Graph Algorithms II

Depth First Search

Depth First Search (DFS)

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- BFS : First Discovered – First Explored

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The algorithm keeps track of

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p[v] = u iff u discovered v

Different from BFS

d[v] = "time" when v is discovered

f[v] = "time" when v's exploration is completed

To keep track of "time", it uses a counter which is incremented by 1

- whenever a new node is discovered
- whenever it finishes exploring a node

DFS(G**)** /* G = (V, E) */

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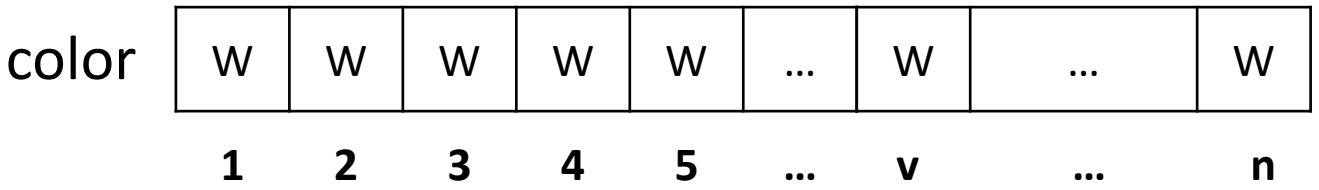
For each v ∈ V **do**

color[v] ← white ;

End For



DFS(G**)** /* G = (V, E) */

For each v ∈ V **do**

color[v] ← white ; d[v] ← ∞ ;

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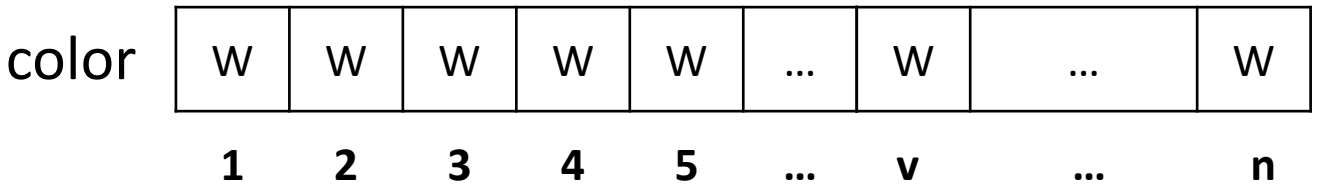

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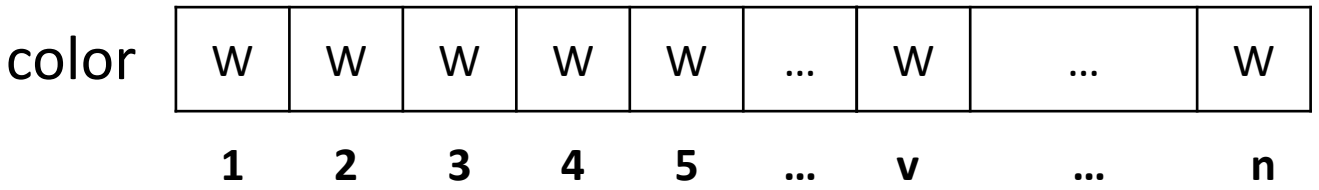

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For each v ∈ V **do**

color[v] ← white ; d[v] ← ∞ ; f[v] ← ∞ ; p[v] ← NIL

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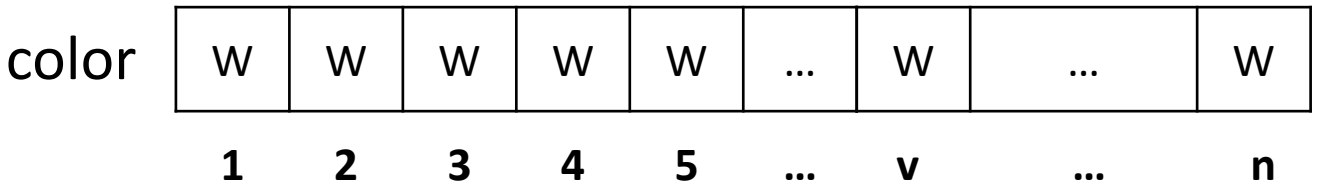

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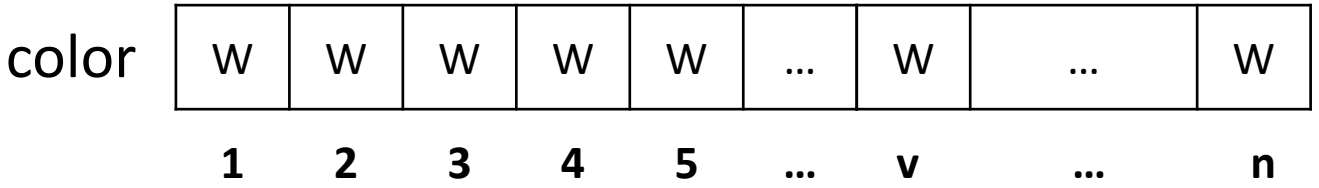

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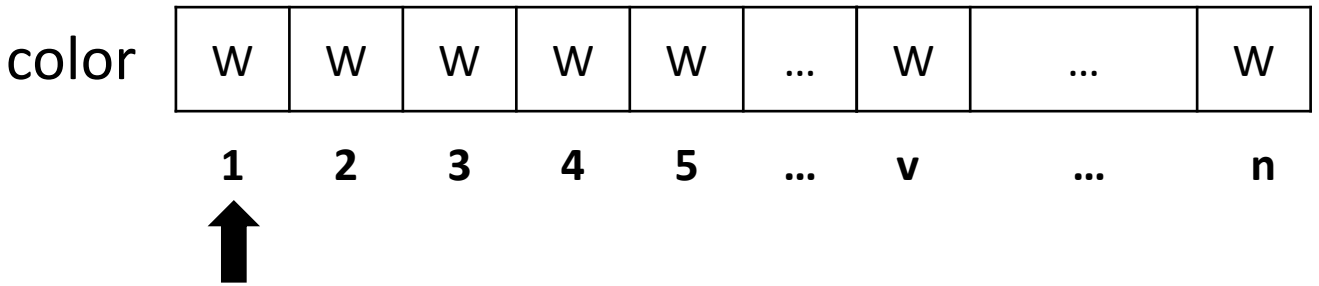
If color[v] = white



