Graph Algorithms I

Breadth First Search

Graphs

Graph G = (V, E)

V: Set of Nodes (Vertices)

E: Set of Edges

Graphs

Graph G = (V, E)

V: Set of Nodes (Vertices)

E: Set of Edges

Let |V| = n

Let |E| = m

Undirected graph:

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Each edge is an **unordered** pair (u,v)
i.e. (u, v) = (v, u)

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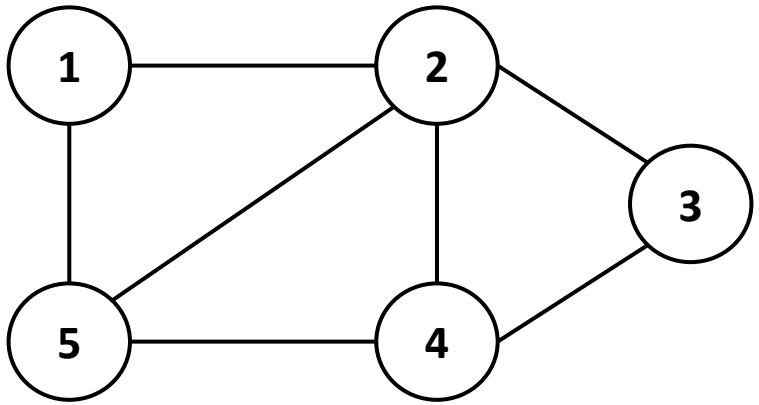

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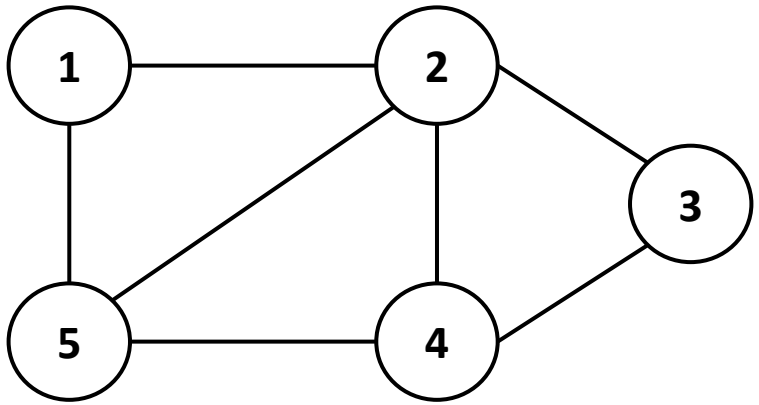

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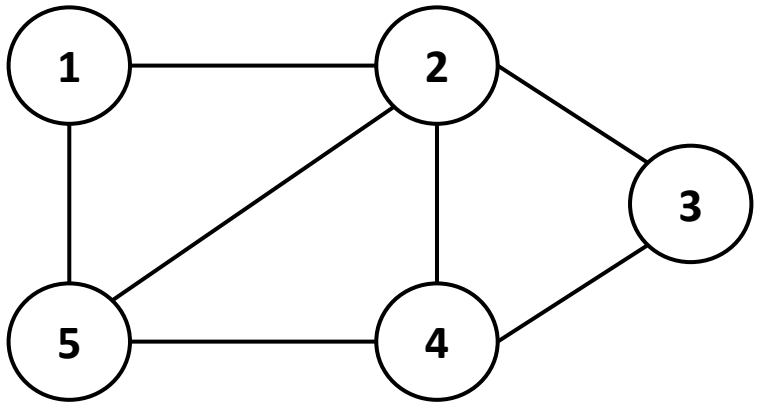

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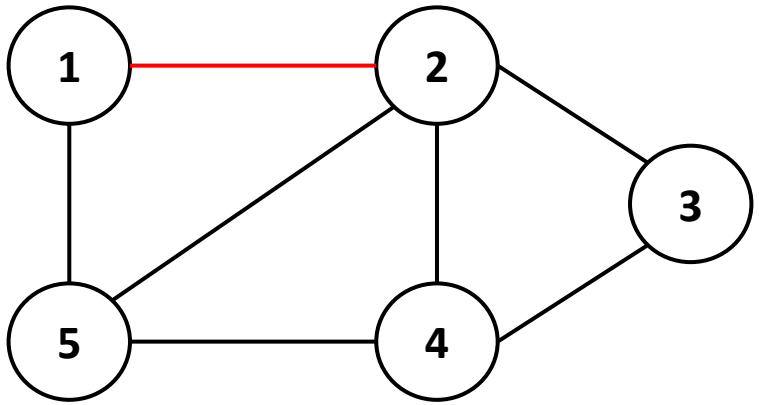

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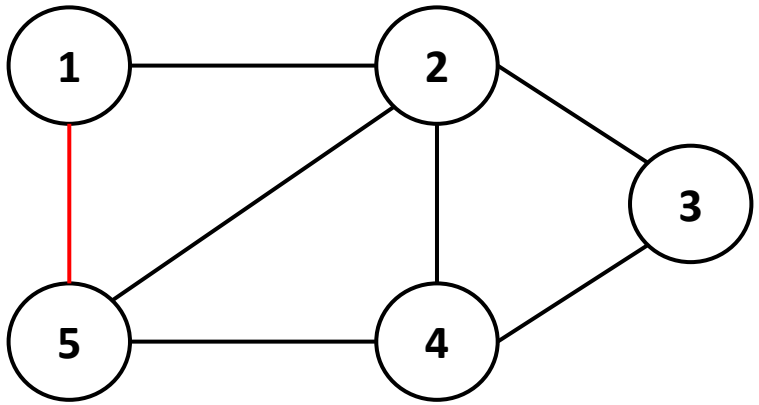

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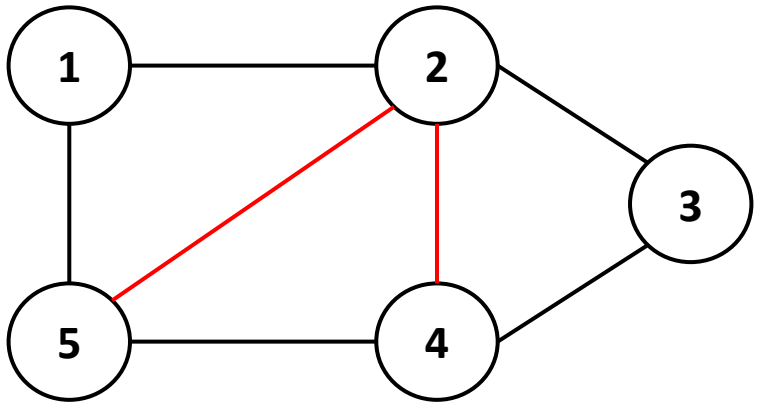

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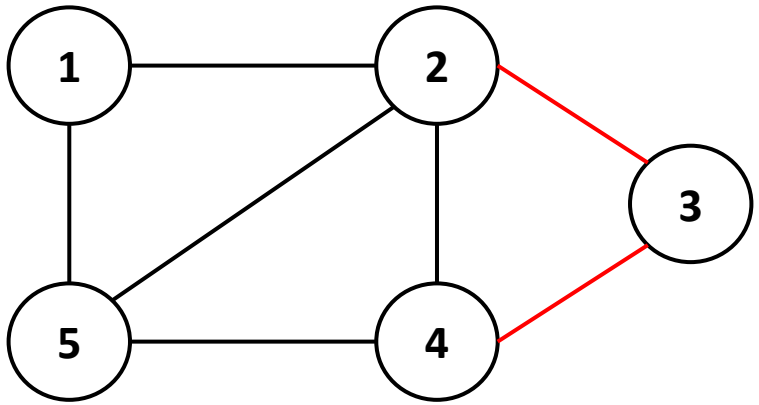

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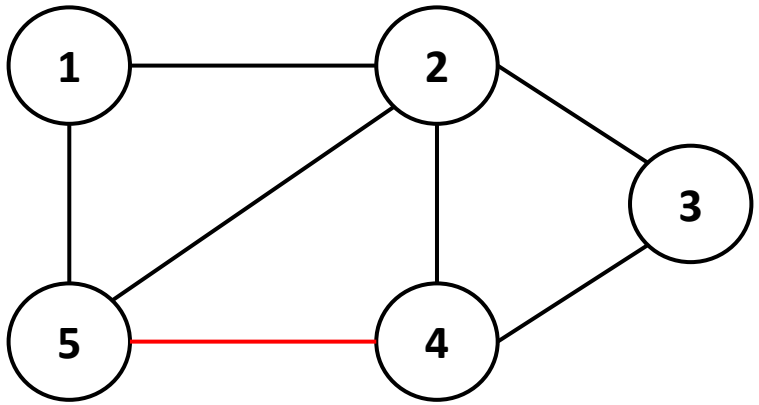

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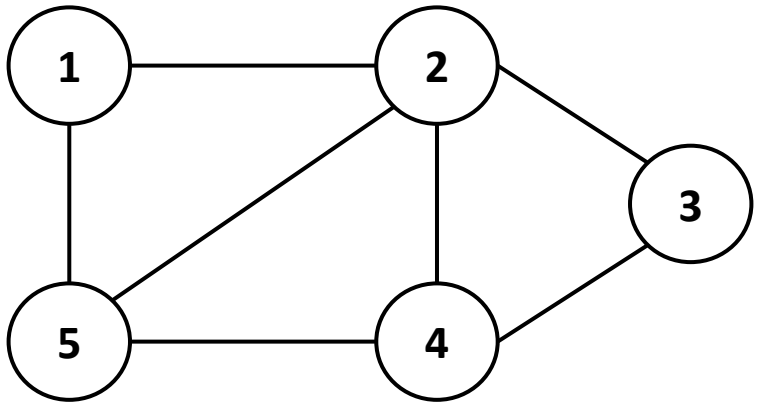

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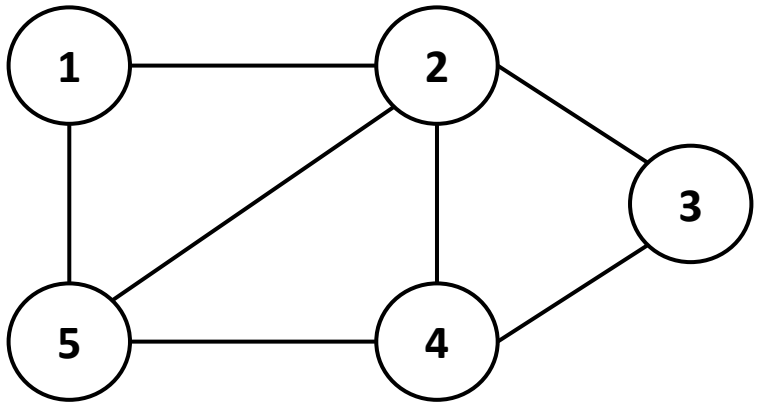

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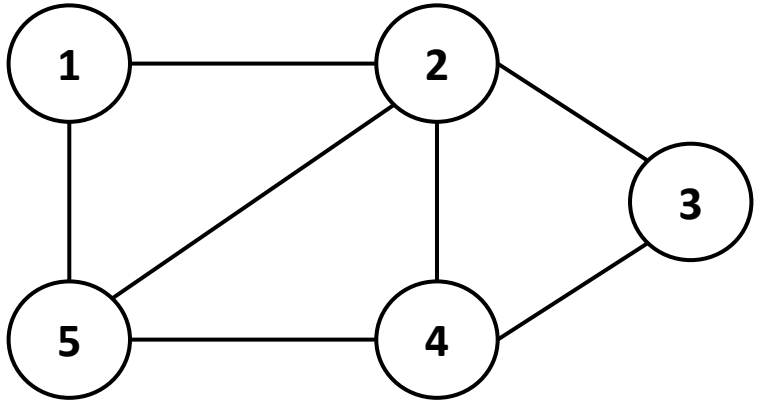

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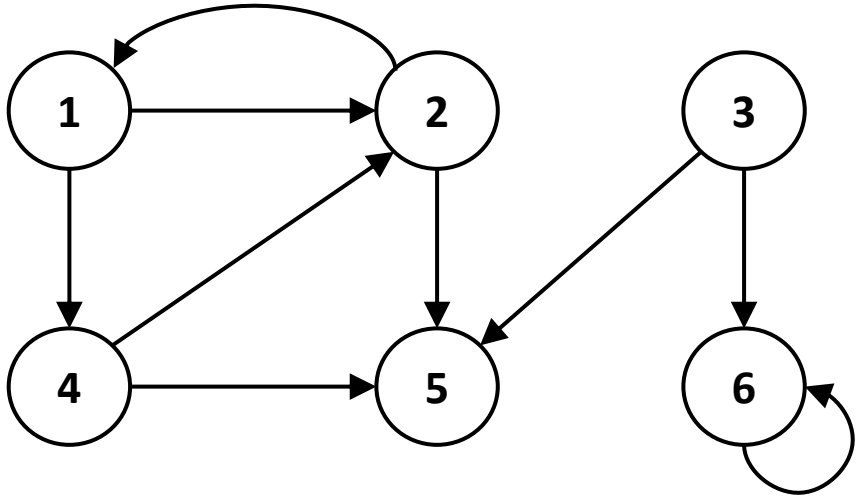

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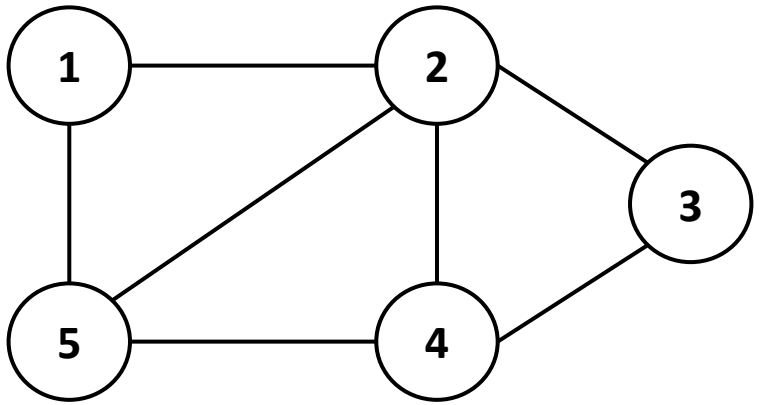

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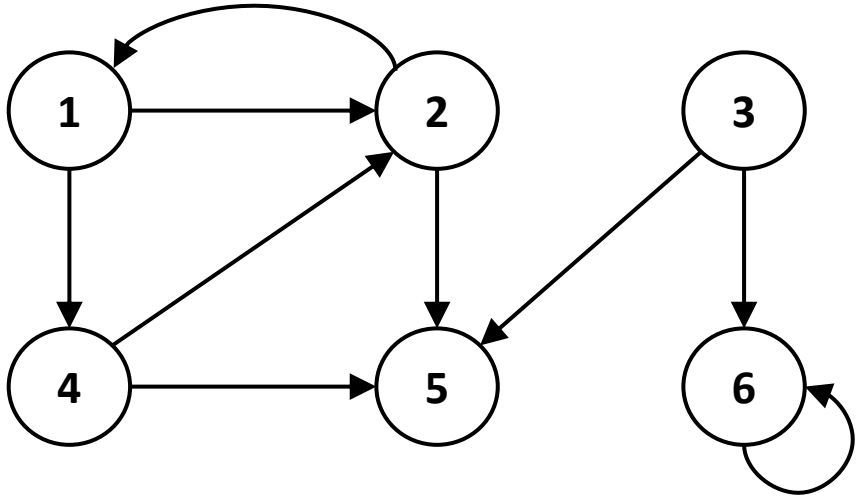

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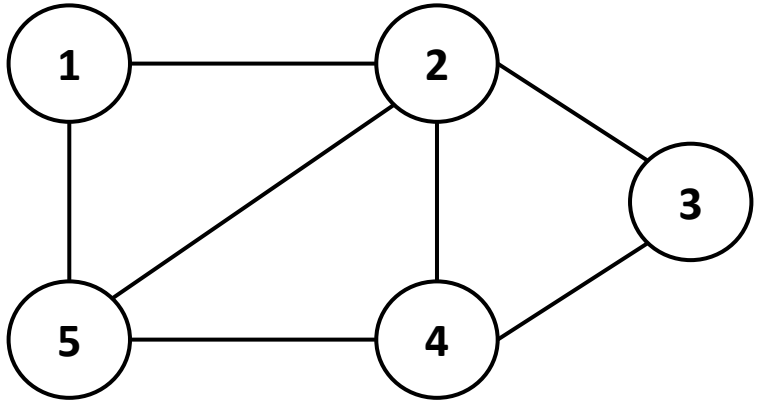

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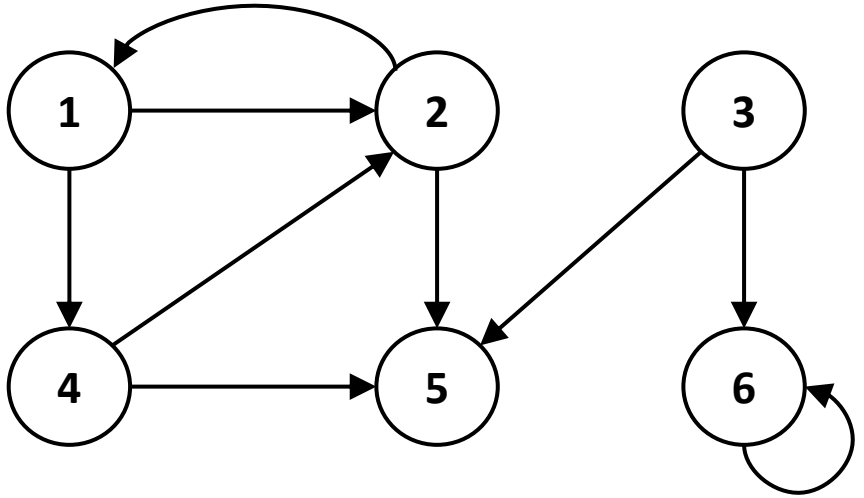

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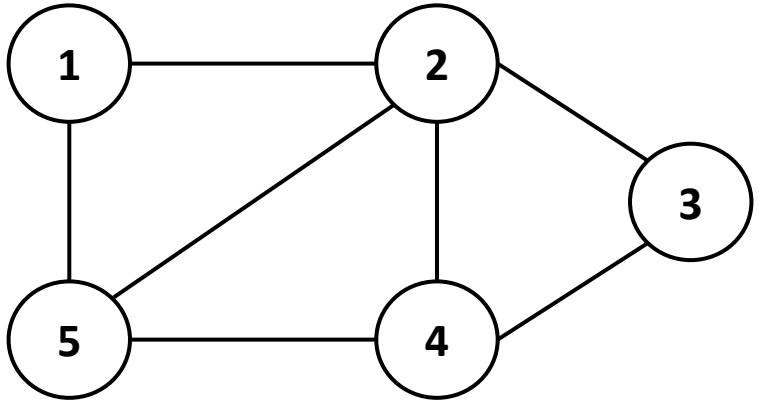

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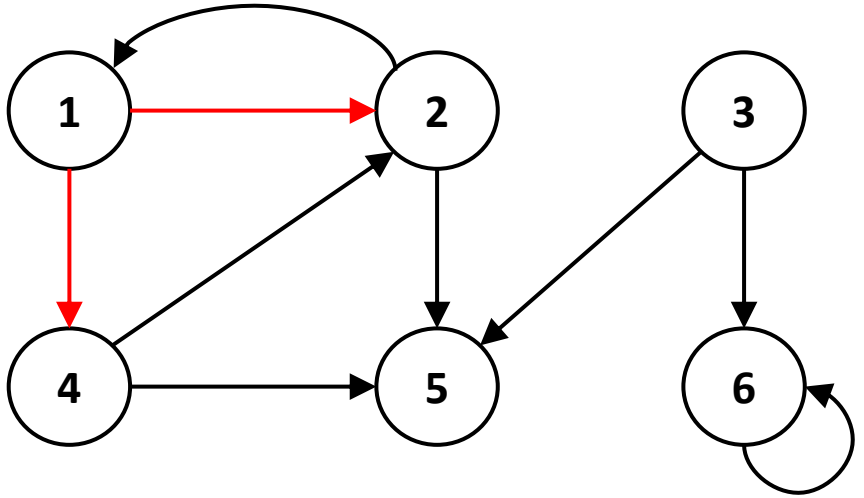

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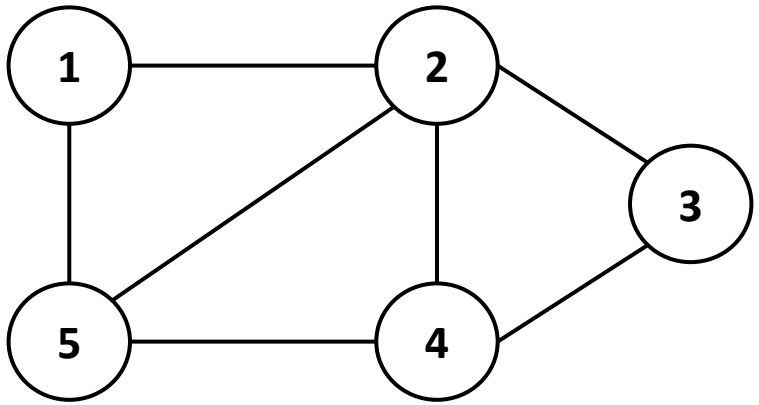

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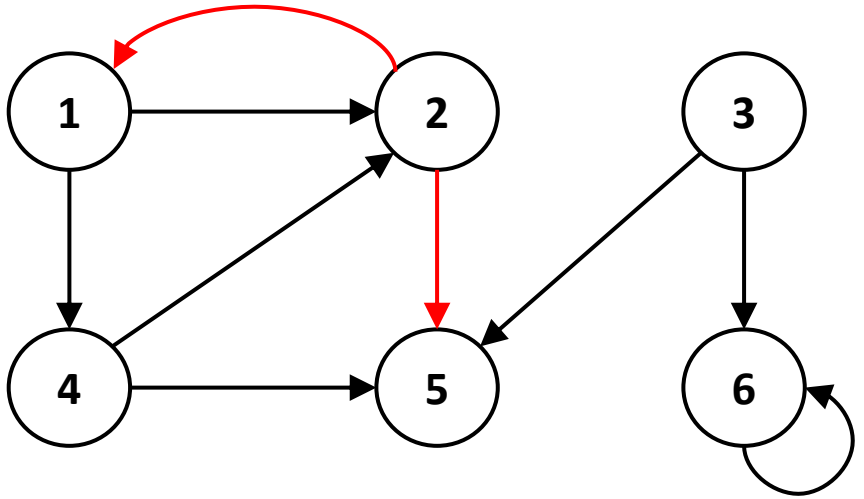

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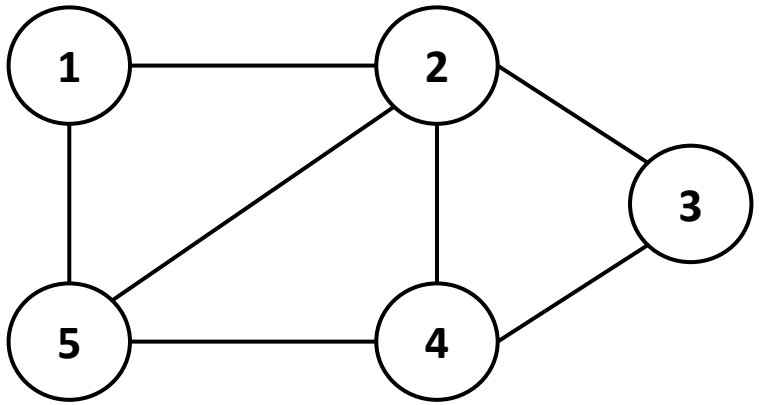

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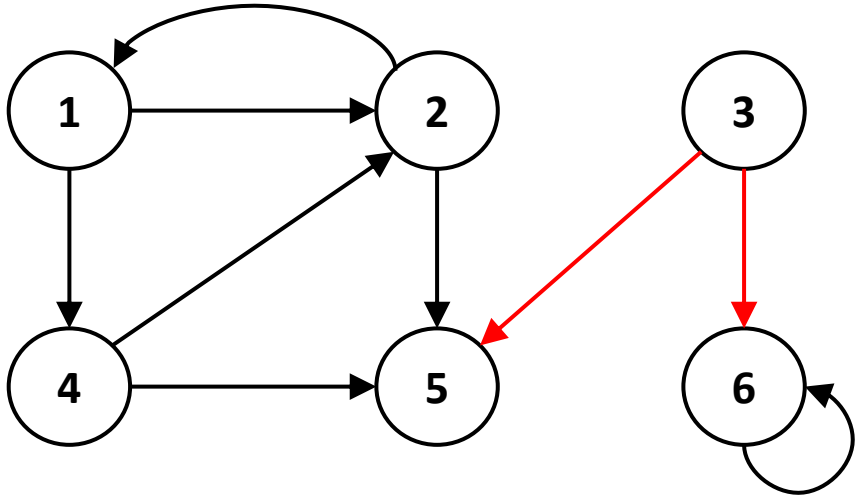

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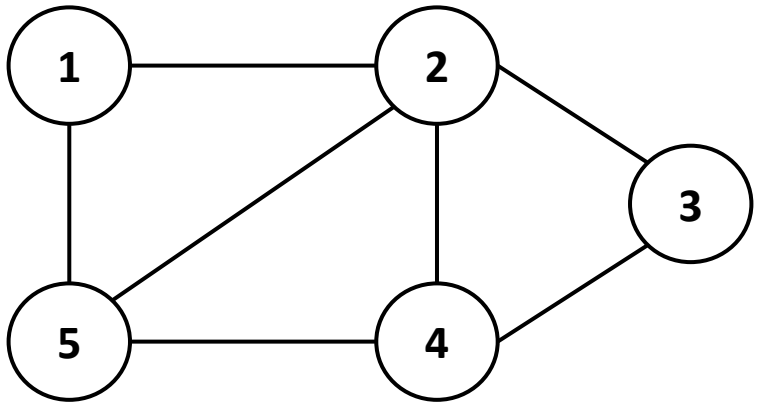

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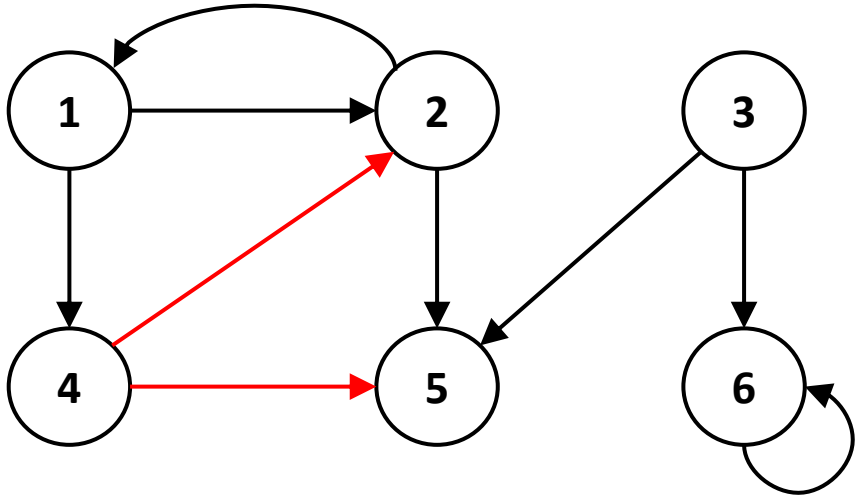

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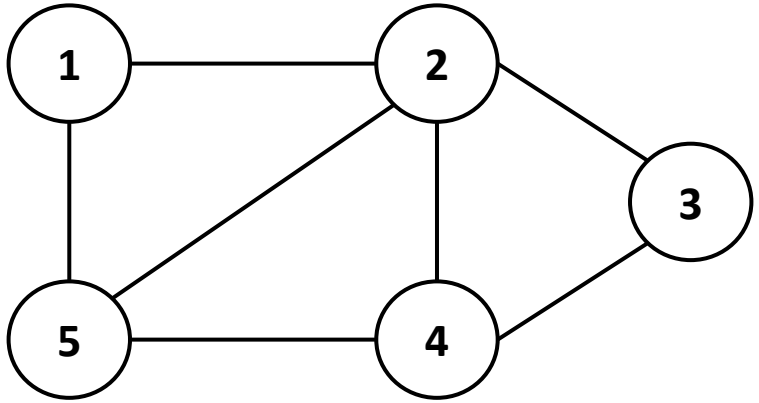

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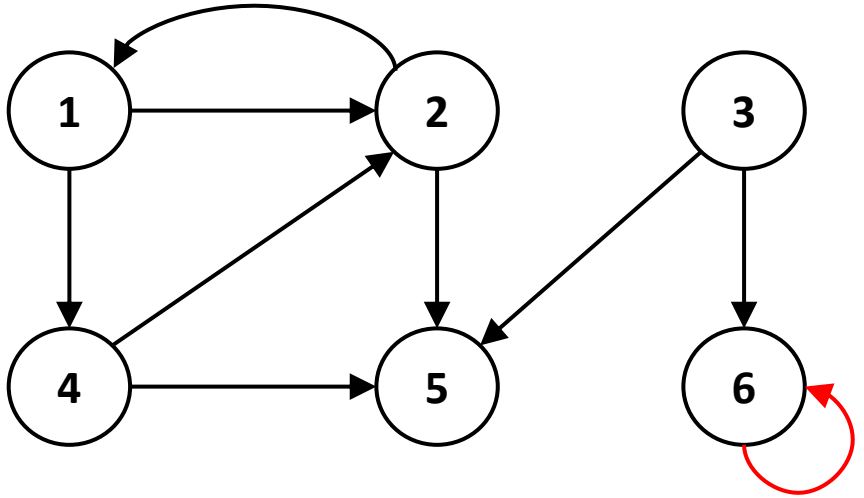

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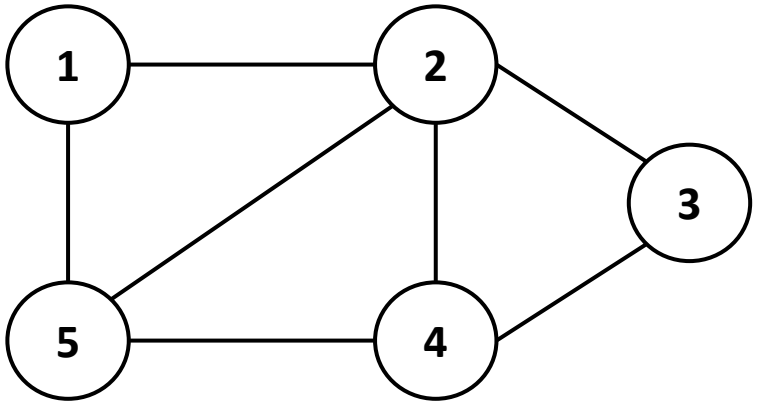
Representing graphs

Representing graphs

Adjacency List of G = (V, E):

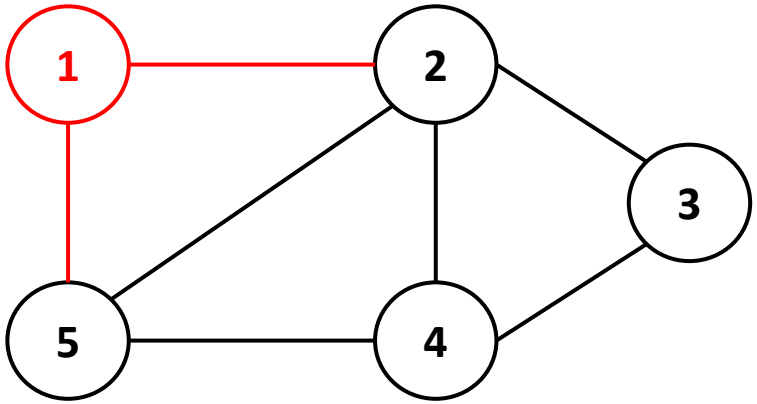
Array **Adj**[1…n] of n lists, one for each node u ∈ V,

Adj[u] : list of all nodes v such that (u,v) ∈ E



Adjacency list

1	
2	
3	
4	
5	



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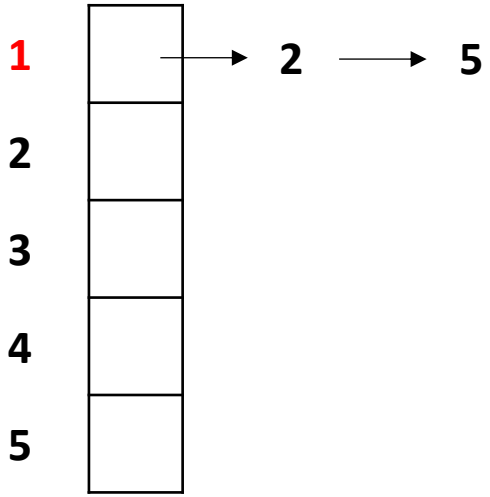

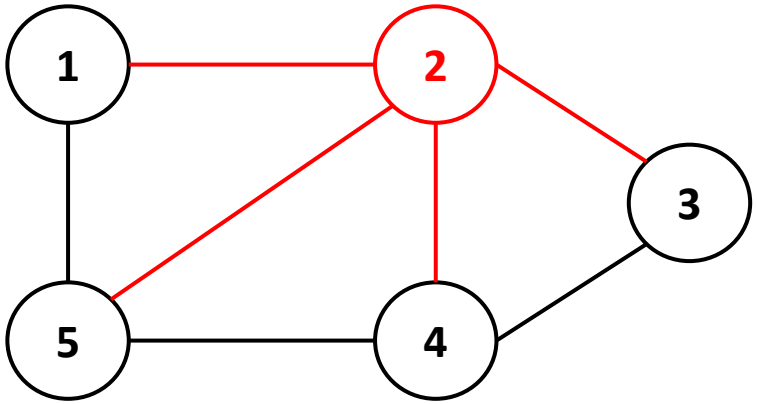

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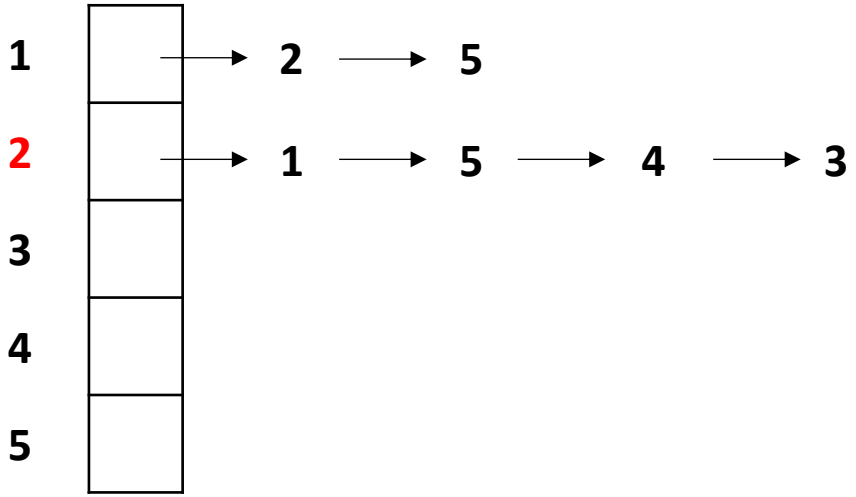

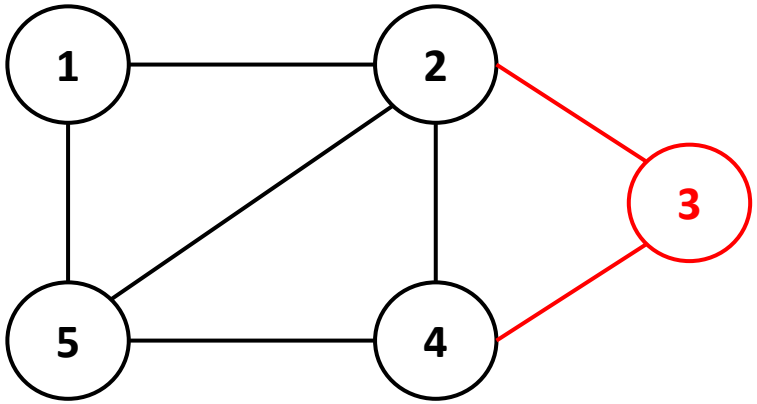

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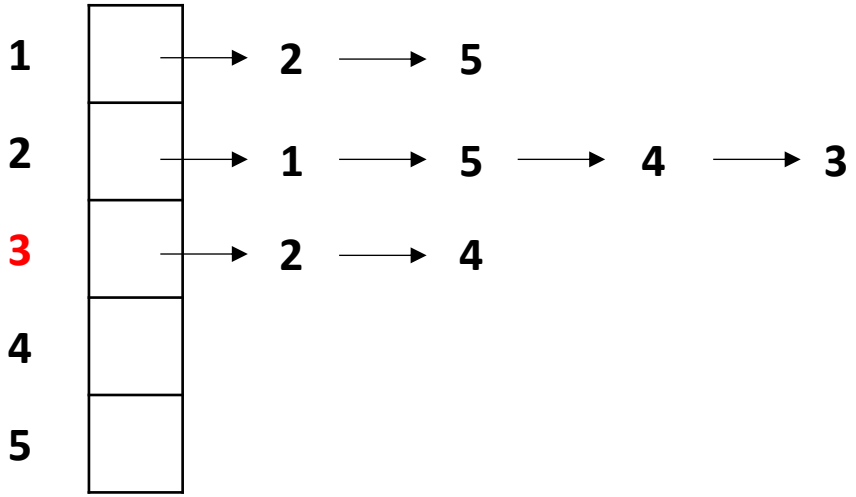

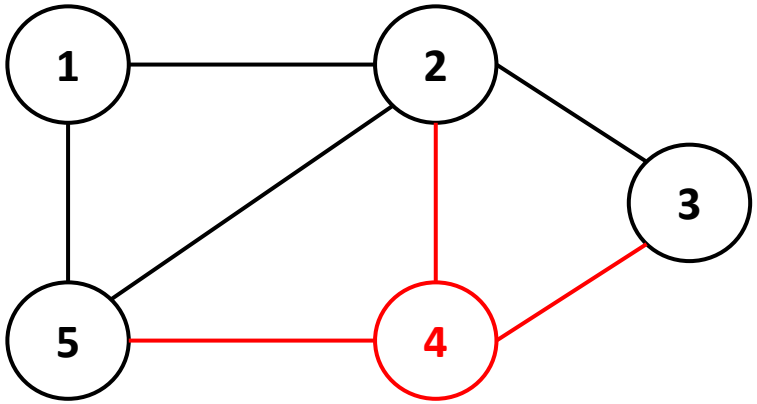

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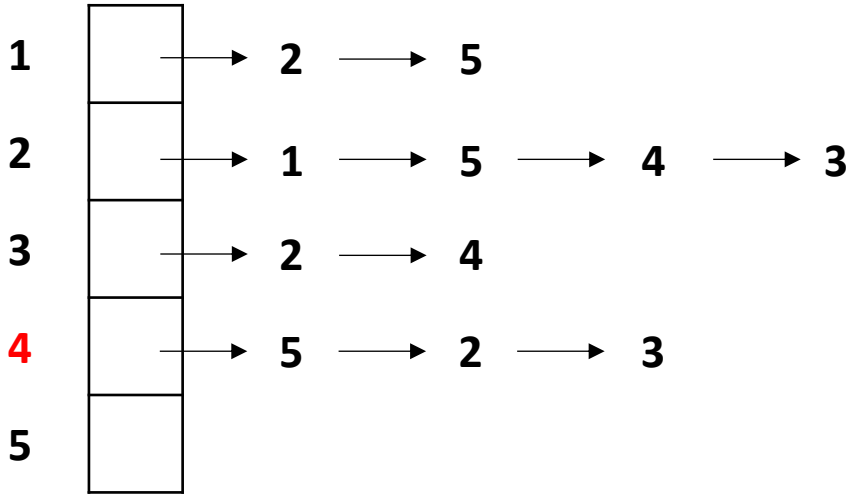
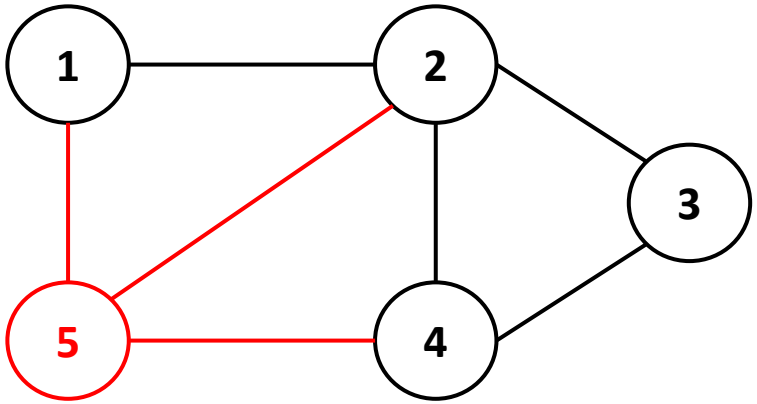
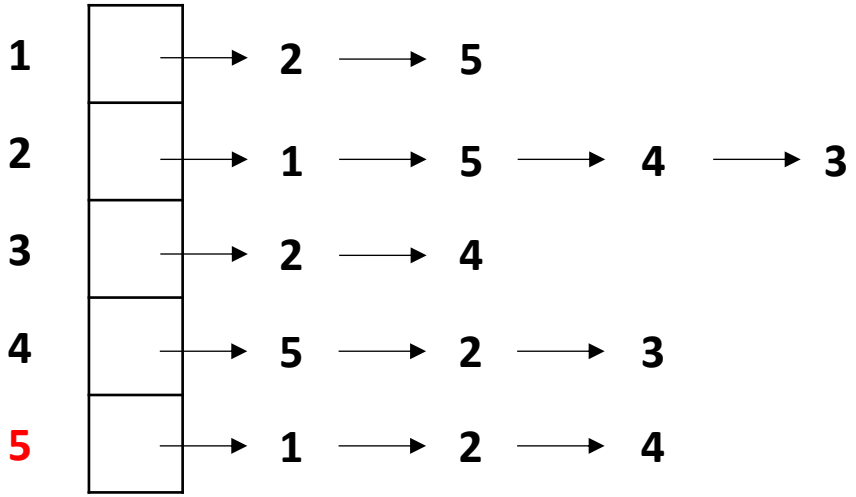
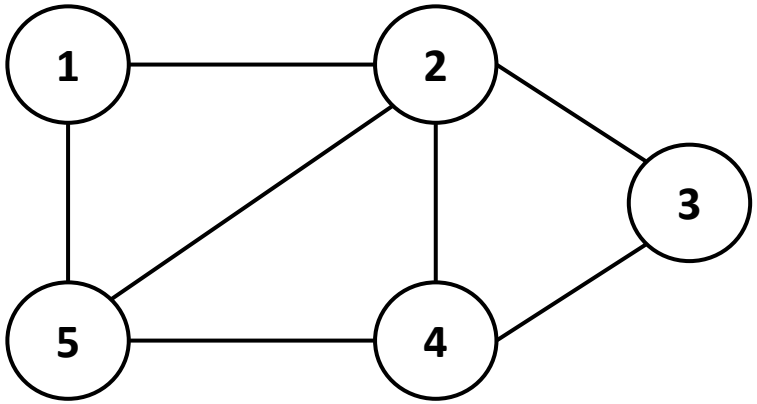


