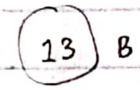
Problem 8·1.

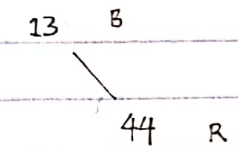
a). [13, 44, 37, 7, 22, 16]

- inserting 13

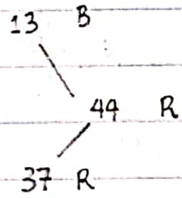


- inserting 44

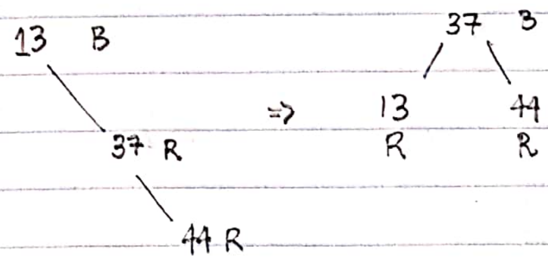
using right subtree property since 44 > 13.



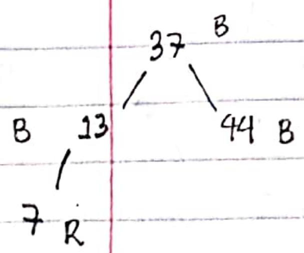
- inserting 37



to resolve violation - rotate 37 with root of tree.

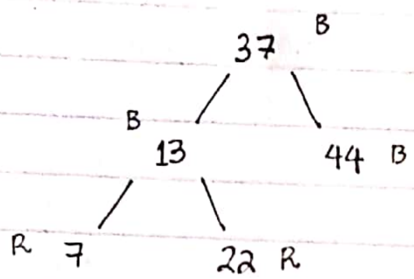


- inserting 7

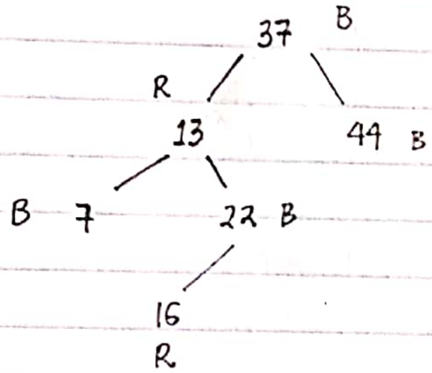


color violations are solved by changing the left & right subtree to black.

- inserting 22



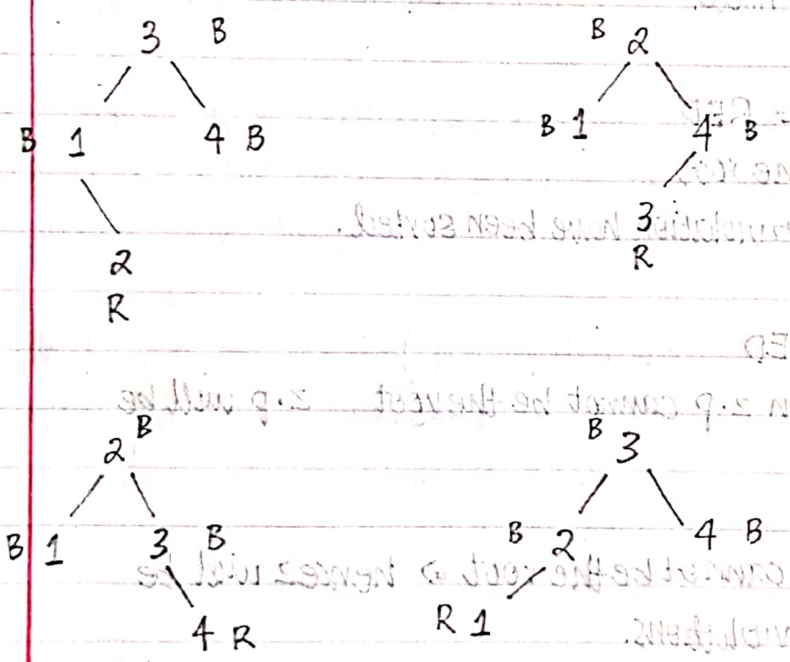
- inserting 16



b. All valid RB Trees that store the values [1, 2, 3, 4].

→ 4! = 24 possible arrangements in which [1, 2, 3, 4] can be inserted into a red black tree.

⇒ out of this, only 4 trees are valid red black trees.



c. Bonus.

Consider a RB tree formed by inserting n nodes. Prove that if $n > 1$, the tree contains at least one red node.

There are different ways to prove that with $n > 1$, a red black tree will have at least 1 red node.

To summarize the cases.

⇒ if z and $z.p.p$ = RED
then z cannot be the root
thus z is red after violation have been sorted.

⇒ if z and $z.p$ = RED
and if after rotation $z.p$ cannot be the root, $z.p$ will be red after the fixup.

⇒ if z is red then z cannot be the root ⇒ hence z will be red after fixing violations.

Proof by induction

Base case: $N = 2$

Hypothesis: a tree with n nodes, with $1 < n \leq N$, there is at least 1 red node

Inductive: for a red black tree with $N+1$ nodes, there is at least 1 red node.

⇒ trivial case = Red node $N+1$ is the child of a black node

⇒ if $N+1$ is inserted as the child of a red node - then we can look at different fixing strategies.

1. if X remains red after recoloring the parent, g. parent & uncle and when X is moved to Grand parent X - we shall have at least 1 red node

2. the parent of X is red after rotation of X about the parent & g. parent. the parent remains red after recoloring X & G.
we will still have a red node.

3. if parent(x) is rotated about g. parent(x) - X remains red
then X = Red after recoloring ~~&~~ parent(x) & g. parent(x).
at least 1 red node.

thus, in all cases of fixing the RB tree with nodes $n > 1$, there exists at least 1 rednode