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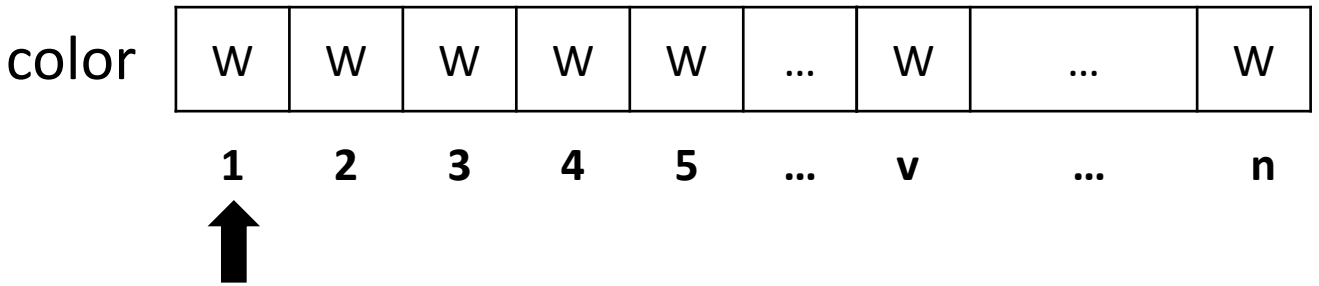

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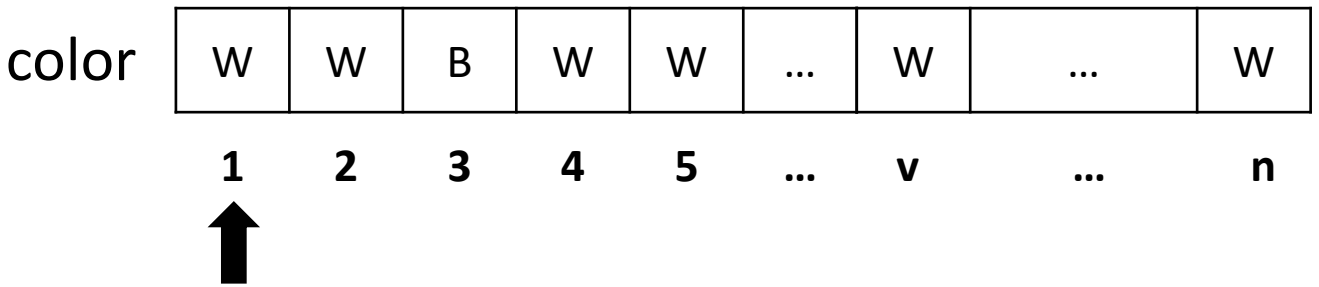

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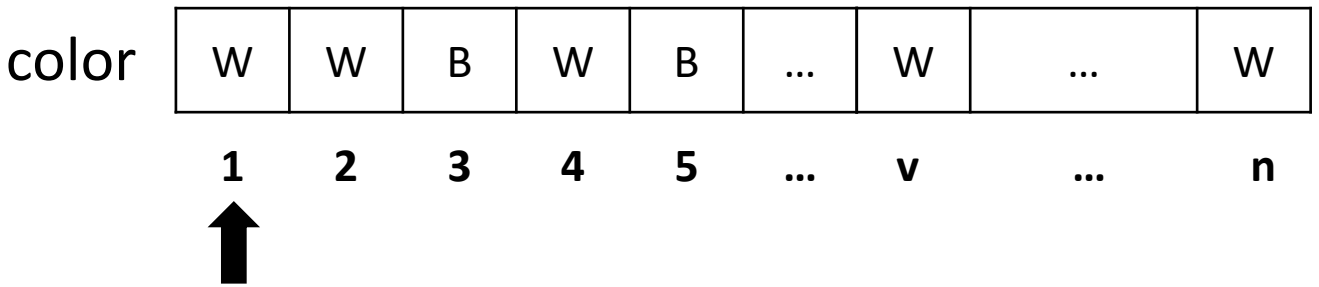

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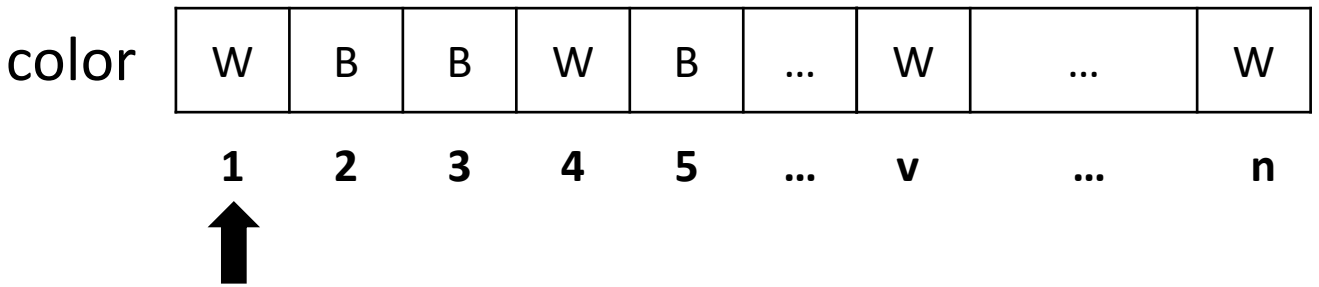