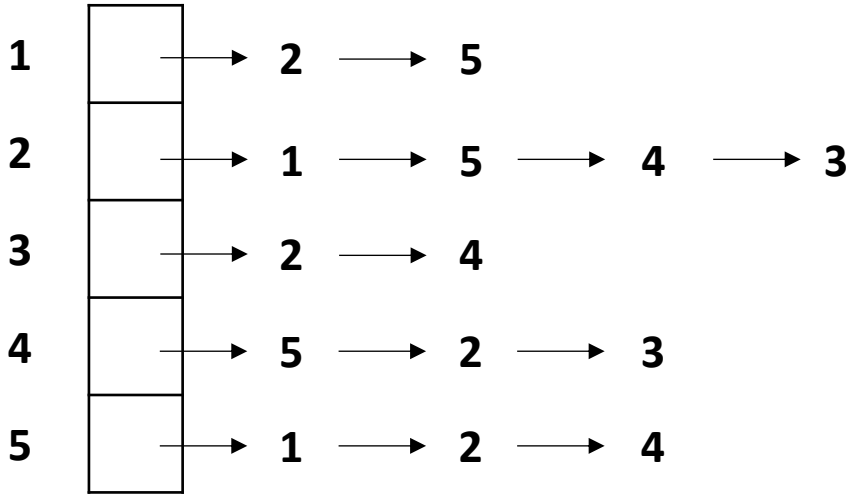


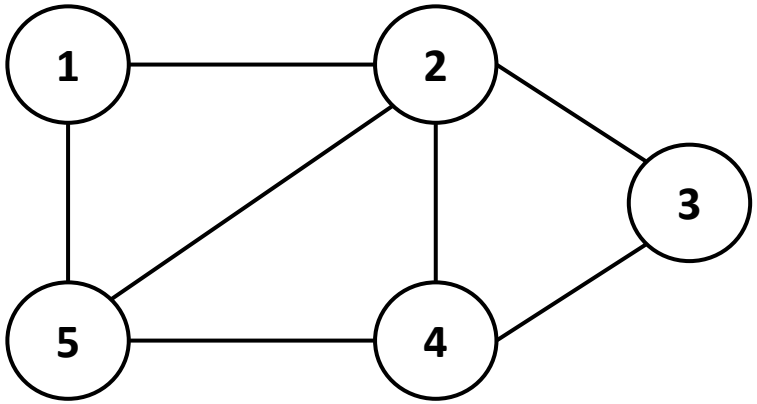
Adjacency list



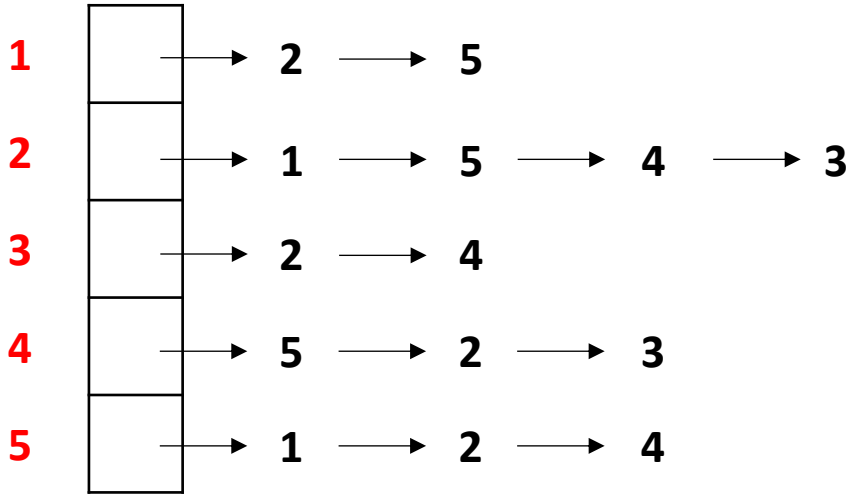


Adjacency list



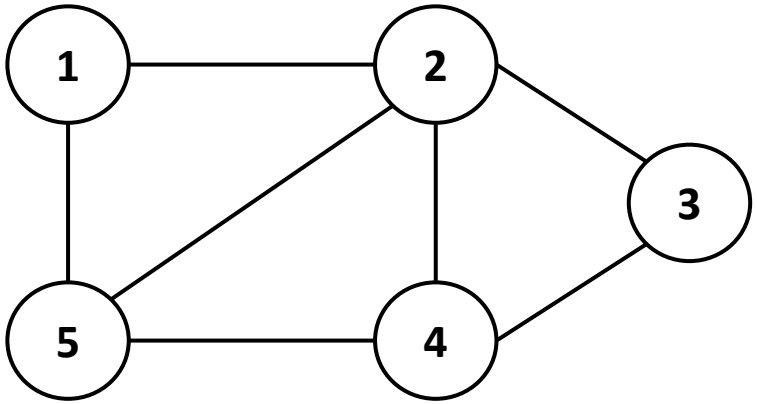


Adjacency list

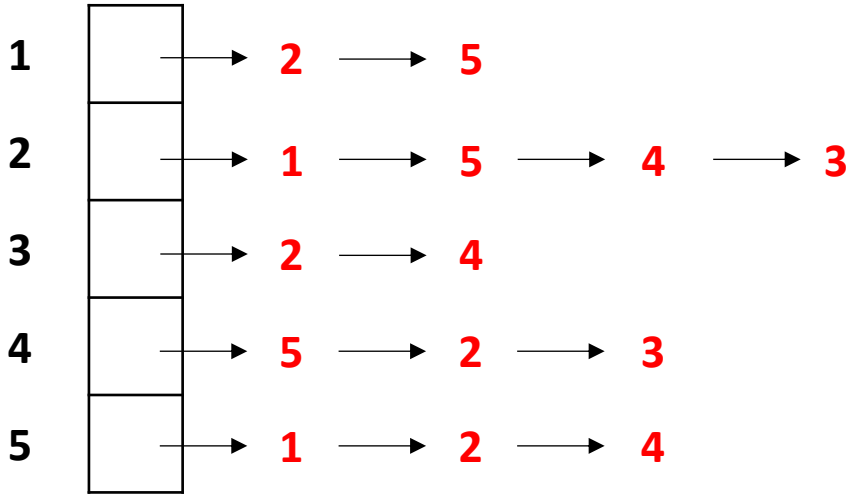


number of nodes

Size: Θ (n)

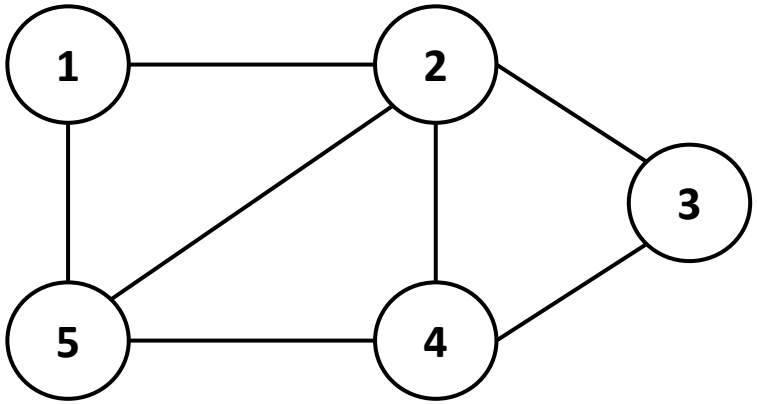


Adjacency list

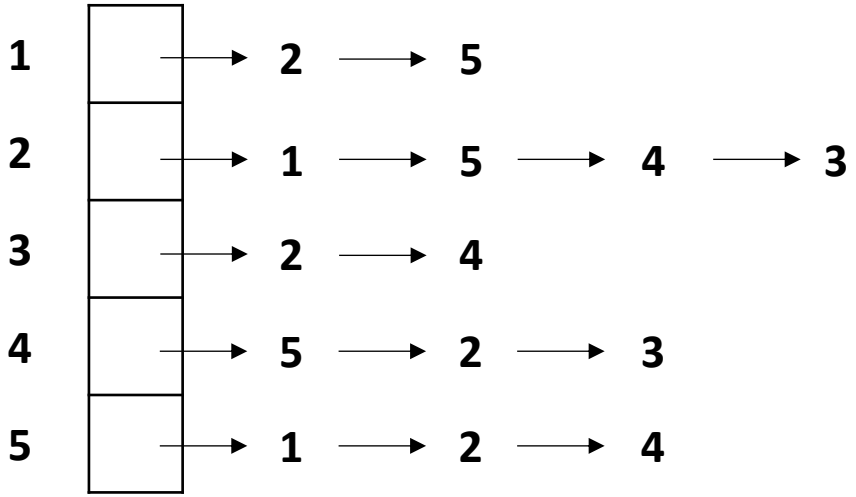


number of nodes number of edges

Size: Θ (n + m)

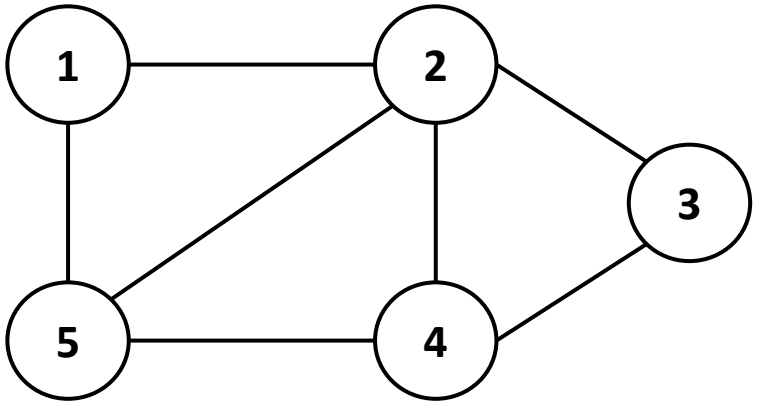


Adjacency list

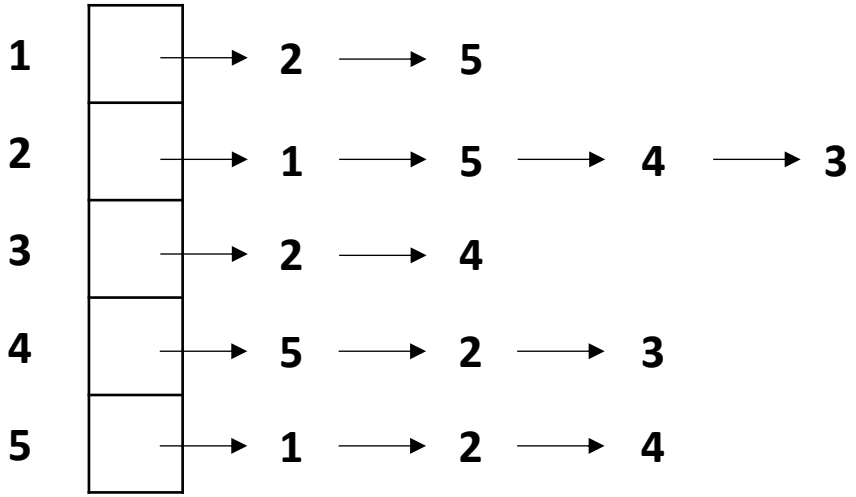


number of nodes number of edges

Size: $\Theta (n + m)$



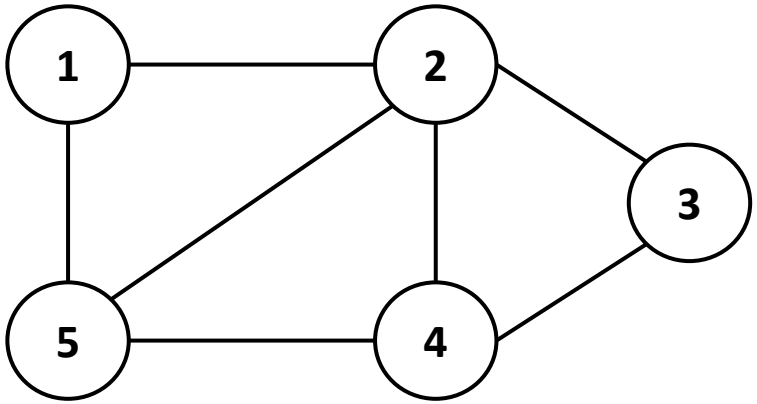
Adjacency list



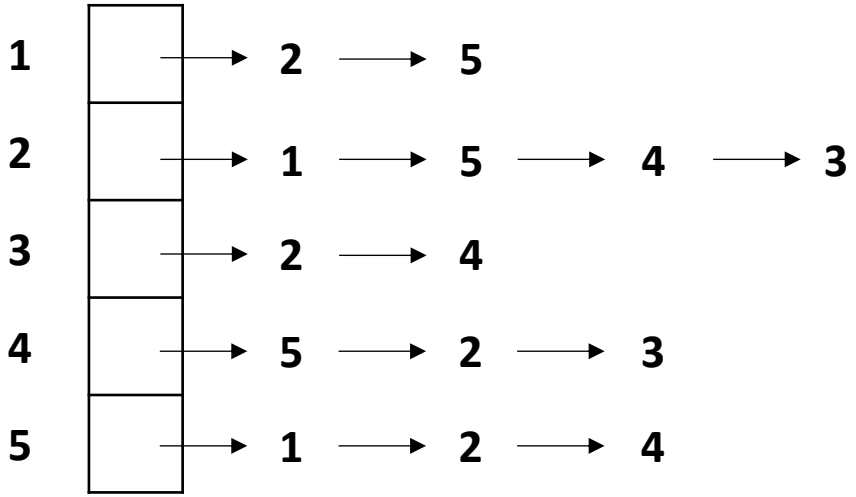
number of nodes number of edges

Size: $\Theta(n + m)$

Note that $m \leq$



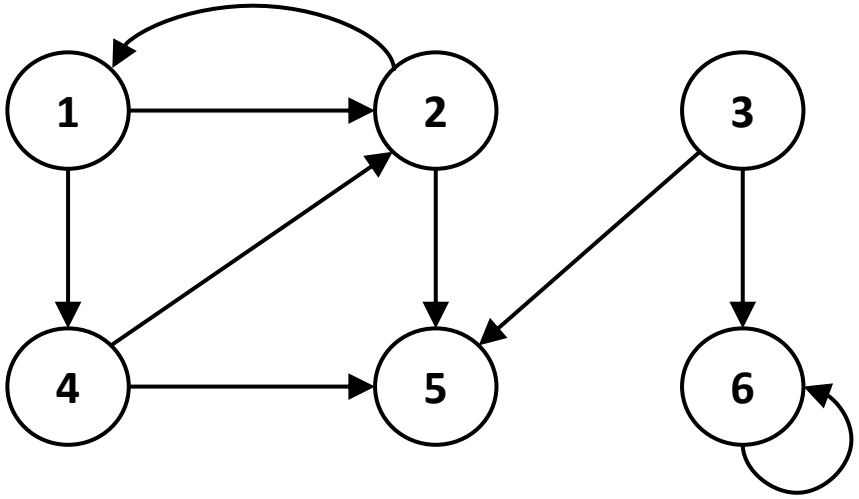
Adjacency list



number of nodes number of edges

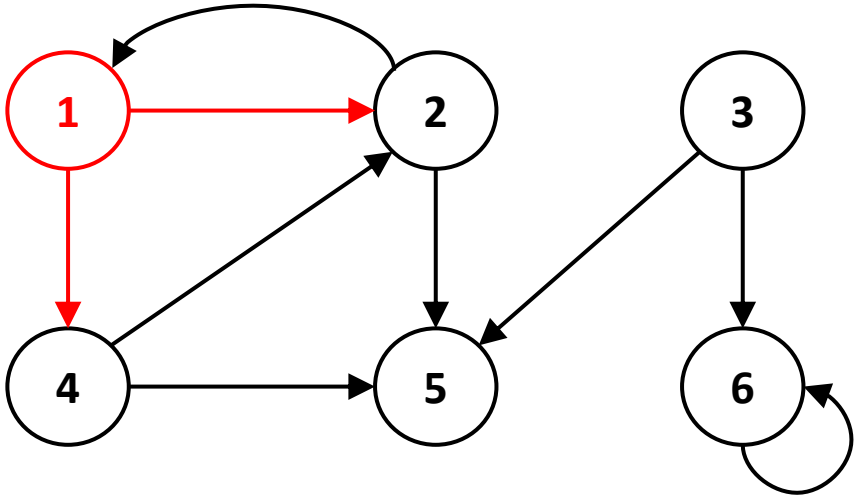
Size: $\Theta (n + m)$

Note that $m \leq n^2$

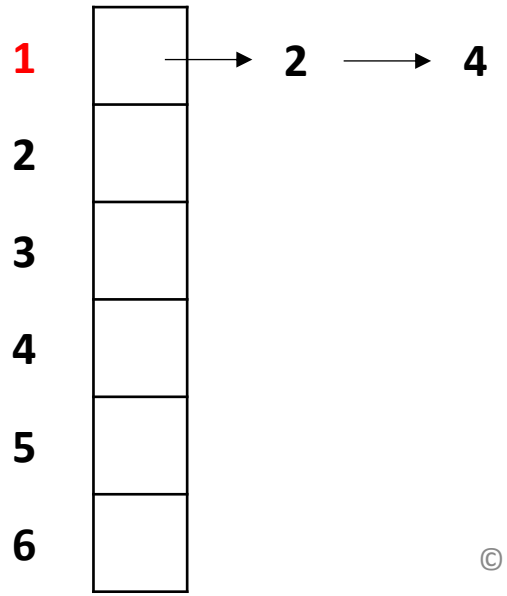


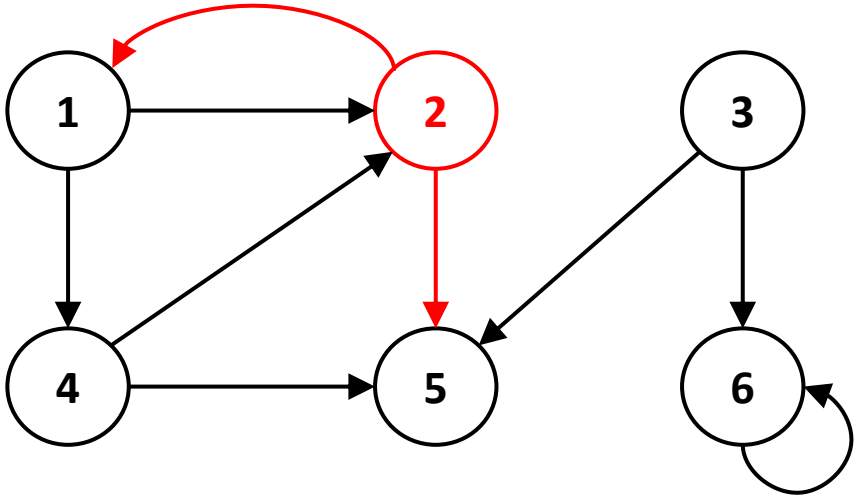
Adjacency list

1	
2	
3	
4	
5	
6	

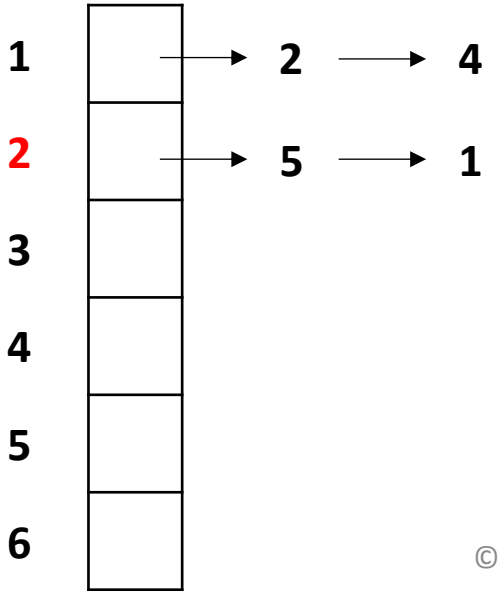


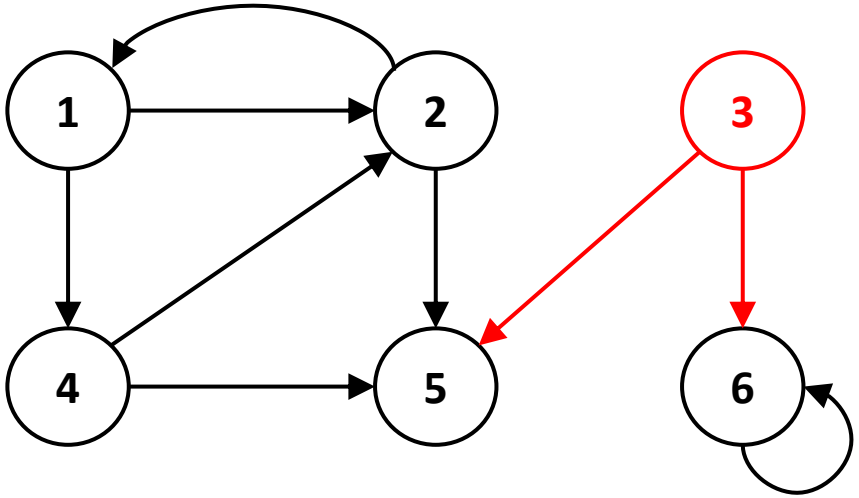
Adjacency list



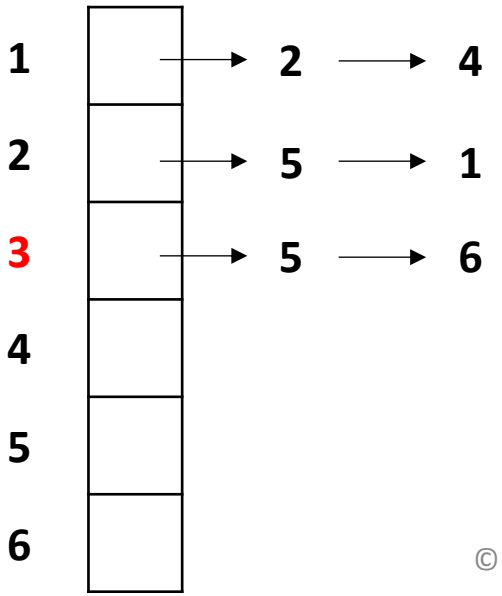


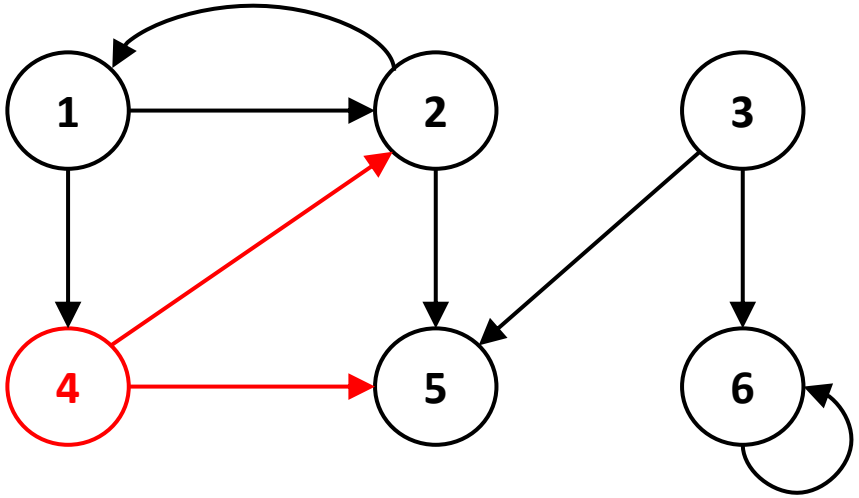
Adjacency list



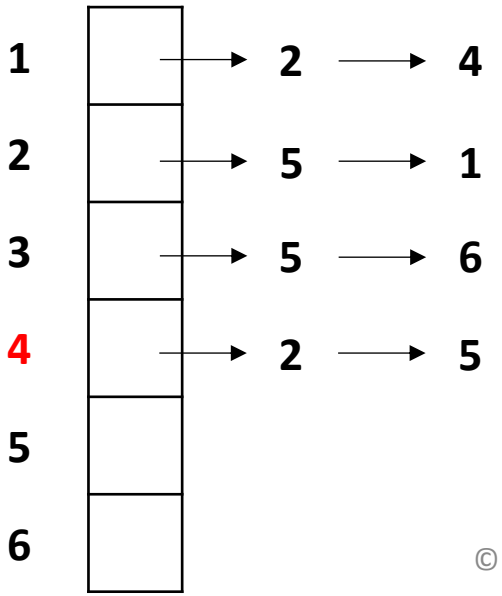


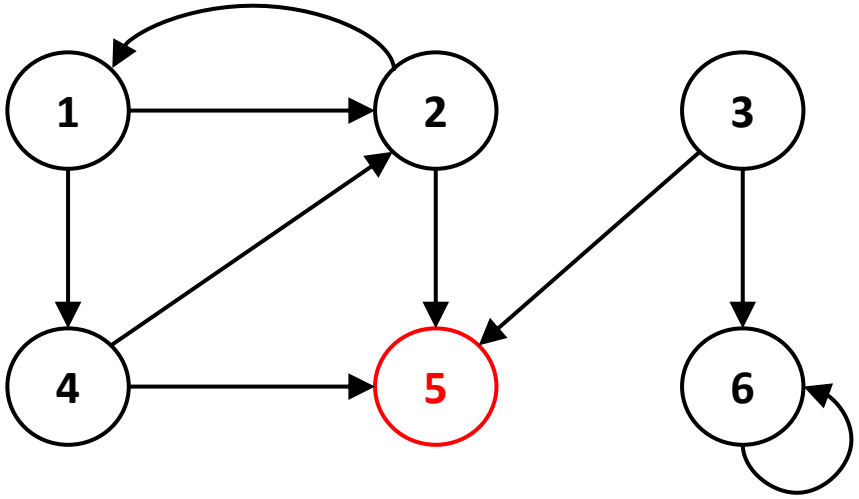
Adjacency list





Adjacency list





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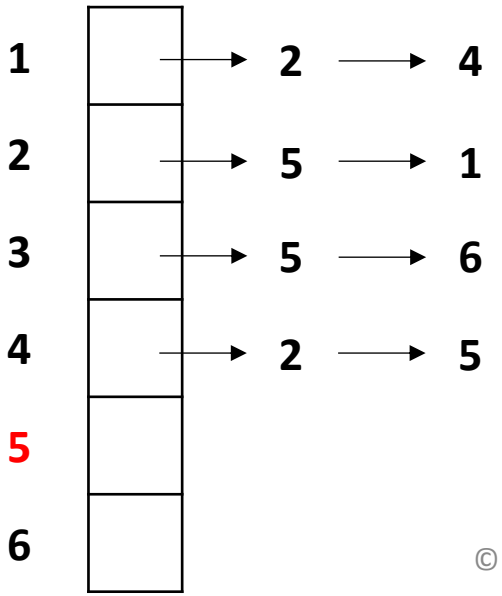

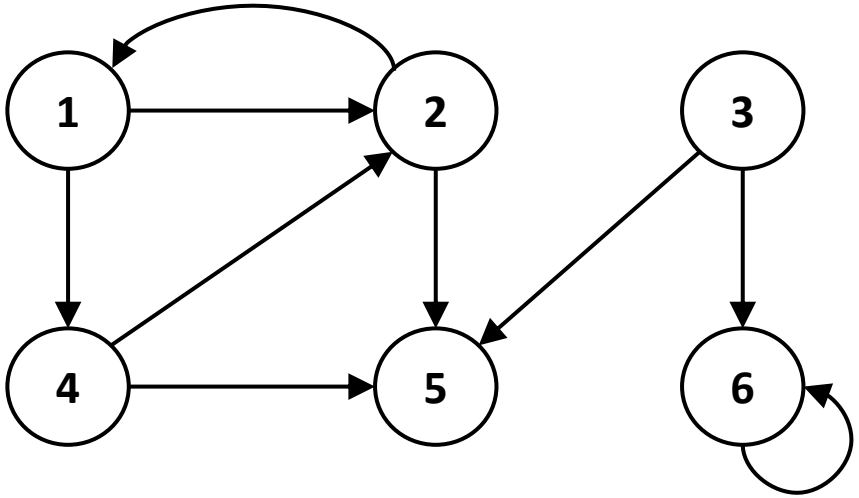

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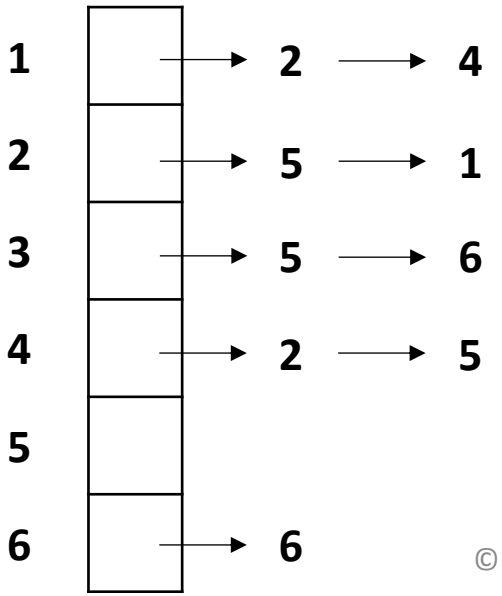

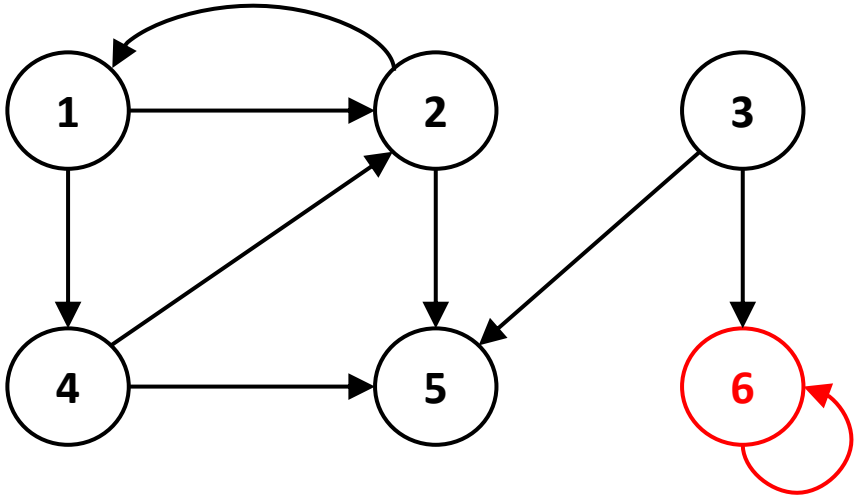

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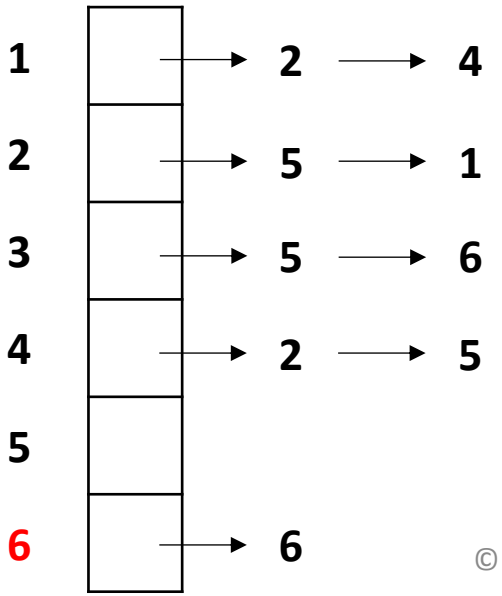
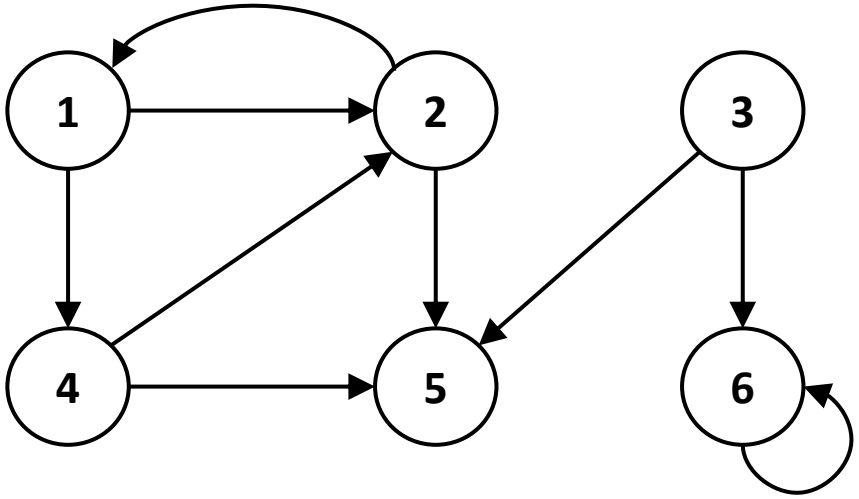
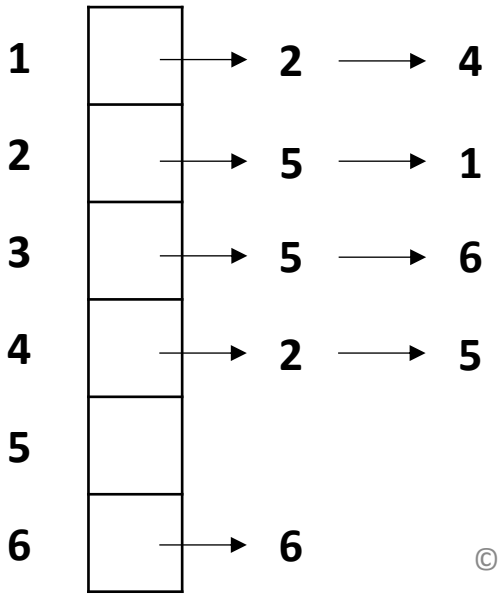




Adjacency list



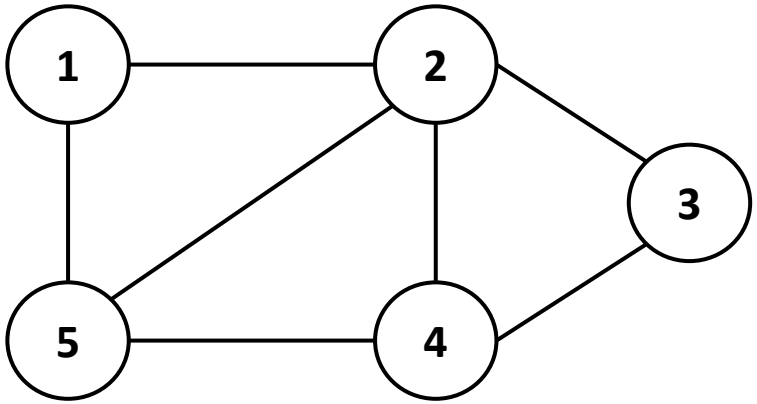
Size: $\Theta (n + m)$

Representing graphs

Adjacency Matrix of G = (V, E):

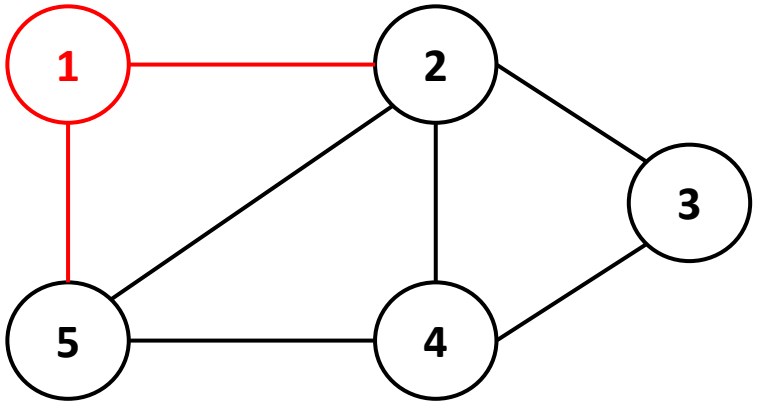
n x n matrix A such that for $1 \leq i,j \leq n$:

$$A[i, j] = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{otherwise} \end{cases}$$



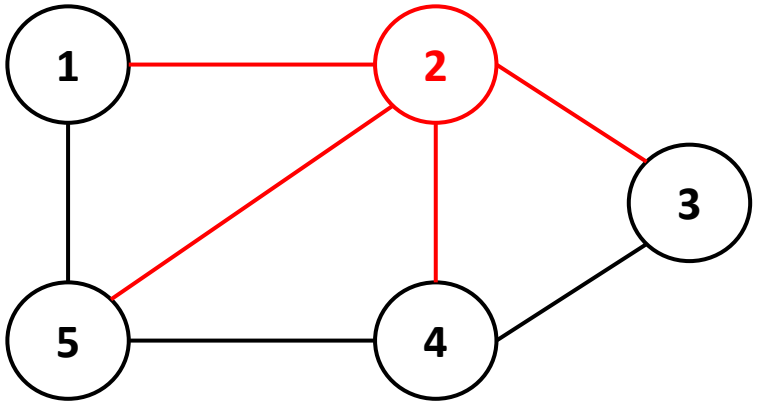
Adjacency matrix

	1	2	3	4	5
1					
2					
3					
4					
5					



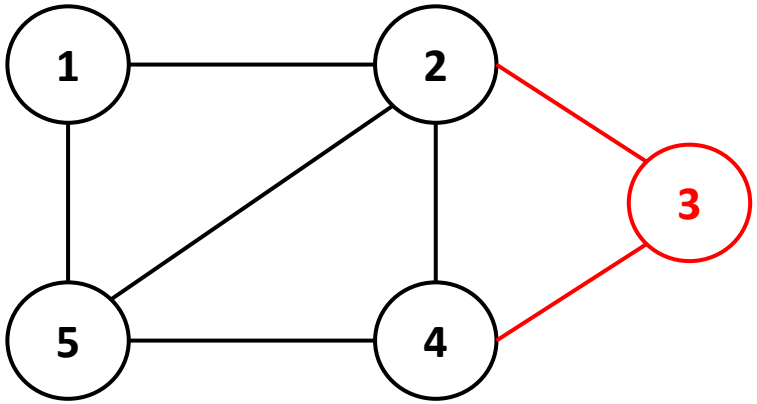
Adjacency matrix

	1	2	3	4	5
1	0	1	0	0	1
2					
3					
4					
5					



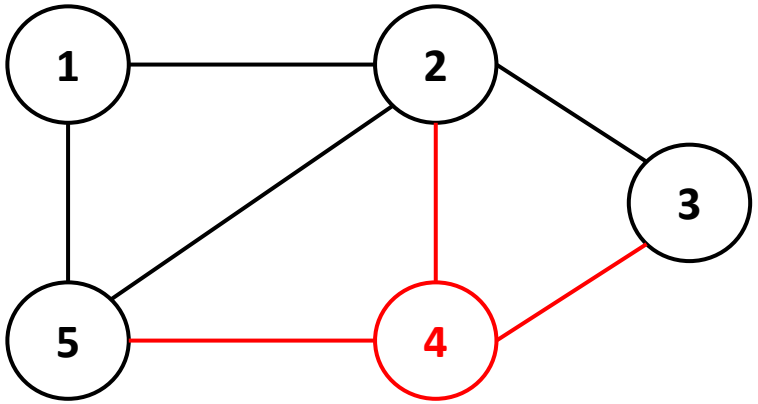
Adjacency matrix

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3					
4					
5					



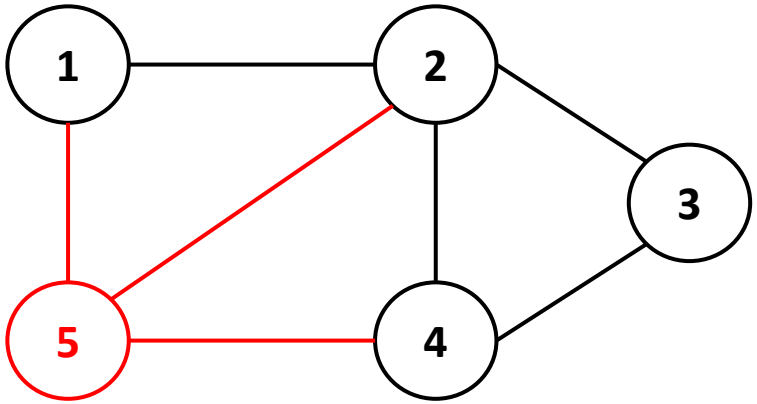
Adjacency matrix

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4					
5					



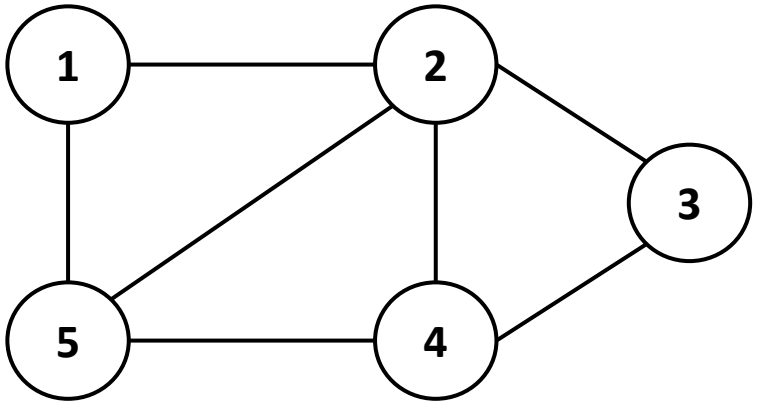
Adjacency matrix

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5					



Adjacency matrix

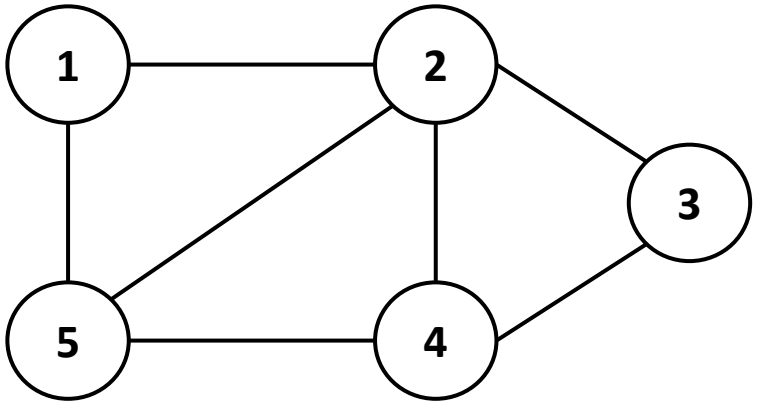
	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0



Adjacency matrix

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

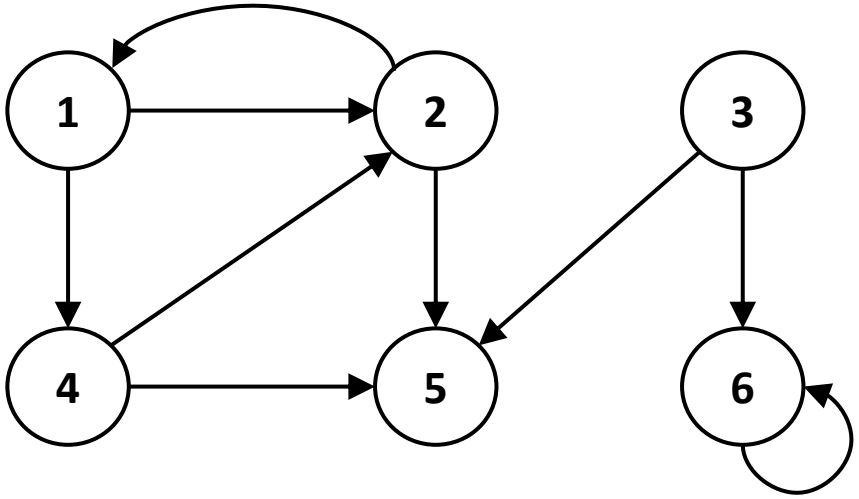
Adjacency matrix of an undirected graph is symmetric about its diagonal



Adjacency matrix

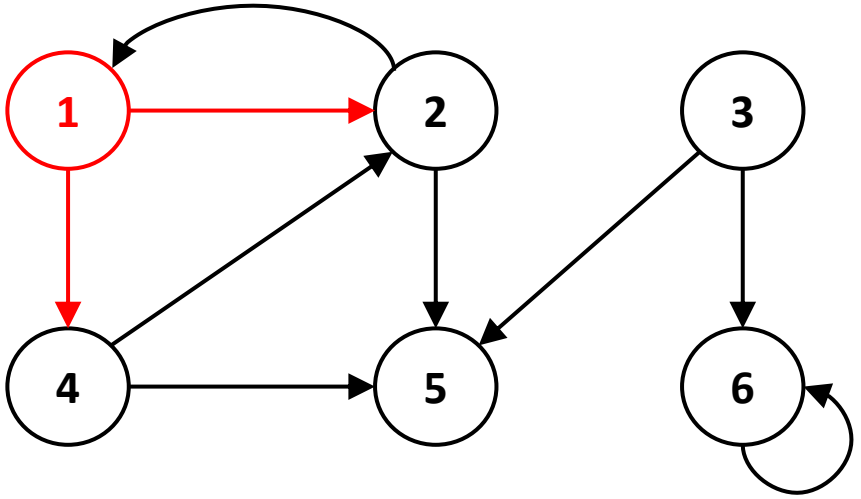
	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Size: $O(n^2)$



