Supply Chain Planning

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This document explains the algorithm to solve supply chain planning problem. At first, the problem is defined and then it is formulated by a mixed integer linear program. At the end, its complexity is briefly discussed.

Throughout this document, the power set of set P is referred to as 2^P, which is the set of all subsets of P, i.e. $\{P' \mid P' \subseteq P\}$. Disjoint union is denoted by $\dot\cup$. The definition of *supply chain* is as follows.

Definition 1 (Supply Chain) *Let P be a set of* products *with $o : P \to \mathbb{N}$ as its order function such that $o(p)$ is the number of ordered product $p \in P$. Moreover, C is a set of* components *with stock function defined by $s : C \to \mathbb{N}_0$. A product or a component is called* entity *and the set of all entities $\{P \,\dot\cup\, C\}$ is referred to as E. Let $C_s \subseteq C$ the component that are directly supplied by a supplier. Each product or component is chained to the network by function $ch : E \to 2^C$. Then $SC = (E = \{P \,\dot\cup\, C\}, o, s, ch, C_s)$ is a supply chain.*

Chain function ch specifies how an entity is connected to other entities in the chain. Formally speaking, entity e is directly supplied by all entities in $ch(e)$. The type of supply is recognized by partitioning entities into sets $\mathcal{A}$ and $\mathcal{O}$ corresponding *and* and *or* delivery, respectively. The set of entities that are suppied by entity e is denoted by $\delta(e)$, i.e. $\{e' \mid e \in ch(e)\}$.

Entities can flow through the chain down from suppliers and inventories as the source and be delivered as the products. An assignment of the flows to the chain that satisfies some constraints is called a *plan*.

Definition 2 (Plan) *A* plan *is an assignment of flows going out of suppliers (s_c for $c \in C_s$), going out of inventories (v_c for $c \in C$), passing through edges $y_{e,f}$ for all $e \in E$, for all $f \in ch(e)$ and being delivered as an entity (x_e for*

$p \in P$) that satisfy the following constrains

$$
\begin{aligned}
0 \le x_p \le o(p), &\qquad \forall\ p \in P && (1)\\
0 \le v_c \le s(c), &\qquad \forall\ c \in C && (2)\\
x_e = y_{e,f}, &\qquad \forall\ e \in \mathcal{A}, \forall\ f \in ch(e) && (3)\\
x_e \le y_{e,f} + (1 - z_{e,f})\bar{x}_e, &\qquad \forall\ e \in \mathcal{O}.\ \forall\ f \in ch(e) && (4)\\
x_e \ge y_{e,f} - (1 - z_{e,f})\bar{x}_e, &\qquad \forall\ e \in \mathcal{O}.\ \forall\ f \in ch(e) && (5)\\
\sum_{f \in \delta(c)} y_{c,f} = x_c + s_c + v_c, &\qquad \forall\ c \in C_s && (6)\\
\sum_{f \in \delta(c)} y_{c,f} = x_c + v_c, &\qquad \forall\ c \in C \setminus C_s && (7)\\
\sum_{f \in ch(e)} z_{e,f} = 1, &\qquad \forall\ e \in \mathcal{O} && (8)\\
z_{e,f} \in \{0,1\}, &\qquad \forall\ e \in \mathcal{O} && (9)\\
s_c \ge 0, &\qquad \forall\ c \in C_s && (10)
\end{aligned}
$$

In Ineqs. (4) and (5), term $\bar{x}_e$ refers to any upper bound on the flow delivered to entity e. It can be easily computed by a bachward search through the chain.

I define two problems related to supply chain. *Supply Chain Planning* problem is a decision problem to check whether a supply chain can satisfy its order.

Problem 1 (Supply Chain Planning (SCP)) *Given a supply chain SC as defined by Def. 1, SCP decides if there is a plan satisfying $x_p = o(p)$ for all $p \in P$.*

The optimization version of Prob. 1 is defined next.

Problem 2 (Maximal Supply Chain Plan (MSCP)) *Given a supply chain SC as defined by Def. 1 and an affine function r on flow variables defined in described in Def. 2, MSCP is the following optimization problem:*

$$
\begin{aligned}
&\max\ r(x, y, v, s)\\
\text{subject to}&\\
&\text{constrains (1) to (10)}
\end{aligned}
$$

It is straightforward to prove Prob. 1 is NP-complete. One can easily encode subset sum problem as *SCP* problem.