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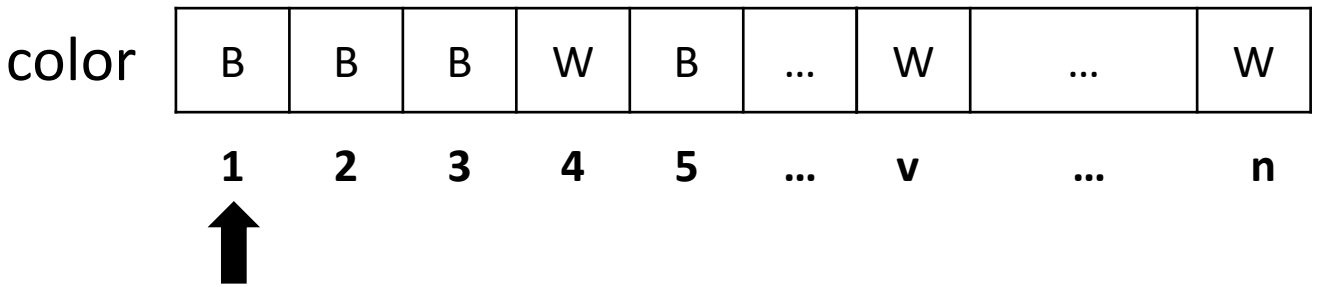

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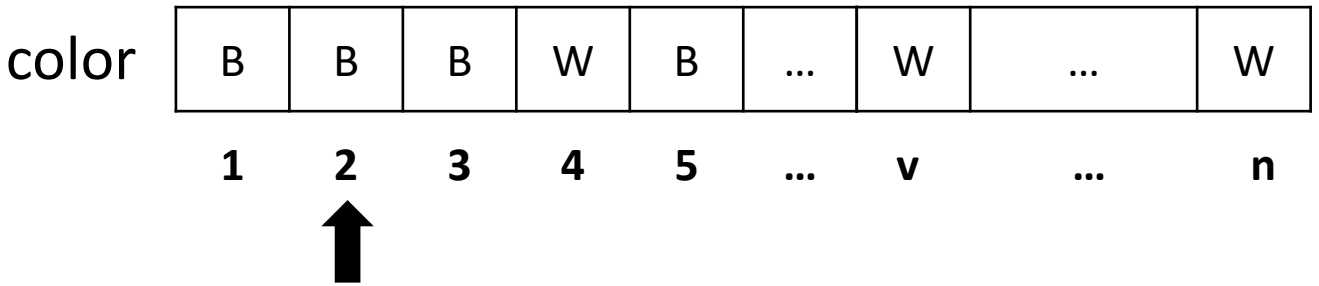

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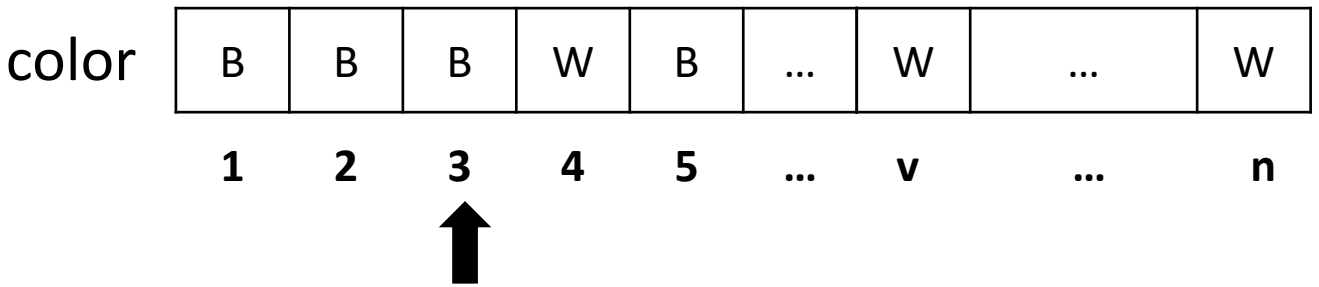

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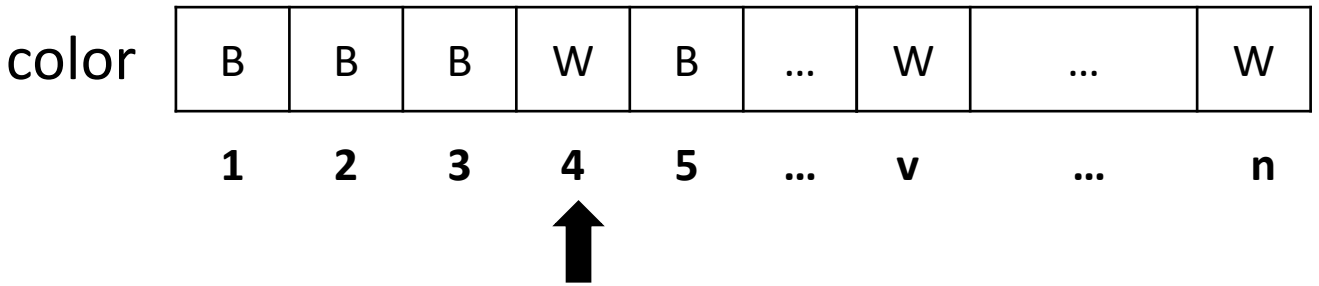