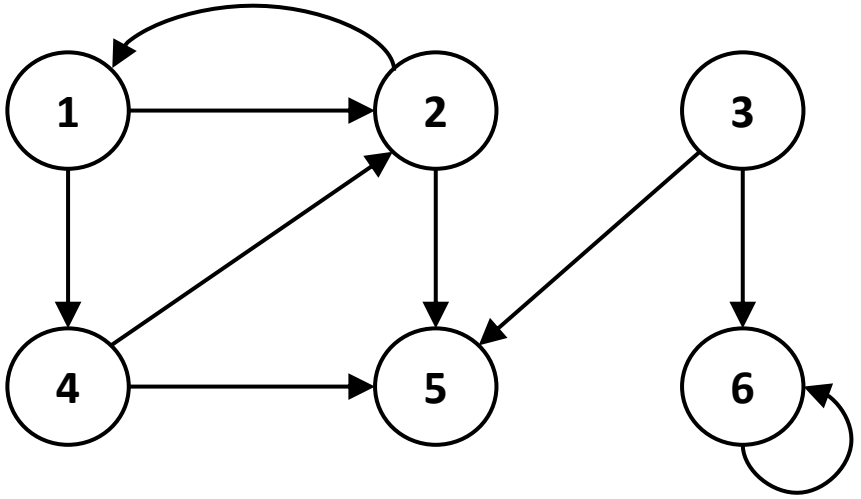
Adjacency matrix

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						



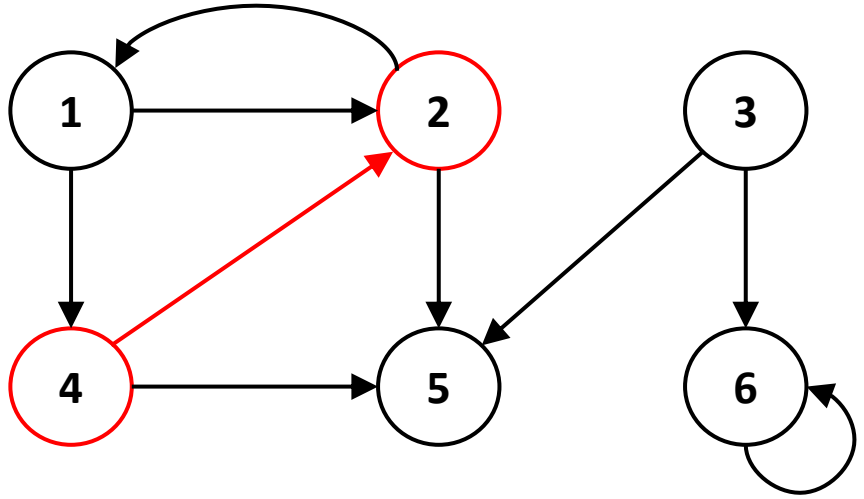
Adjacency matrix

	1	2	3	4	5	6
1	0	1	0	1	0	0
2						
3						
4						
5						
6						



Adjacency matrix

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	1	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	1	0
5	0	0	0	0	0	0
6	0	0	0	0	0	1

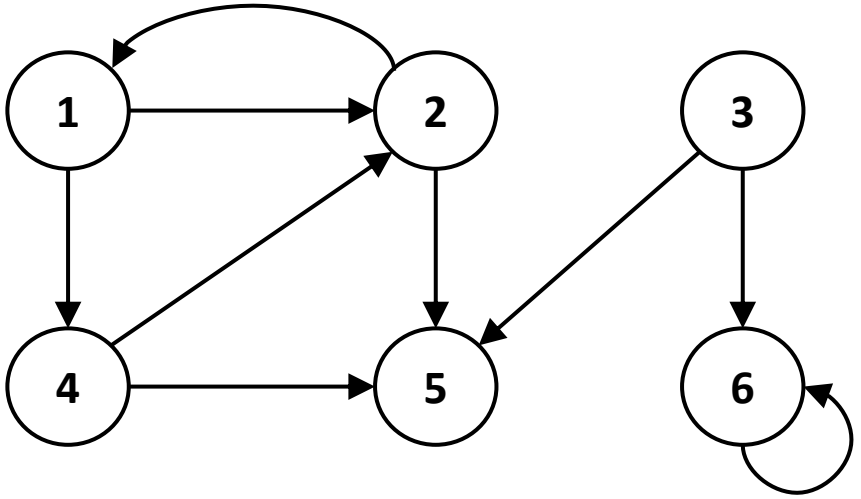


Adjacency matrix

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	1	0	0	**0**	1	0
3	0	0	0	0	1	1
4	0	**1**	0	0	1	0
5	0	0	0	0	0	0
6	0	0	0	0	0	1

A[2, 4] ≠ A[4, 2]

Adjacency matrix of a directed graph is not necessarily symmetric about its diagonal

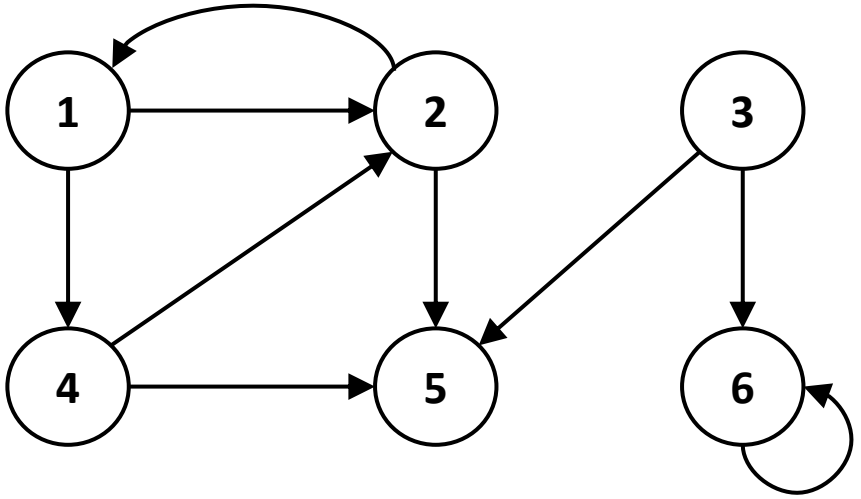


Adjacency matrix

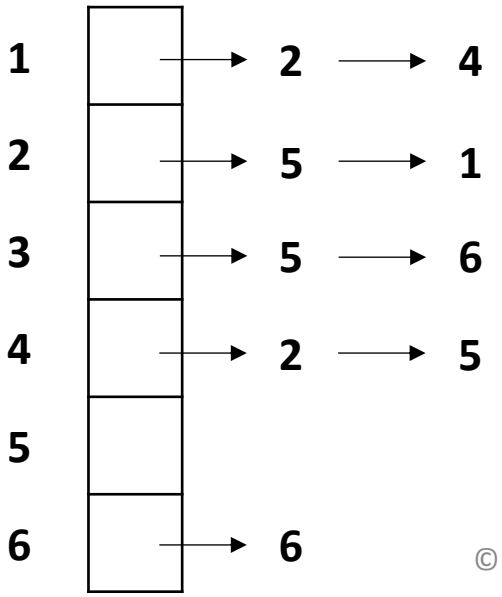
	1	2	3	4	5	6
1	0	1	0	1	0	0
2	1	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	1	0
5	0	0	0	0	0	0
6	0	0	0	0	0	1

Size: $\Theta(n^2)$

When to use one representation over the other?

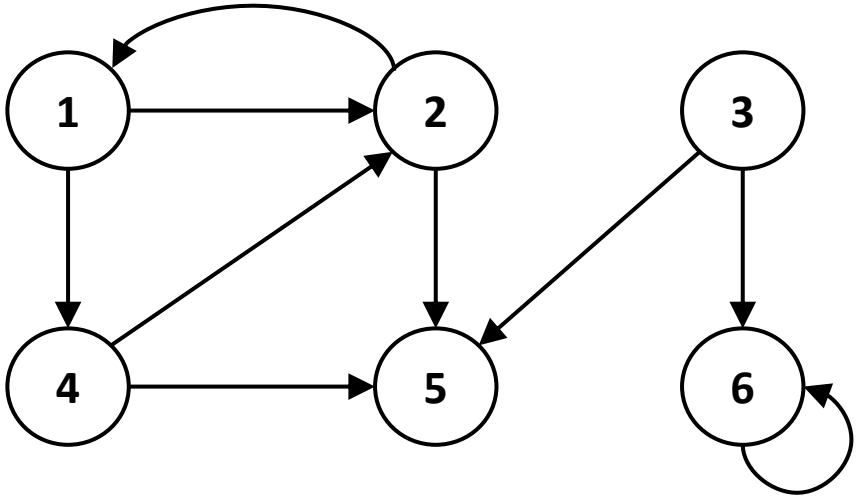


Adjacency list Size: $\Theta (n + m)$

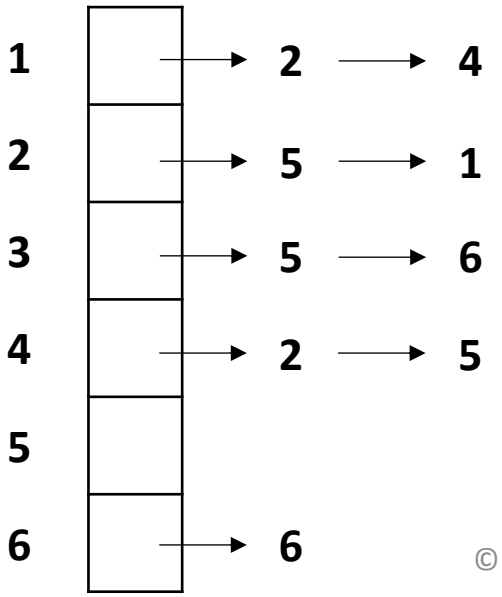


Adjacency matrix Size: $\Theta (n^2)$

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	1	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	1	0
5	0	0	0	0	0	0
6	0	0	0	0	0	1



Adjacency list Size: $\Theta(n + m)$



edge (3,6) is in G?

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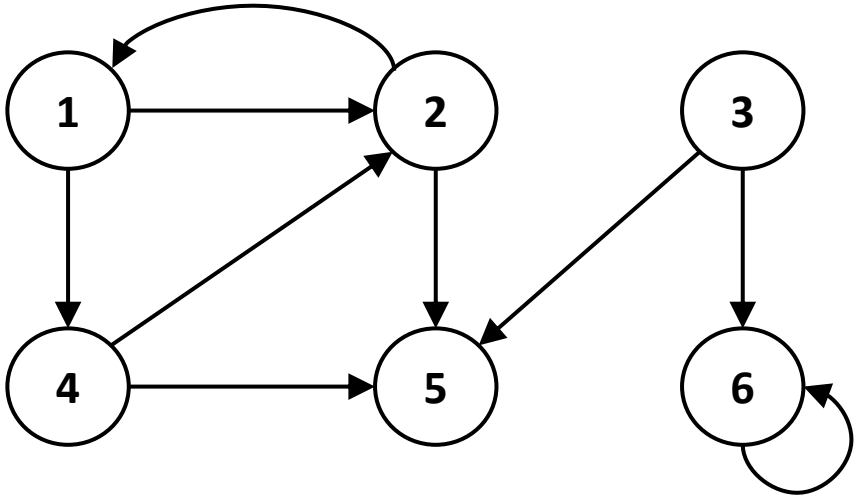

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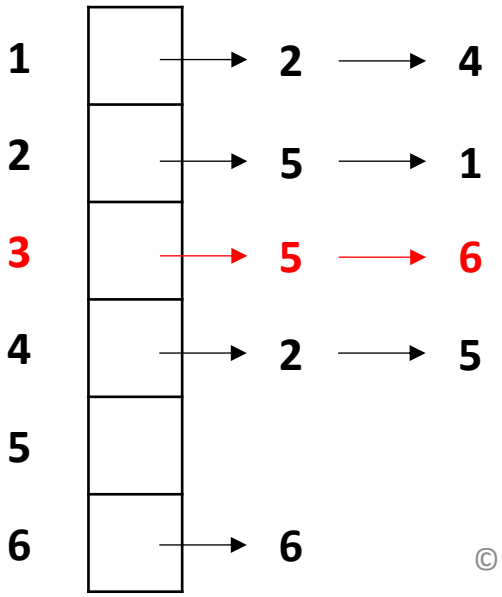

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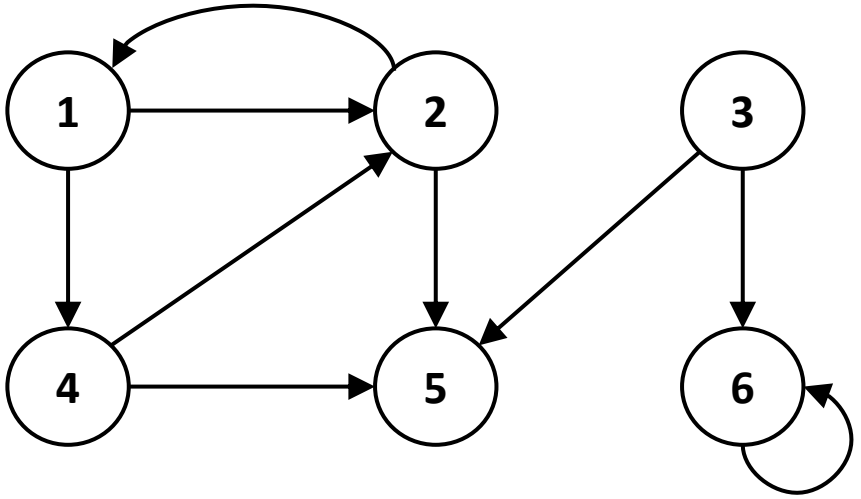

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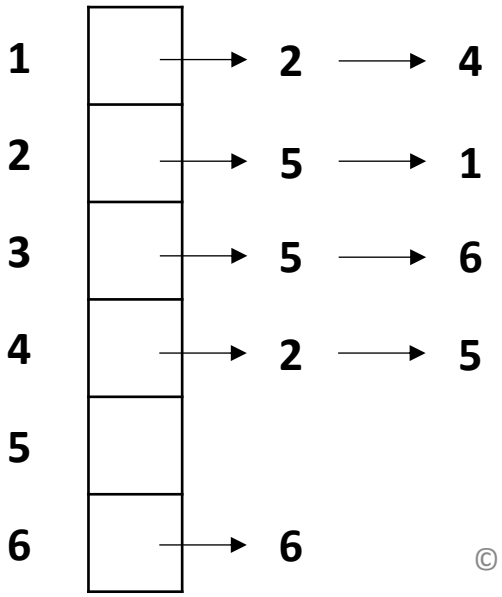

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A[3,6] = 1 ?

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1	0	1	0	1	0	0
2	1	0	0	0	1	0
3	0	0	0	0	1	**1**
4	0	1	0	0	1	0
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- For sparse graphs ($m << n^2$), Adjacency List is more space-efficient.

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- Finding whether a certain $(u, v) \in E$:
  - Is O(1) with Adjacency Matrix
  - Is O(n) with Adjacency List

Graph Search

Systematic exploration of graph:

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- Start at some node and explore it

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Graph searches reveal structural properties of G:

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Shortest path info,

etc…

Graph Search

Two basic types of graph searches:

Graph Search

Two basic types of graph searches:

- Breadth First Search (BFS)
- Depth First Search (DFS)

Breadth First Search

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BFS started at a node s

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(1) Explore s (i.e. follow all the edges out of s to discover new nodes)

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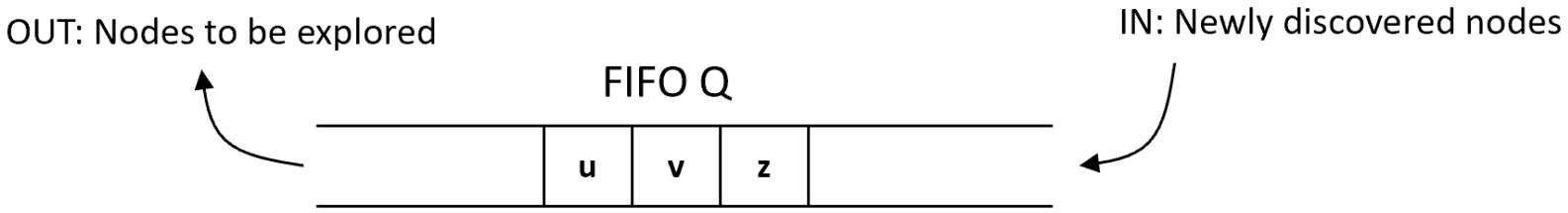

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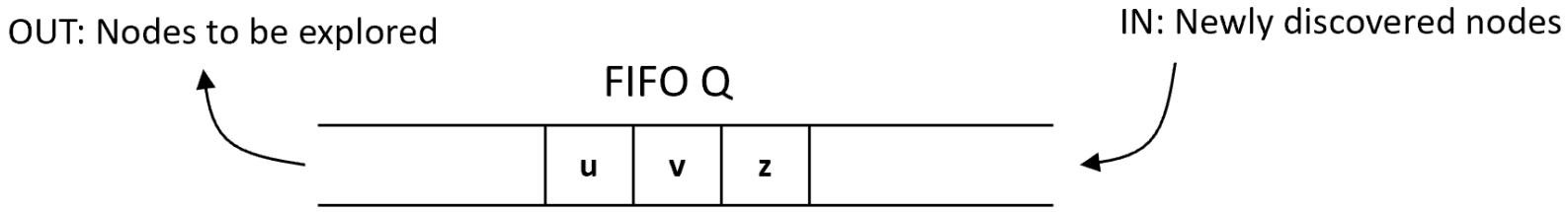

Do (2) until all discovered nodes are explored

BFS algorithm maintains the following information for each node v:

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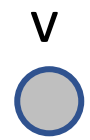

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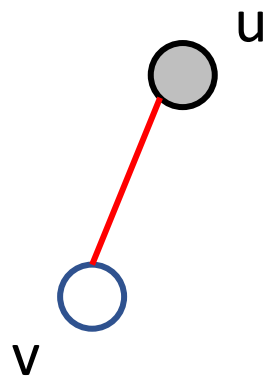

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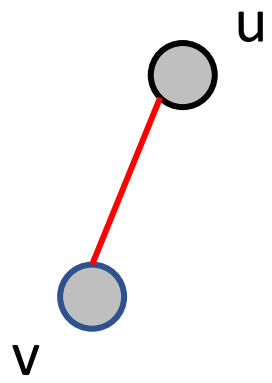

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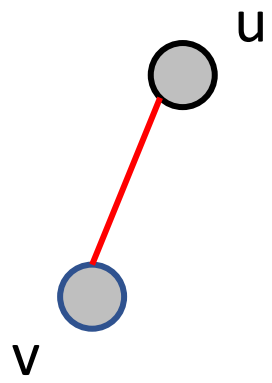

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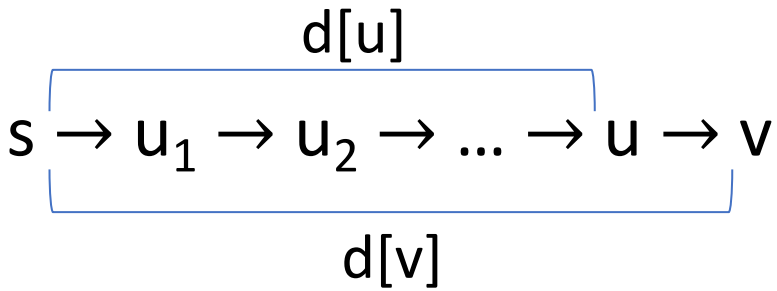

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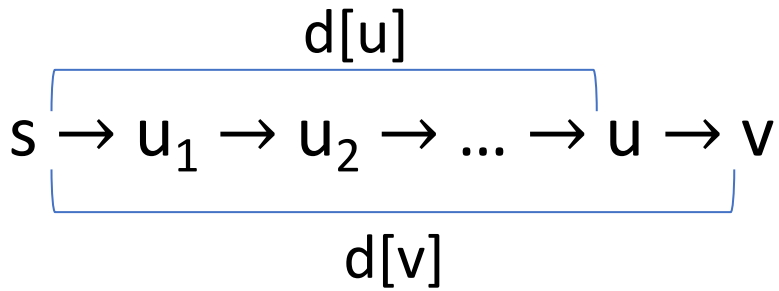

Clearly, d[v] = d[u] + 1

BFS(G, s**)** /* G = (V, E) and s ∈ V */

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            color[v] ← white
            d[v]  ←  ∞
            p[v]  ←  NIL
    Q ← empty ; ENQ(Q, s)                          /*  Q: nodes that are discovered but not yet explored  */
    While Q is not empty  do
            u ← DEQ(Q)                                          /*  Explore u  */
            For each (u, v) ∈ E  do                             /*  Explore edge (u,v)  */
                    If color[v] = white  then do                /*  If v is first discovered  */
                            color[v] ← grey
                            d[v] ← d[u] + 1
                            p[v] ← u
                            ENQ(Q, v)
                    End If
```

```
BFS(G, s)                                          /*  G = (V, E) and s ∈ V  */
    color[s] ← grey ;  d[s] ← 0 ;  p[s] ← NIL
    For each v ∈ V – {s}  do
            color[v] ← white
            d[v]  ←  ∞
            p[v]  ←  NIL
    Q ← empty ; ENQ(Q, s)                          /*  Q: nodes that are discovered but not yet explored  */
    While Q is not empty  do
            u ← DEQ(Q)                                          /*  Explore u  */
            For each (u, v) ∈ E  do                             /*  Explore edge (u,v)  */
                    If color[v] = white  then do                /*  If v is first discovered  */
                            color[v] ← grey
                            d[v] ← d[u] + 1
                            p[v] ← u
                            ENQ(Q, v)
                    End If
            End For
```

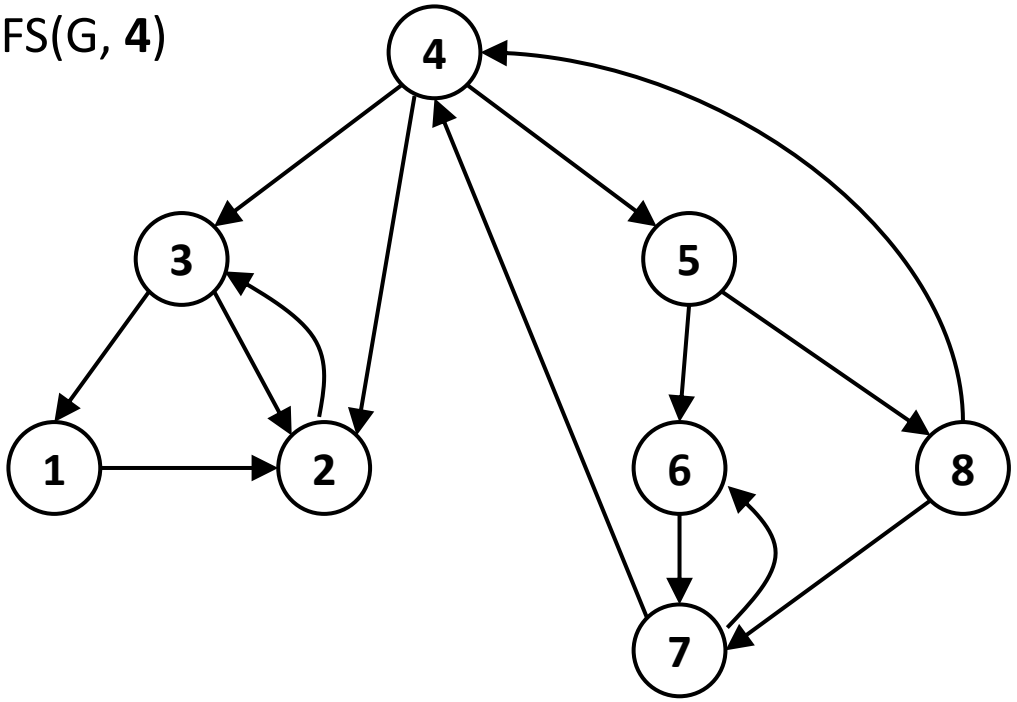
```
BFS(G, s)                                          /*  G = (V, E) and s ∈ V  */
    color[s] ← grey ; d[s] ← 0 ; p[s] ← NIL
    For each v ∈ V – {s}  do
            color[v] ← white
            d[v] ← ∞
            p[v] ← NIL
    Q ← empty ; ENQ(Q, s)                          /*  Q: nodes that are discovered but not yet explored  */
    While Q is not empty  do
            u ← DEQ(Q)                                          /*  Explore u  */
            For each (u, v) ∈ E  do                             /*  Explore edge (u,v)  */
                    If color[v] = white  then do                /*  If v is first discovered  */
                            color[v] ← grey
                            d[v] ← d[u] + 1
                            p[v] ← u
                            ENQ(Q, v)
                    End If
            End For
            color[u] ←                                          /*  Done exploring u  */
```


```
BFS(G, s)                                          /*  G = (V, E) and s ∈ V  */
    color[s] ← grey ; d[s] ← 0 ; p[s] ← NIL
    For each v ∈ V – {s}  do
            color[v] ← white
            d[v] ← ∞
            p[v] ← NIL
    Q ← empty ; ENQ(Q, s)                          /*  Q: nodes that are discovered but not yet explored  */
    While Q is not empty  do
            u ← DEQ(Q)                                          /*  Explore u  */
            For each (u, v) ∈ E  do                             /*  Explore edge (u,v)  */
                    If color[v] = white  then do                /*  If v is first discovered  */
                            color[v] ← grey
                            d[v] ← d[u] + 1
                            p[v] ← u
                            ENQ(Q, v)
                    End If
            End For
            color[u] ← black                                    /*  Done exploring u  */
```

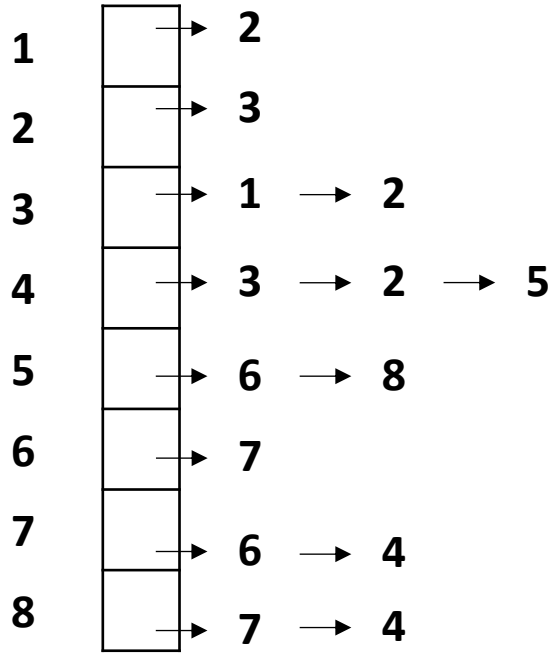

```
BFS(G, s)                                          /*  G = (V, E) and s ∈ V  */
    color[s] ← grey ; d[s] ← 0 ; p[s] ← NIL
    For each v ∈ V – {s}  do
            color[v] ← white
            d[v] ← ∞
            p[v] ← NIL
    Q ← empty ; ENQ(Q, s)                          /*  Q: nodes that are discovered but not yet explored  */
    While Q is not empty  do
            u ← DEQ(Q)                                          /*  Explore u  */
            For each (u, v) ∈ E  do                             /*  Explore edge (u,v)  */
                    If color[v] = white  then do                /*  If v is first discovered  */
                            color[v] ← grey
                            d[v] ← d[u] + 1
                            p[v] ← u
                            ENQ(Q, v)
                    End If
            End For
            color[u] ← black                                    /*  Done exploring u  */
    End While
End BFS
```


