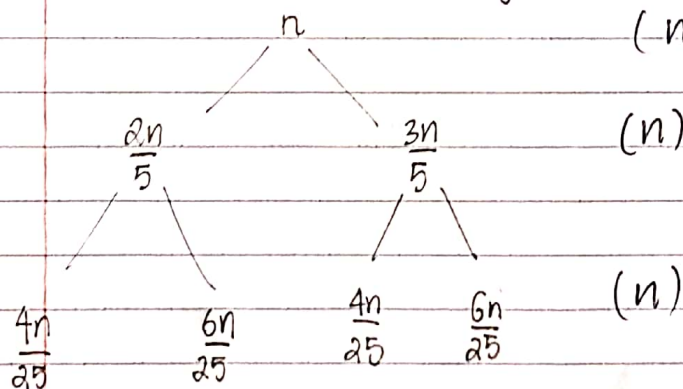
$$n^{\log_{10} 11} = n^{1.04}$$

since $f(n)$ is polynomially larger than $n^{\log_b a}$

$$T(n) = \Theta(2^n)$$

e) $T(n) = T(2n/5) + T(3n/5) + \Theta n$

using a ~~tree~~ recursion tree.



$$\therefore T(n) = \Theta(n \log n).$$