DFS-Explore(G, u**)** /* Explore node u */

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    For each edge (u, v) ∈ E do            /*   Explore edge (u,v)           */
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            DFS-Explore(G, v)              /*   Explore v                    */
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    color[u]  ←   black                    /* u's exploration is over          */
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End DFS-Explore
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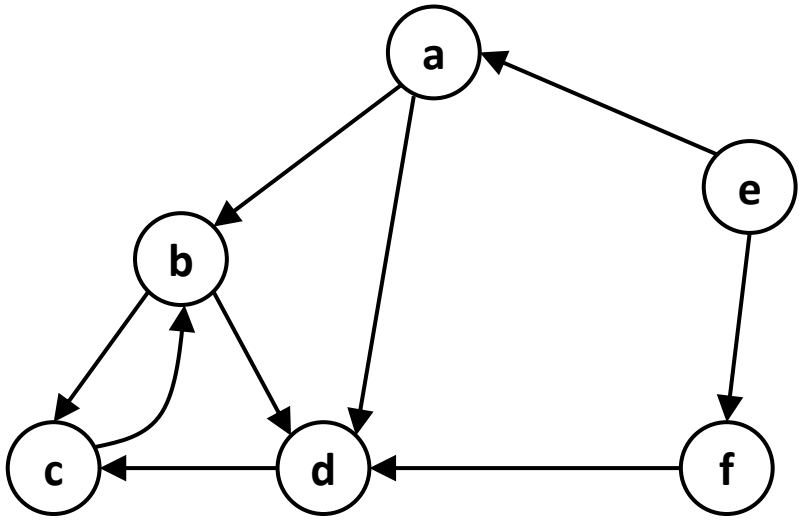
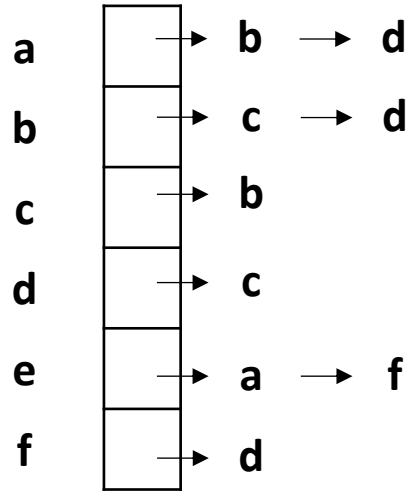
DFS(G)



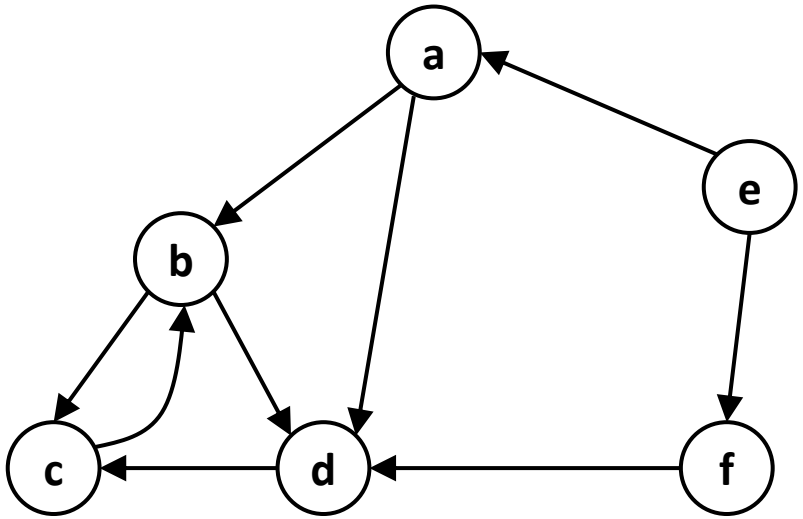
Adj List of G :



Colors of nodes:

a	b	c	d	e	f
W	W	W	W	W	W

DFS(G)



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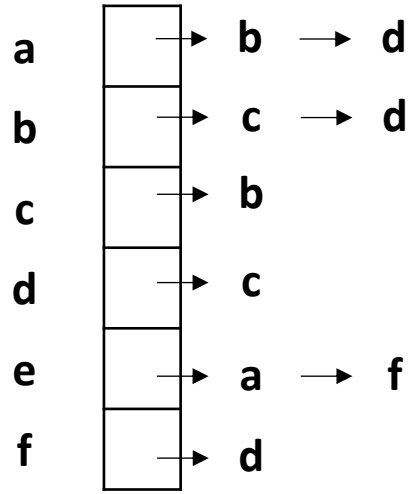

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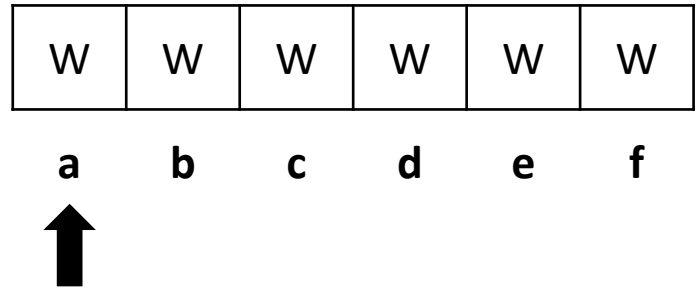
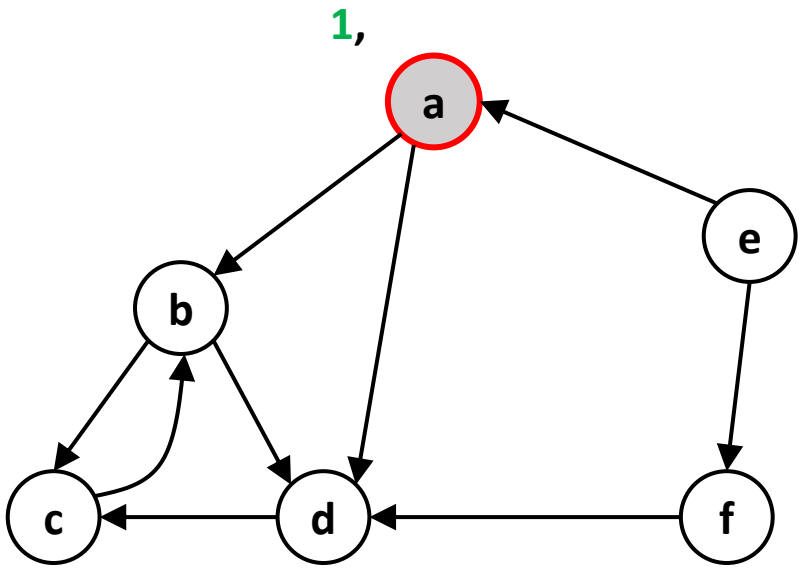


DFS-Explore(G, **a)**

DFS(G)