```
BFS(G, s)                                          /*  G = (V, E) and s ∈ V  */
    color[s] ← grey ; d[s] ← 0 ; p[s] ← NIL
    For each v ∈ V – {s}  do
            color[v] ← white
            d[v] ← ∞
            p[v] ← NIL
    Q ← empty ; ENQ(Q, s)          /*  Q: nodes that are discovered but not yet explored  */
    While Q is not empty  do
            u ← DEQ(Q)                                 /*  Explore u  */
        For each (u, v) ∈ E  do                        /*  Explore edge (u,v)  */
            If color[v] = white  then do               /*  If v is first discovered  */
                color[v] ← grey
                d[v] ← d[u] + 1
                p[v] ← u
                ENQ(Q, v)
            End If
        End For
            color[u] ← black                           /*  Done exploring u  */
    End While
End BFS
```


BFS(G, **4**)

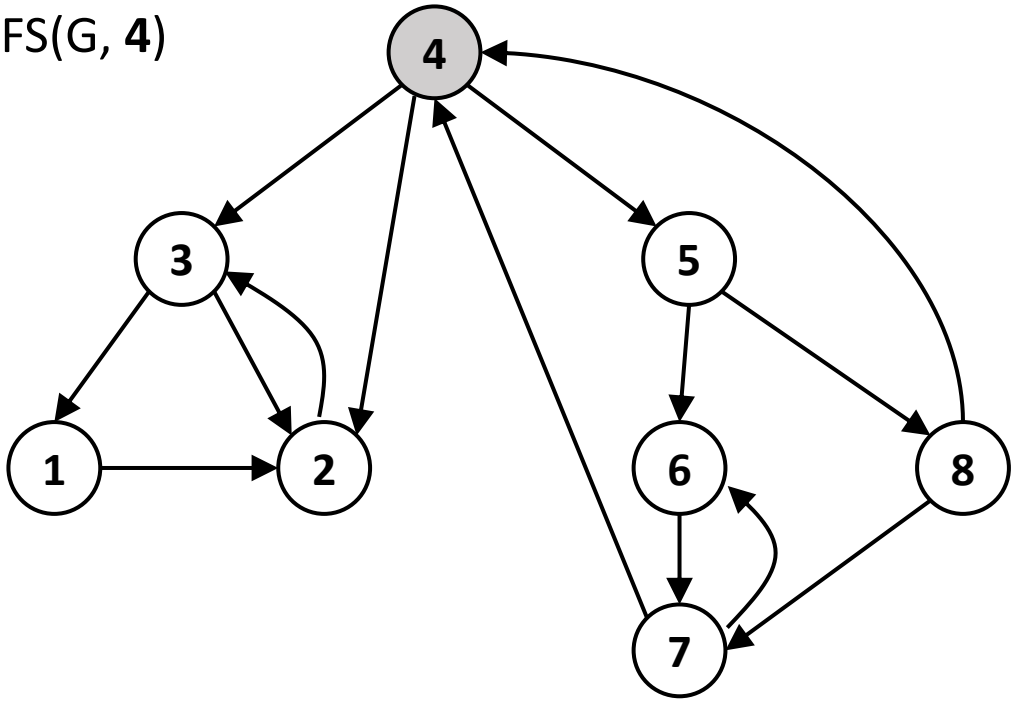


Adj List of G :



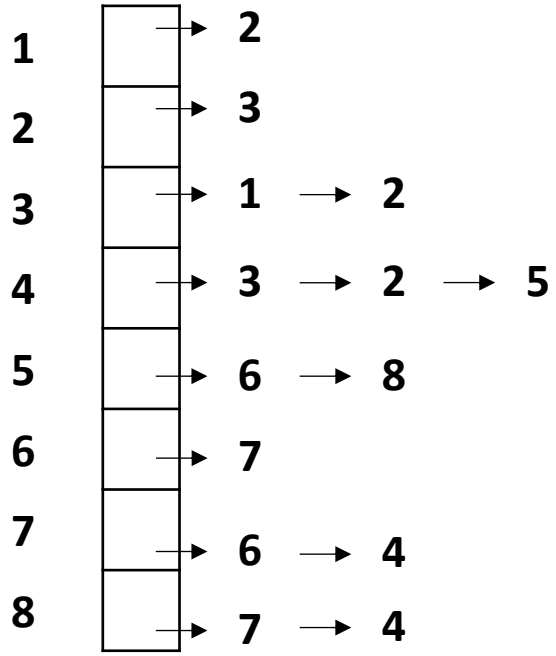
Contents of Q :

BFS(G, **4**)



4

Adj List of G :

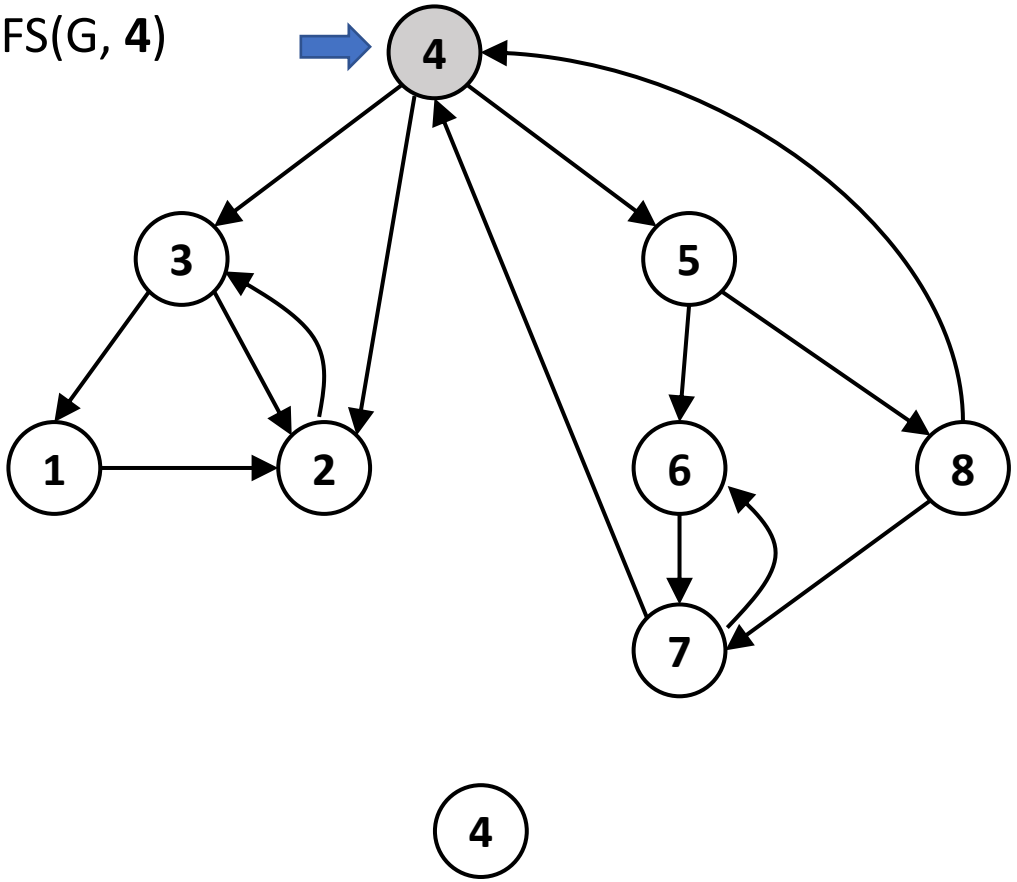


Contents of Q :

d = 0

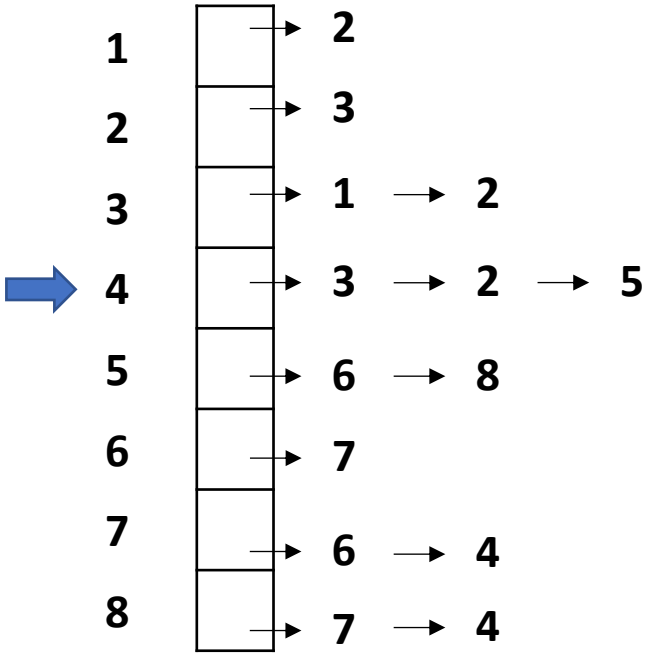
4

BFS(G, **4**)



4

Adj List of G :

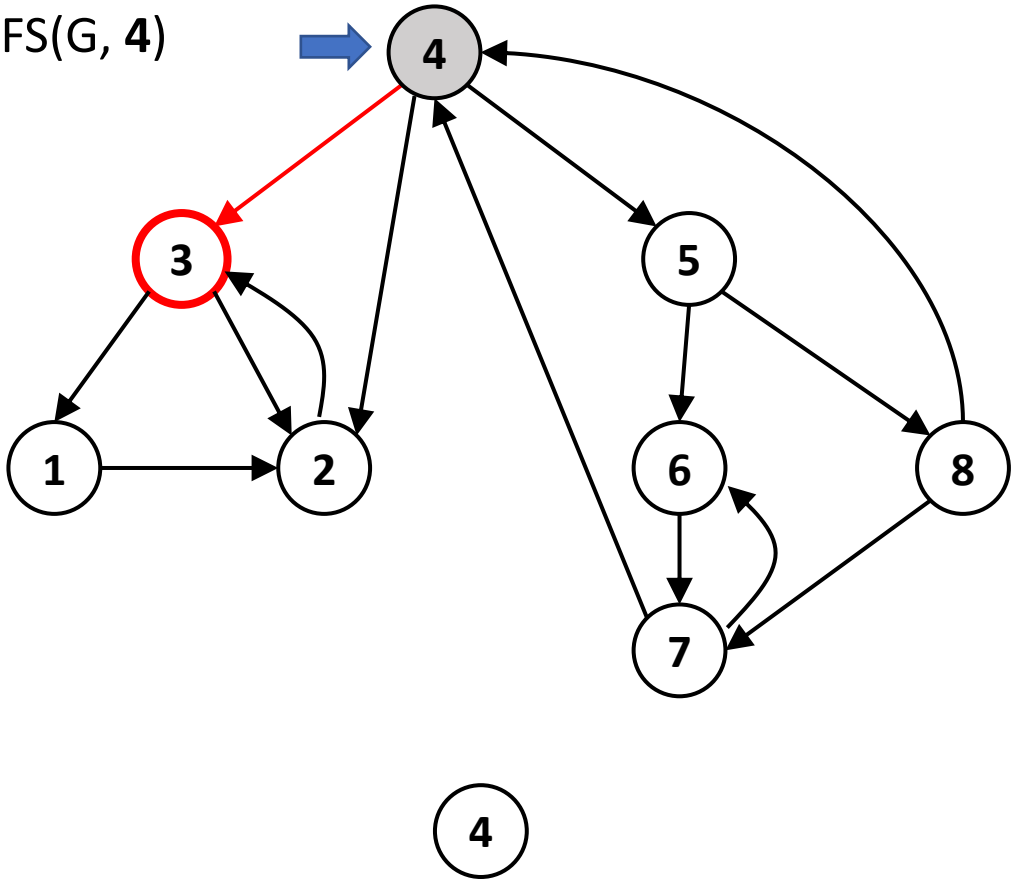


Contents of Q :

d = 0

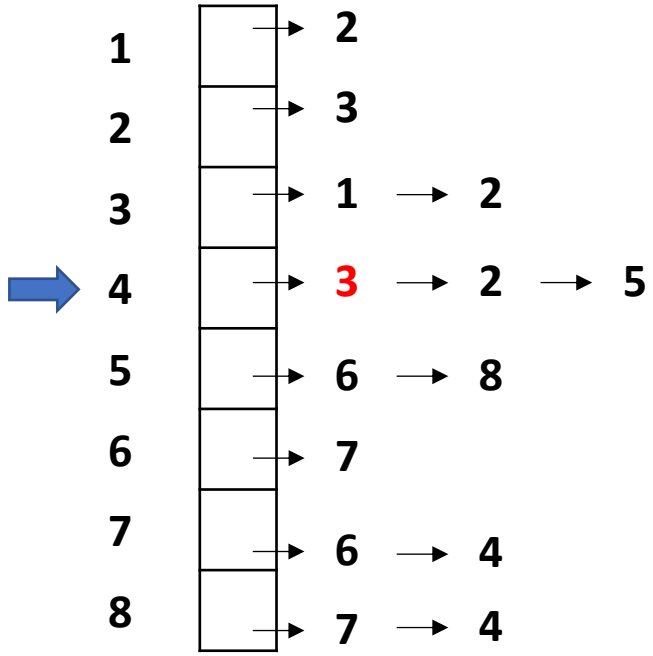
4

BFS(G, **4**)



4

Adj List of G :

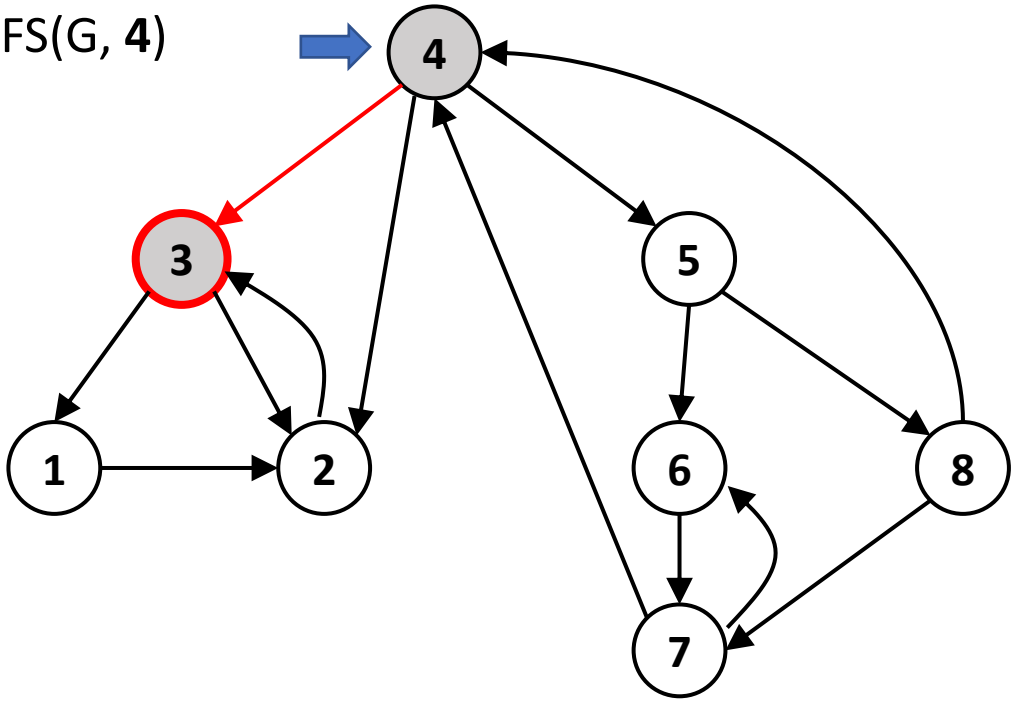


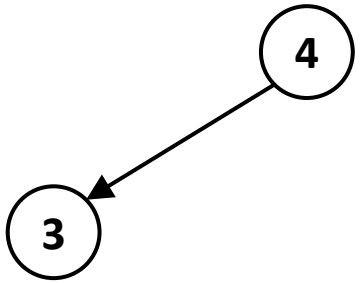
Contents of Q :

d = 0

4

BFS(G, **4**)





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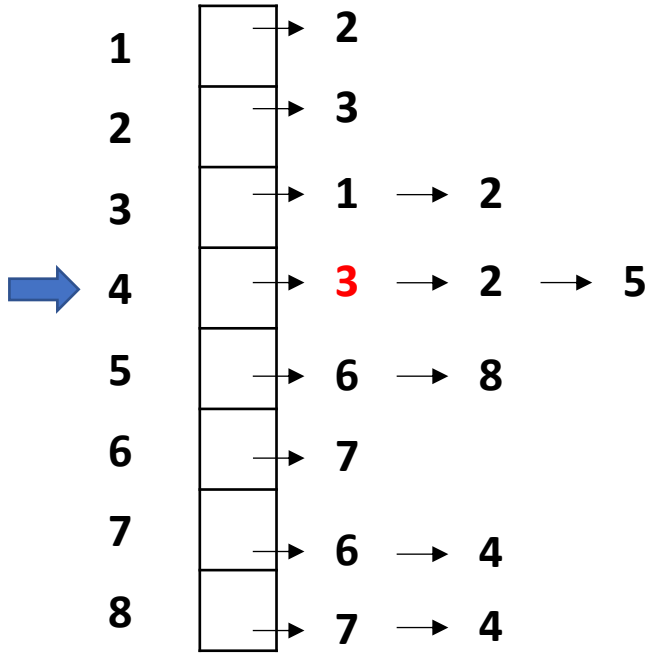


Contents of Q :

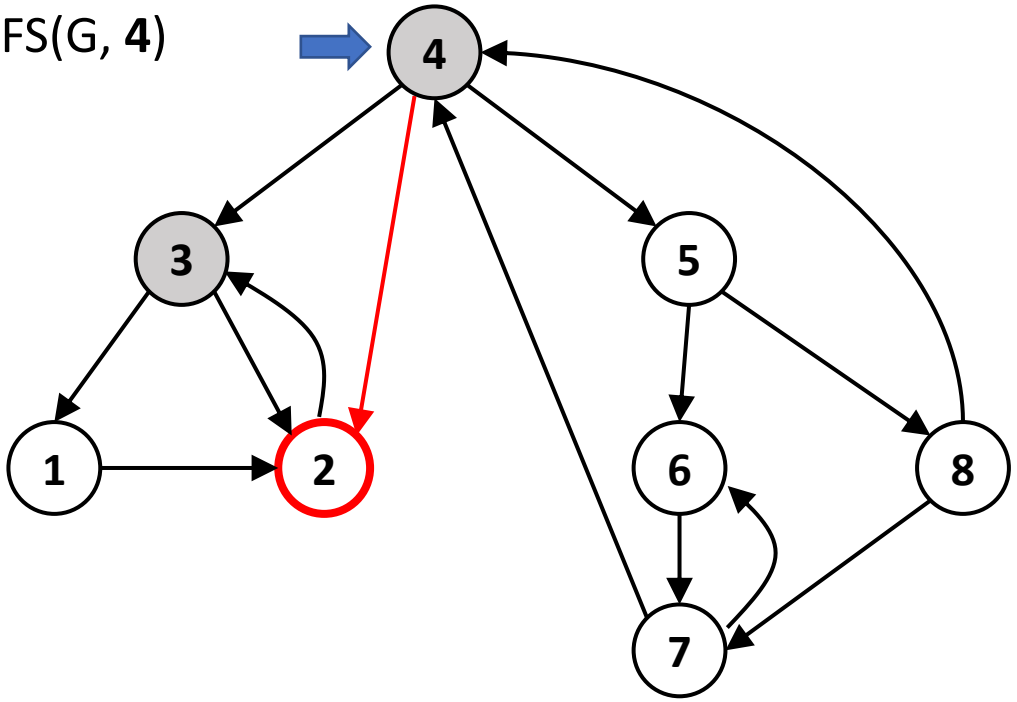
d = 0

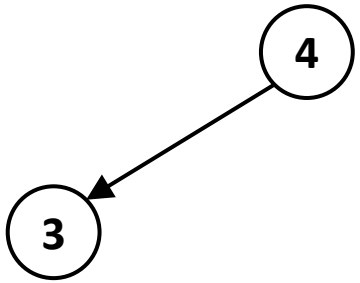
4

d = 1

3

BFS(G, **4**)





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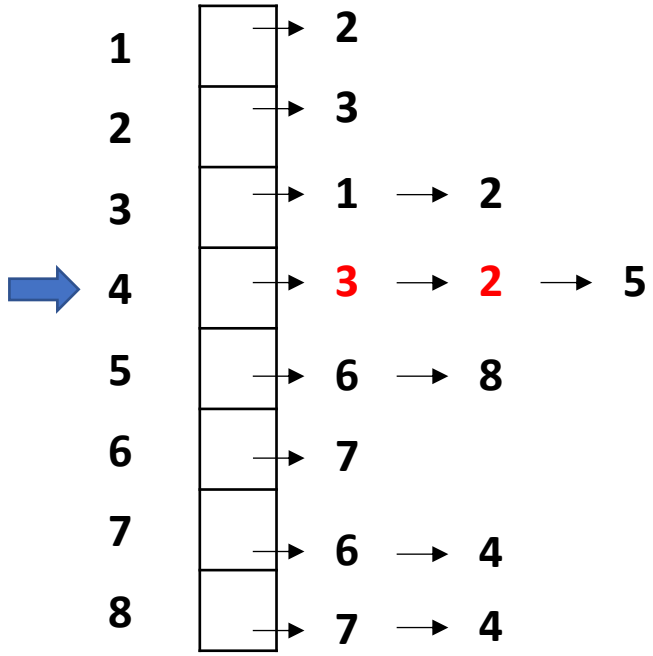


Contents of Q :

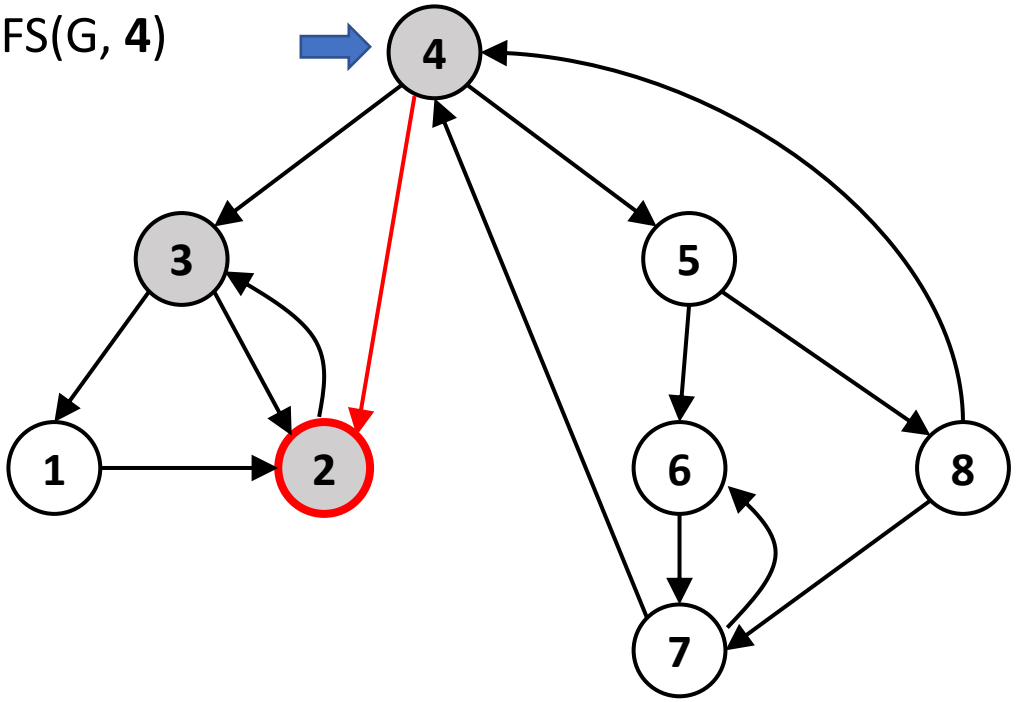
d = 0

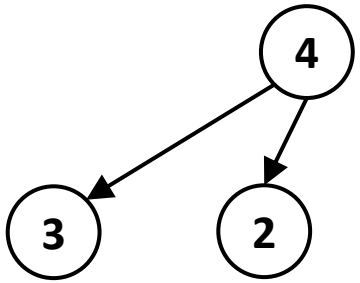
4

d = 1

3

BFS(G, **4**)





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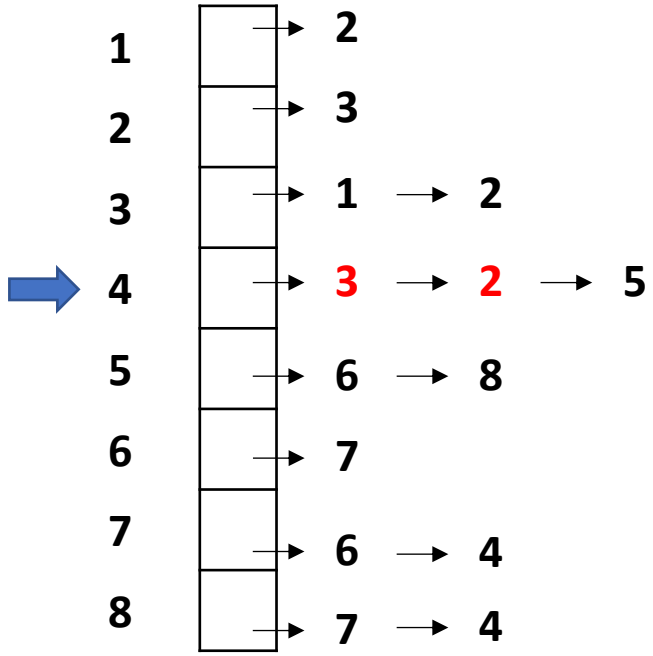


Contents of Q :

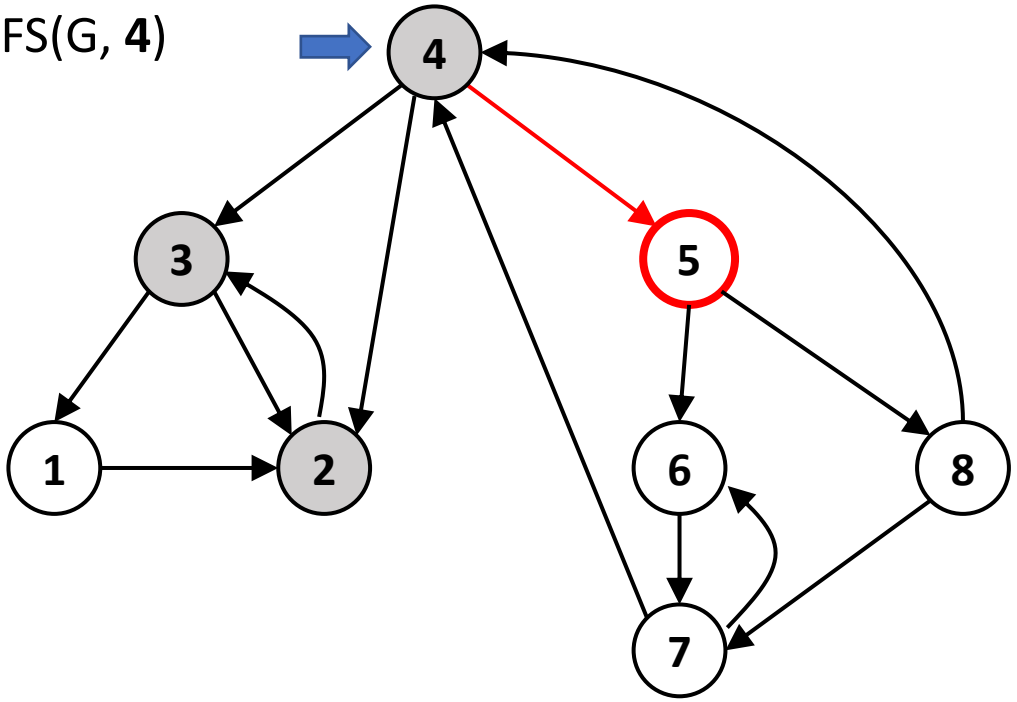
d = 0

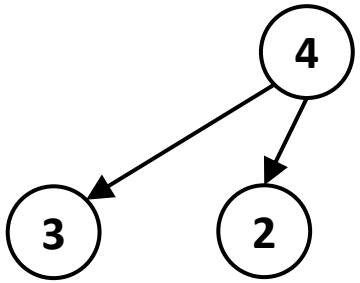
4

d = 1 1

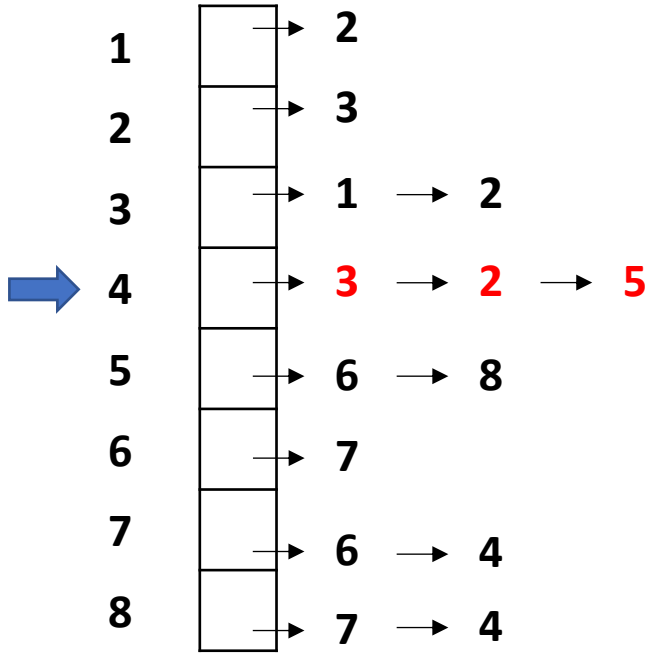
3	2

BFS(G, **4**)





Adj List of G :



Contents of Q :

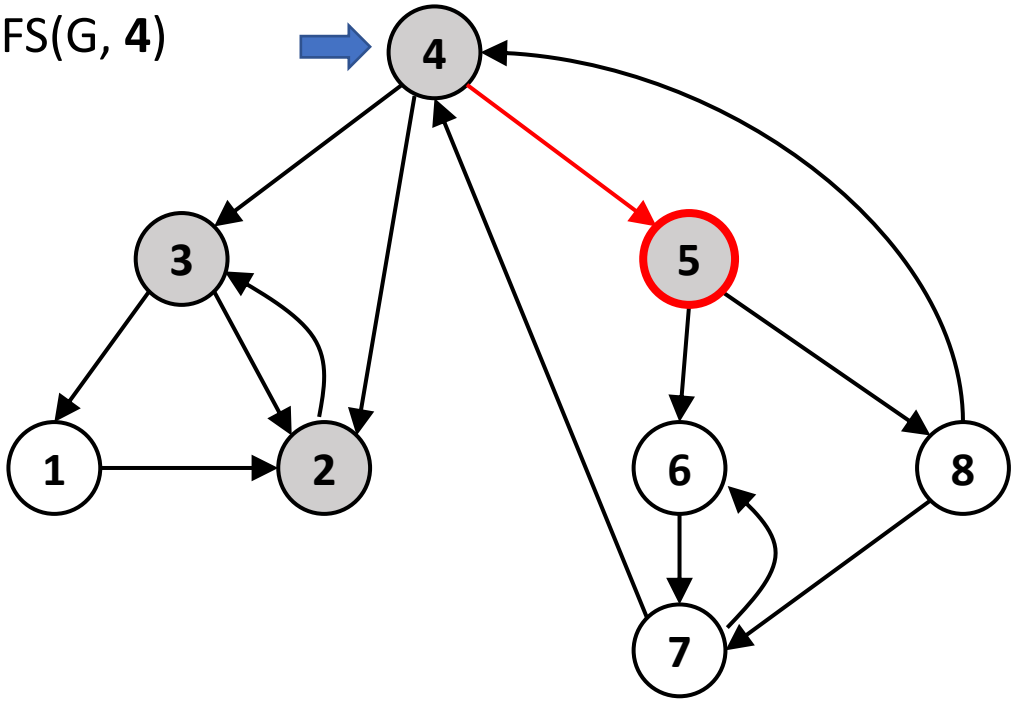
d = 0

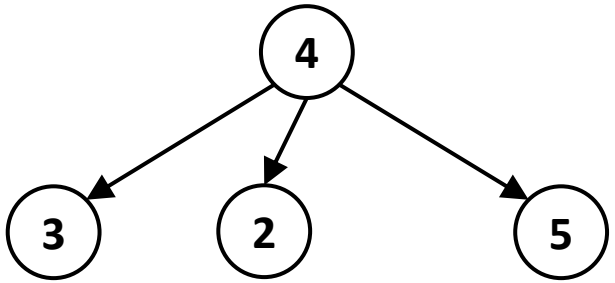
4

d = 1 1

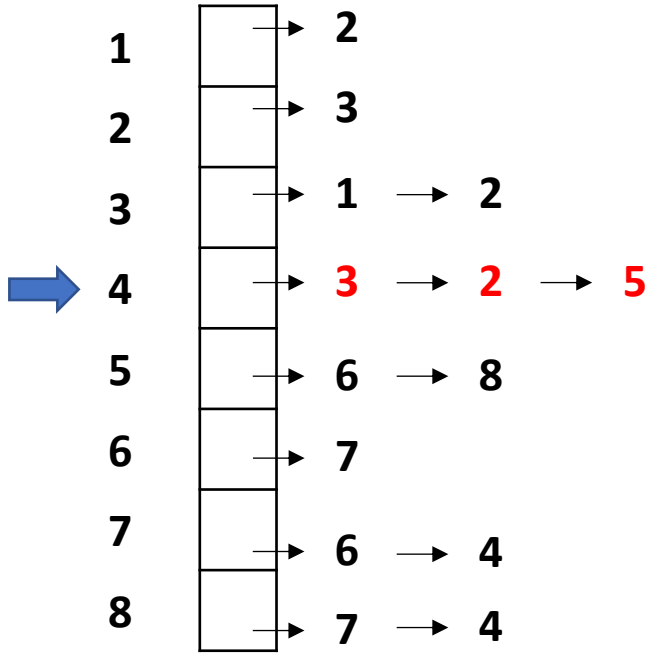
3	2

BFS(G, **4**)





Adj List of G :



Contents of Q :

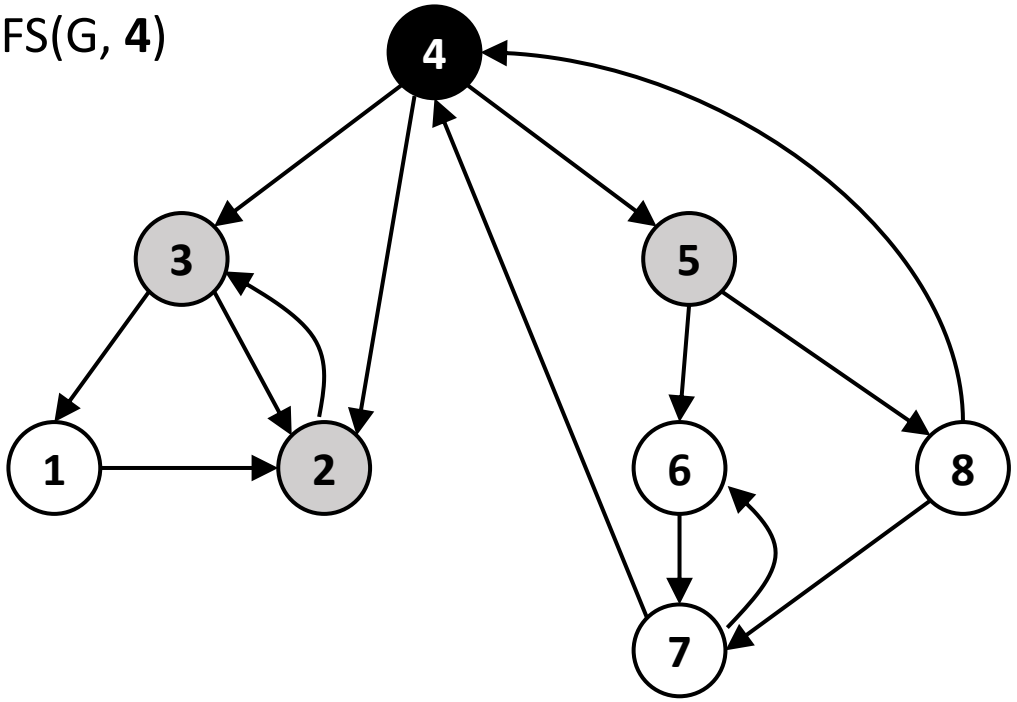
d = 0

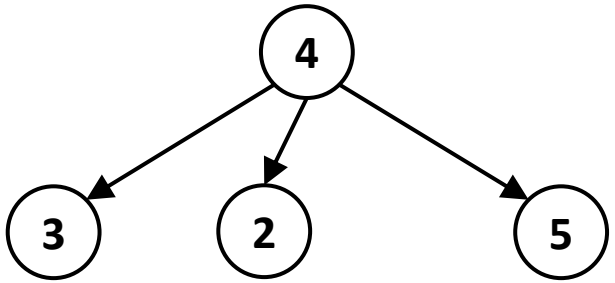
4

d = 1 1 1

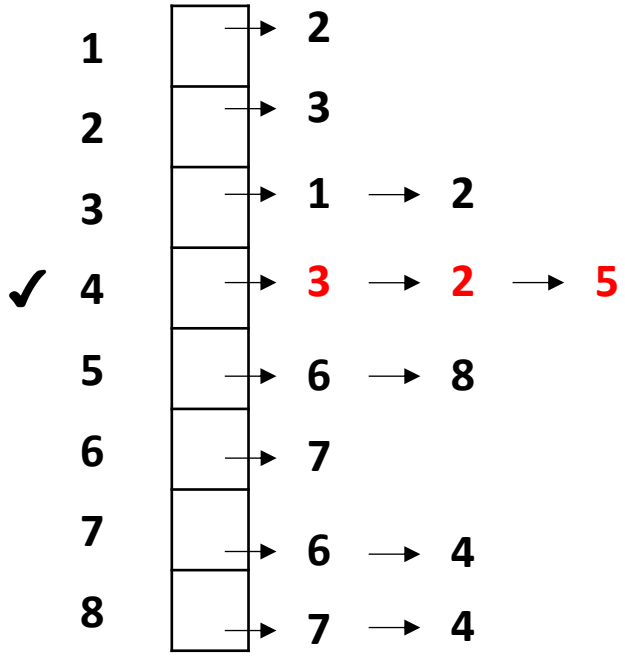
3	2	5

BFS(G, **4**)





