Translating prior predictive distributions into priors for model parameters

SURPH away day

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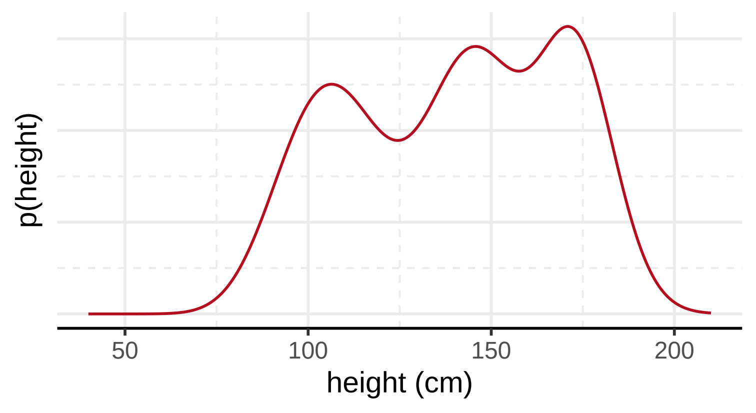
An example

- You find yourself modelling human growth, measured in centimetres, for a uniformly random sample of adolescents

Possible prior predictive

Possible prior predictive $t(Y) =$



Model (1)

- *"The Preece-Baines model is a standard model for this kind of data"*
- The simplest PB model looks like this:

$$Y(\mathtt{age}) = h_1 - \frac{2(h_1 - h_0)}{\exp\left\{s_0(\mathtt{age} - \gamma)\right\}\exp\left\{s_1(\mathtt{age} - \gamma)\right\}}$$

Model (2)

$$Y(\mathsf{age}) = h_1 - \frac{2(h_1 - h_0)}{\exp\{s_0(\mathsf{age} - \gamma)\}\exp\{s_1(\mathsf{age} - \gamma)\}}$$

- How should you "translate" p(Y) into a joint prior for $\theta = (h_1, h_0, s_0, s_1, \gamma)$?
- Best practice is *painful* (especially for more complex models)

General idea (1)

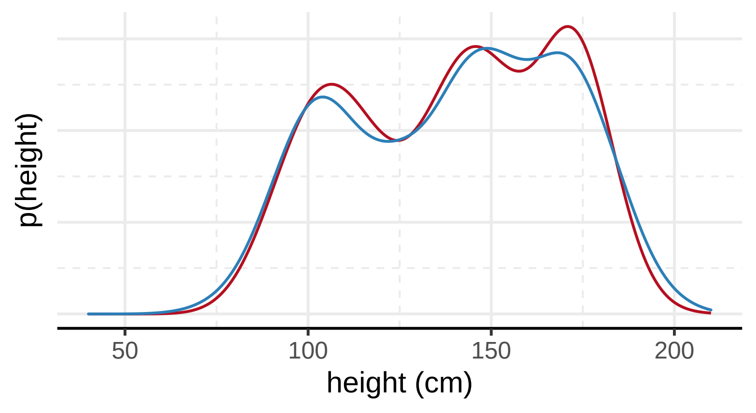
- Pick hyperparameters λ of prior $\mathrm{p}(\theta \mid \lambda)$ by minimising some discrepancy

$$D(\lambda) = \int d\left(\mathrm{p}(Y \mid \lambda), t(Y)\right) \mathrm{d}Y$$

- Solution $\lambda^* = \min_\lambda D(\lambda)$

General idea (2)

- Solution $\mathrm{p}(Y \mid \lambda^*)$ and target $t(Y)$:



Challenges

- Optimisation surface
- Numerics
- Inherent, irreducible noise
- Underspecification

Partial solutions

We can partly address these issues:

- Two-stage, multi-objective, gradient-free global optimisation
- Careful numerical implementation and importance sampling
- Regularisation term(s)
- R package implementation with few requirements for users: gitlab.com/andrew-manderson/pbbo