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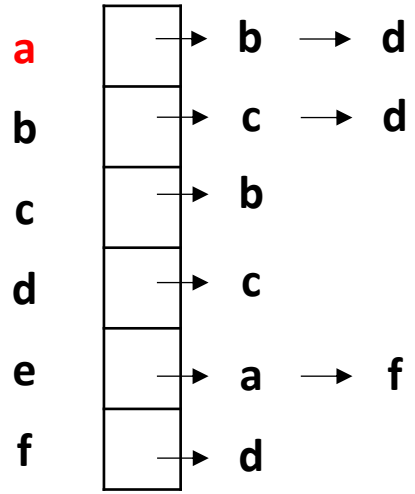

Colors of nodes:

G	W	W	W	W	W
a	b	c	d	e	f

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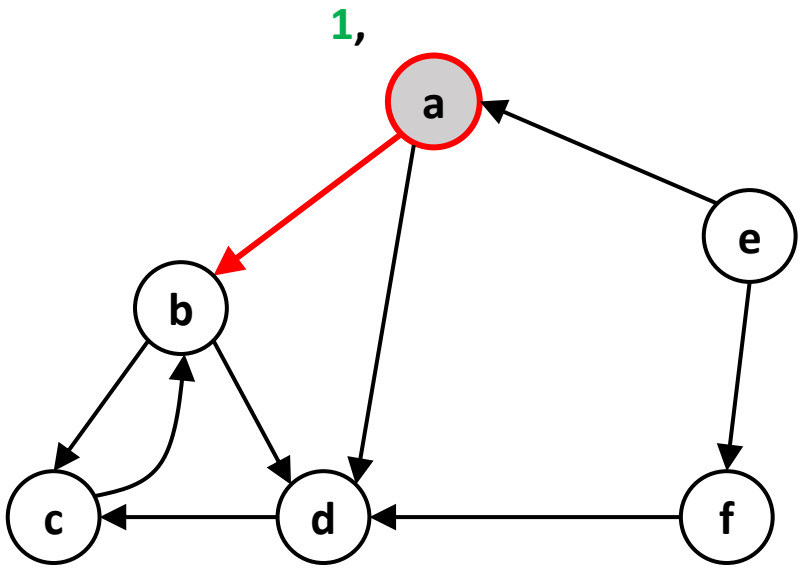
DFS(G)



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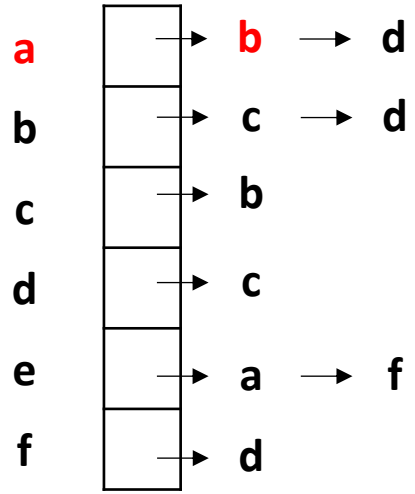

Colors of nodes:

a	b	c	d	e	f
G	W	W	W	W	W

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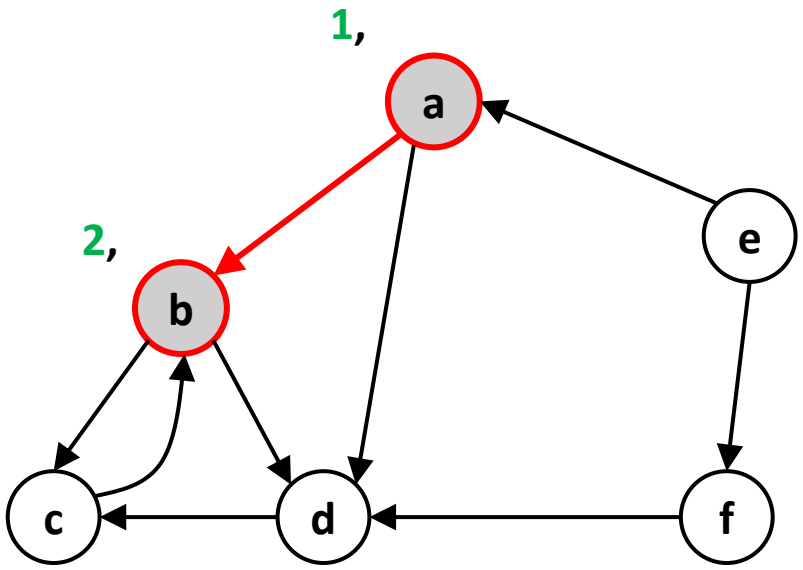

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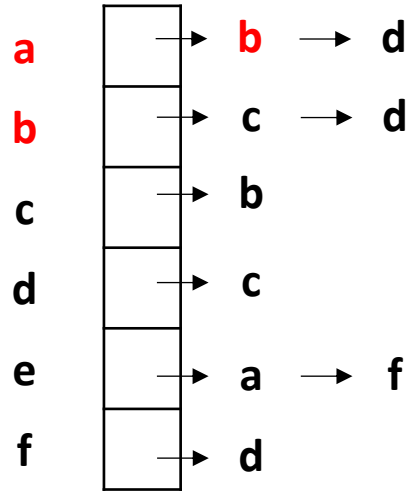

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a	b	c	d	e	f

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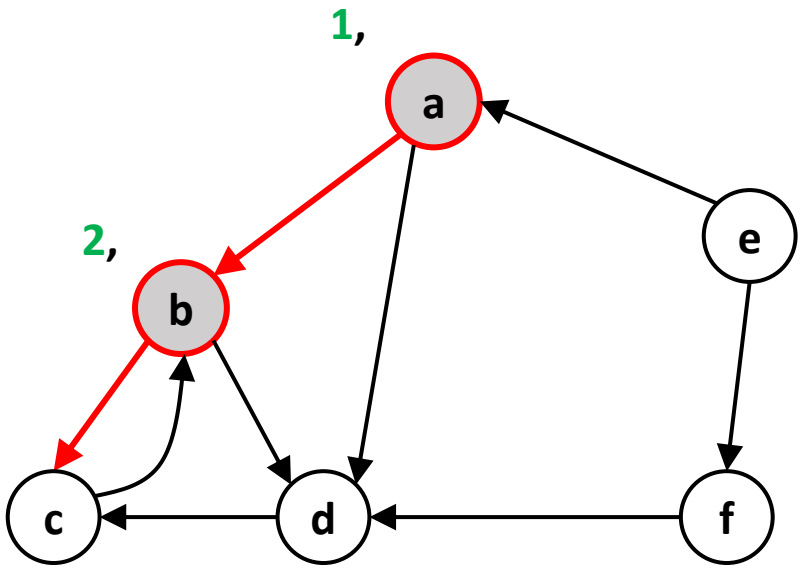