Adj List of G :



Contents of Q :

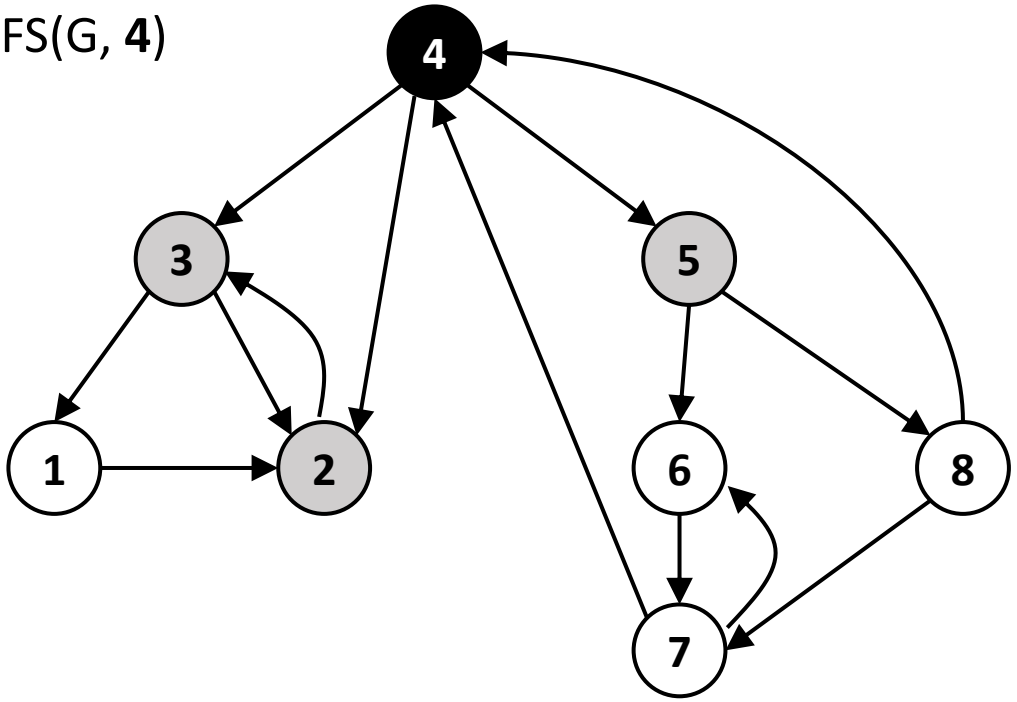
d = 0

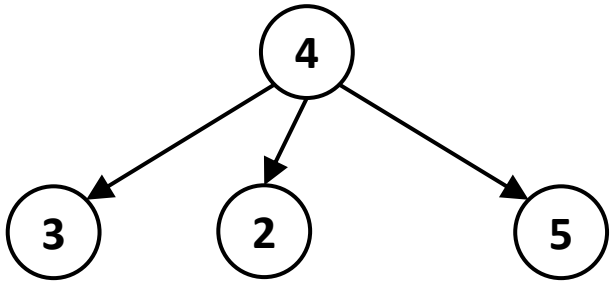
4

d = 1 1 1

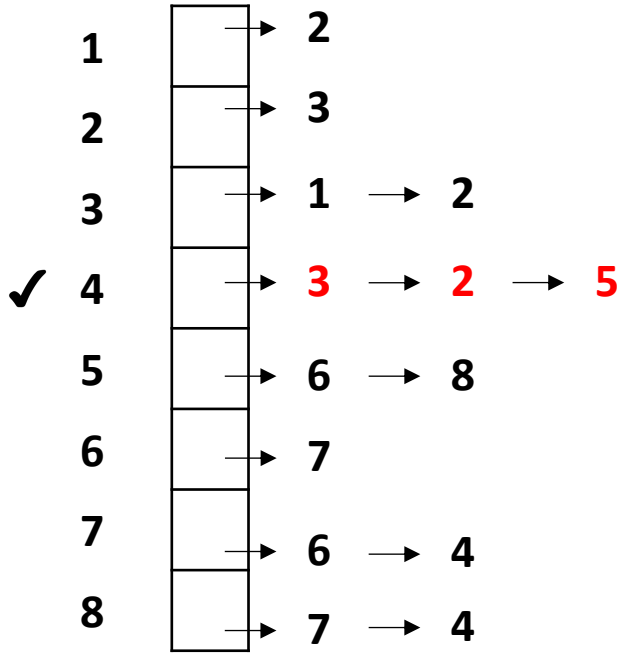
3	2	5

BFS(G, **4**)





Adj List of G :



Contents of Q :

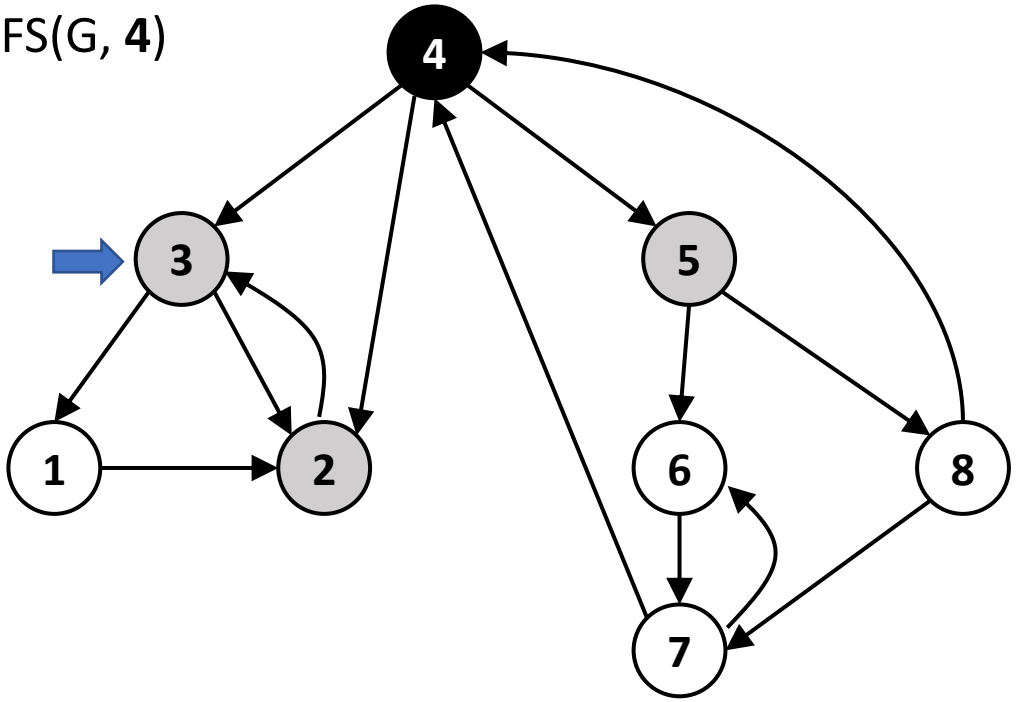
d = 0

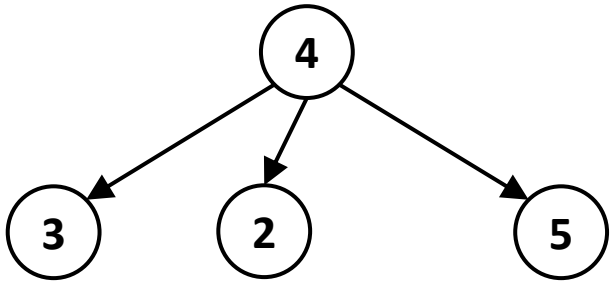
4

d = 1 1 1

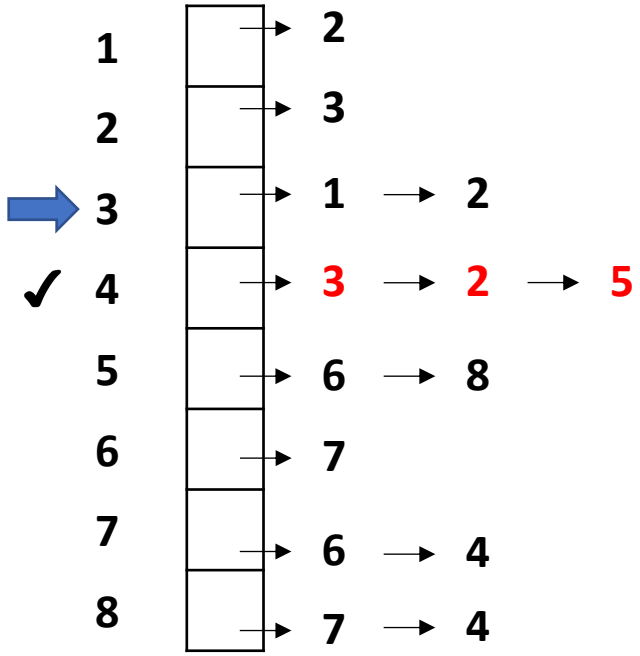
3	2	5

BFS(G, **4**)





Adj List of G :



Contents of Q :

d = 0

4

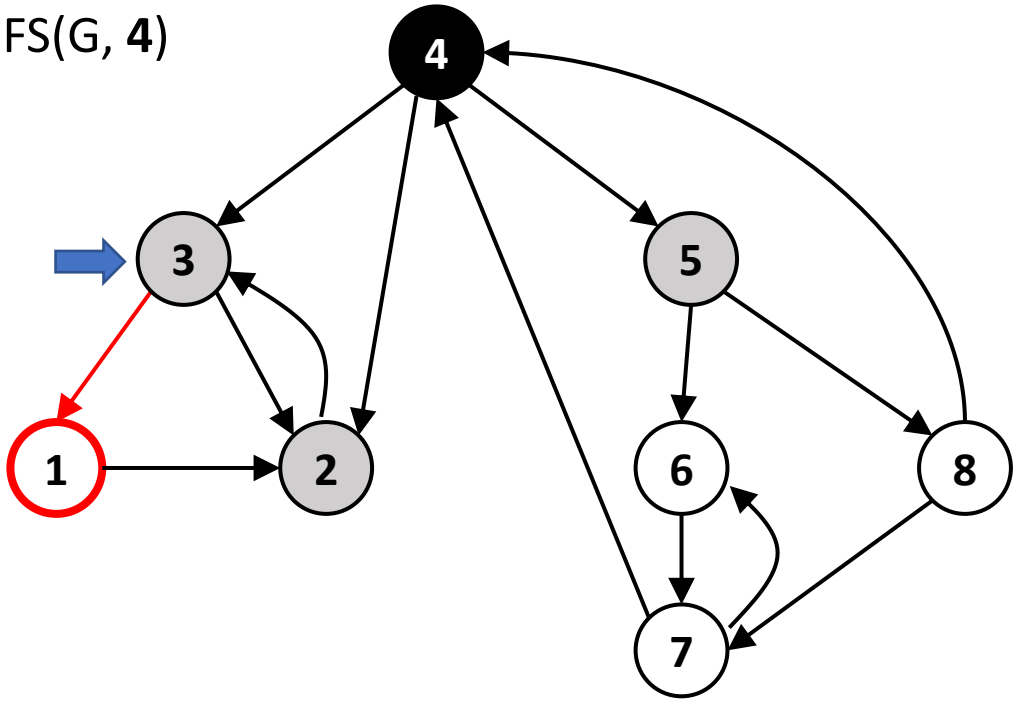
d = 1 1 1

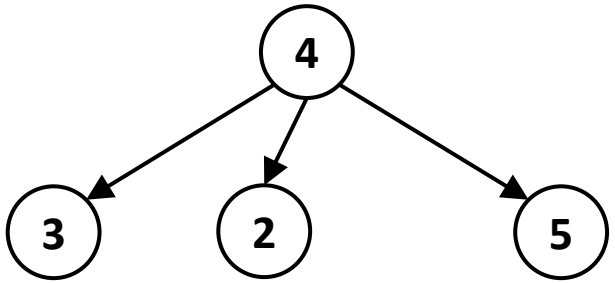
3	2	5

1 1

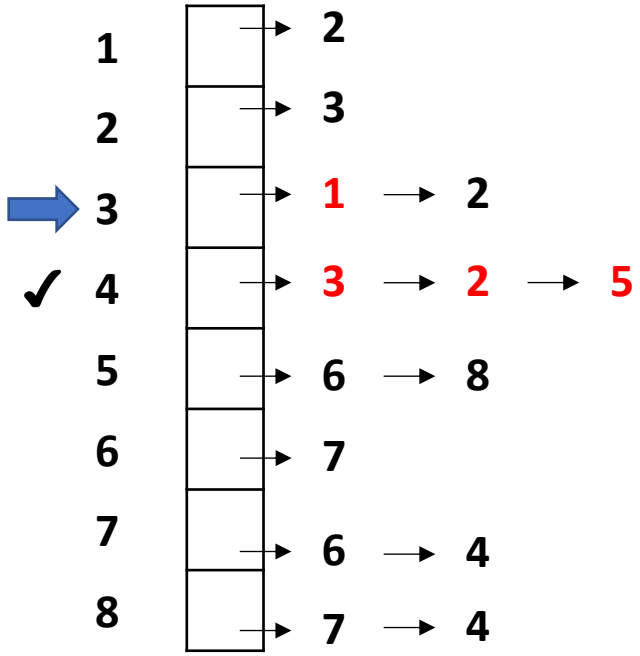
2	5

BFS(G, **4**)





Adj List of G :



Contents of Q :

d = 0

4

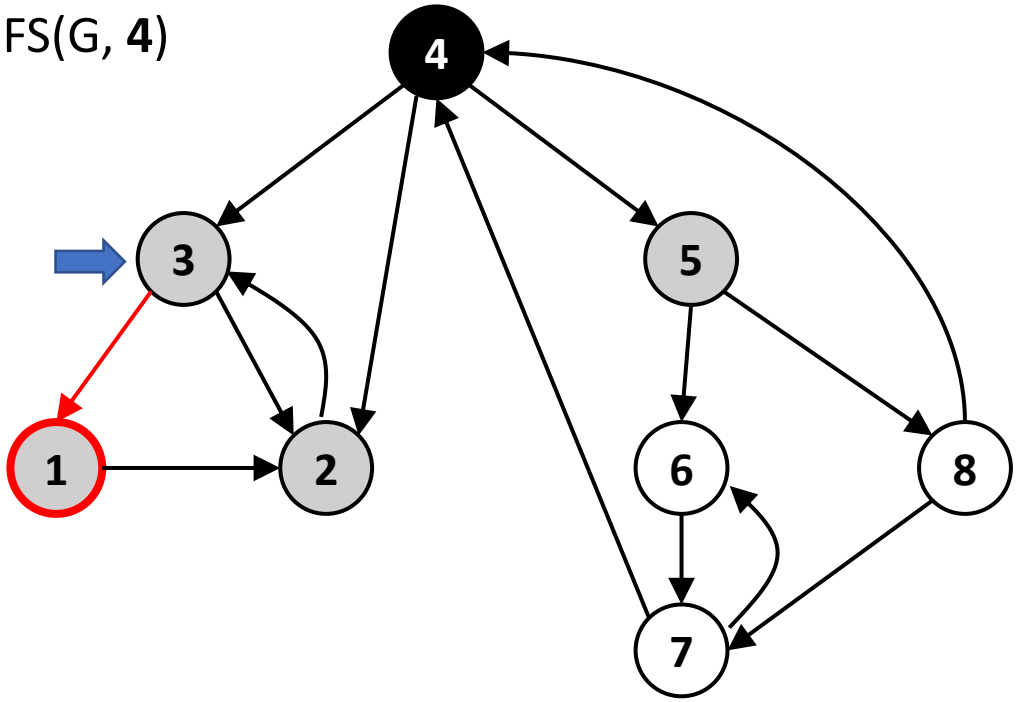
d = 1 1 1

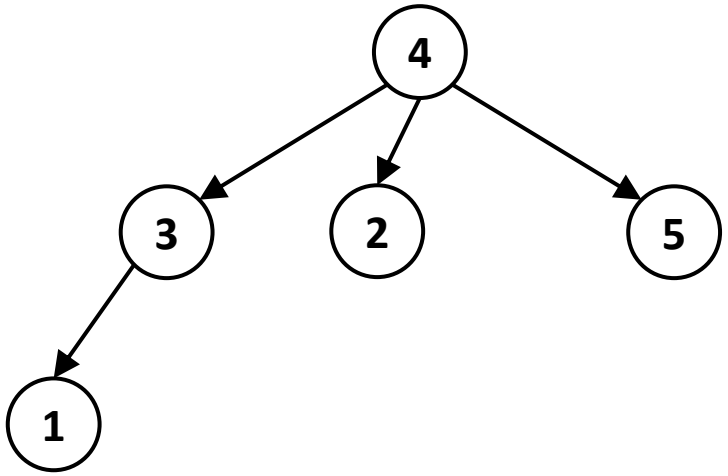
3	2	5

1 1

2	5

BFS(G, **4**)





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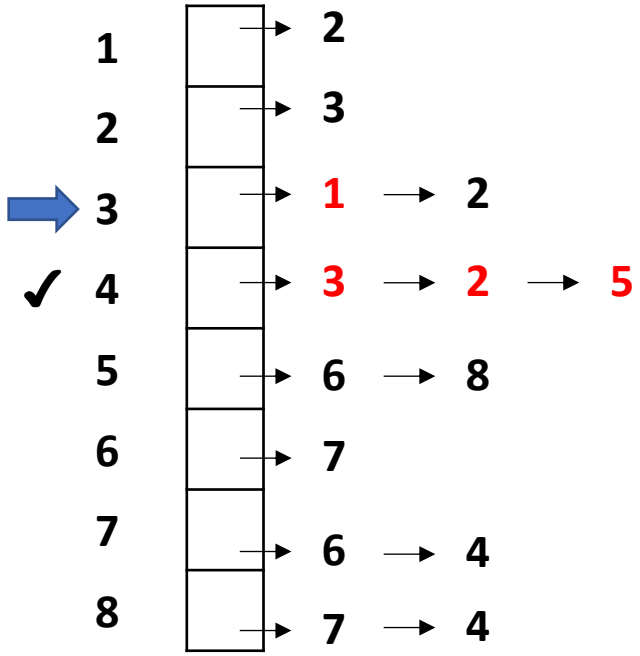


Contents of Q :

d = 0

4

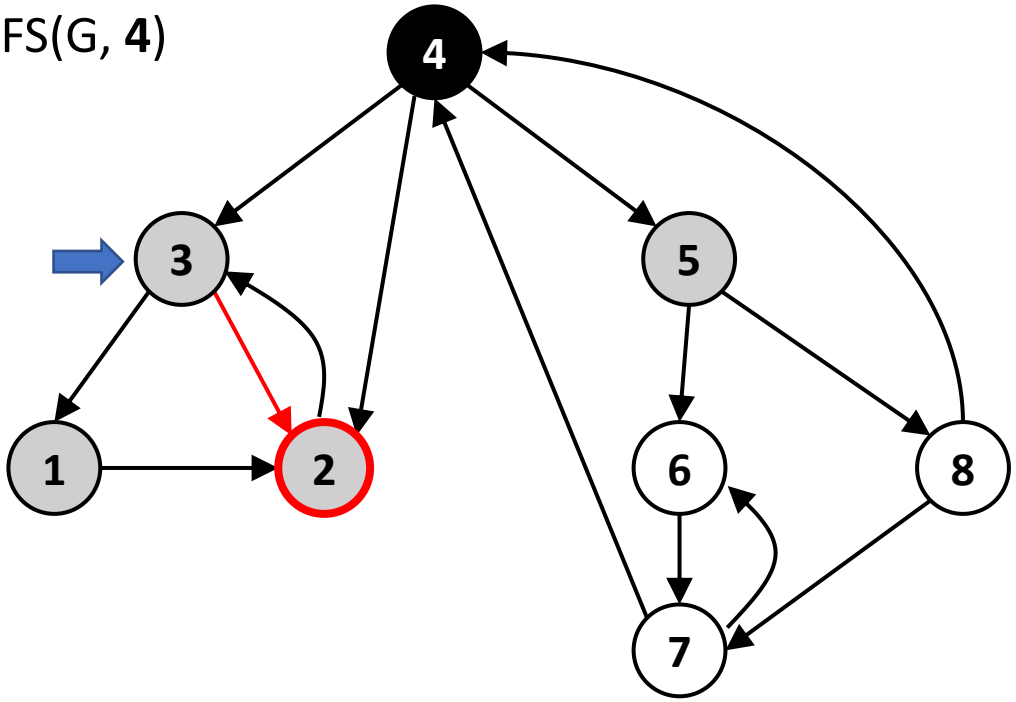
d = 1 1 1

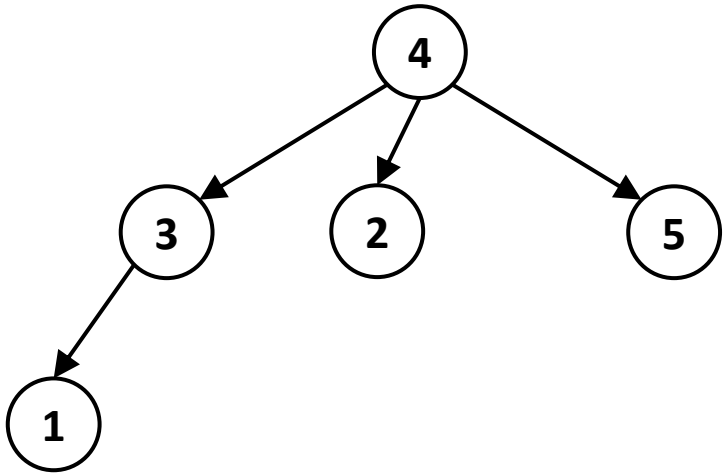
3	2	5

1 1 2

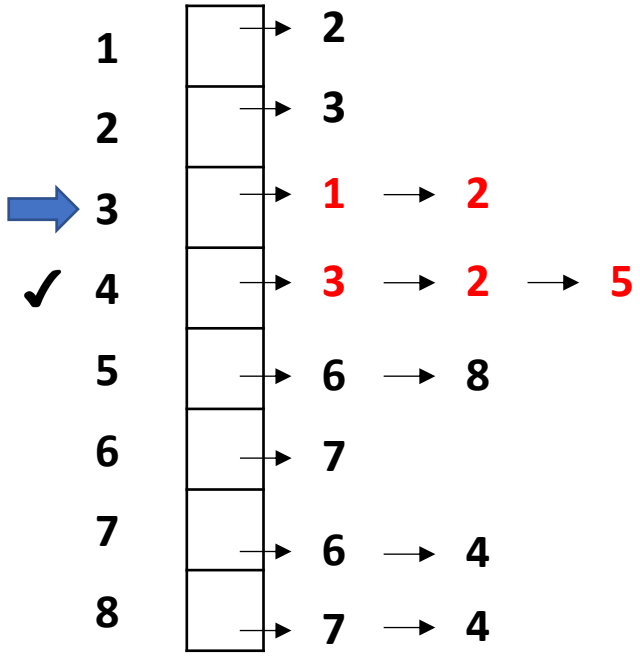
2	5	1

BFS(G, **4**)





Adj List of G :



Contents of Q :

d = 0

4

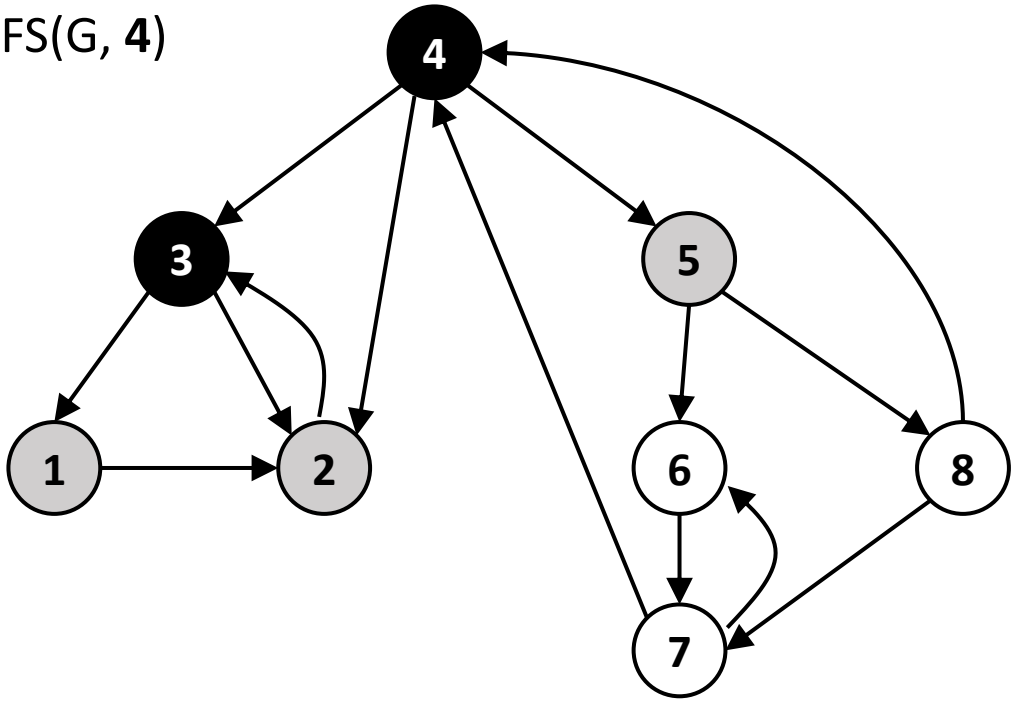
d = 1 1 1

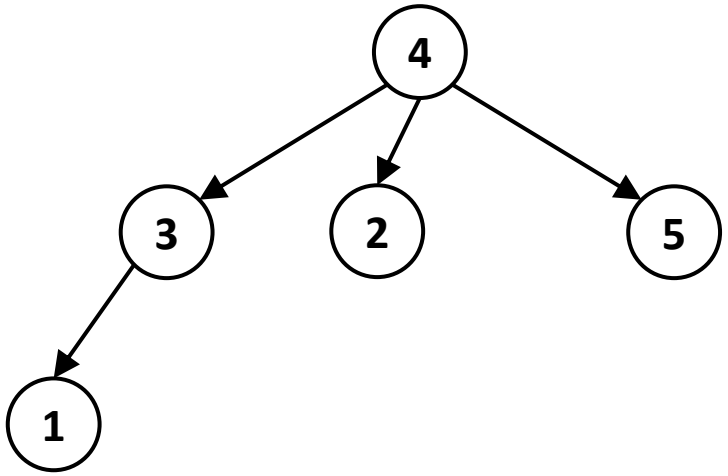
3	2	5

1 1 2

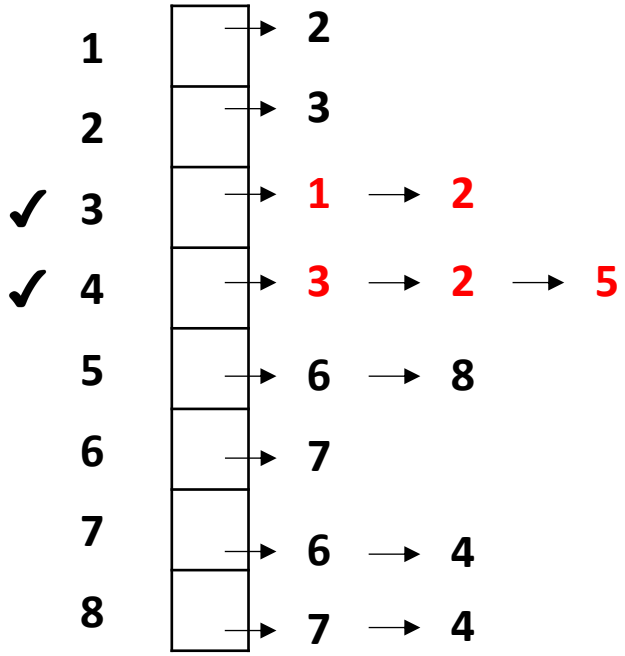
2	5	1

BFS(G, **4**)





Adj List of G :



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4

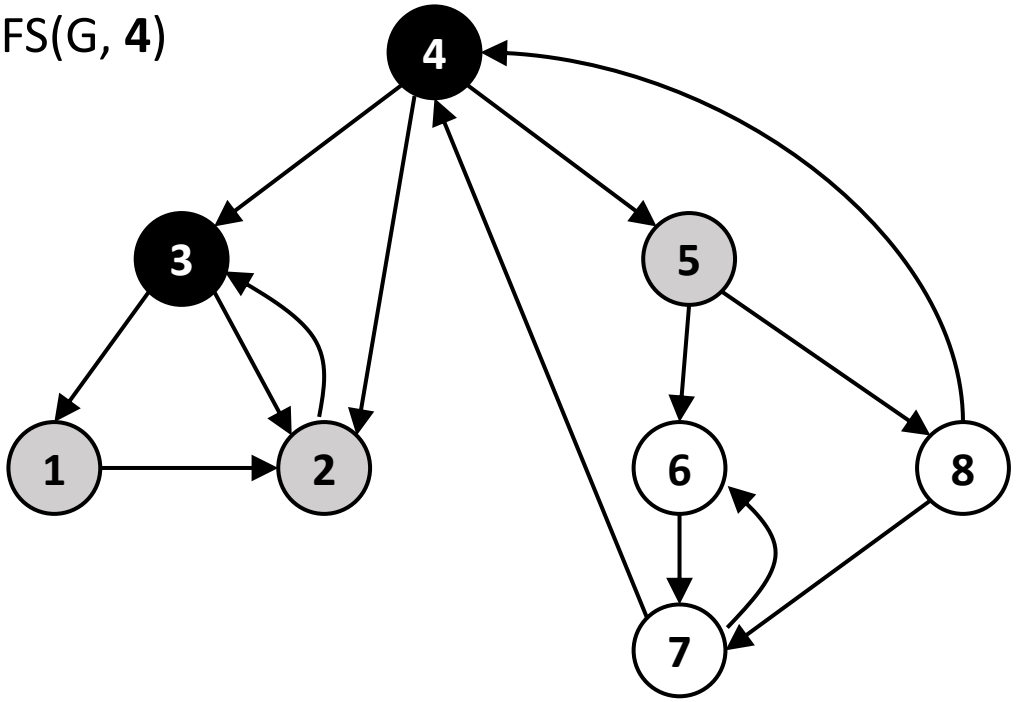
d = 1 1 1

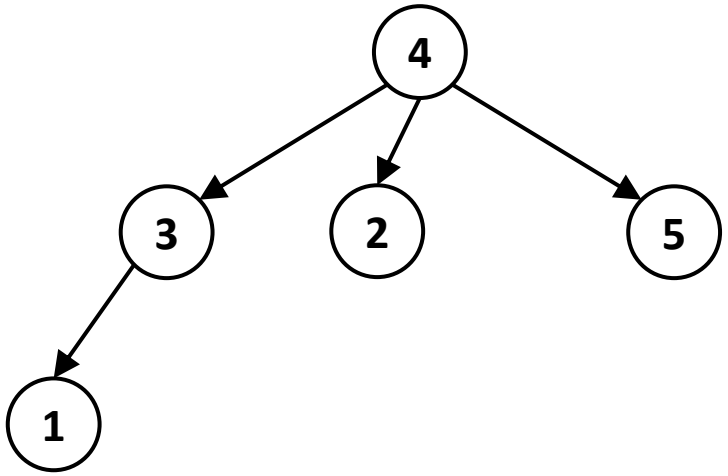
3	2	5

1 1 2

2	5	1

BFS(G, **4**)





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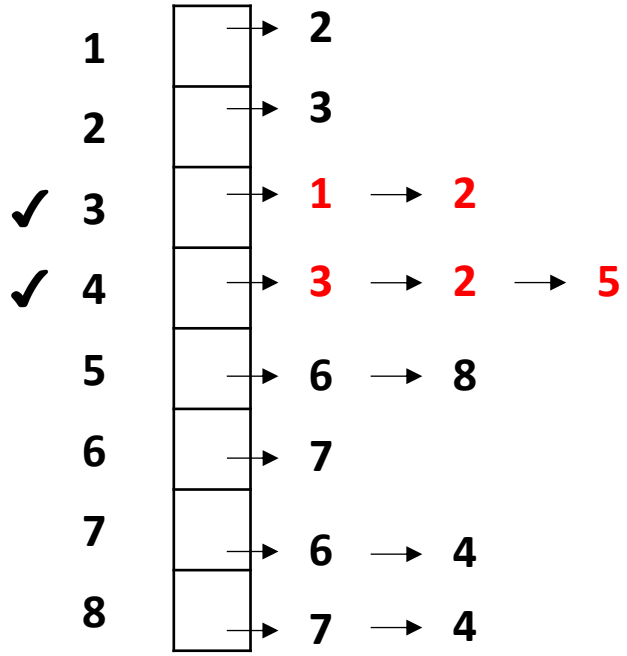

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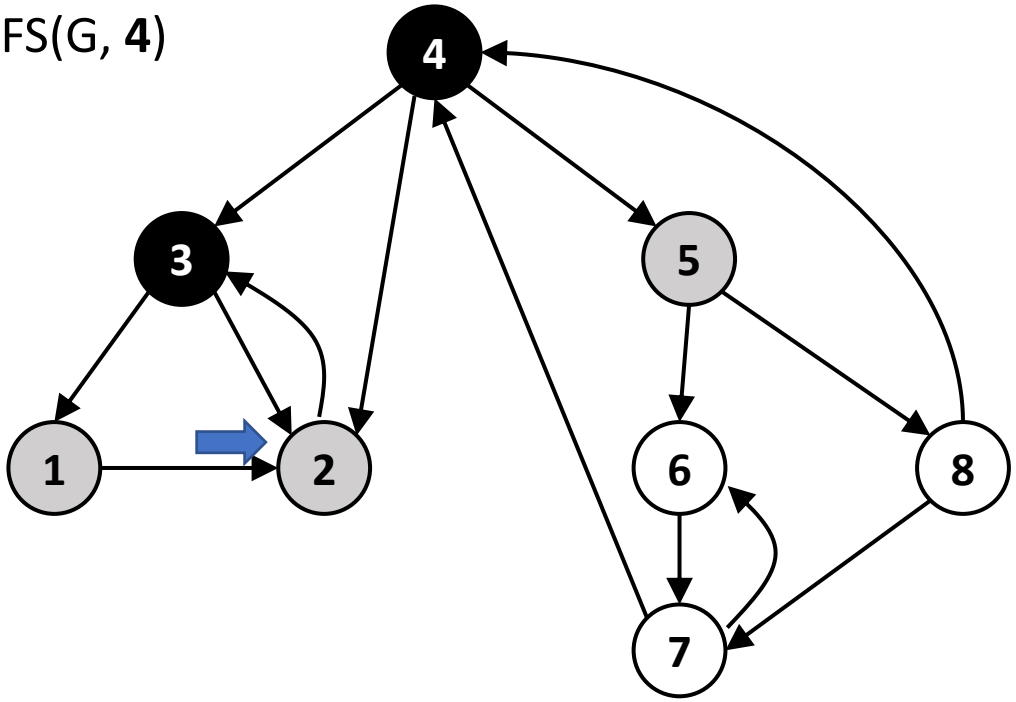

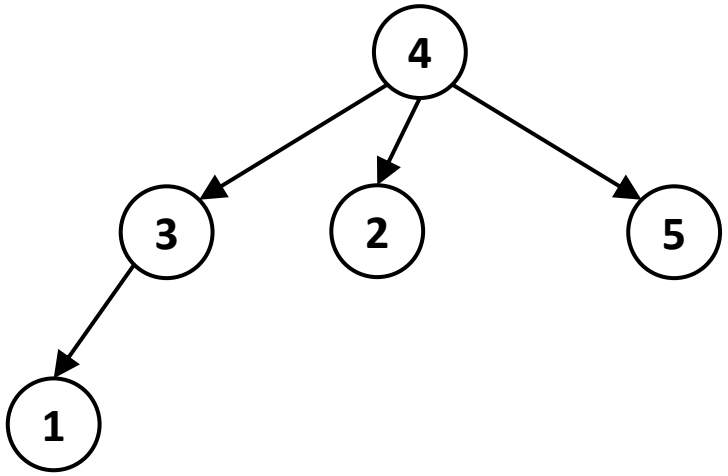

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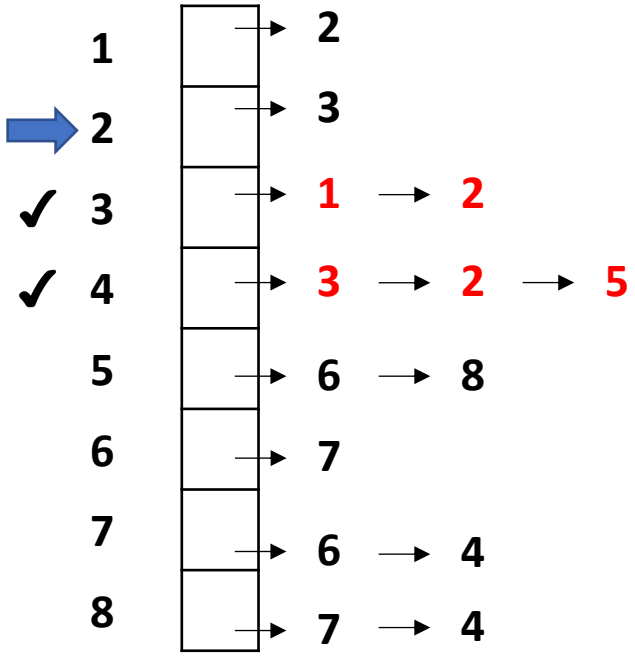

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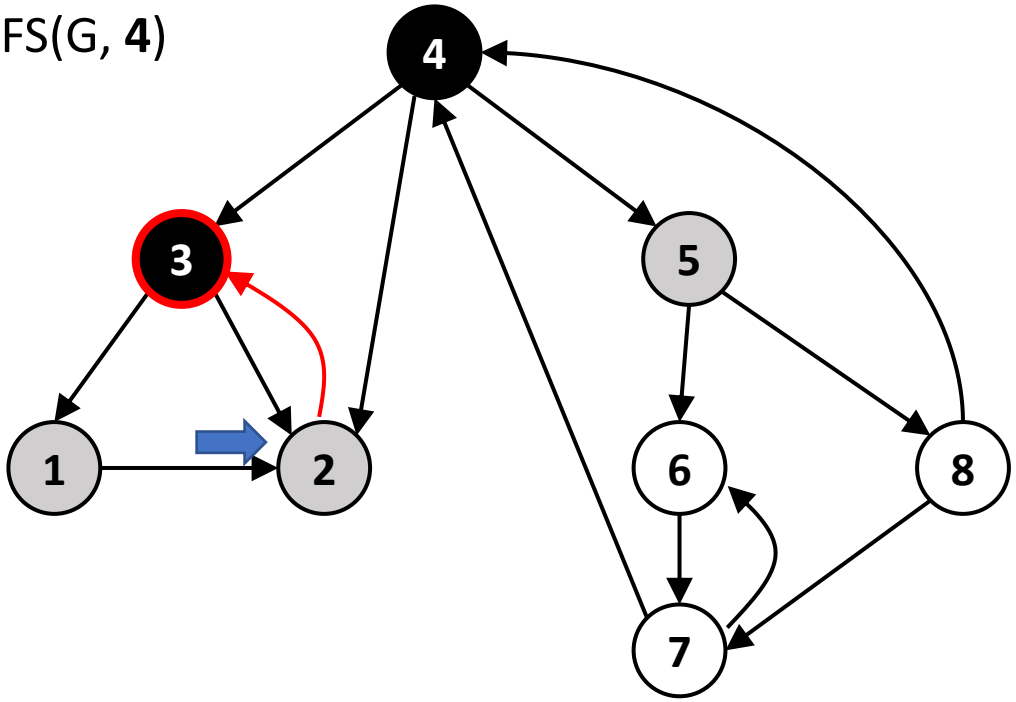
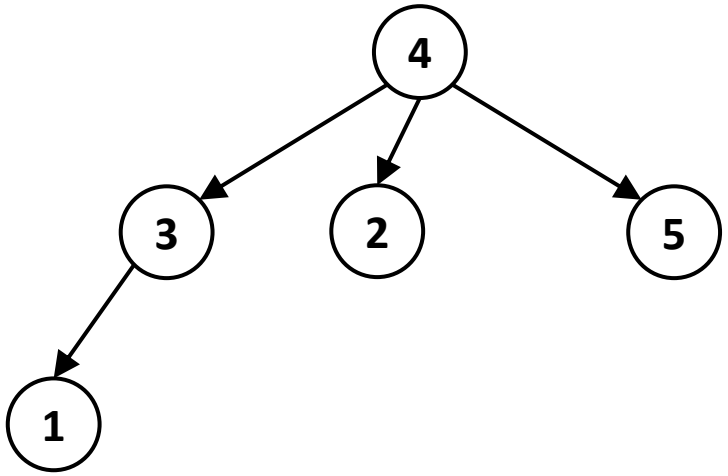
5	1

BFS(G, **4**)





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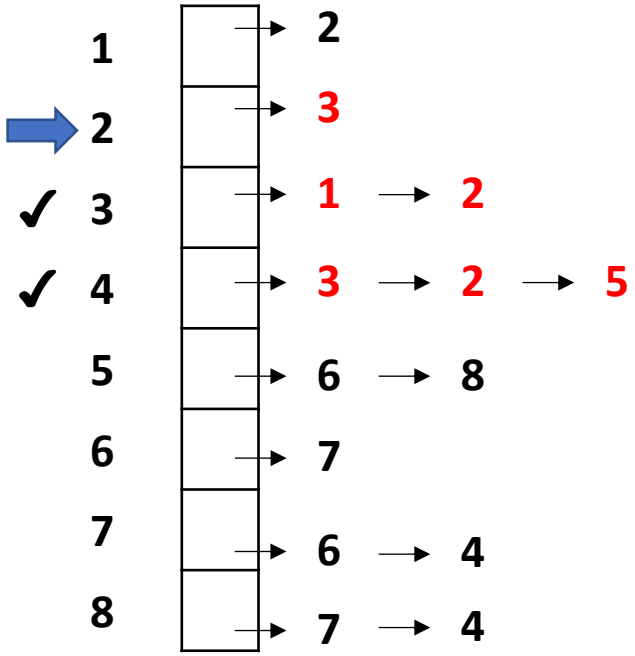

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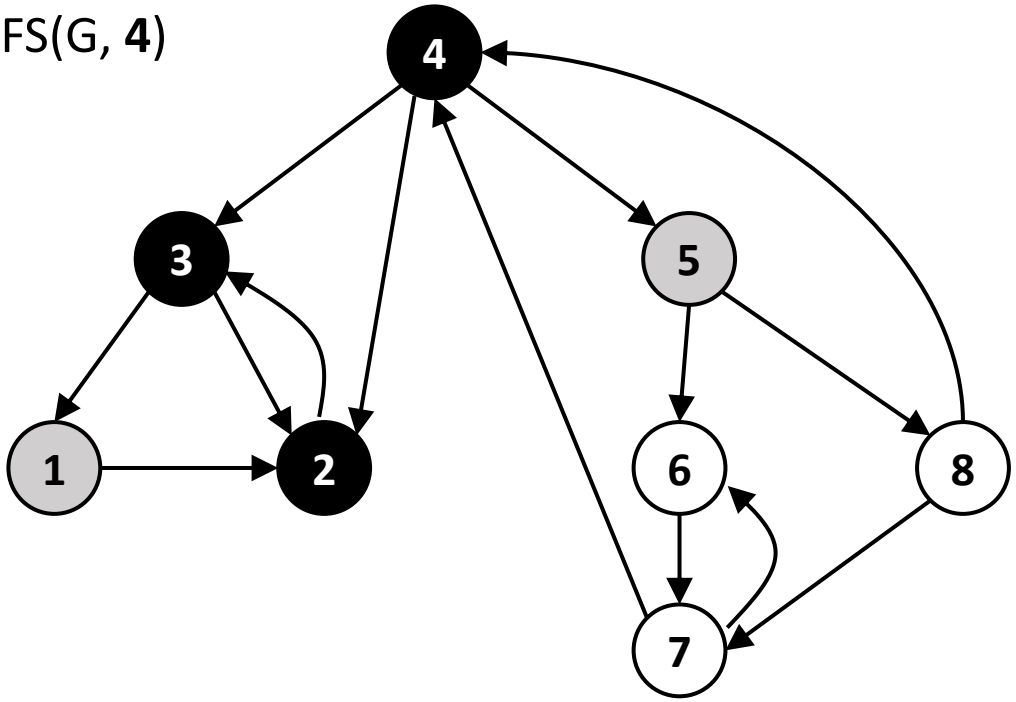
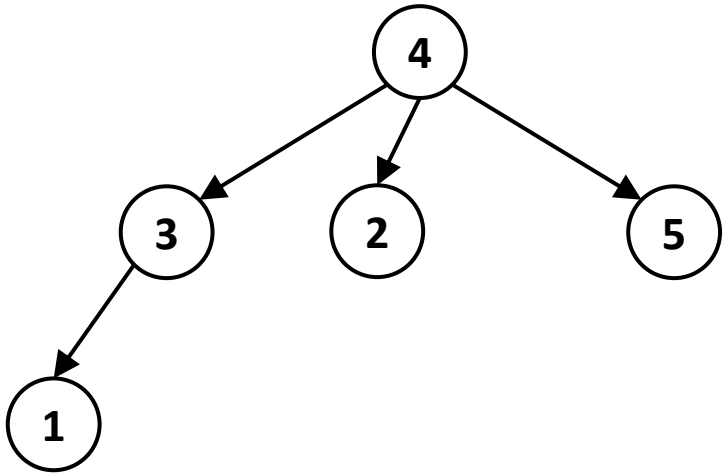
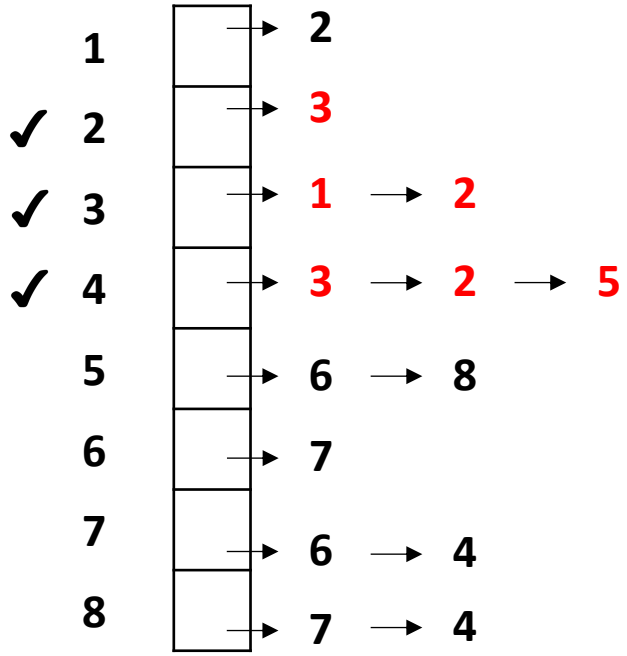




Adj List of G :



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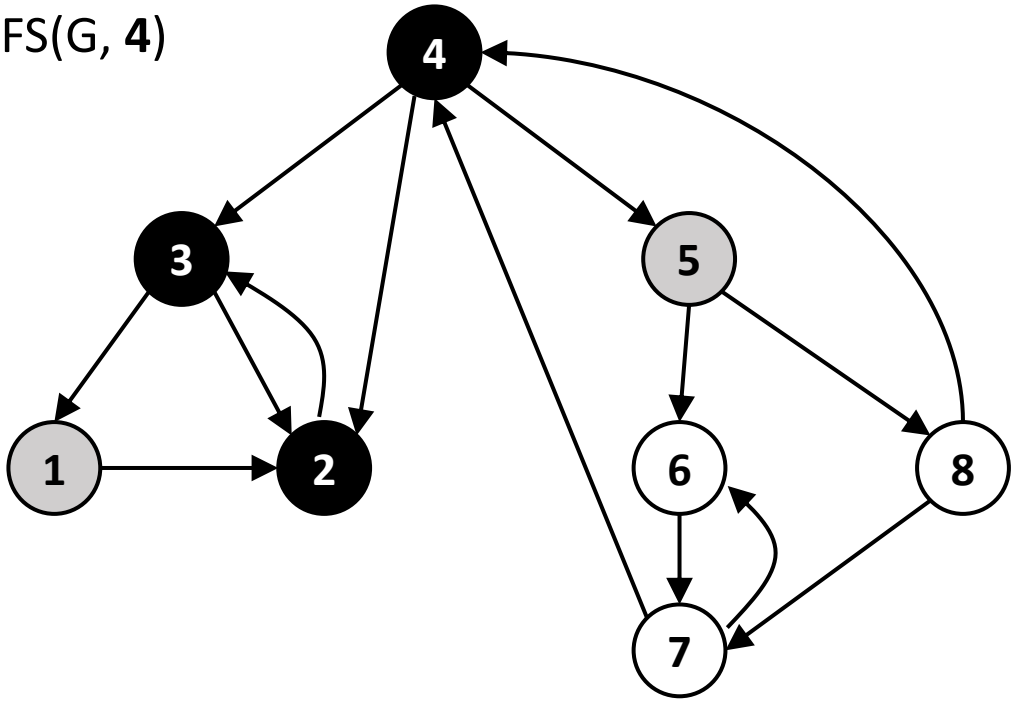
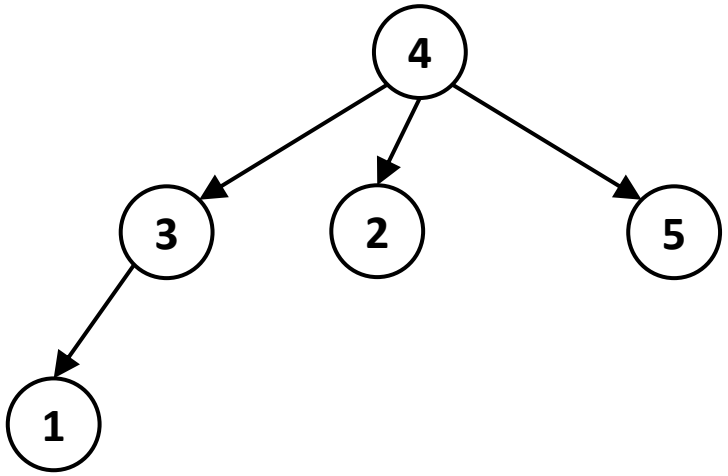
BFS(G, **4**)





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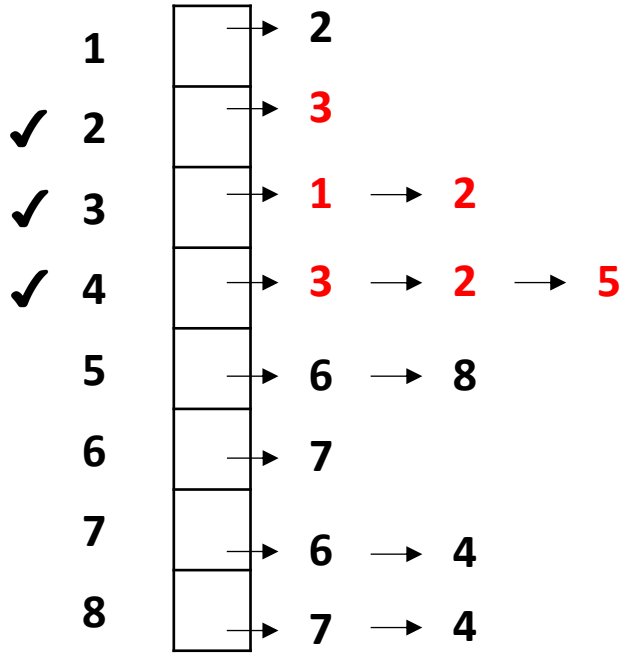

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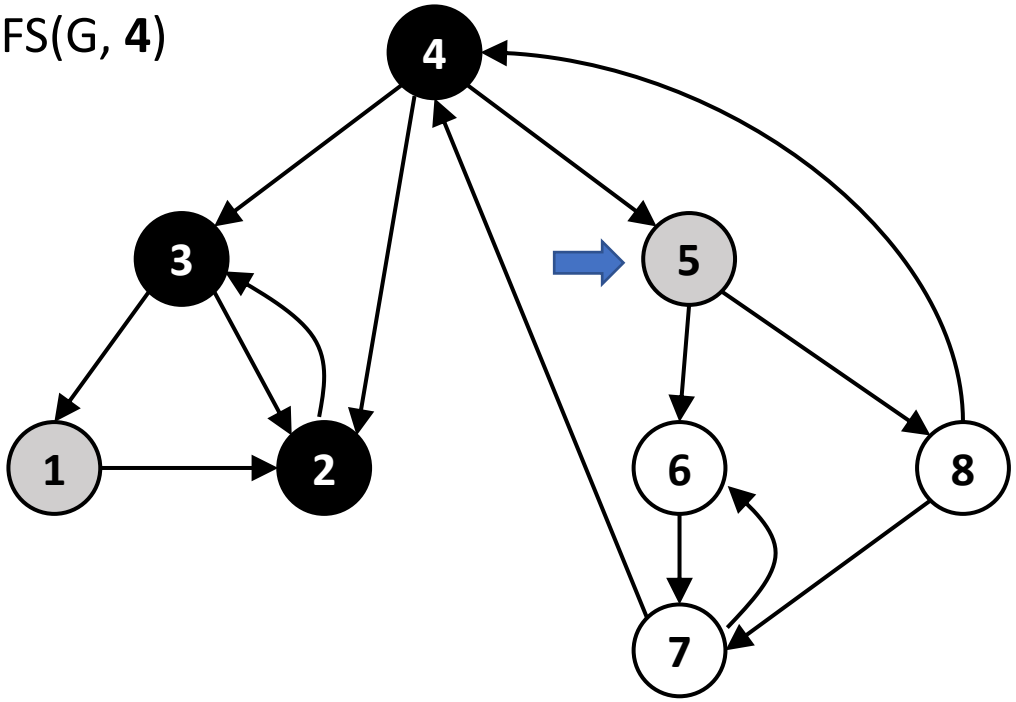