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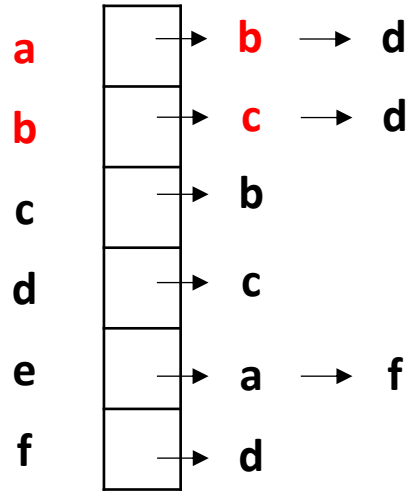

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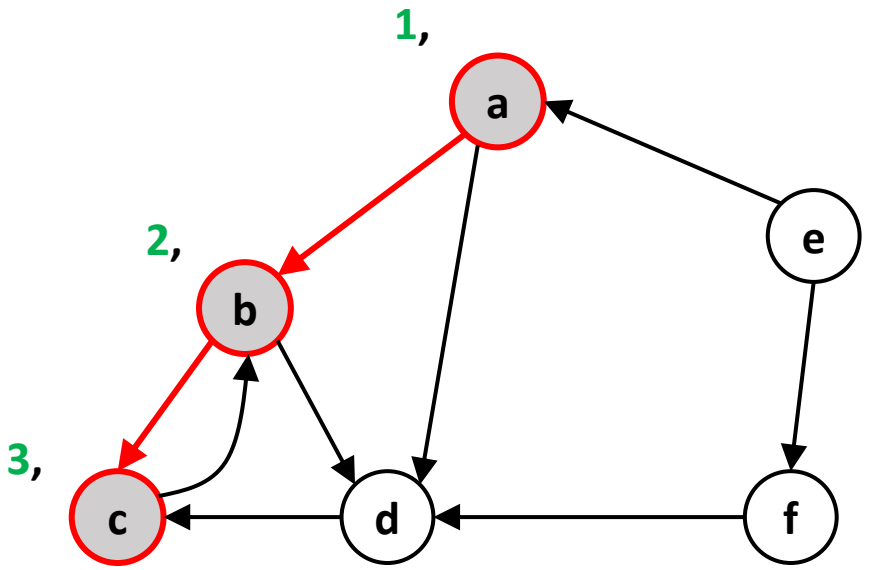
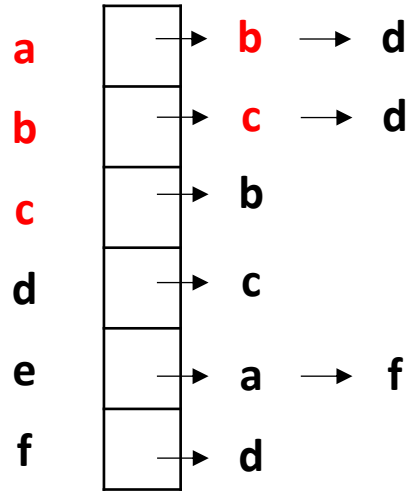
DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

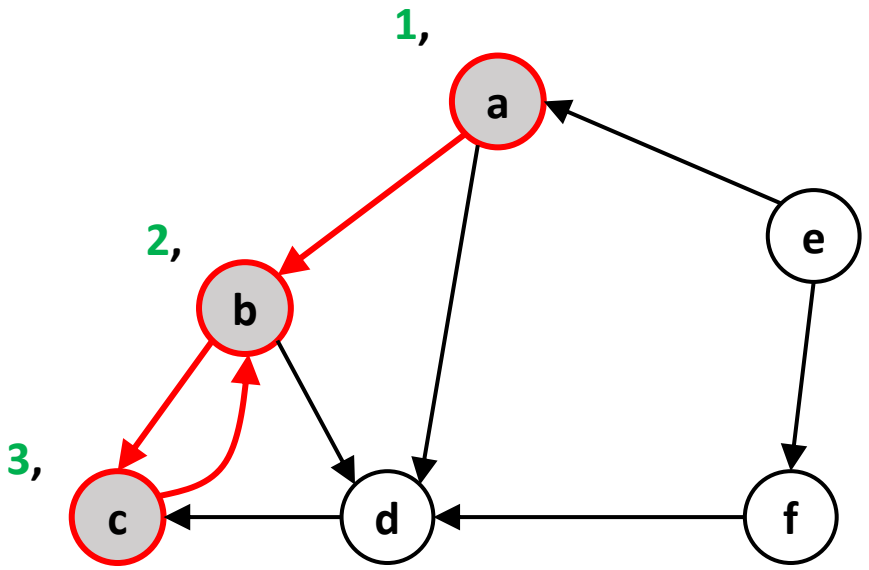


Colors of nodes:

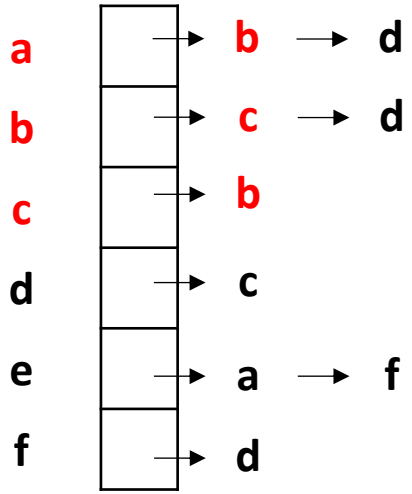
G	G	G	W	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