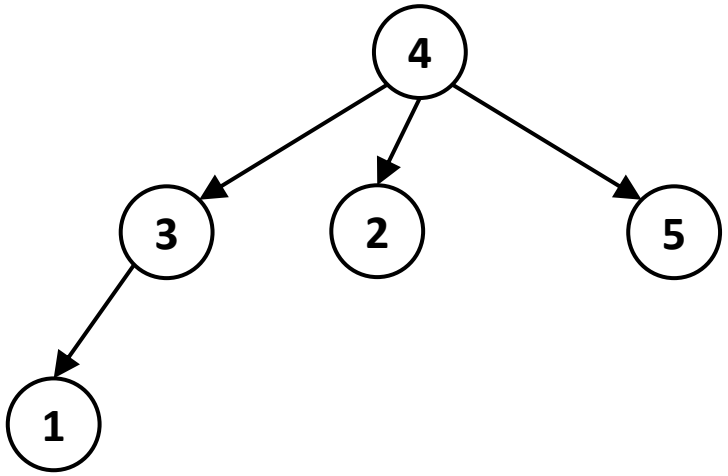
BFS(G, **4**)





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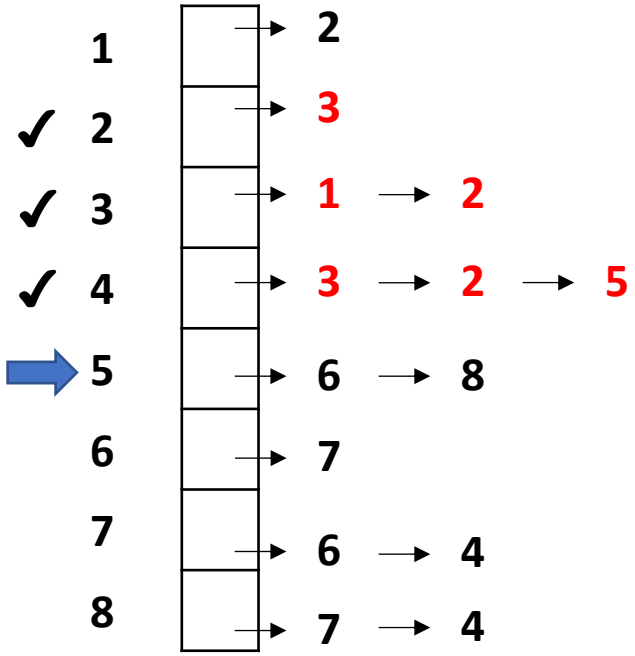

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4

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3	2	5

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2	5	1

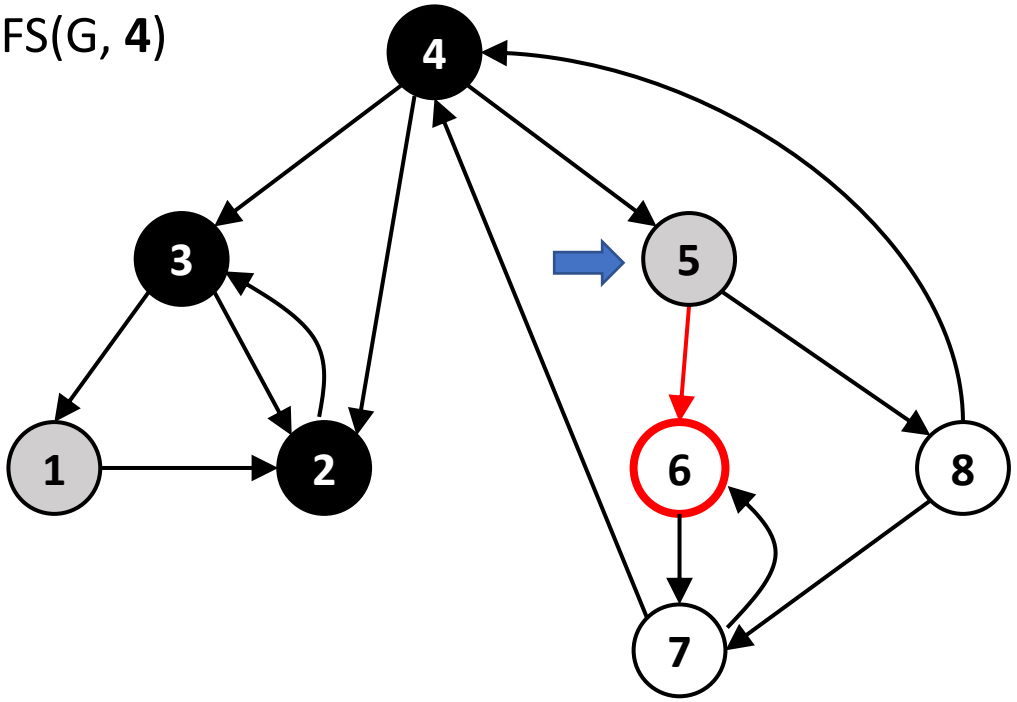
1 2

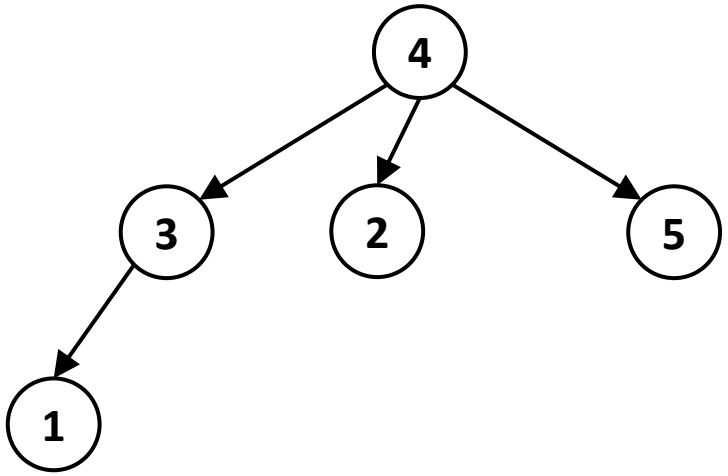
5	1

2

1

BFS(G, **4**)





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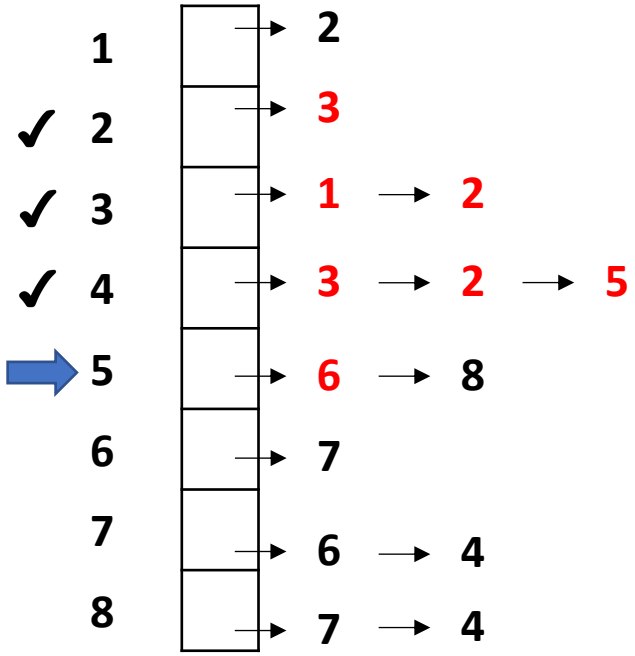


Contents of Q :

d = 0

4

d = 1 1 1

3	2	5

1 1 2

2	5	1

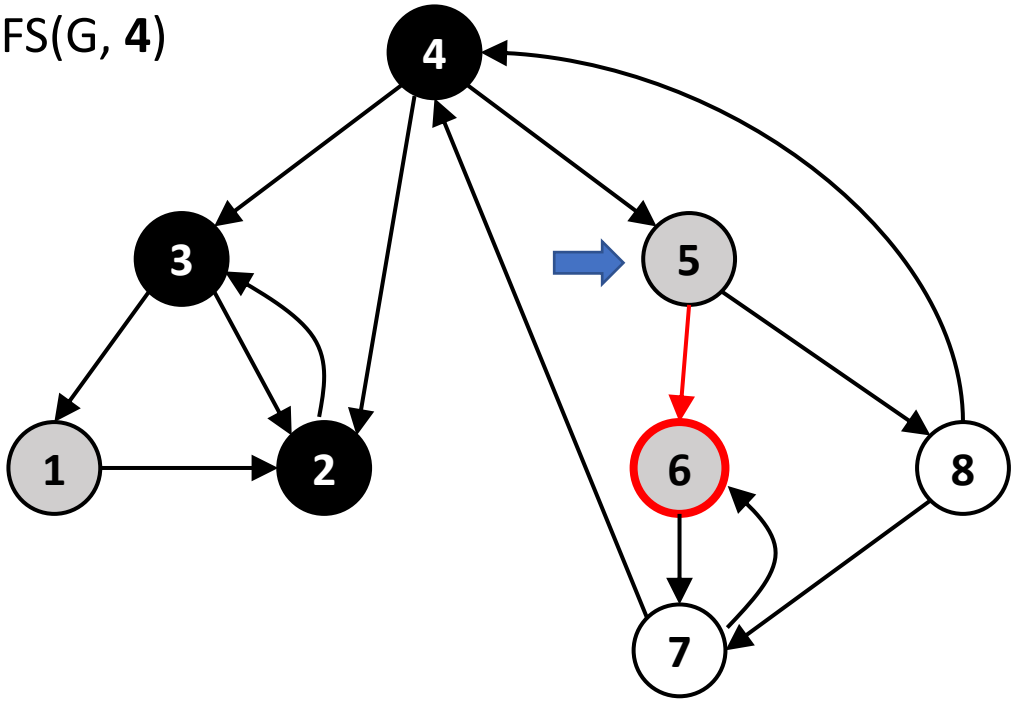
1 2

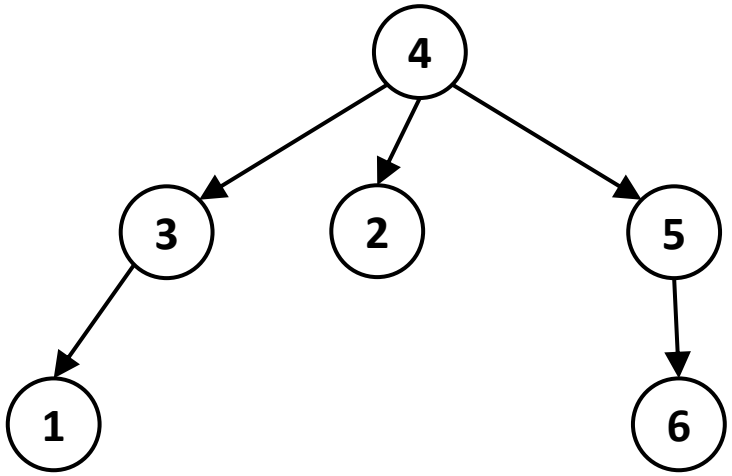
5	1

2

1

BFS(G, **4**)





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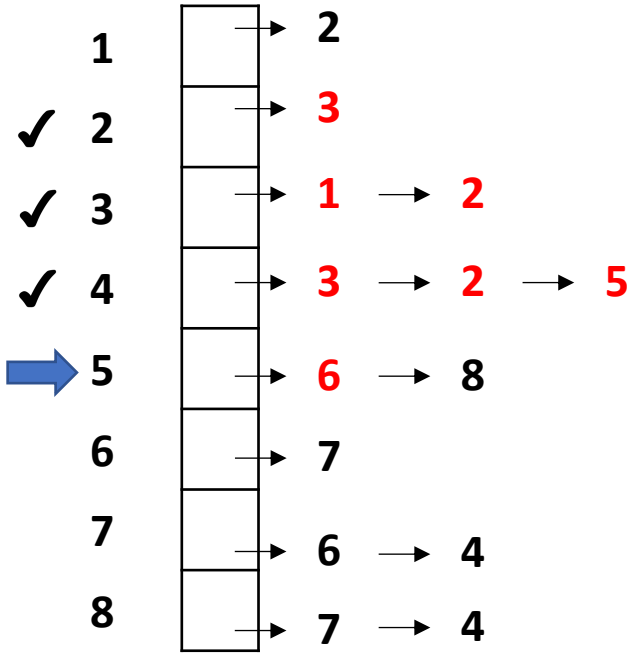


Contents of Q :

d = 0

4

d =

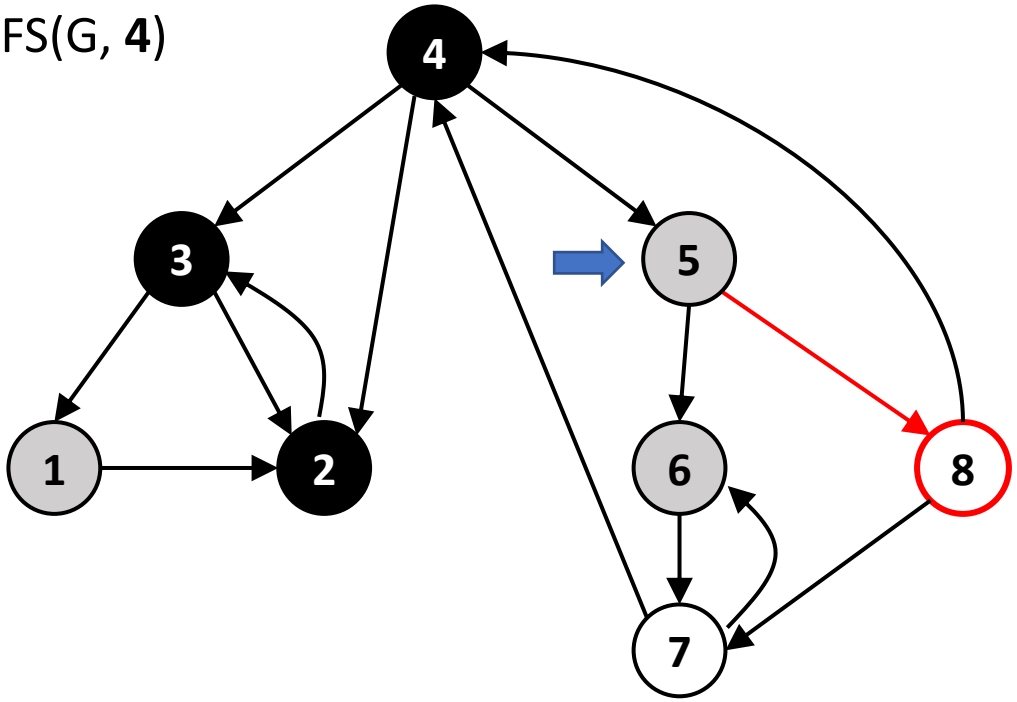
1	1	1
3	2	5

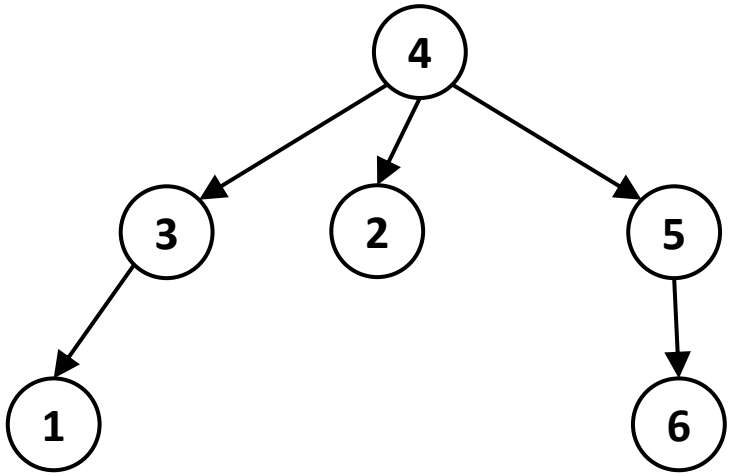
1	1	2
2	5	1

1	2
5	1

2	2
1	6

BFS(G, **4**)





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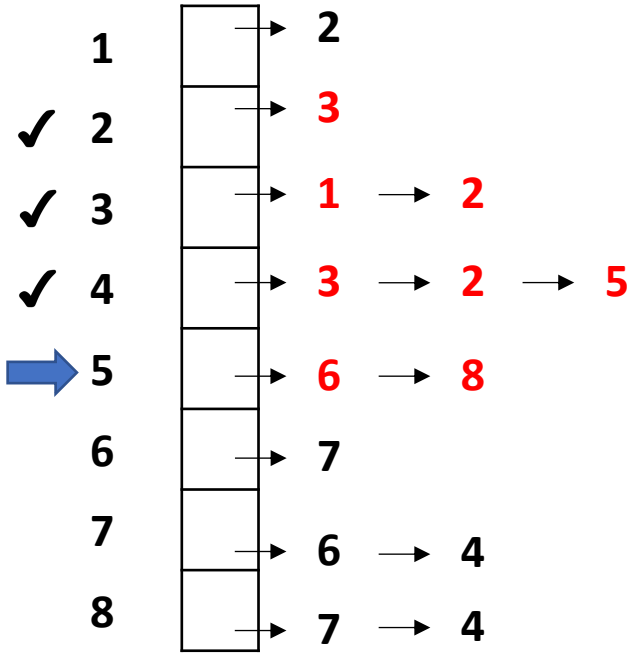

Contents of Q :

d = 0

4

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3	2	5

1 1 2

2	5	1

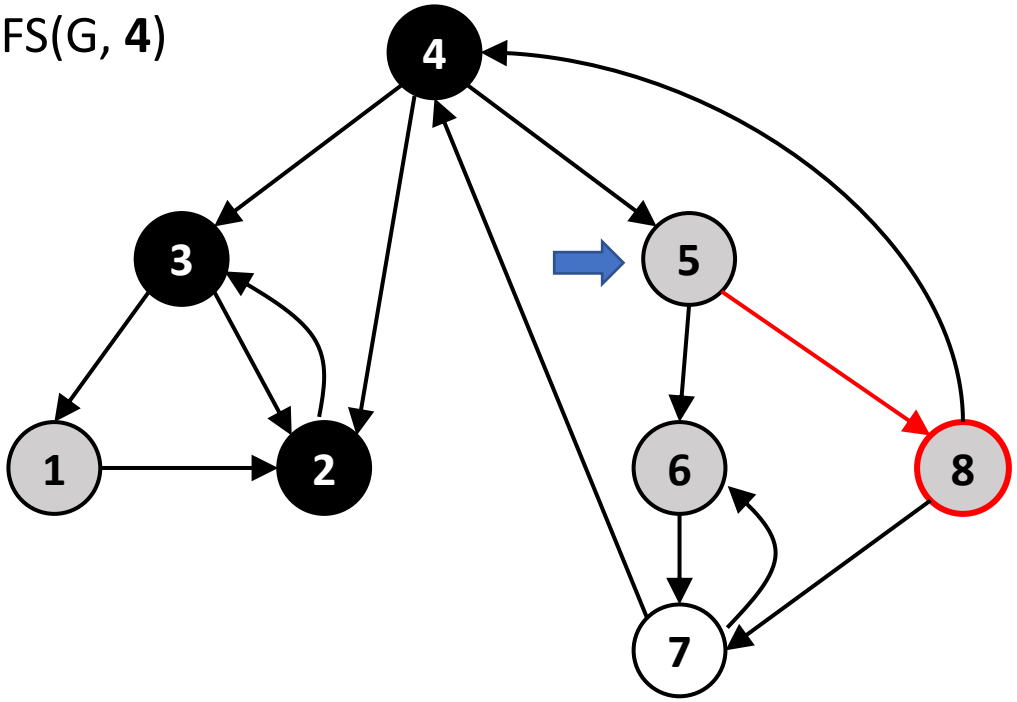
1 2

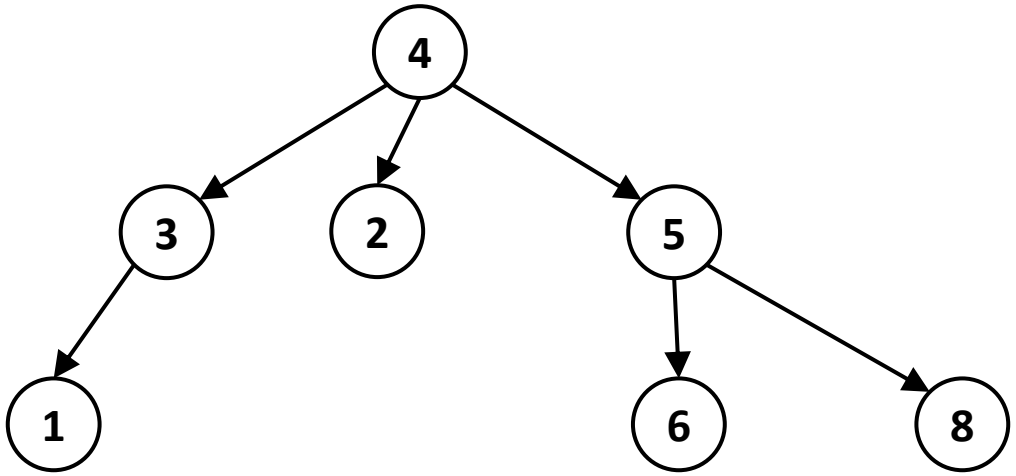
5	1

2 2

1	6

BFS(G, **4**)





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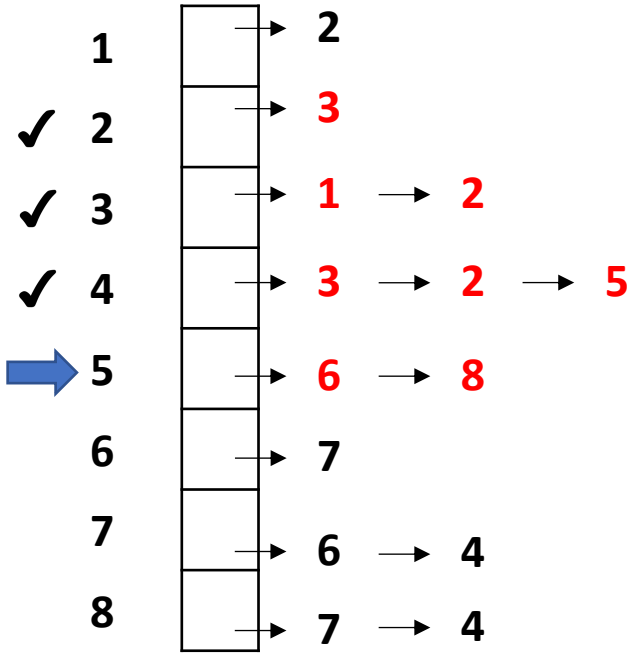


Contents of Q :

d = 0

4

d = 1 1 1

3	2	5

1 1 2

2	5	1

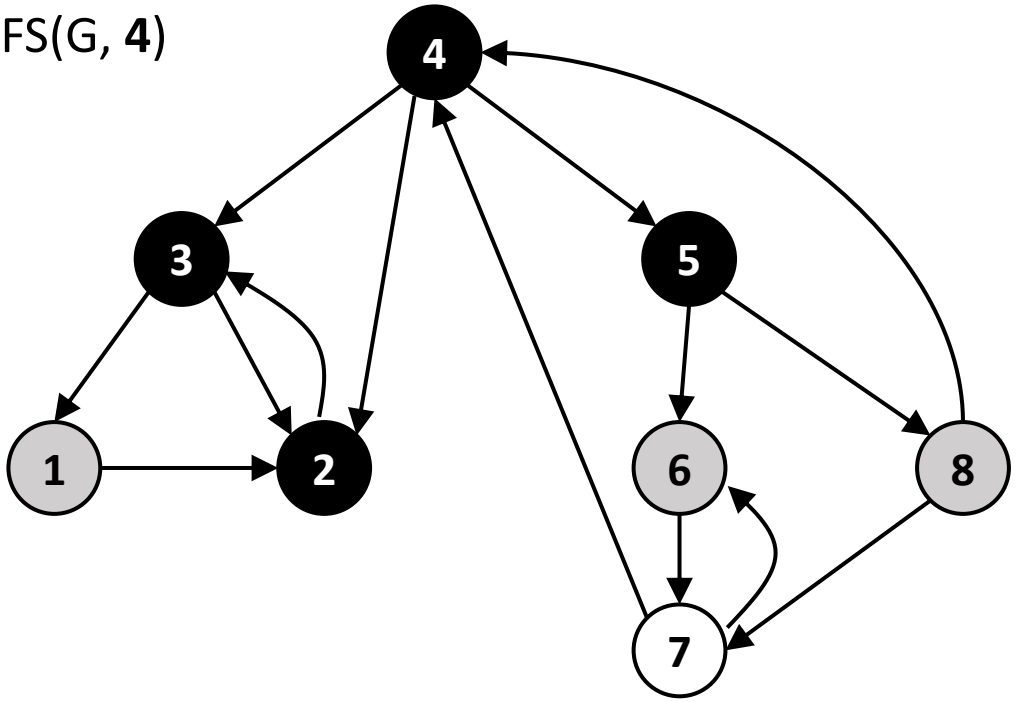
1 2

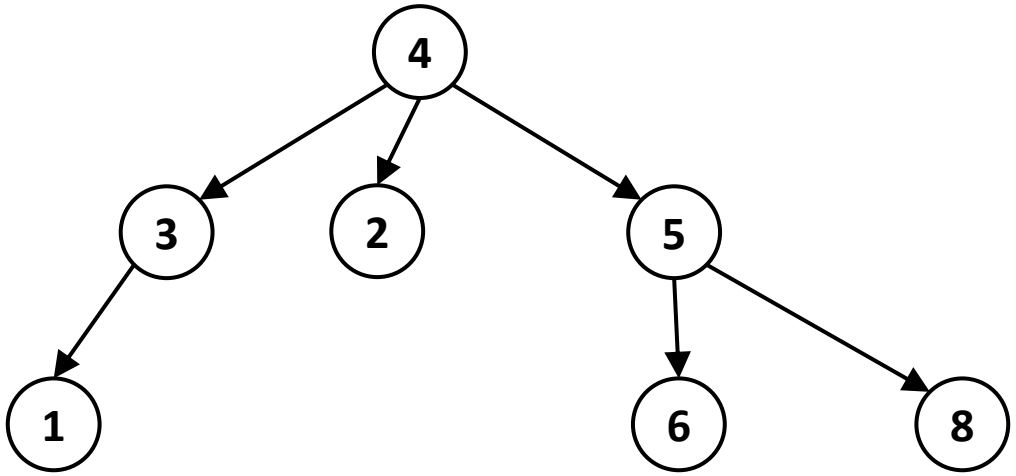
5	1

2 2 2

1	6	8

BFS(G, **4**)





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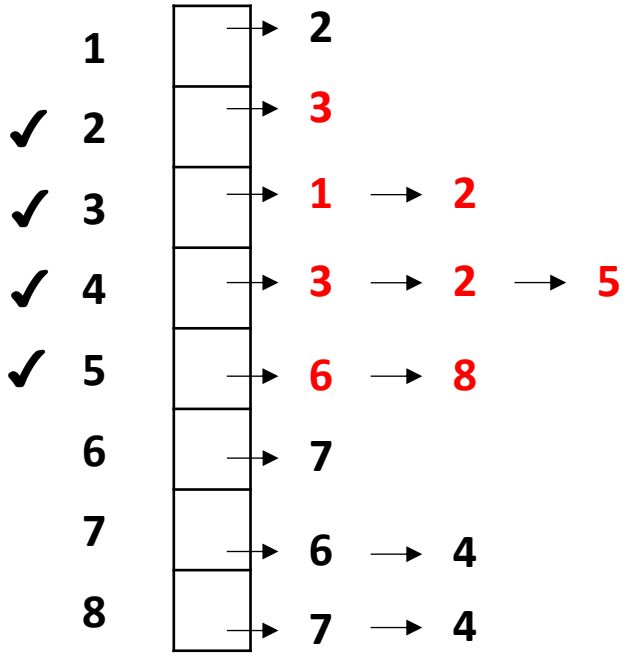

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d = 0

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1	1	2
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1	2
5	1

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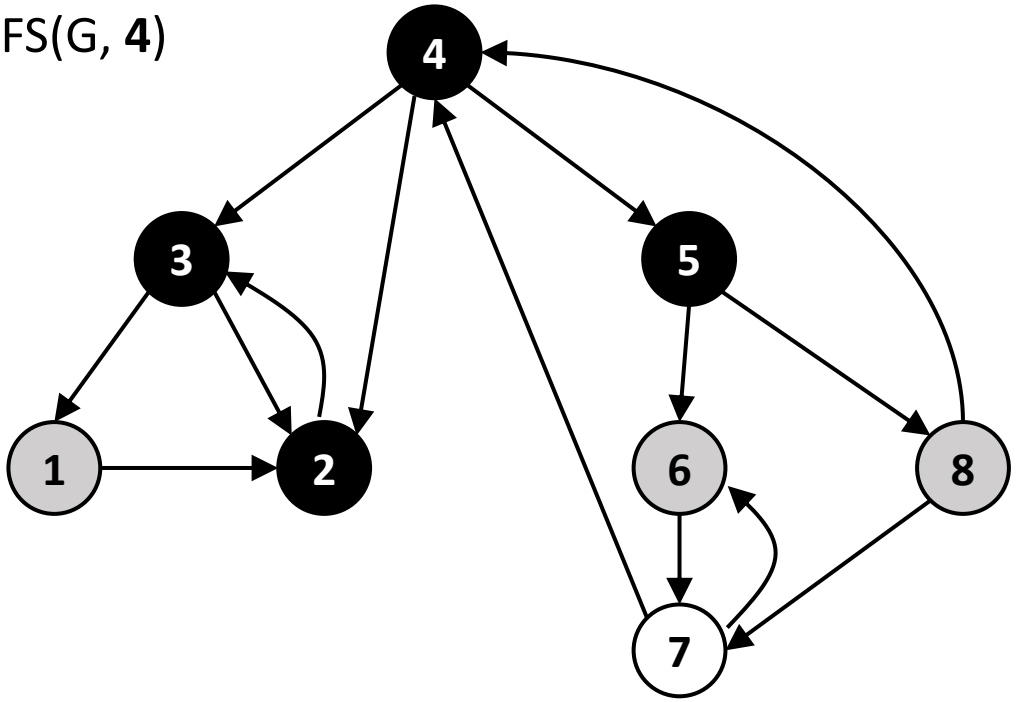
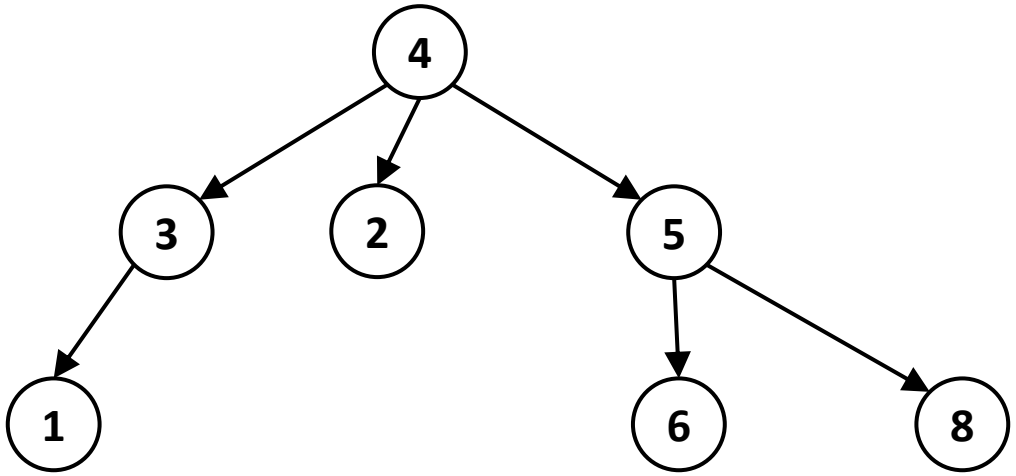
BFS(G, **4**)





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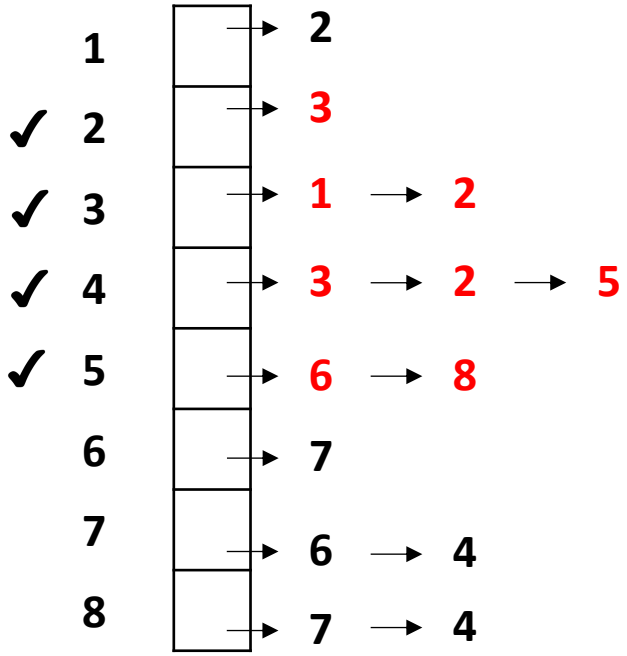

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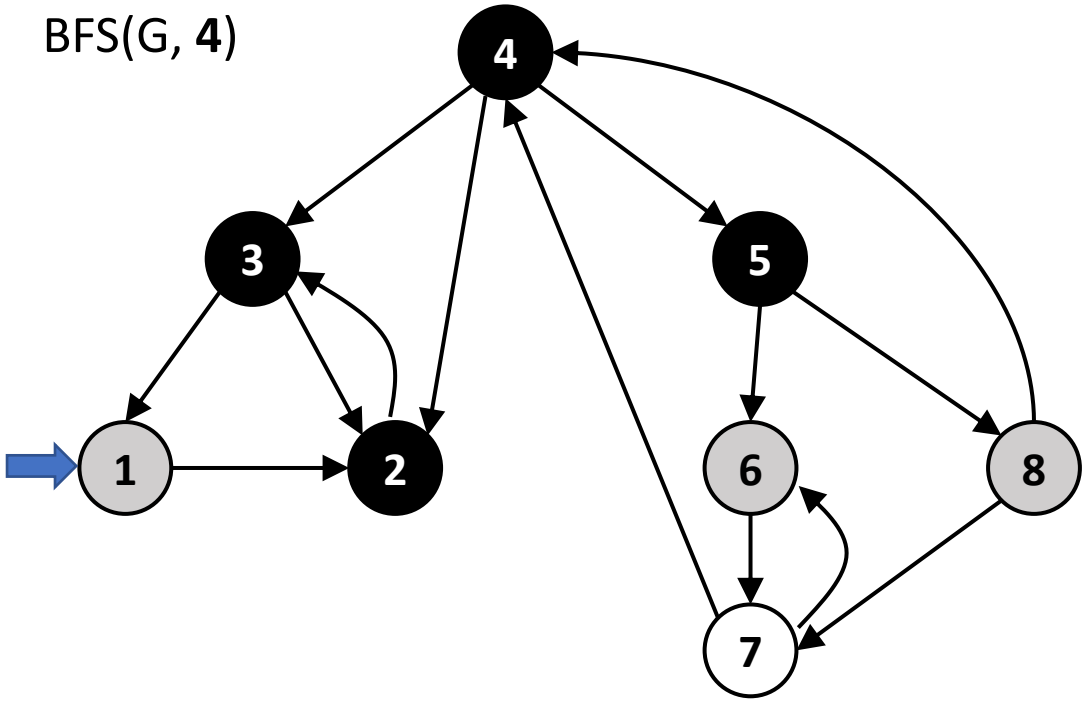
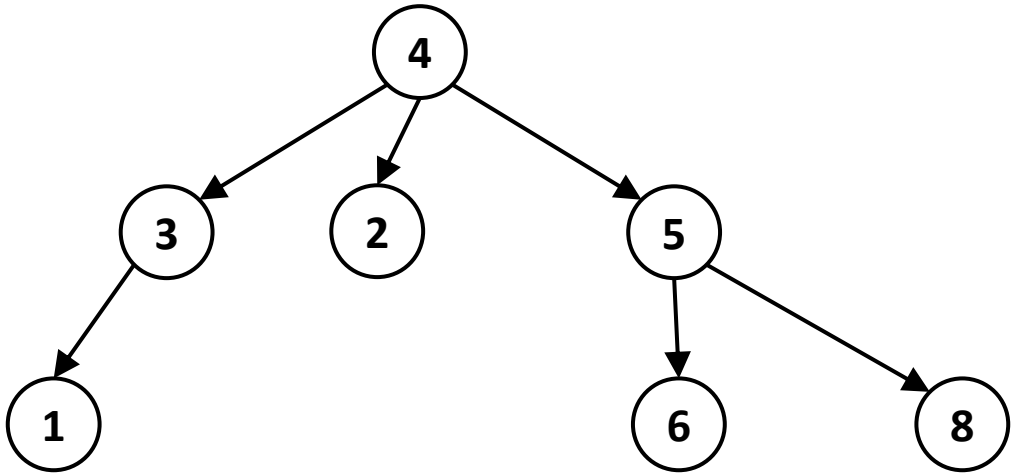
BFS(G, **4**)





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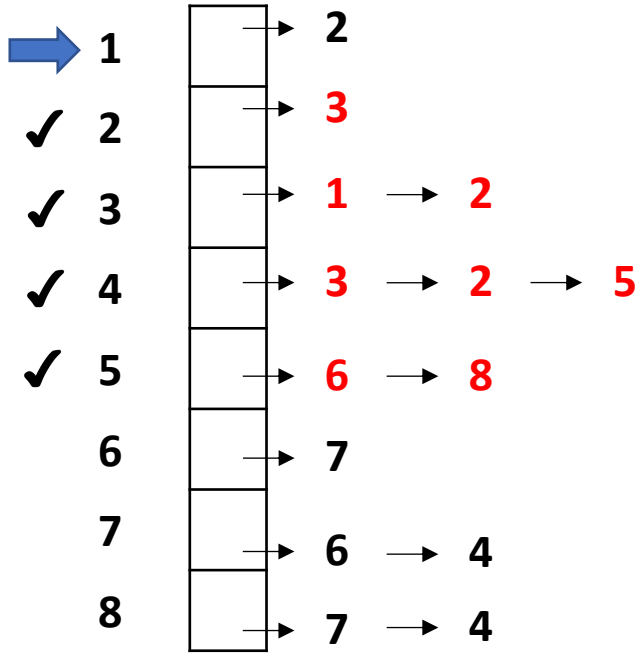

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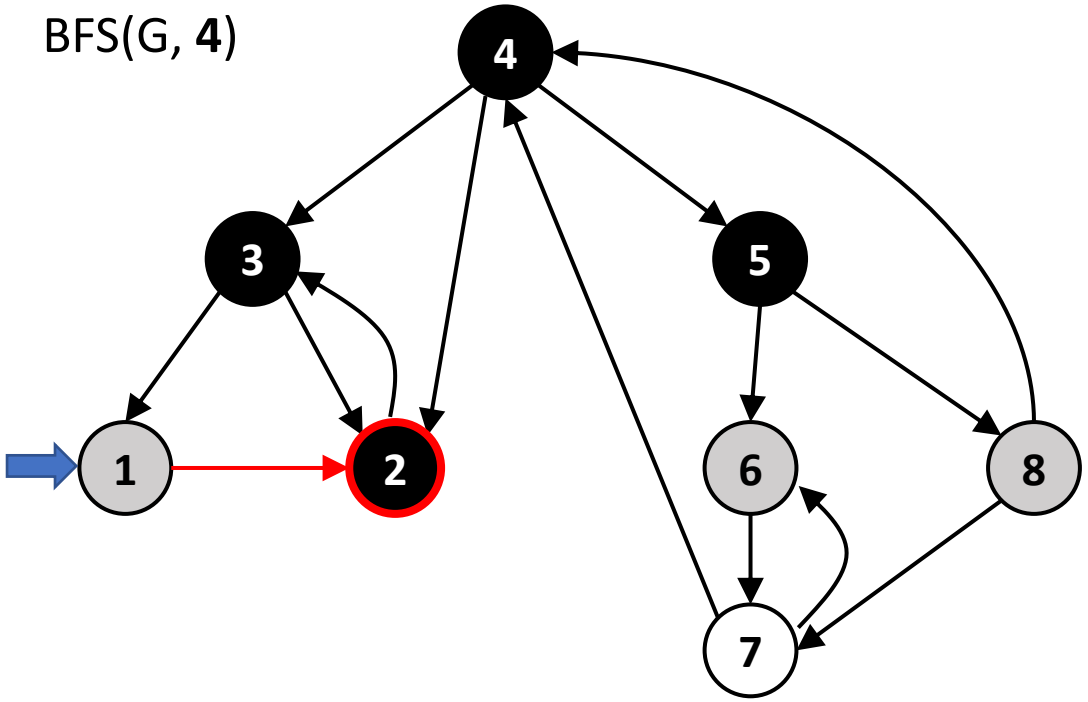
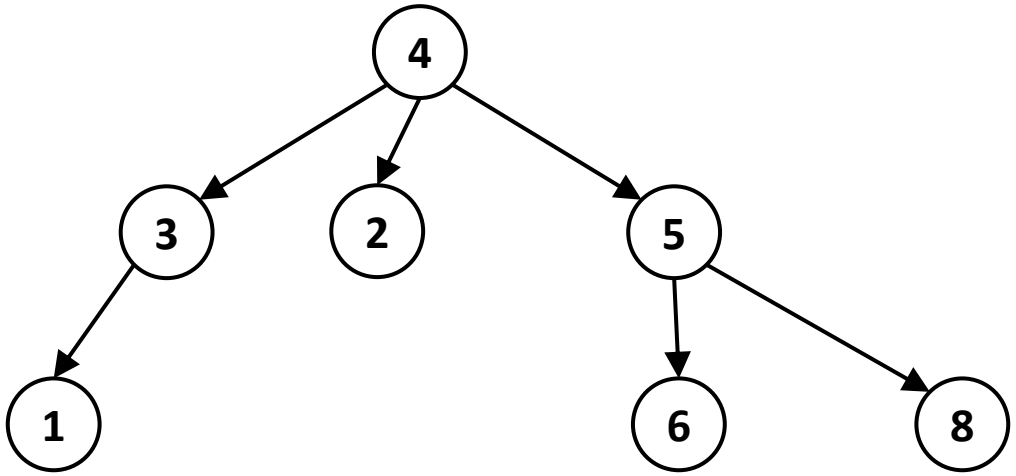
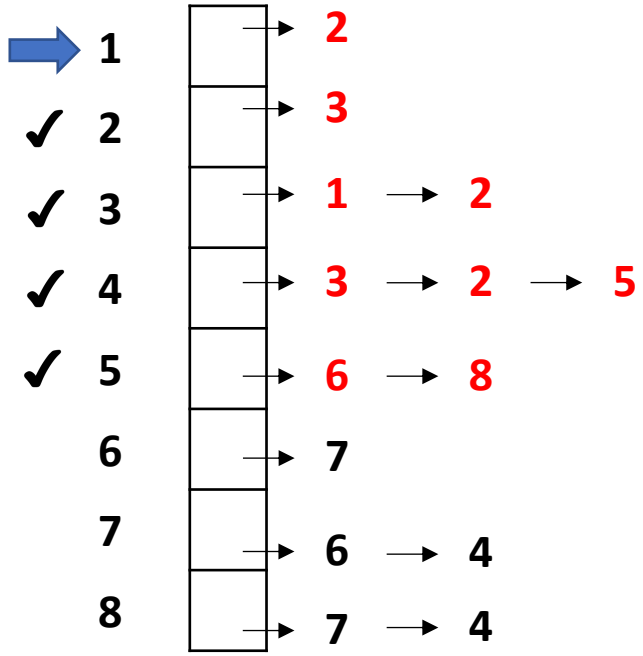
6	8

BFS(G, **4**)





Adj List of G :



Contents of Q :

d = 0

4

d = 1 1 1

3	2	5

1 1 2

2	5	1

1 2

5	1

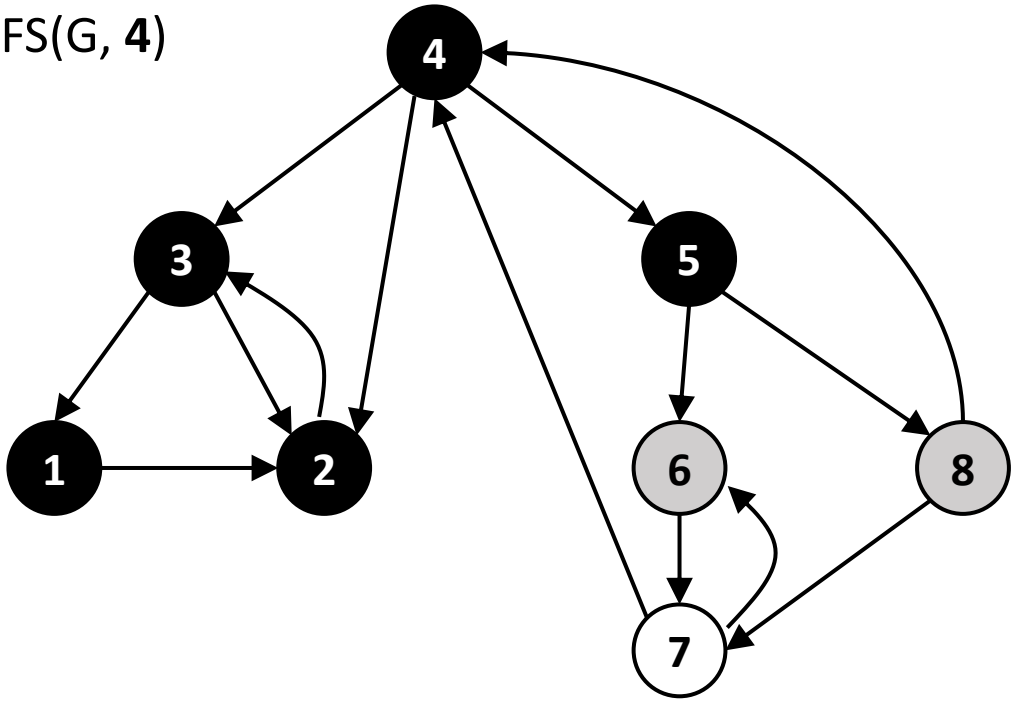
2 2 2

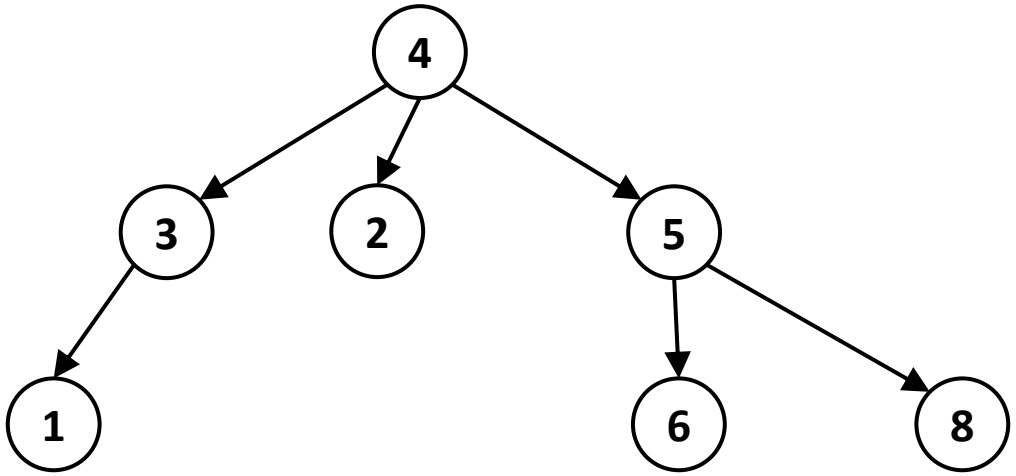
1	6	8

2 2

6	8

BFS(G, **4**)





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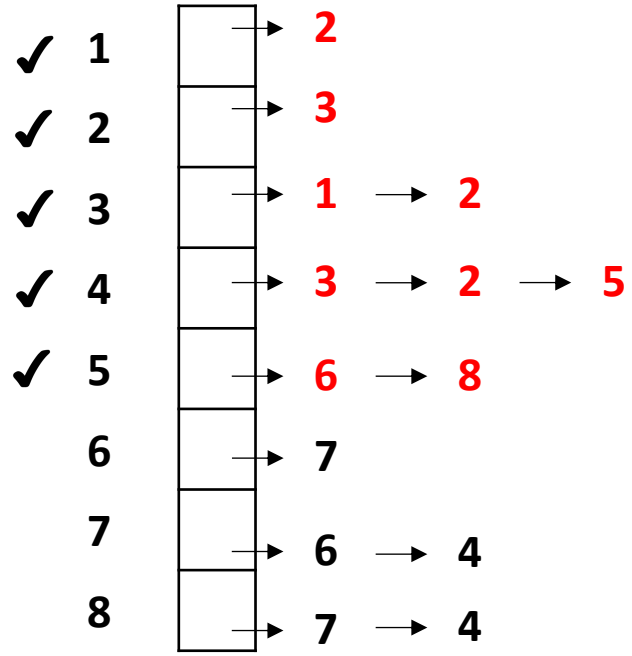

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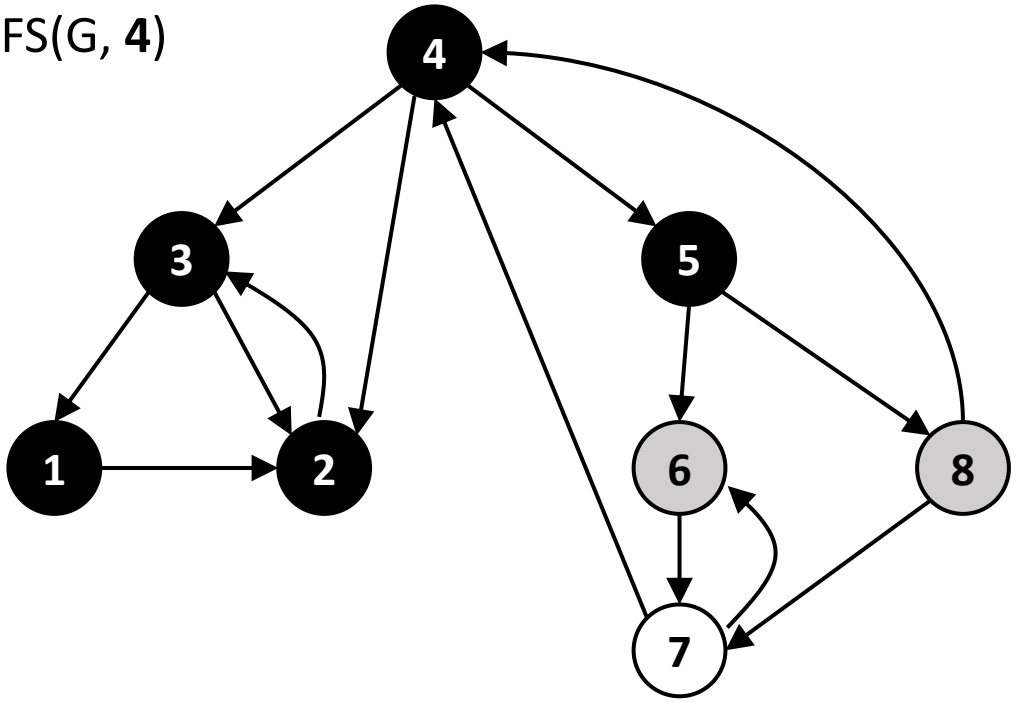
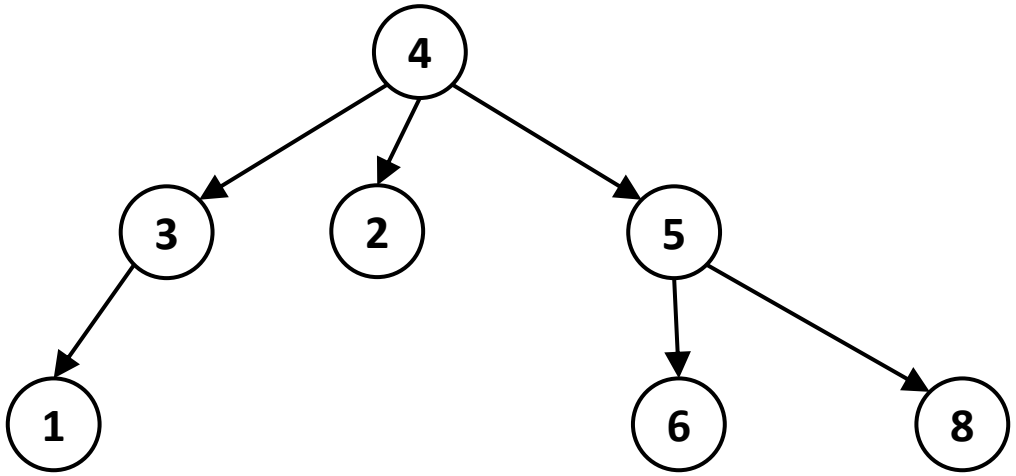
BFS(G, **4**)





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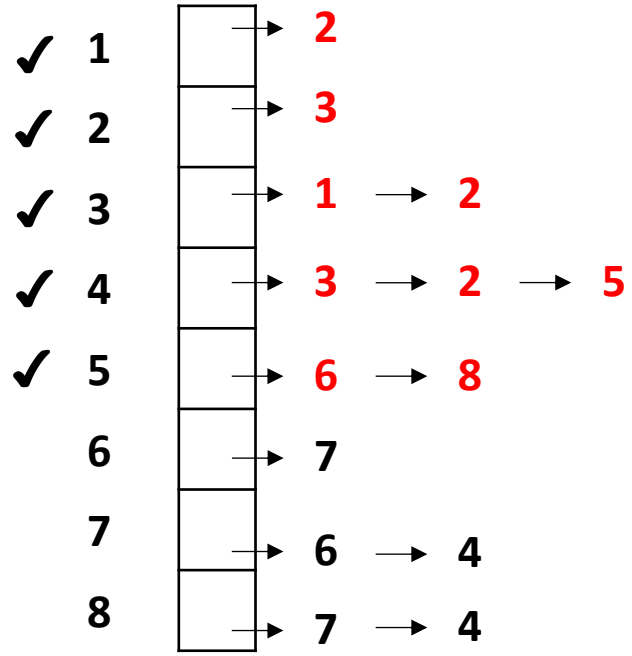

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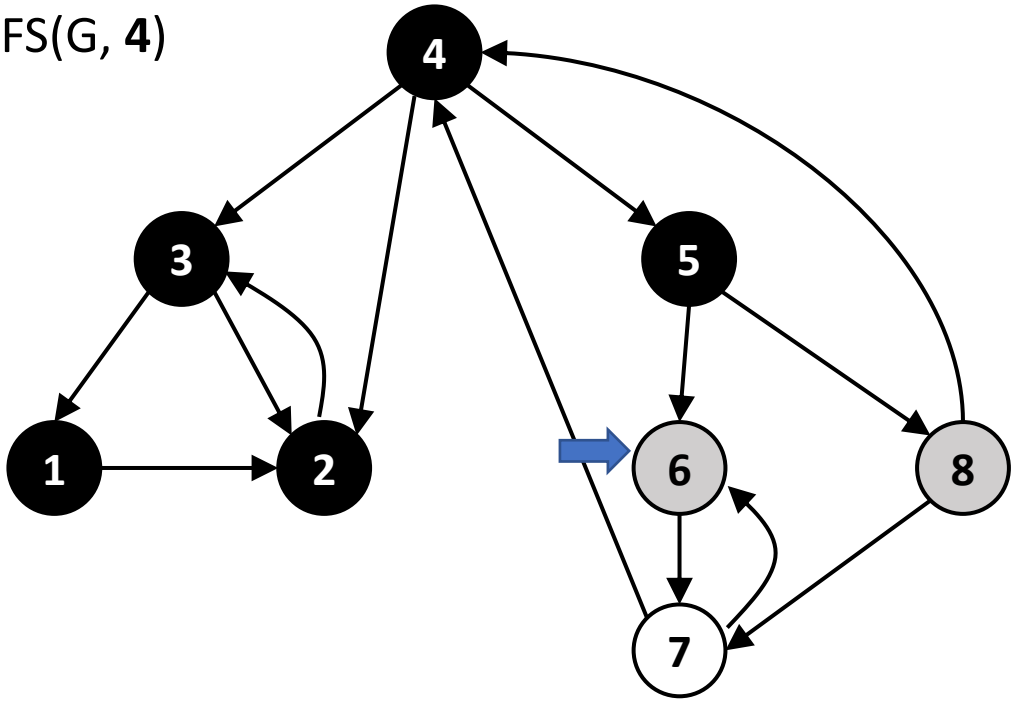
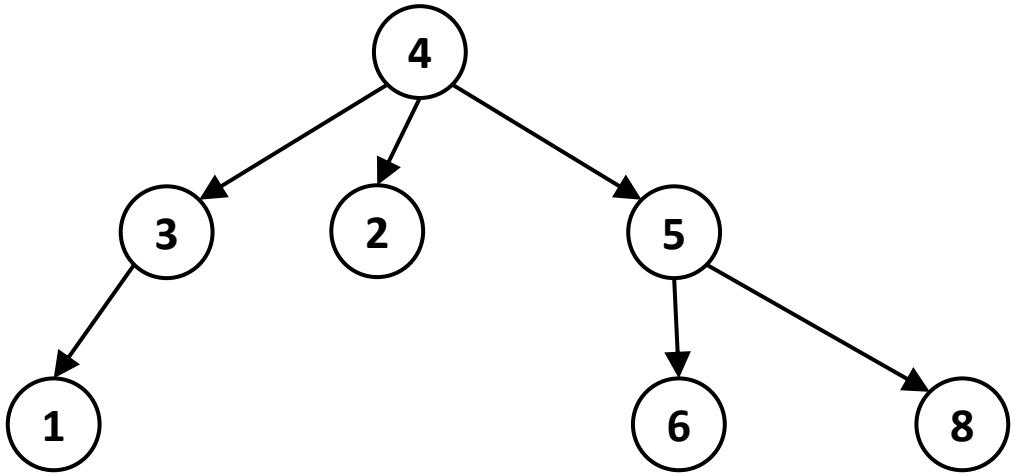
BFS(G, **4**)





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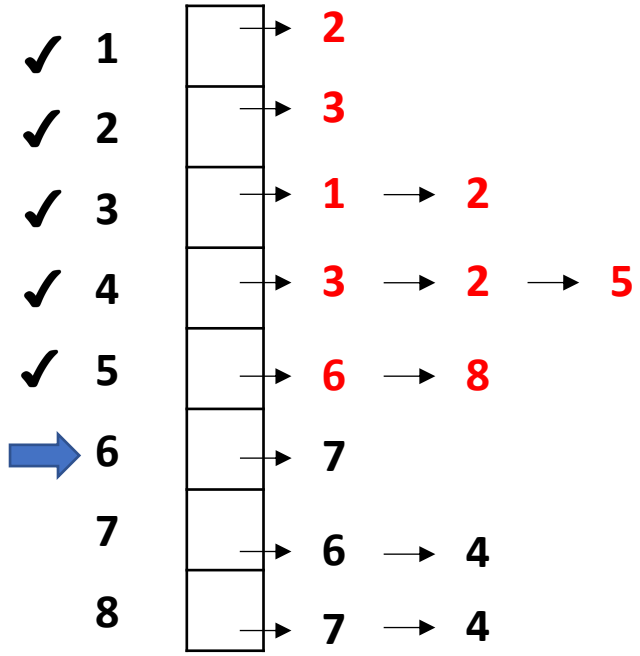

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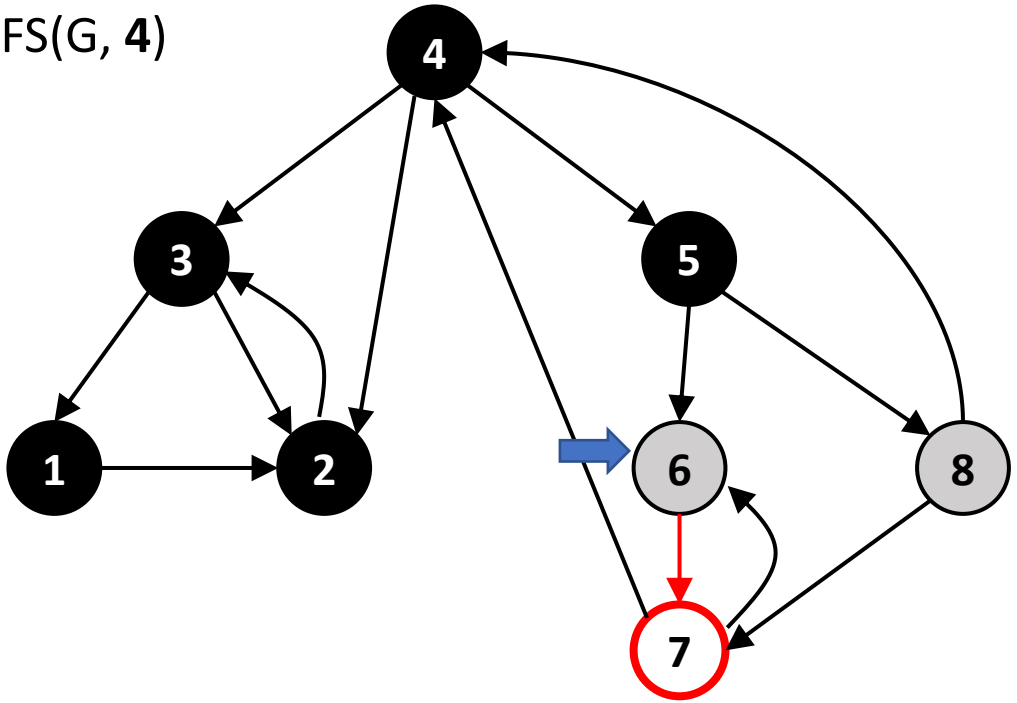
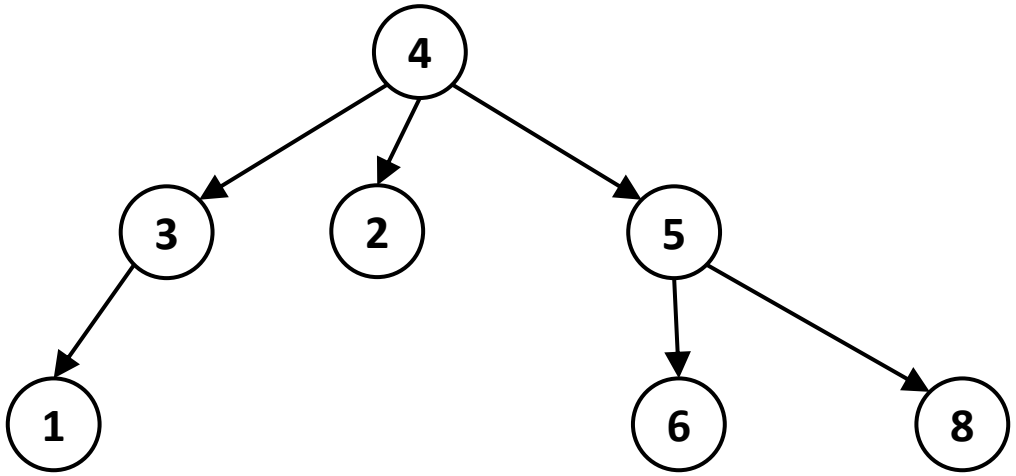
6	8

8

BFS(G, **4**)





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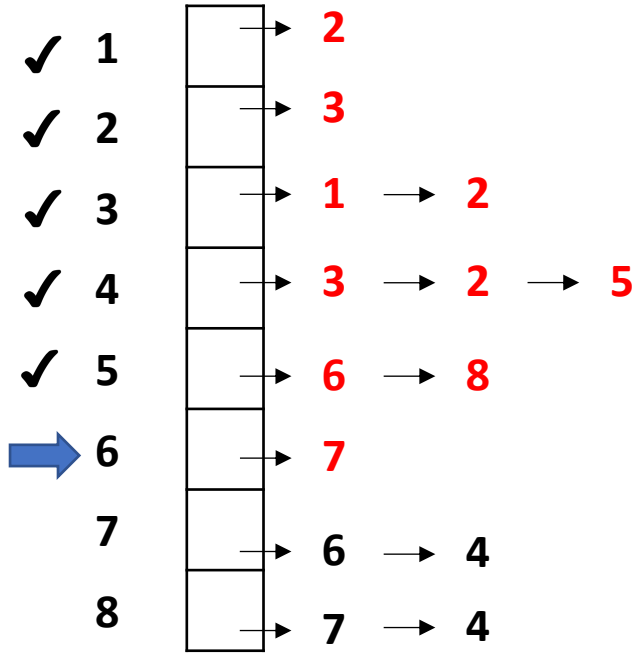

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3	2	5

1 1 2

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1 2

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2 2 2

1	6	8

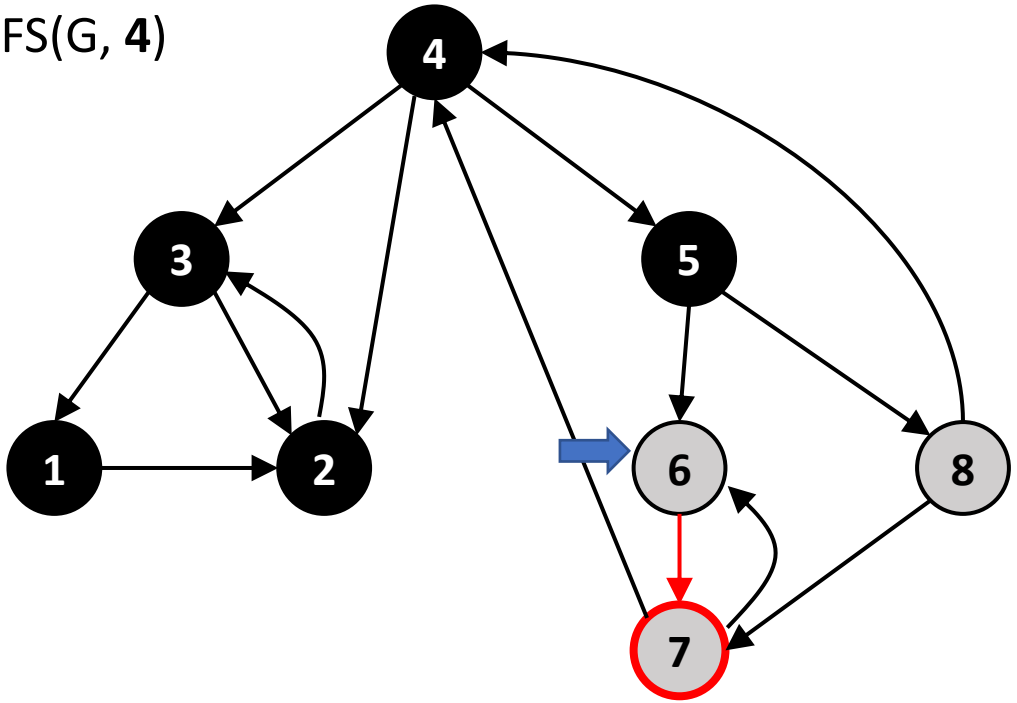
2 2

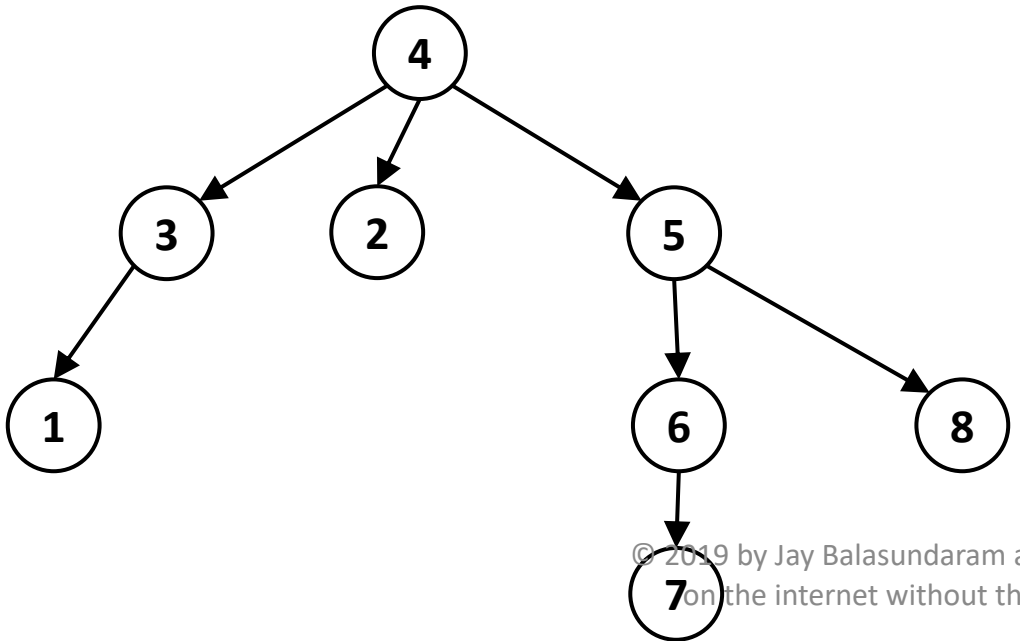
6	8

2

8

BFS(G, **4**)





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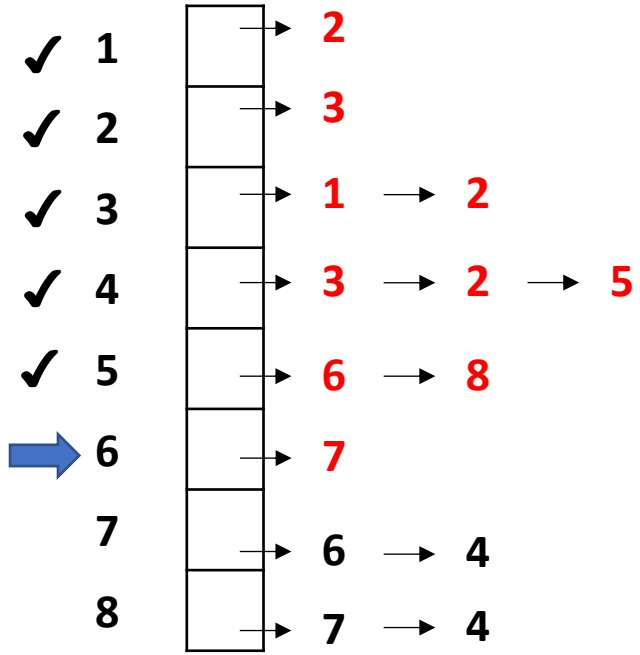

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5	1

2 2 2

1	6	8

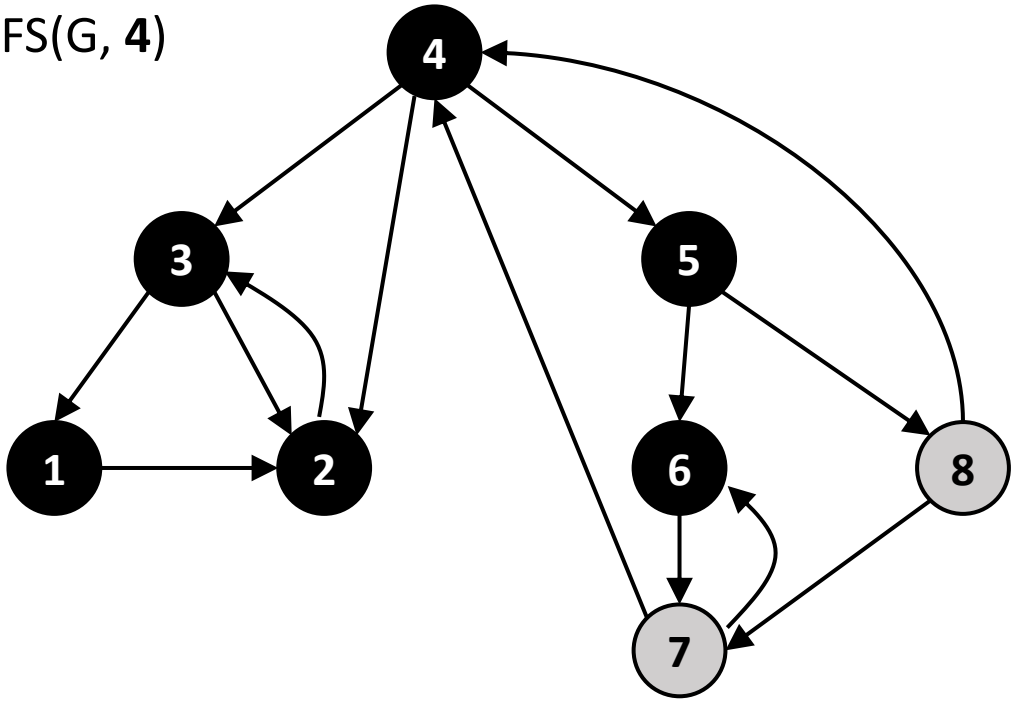
2 2

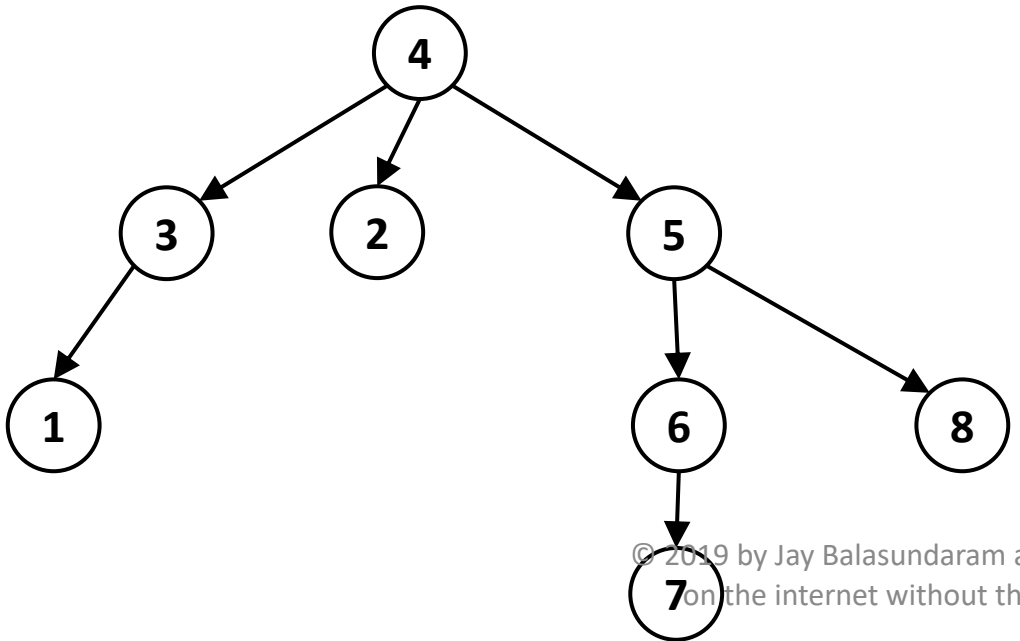
6	8

2 3

8	7

BFS(G, **4**)





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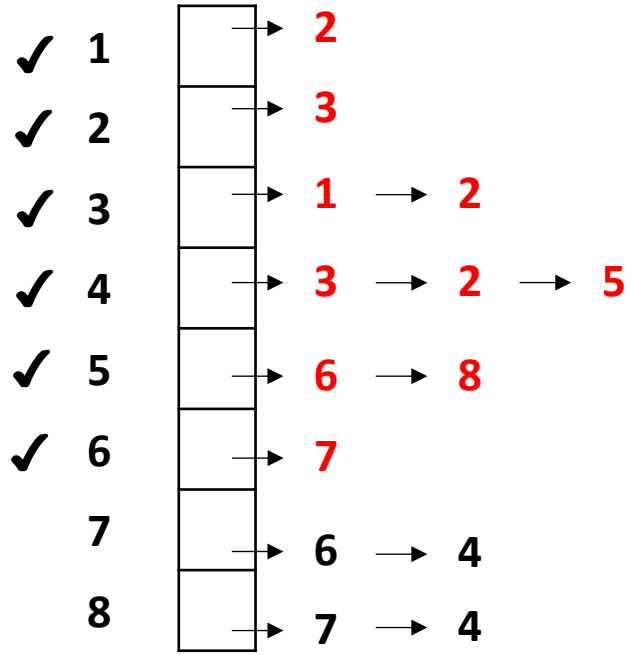

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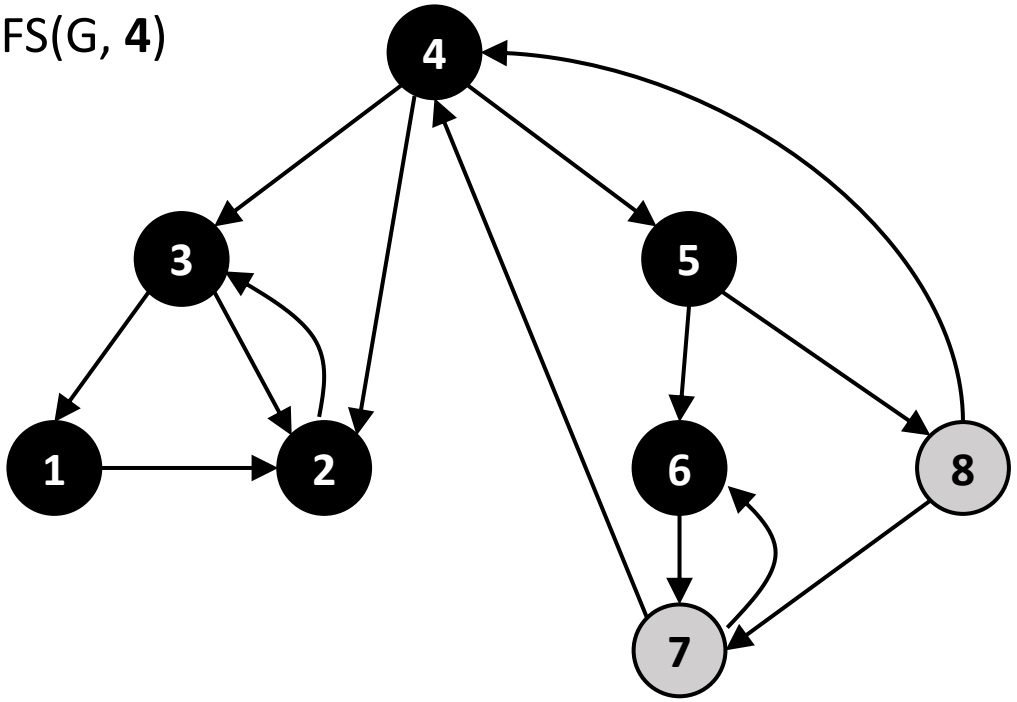
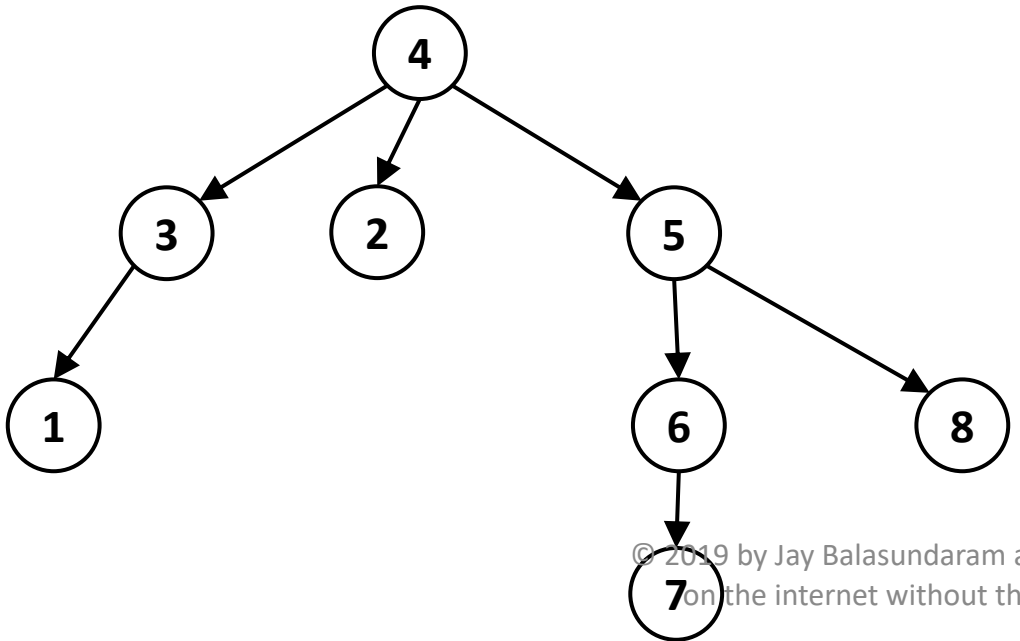
BFS(G, **4**)





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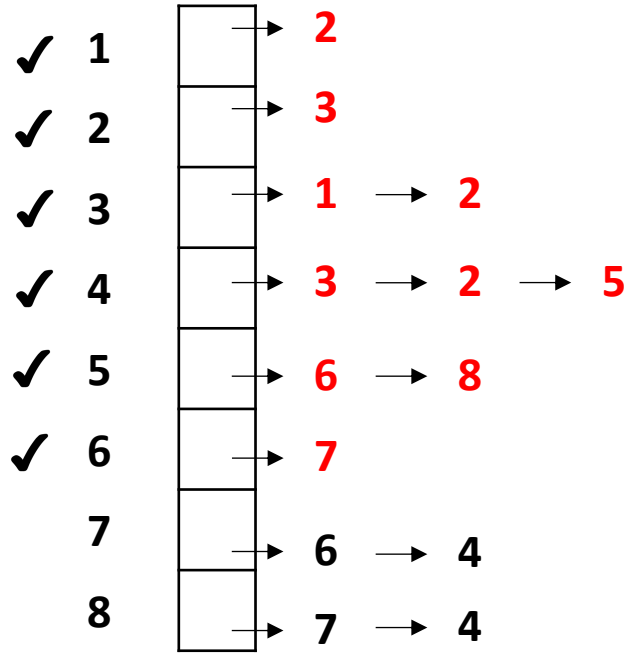

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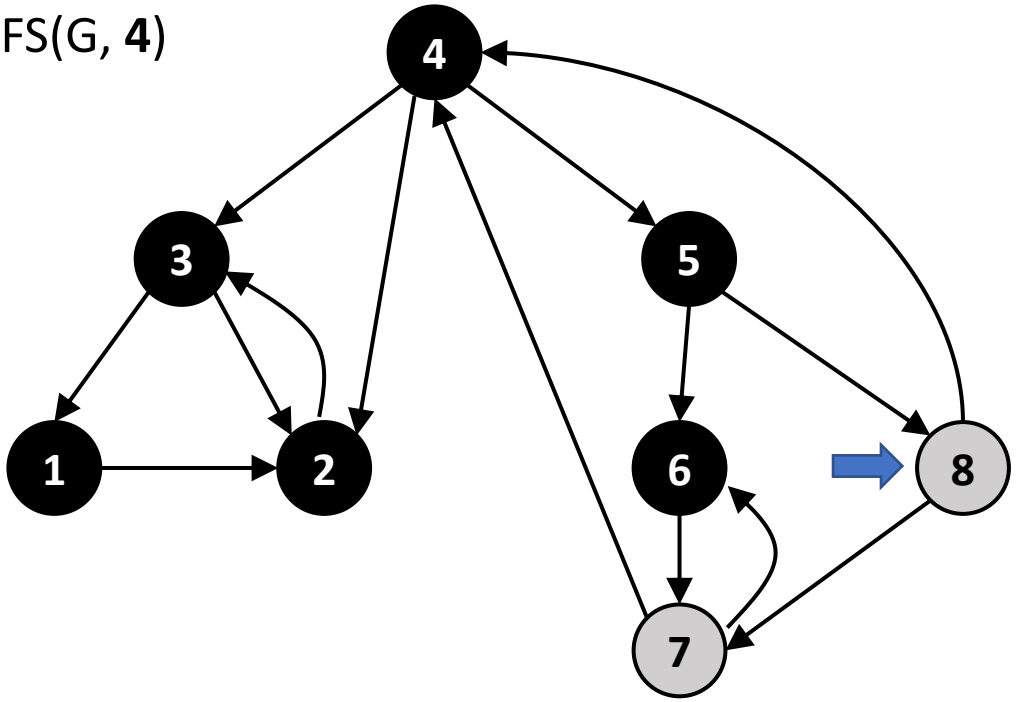
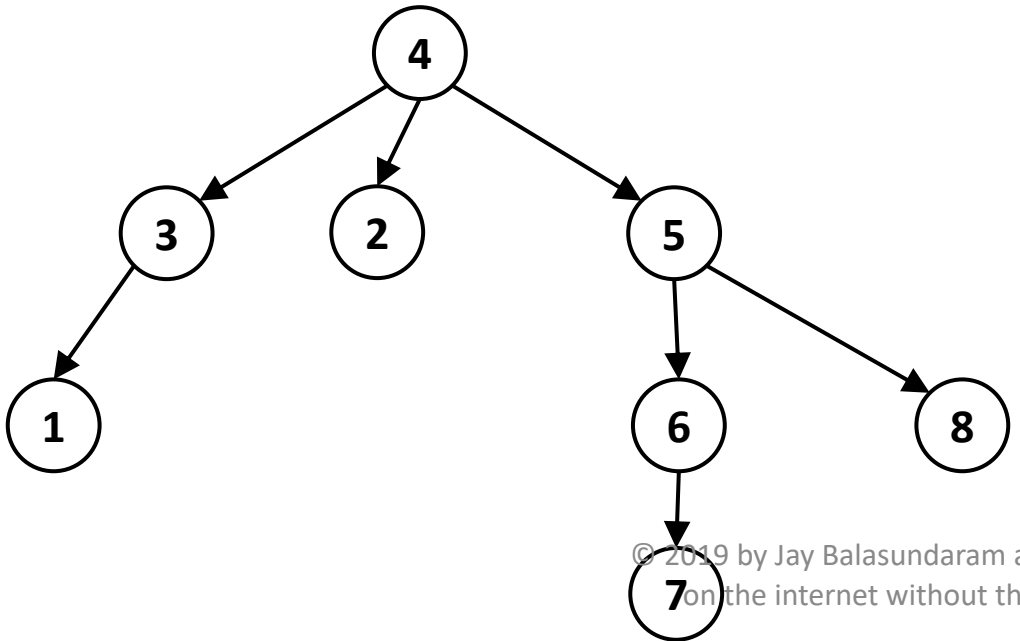
BFS(G, **4**)





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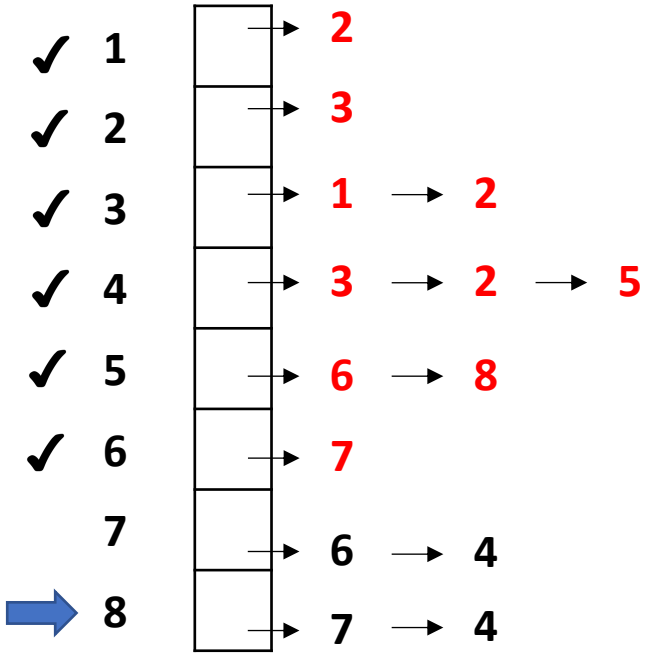

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1 2

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2 2 2

1	6	8

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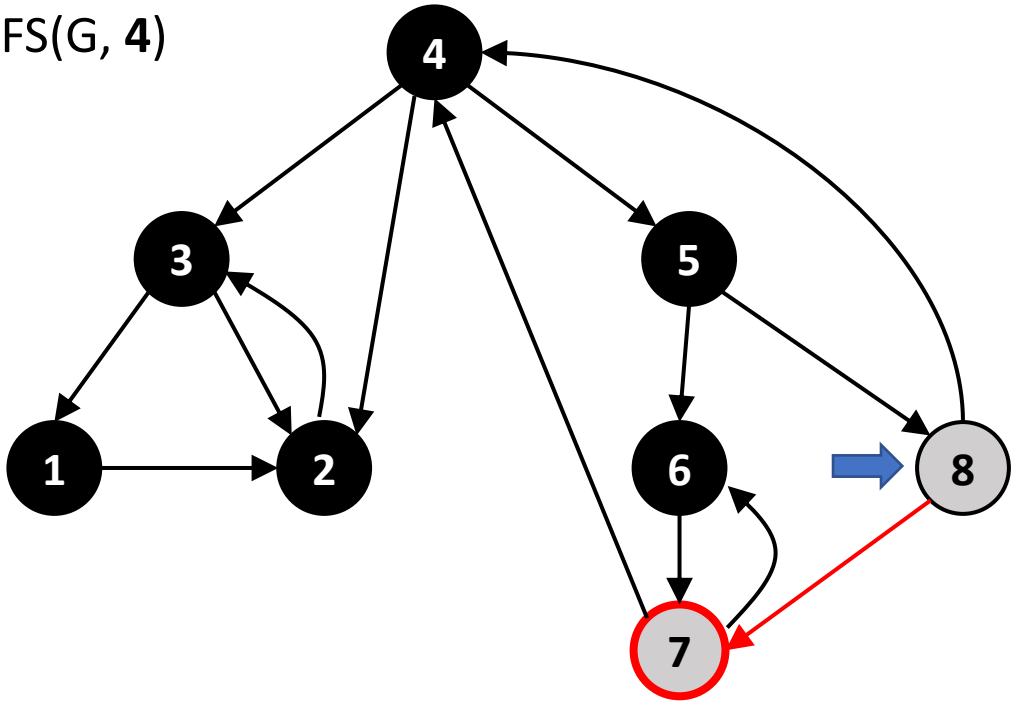
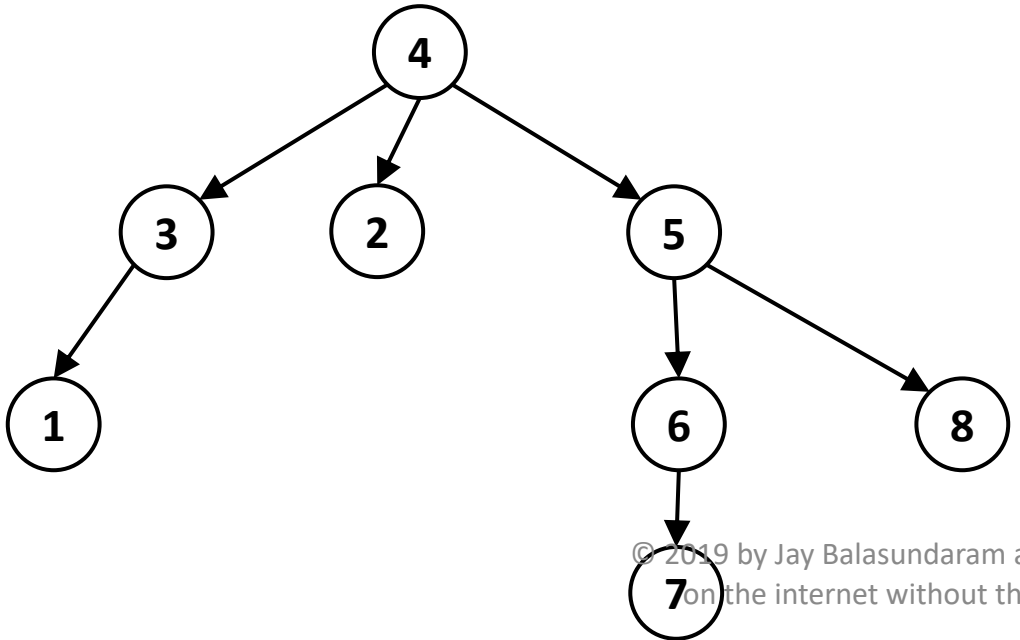
3

7

BFS(G, **4**)





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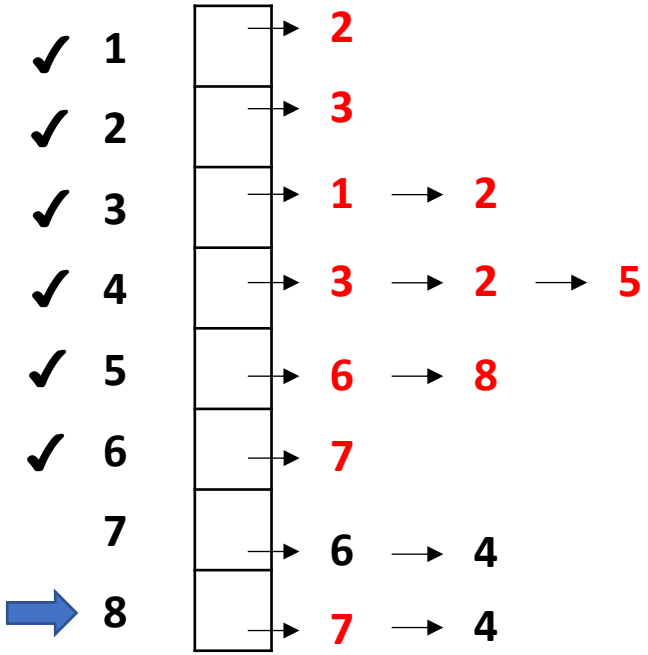