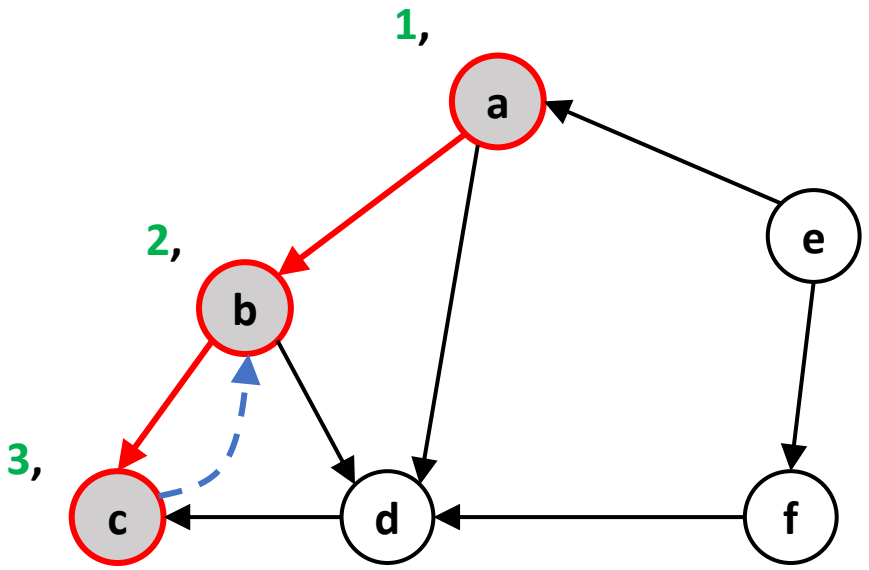


Colors of nodes:

G	G	G	W	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



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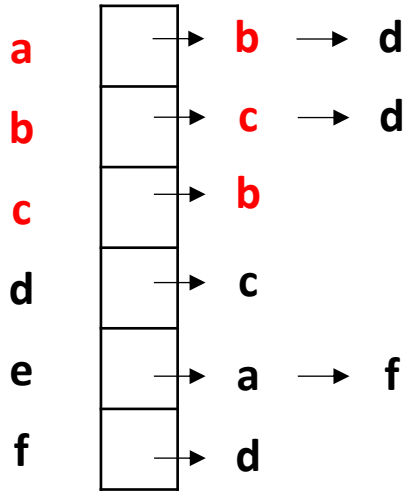
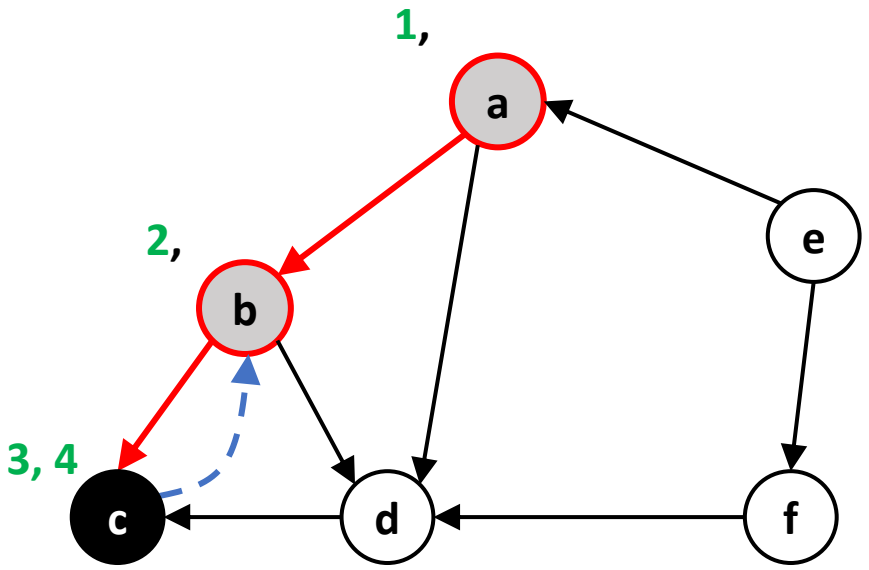


Colors of nodes:

G	G	G	W	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



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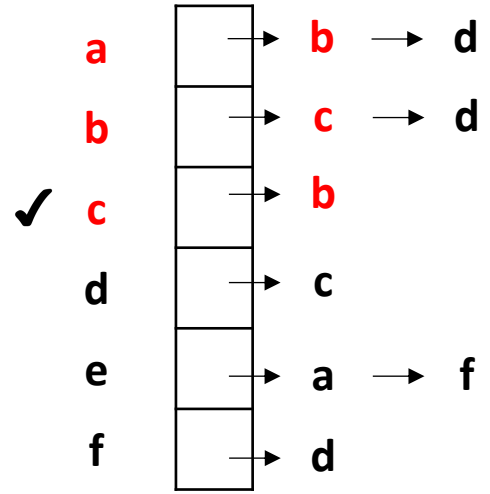
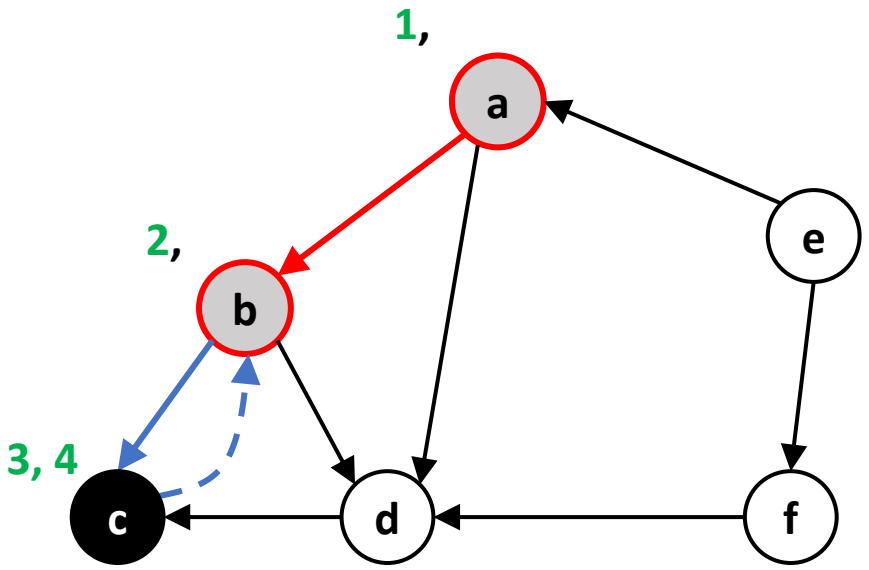