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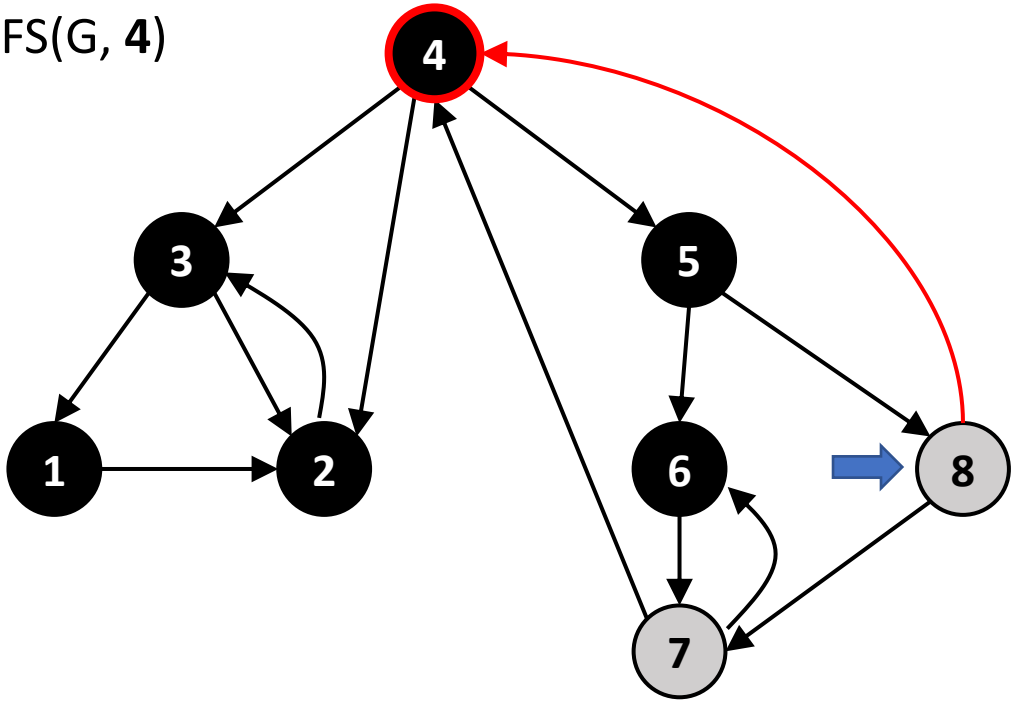
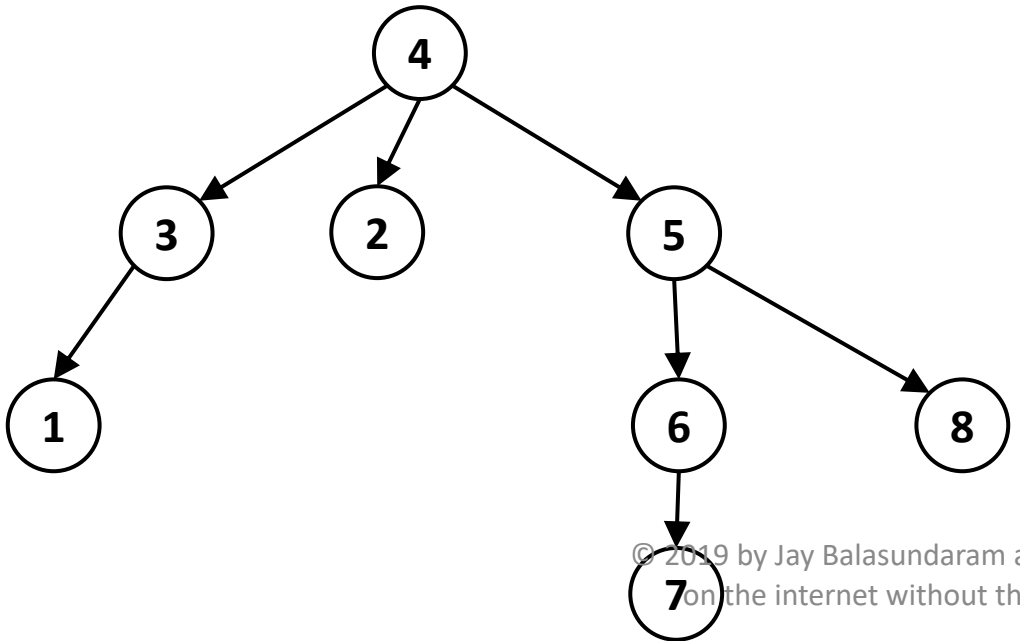
6	8

2	3
8	7

3
7

BFS(G, **4**)





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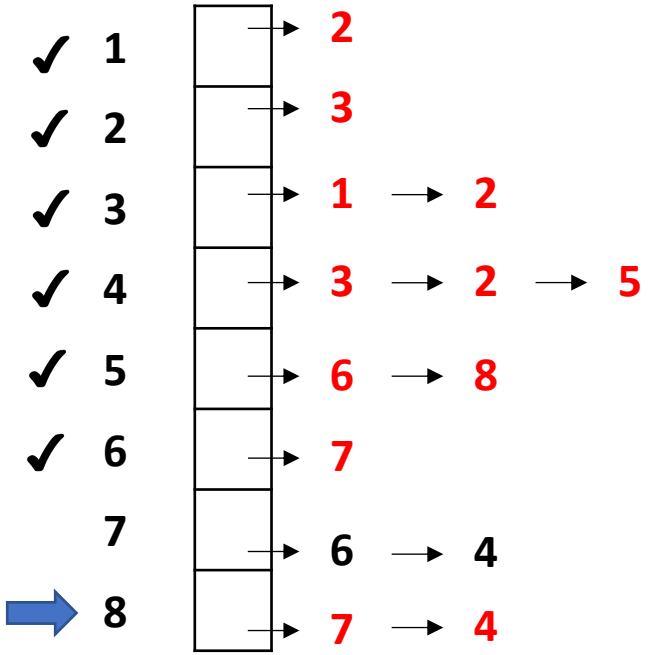

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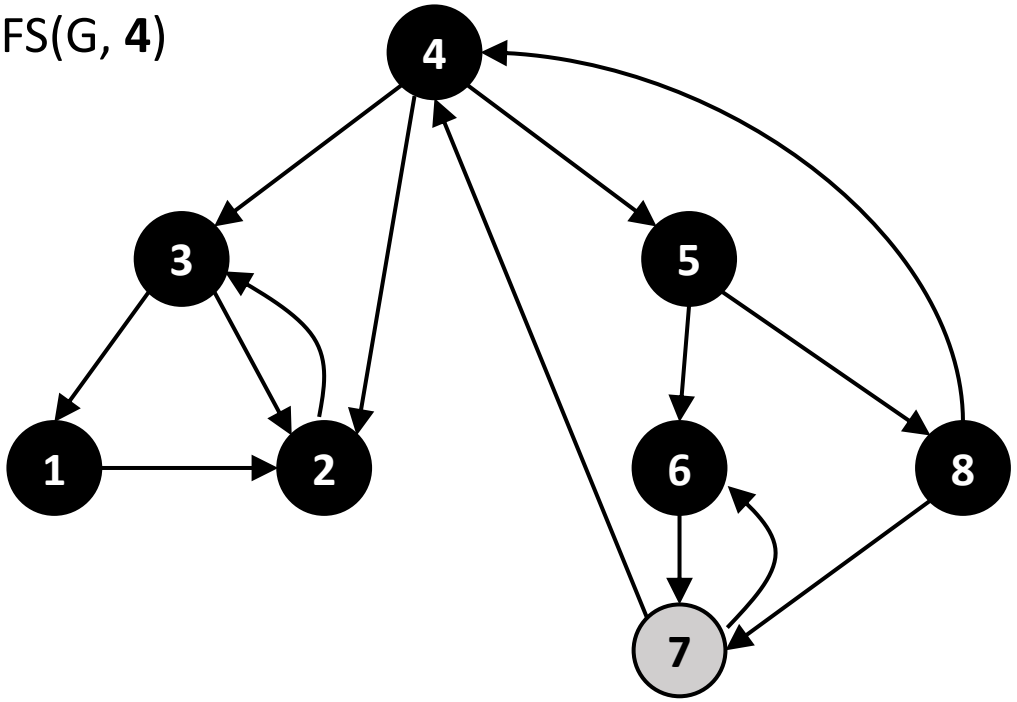
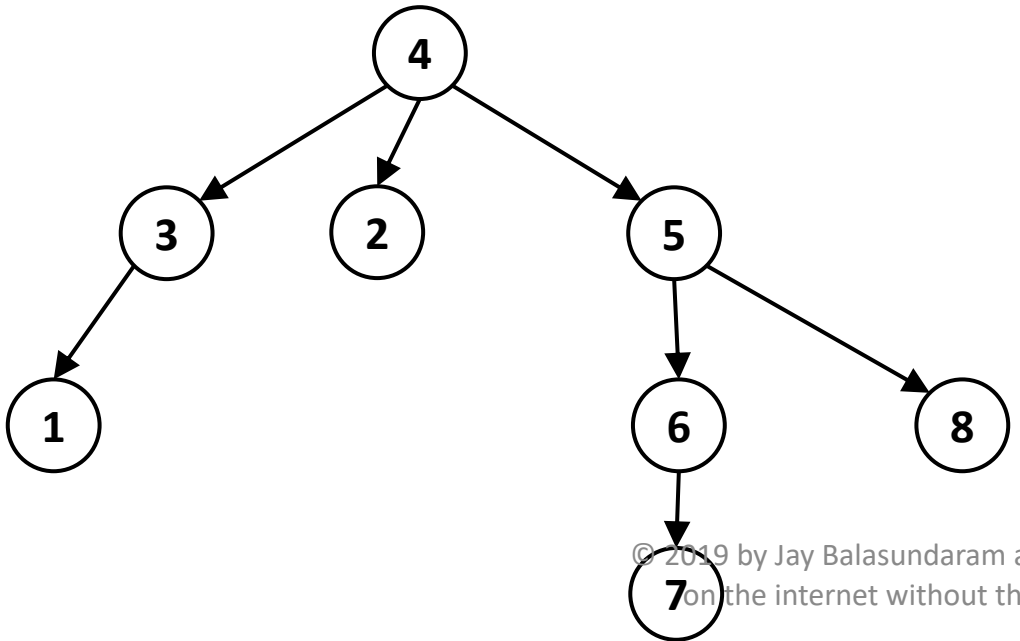
3
7

BFS(G, **4**)





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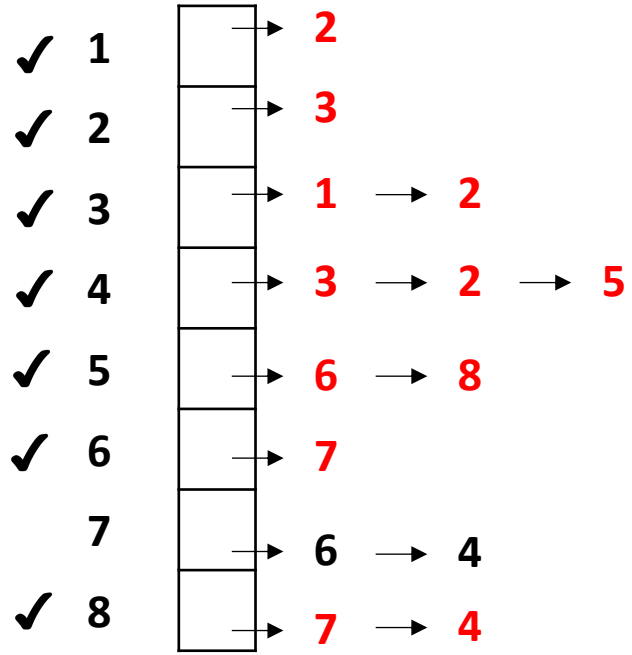

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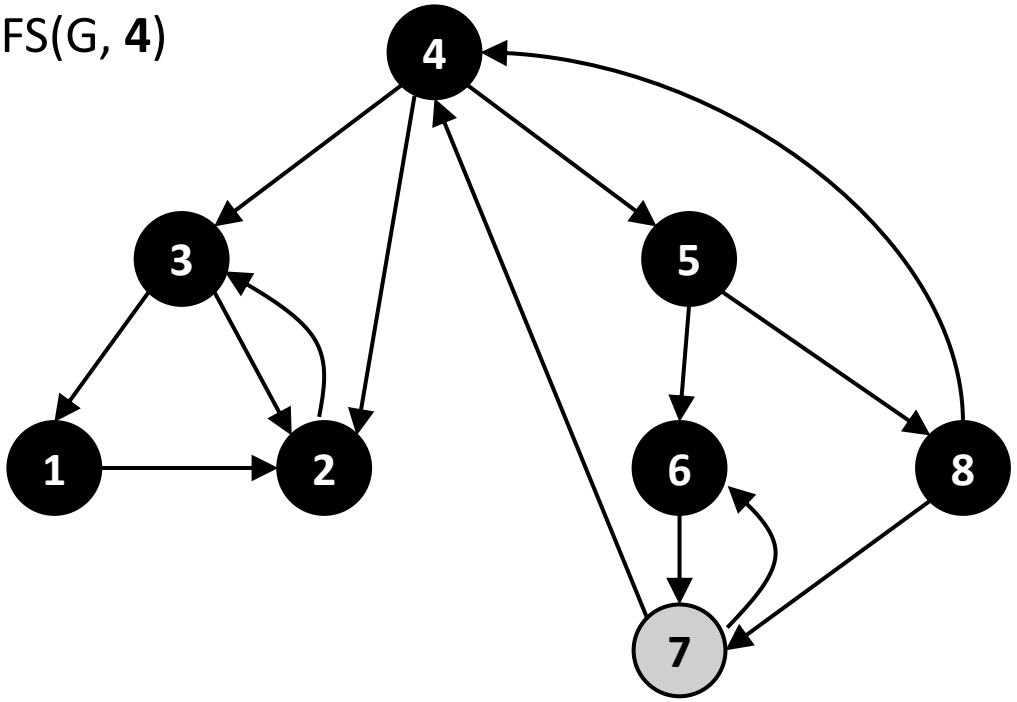
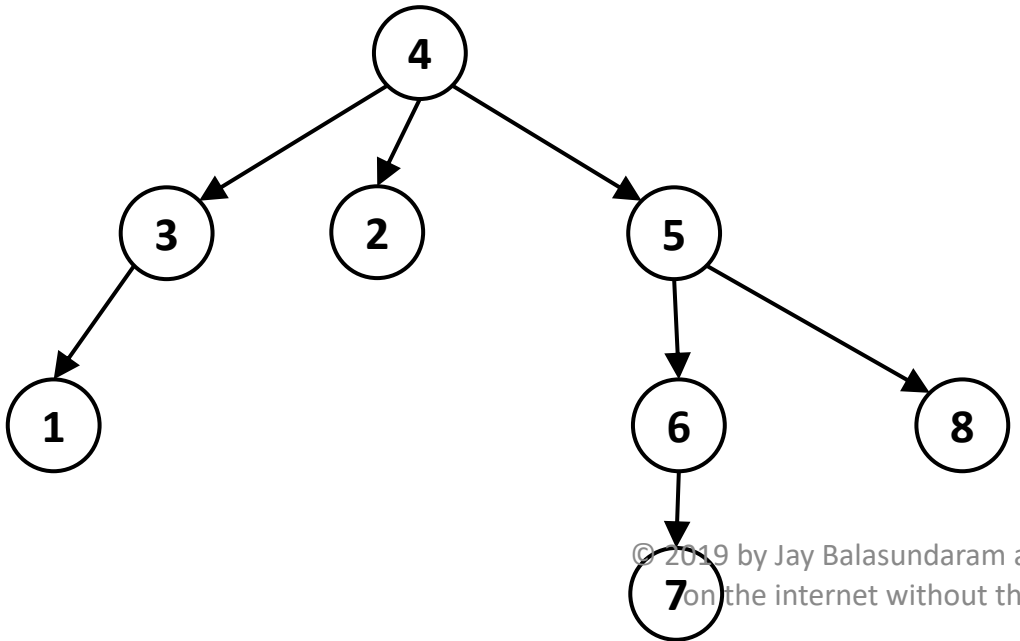
BFS(G, **4**)





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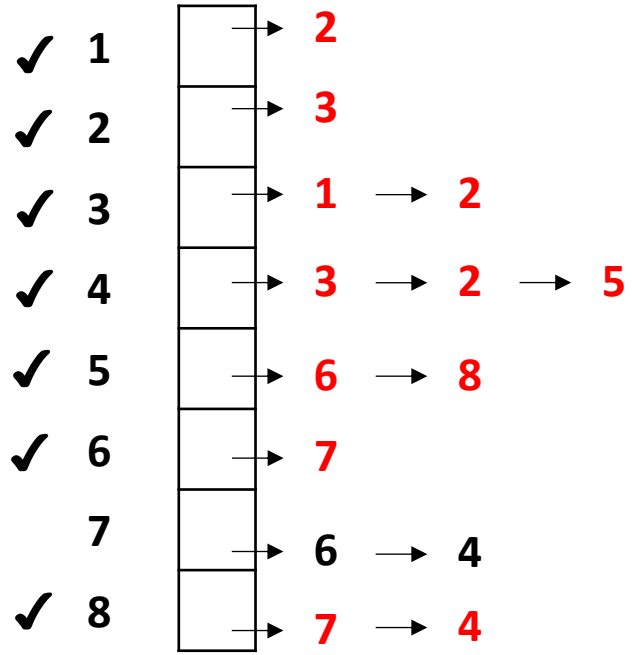

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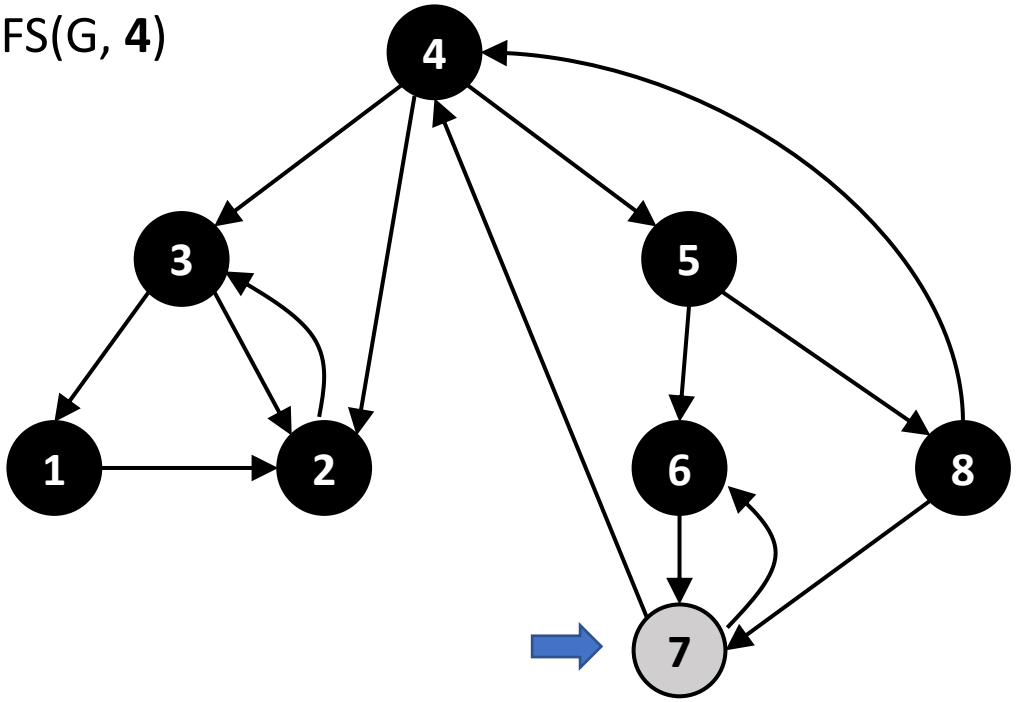
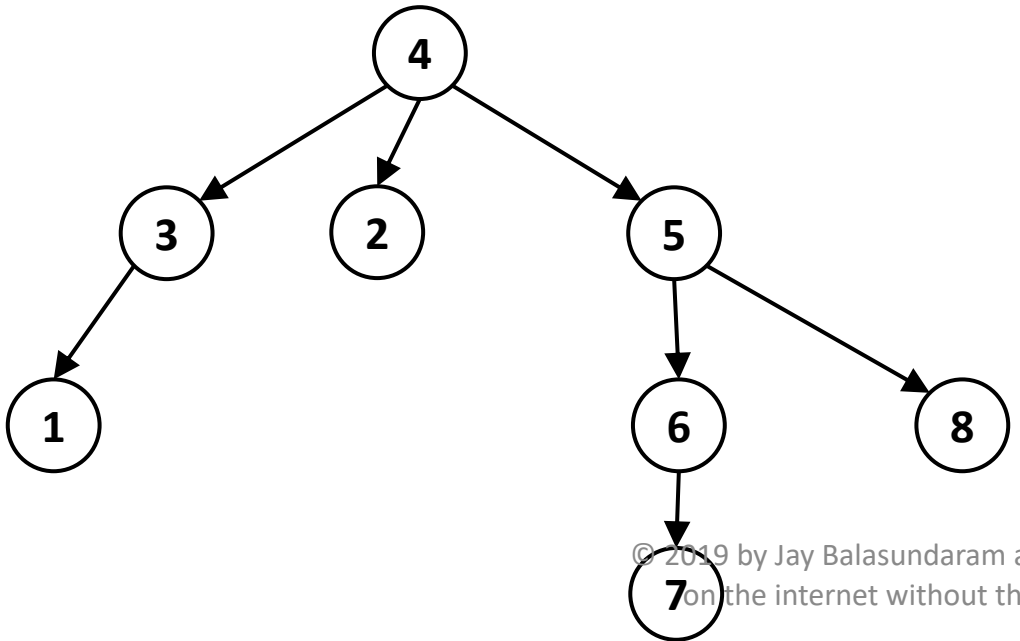
BFS(G, **4**)





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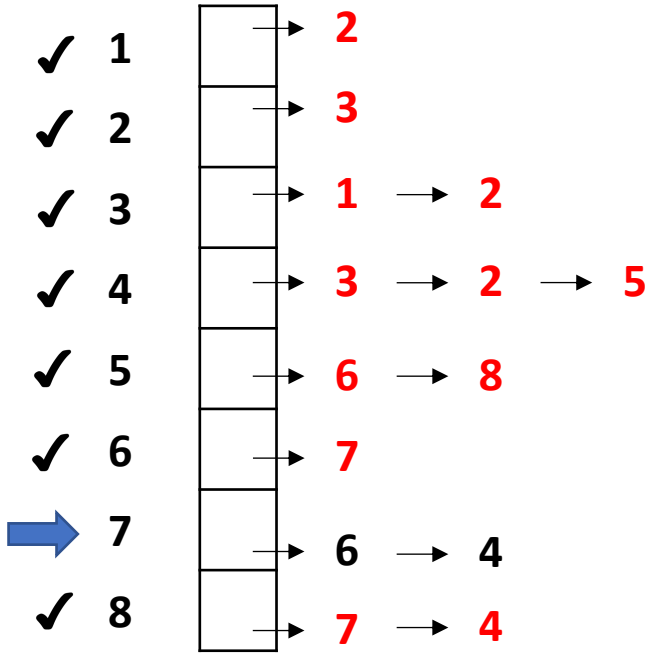

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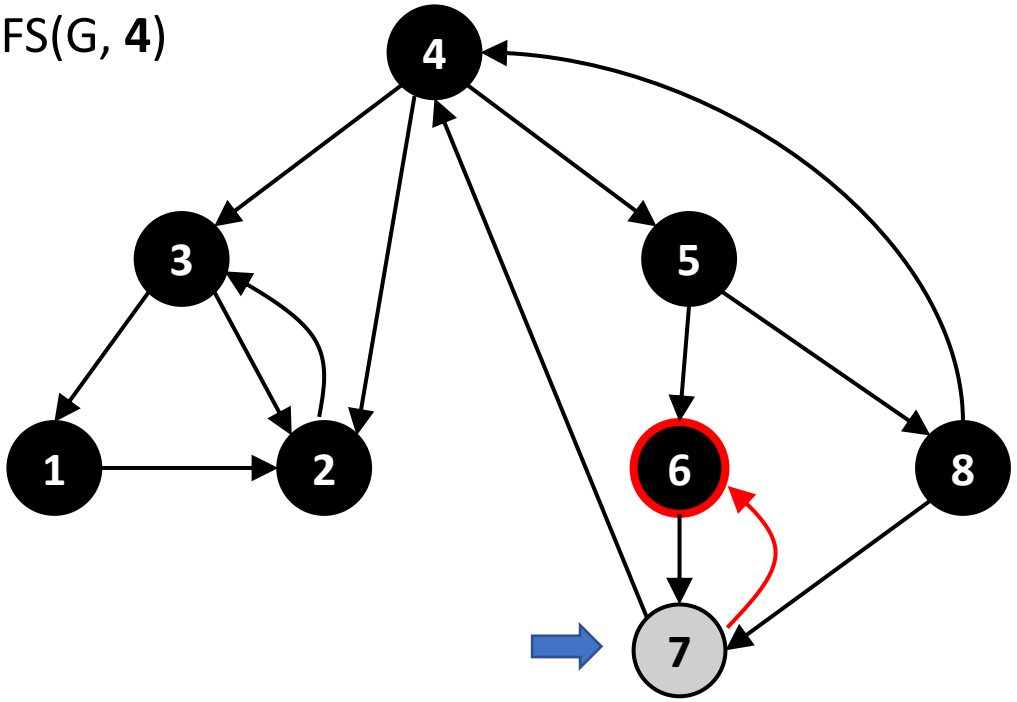
2 3

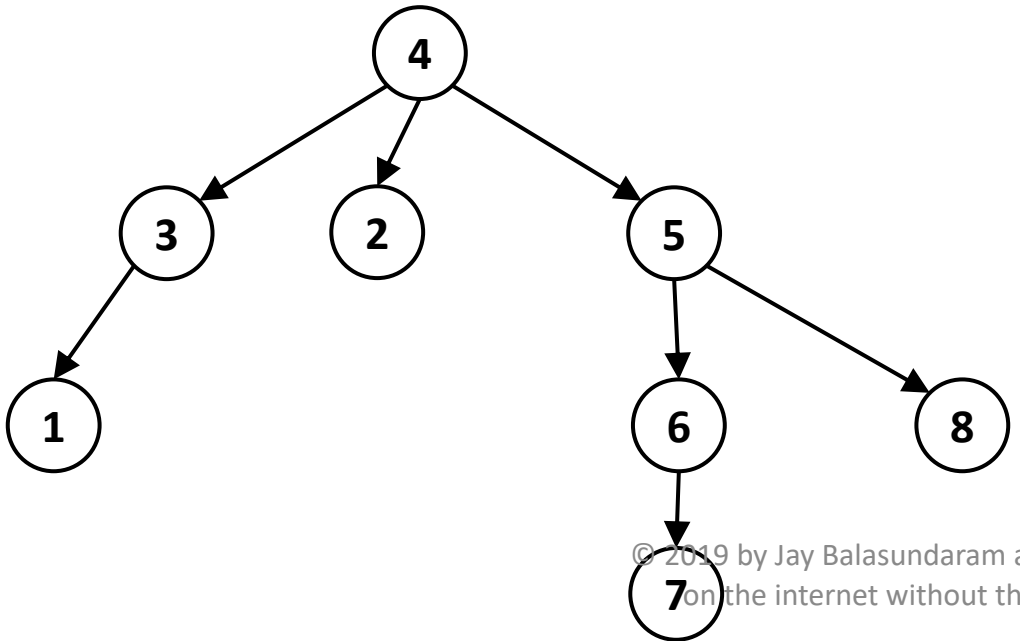
8	7

3

7

BFS(G, **4**)





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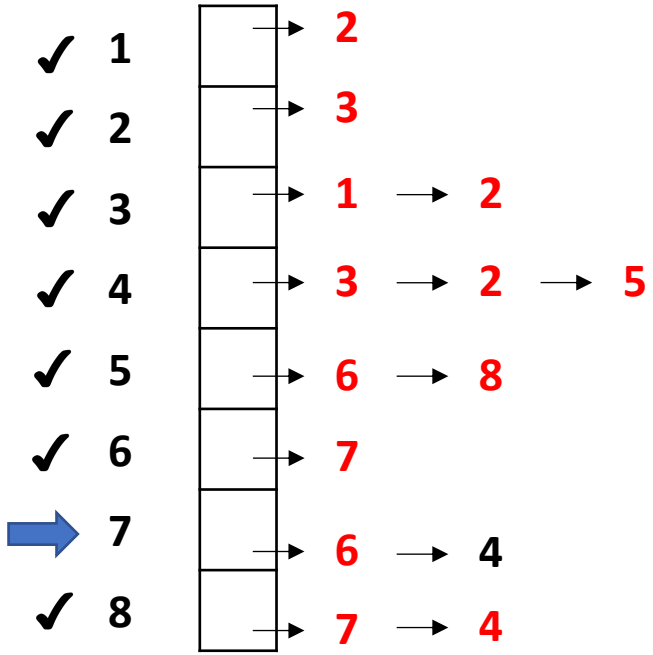

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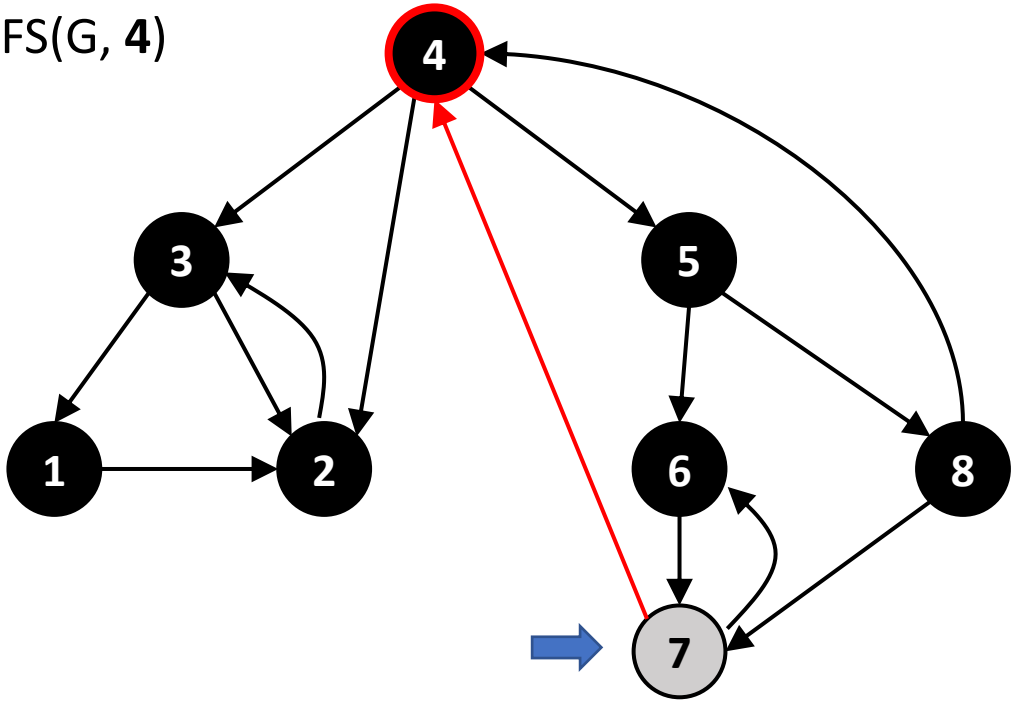
2 3

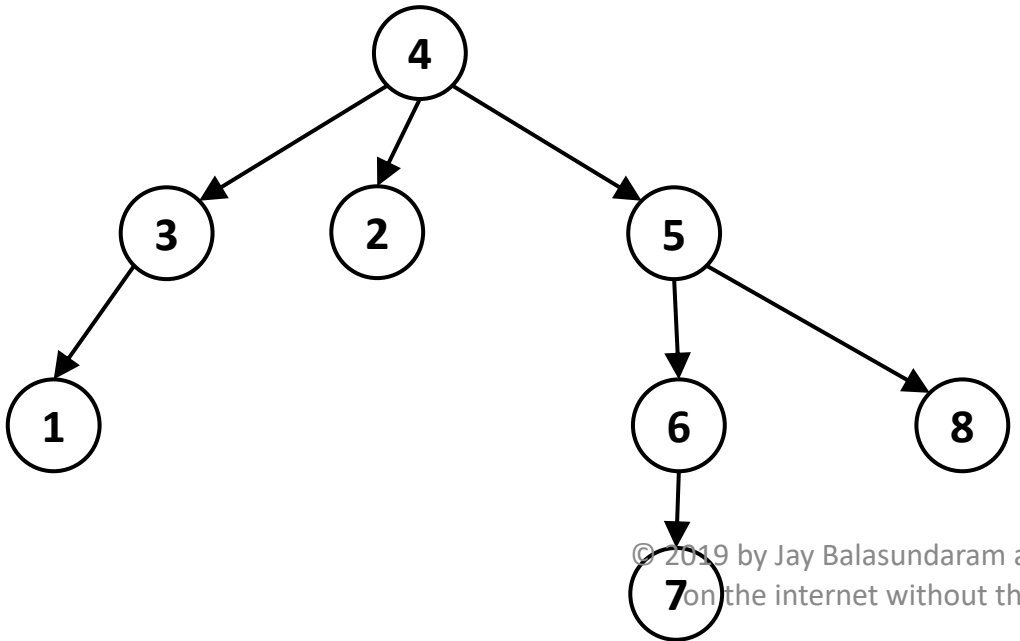
8	7

3

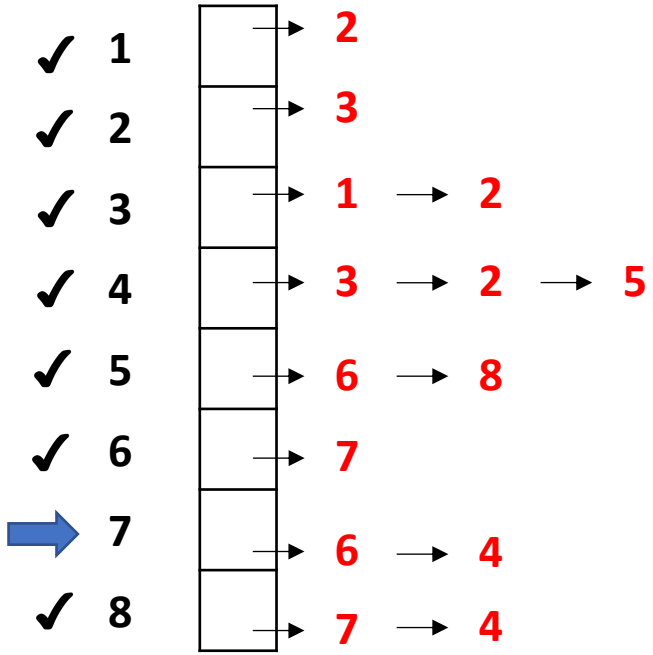
7

BFS(G, **4**)





Adj List of G :



Contents of Q :

d = 0

4

d = 1 1 1

3	2	5

1 1 2

2	5	1

1 2

5	1

2 2 2

1	6	8

2 2

6	8

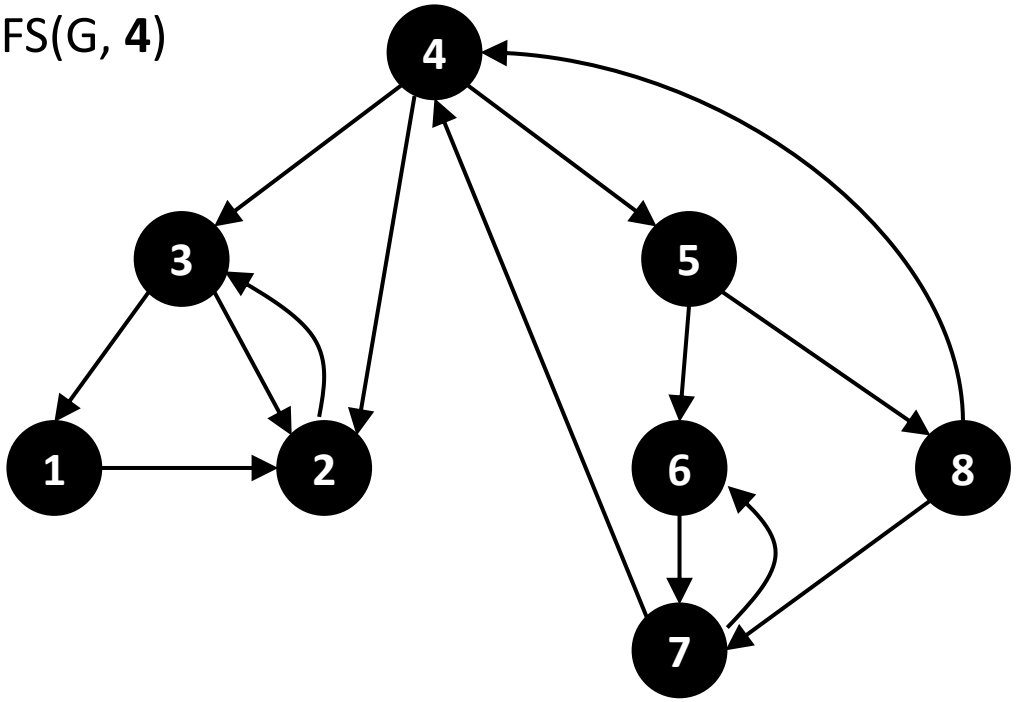
2 3

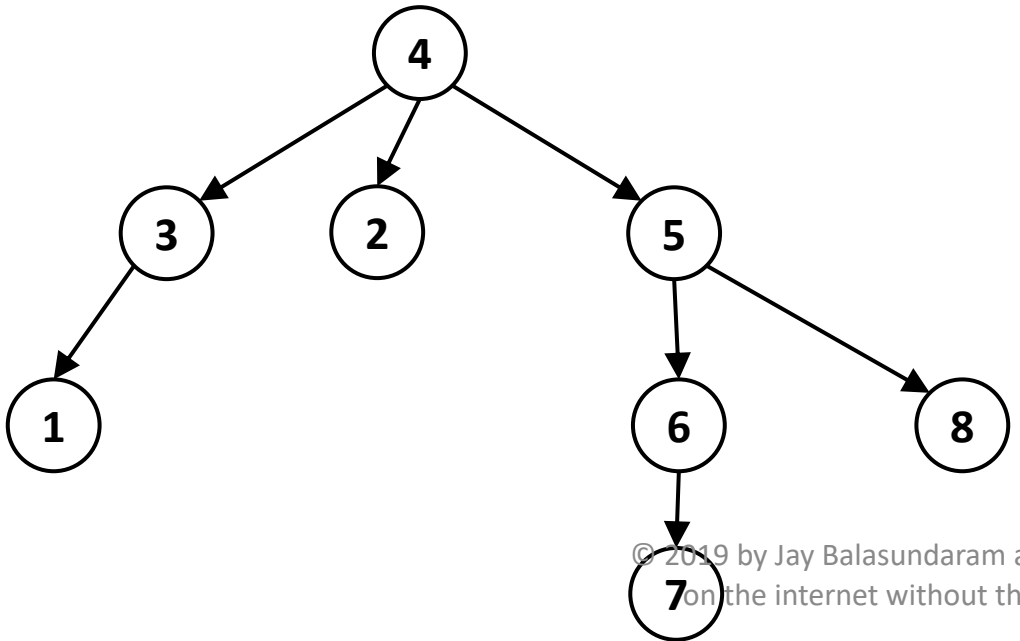
8	7

3

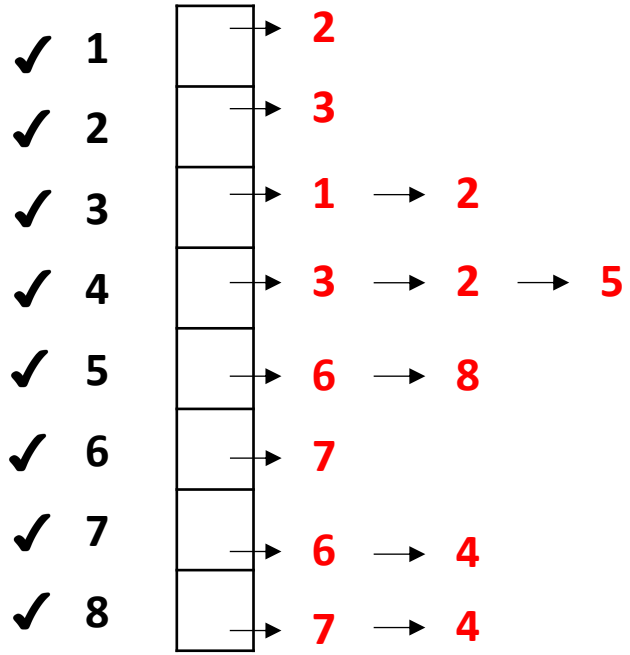
7

BFS(G, **4**)





Adj List of G :



Contents of Q :

d = 0

4

d = 1 1 1

3	2	5

2	5	1

5	1

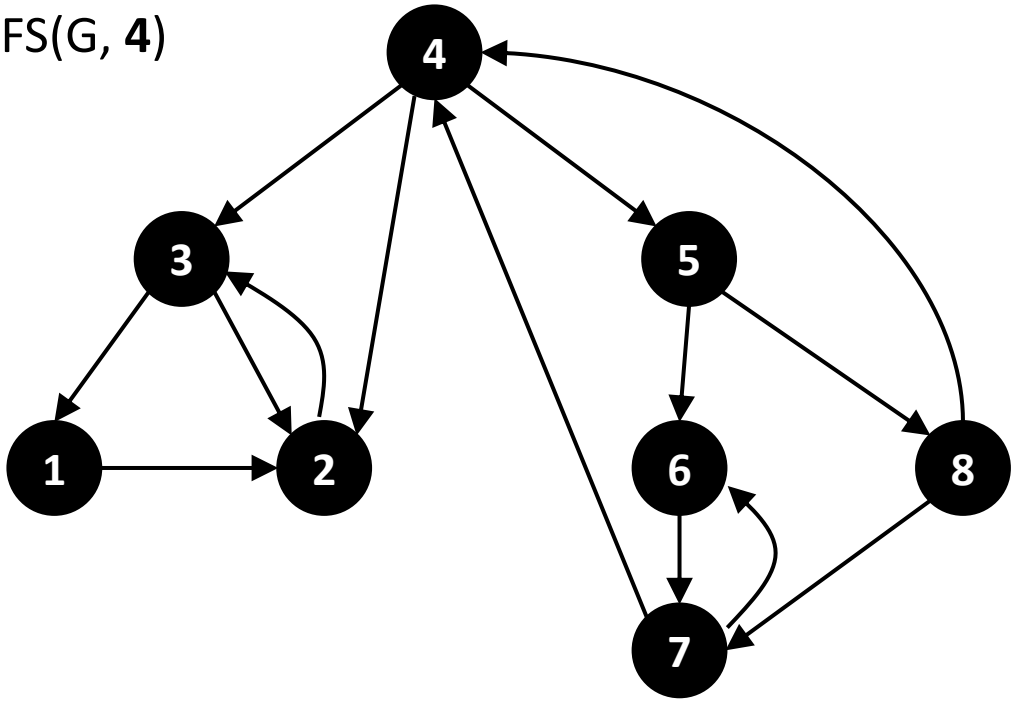
1	6	8

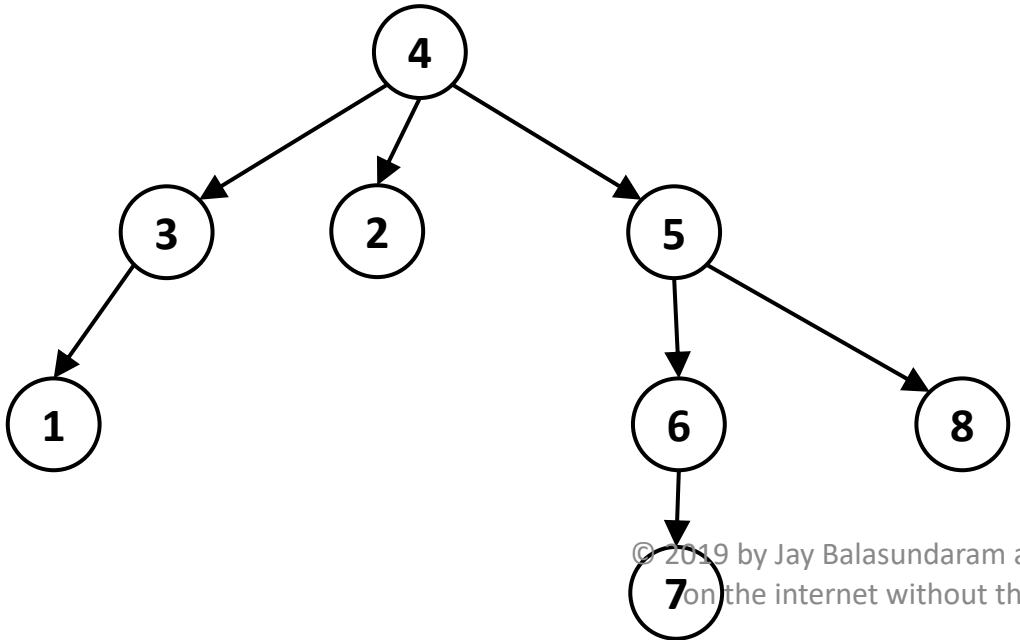
6	8

8	7

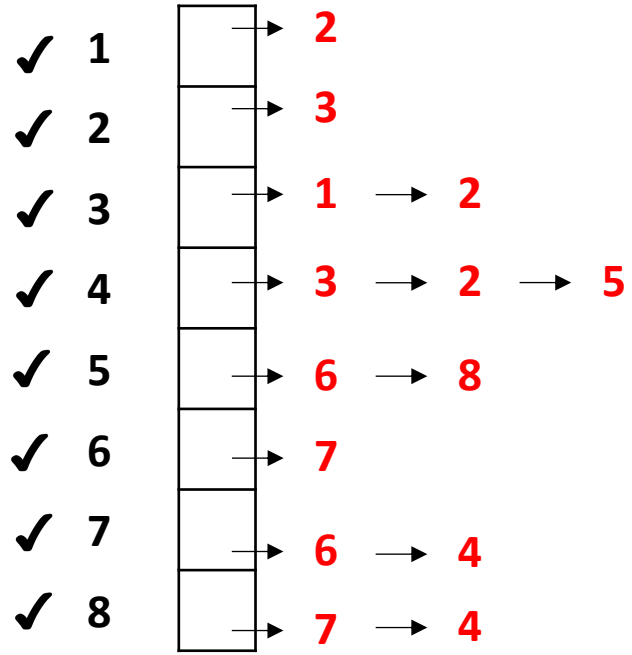
7

BFS(G, **4**)





Adj List of G :



Worst-Case Time Complexity of BFS:

$O(|V| + |E|)$

Contents of Q :

d = 0

4

d = 1 1 1

3	2	5

1 1 2

2	5	1

1 2

5	1

2 2 2

1	6	8

2 2

6	8

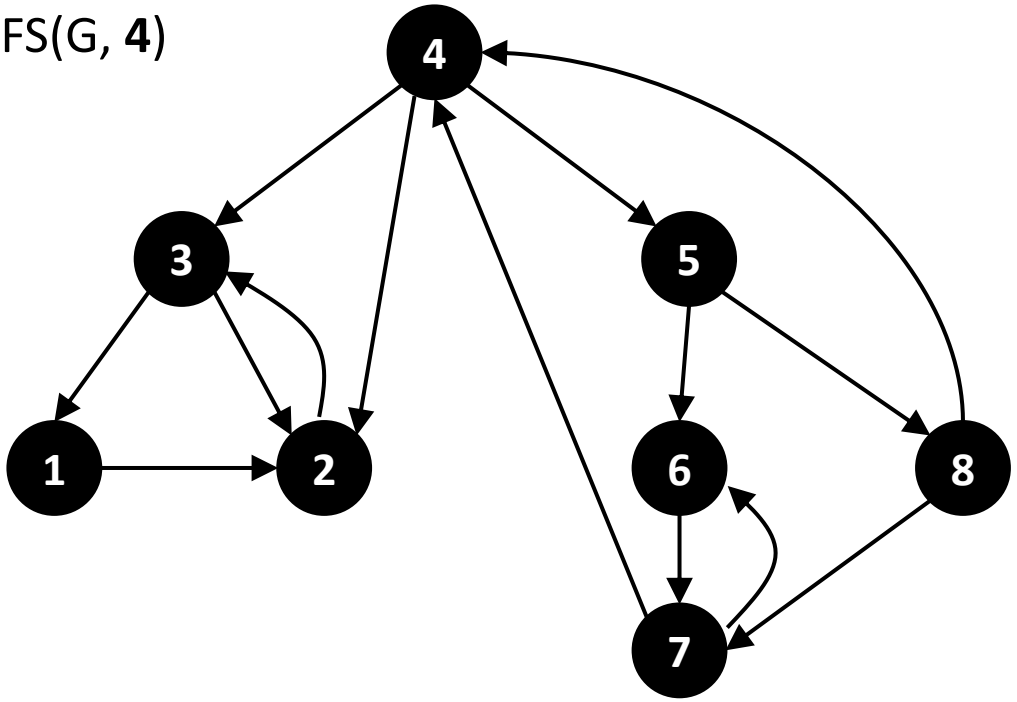
2 3

8	7

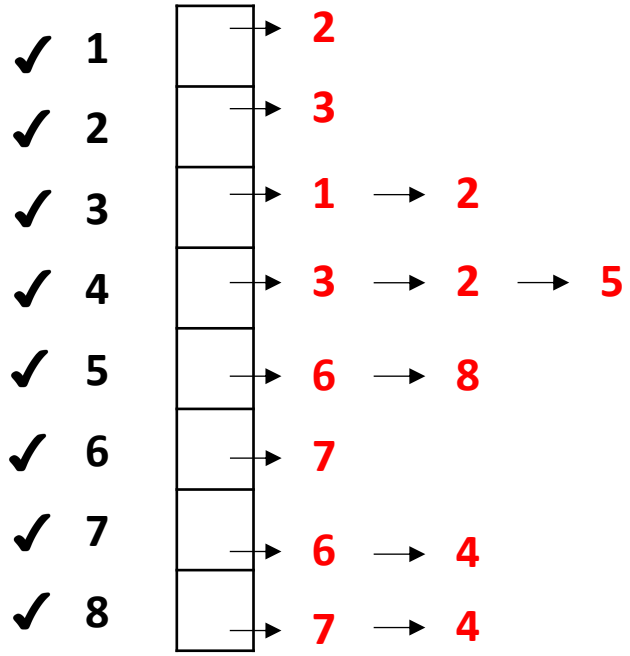
3

7

BFS(G, **4**)



Adj List of G :



Contents of Q :

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4

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3	2	5

1 1 2

2	5	1

1 2

5	1

2 2 2

1	6	8

2 2

6	8

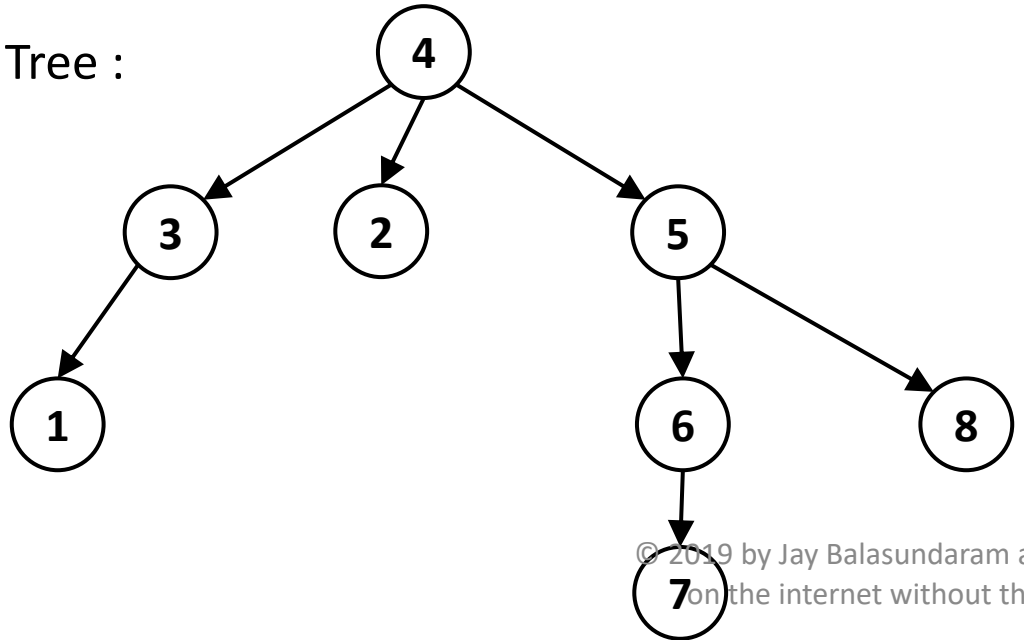
2 3

8	7

3

7

BFS Tree :



Worst-Case Time Complexity of BFS:

$O(|V| + |E|)$