Colors of nodes:

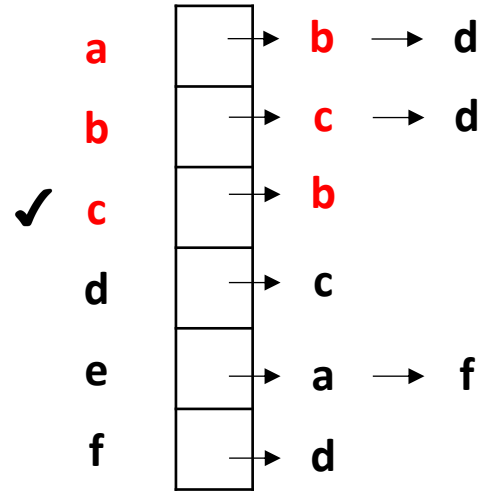
G	G	B	W	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

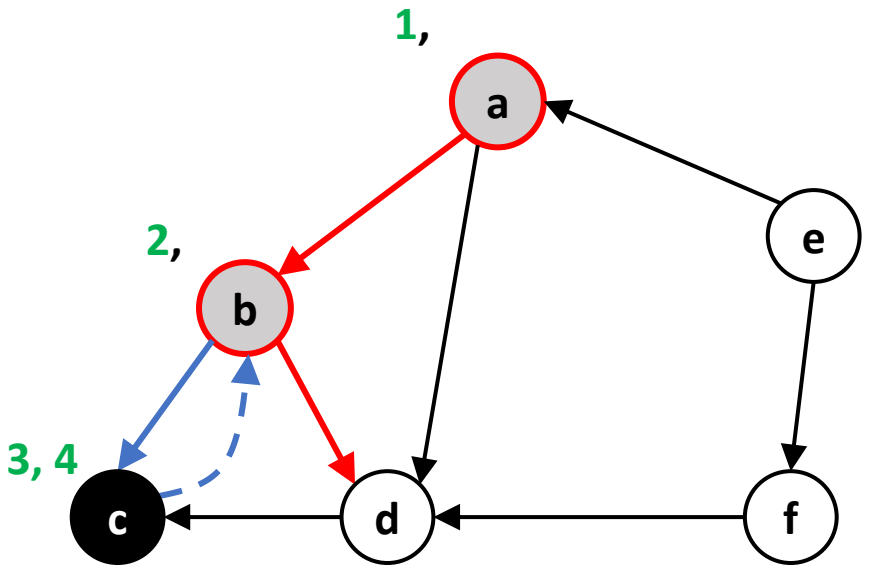


Colors of nodes:

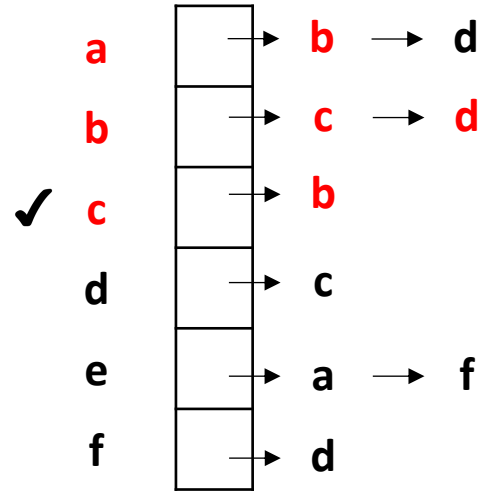
G	G	B	W	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

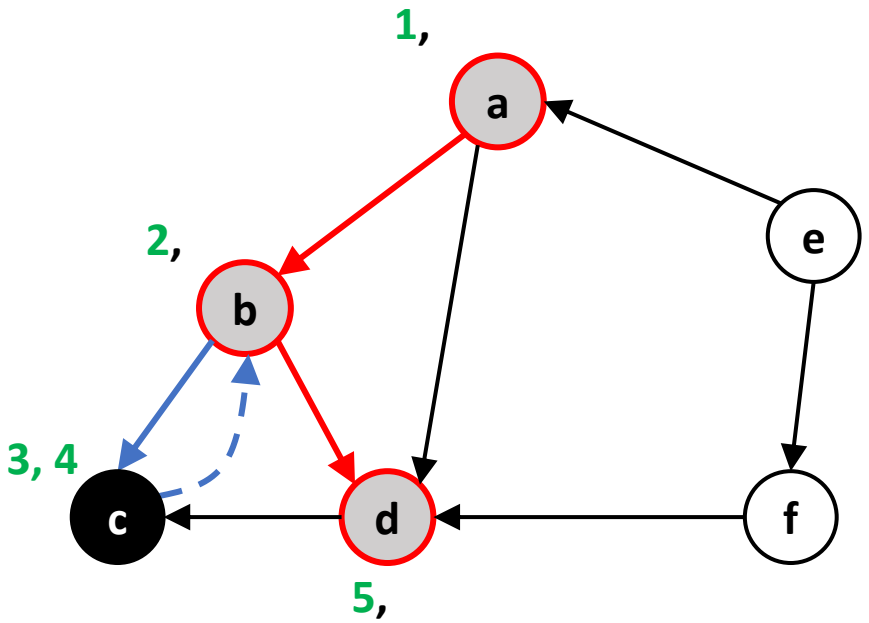


Colors of nodes:

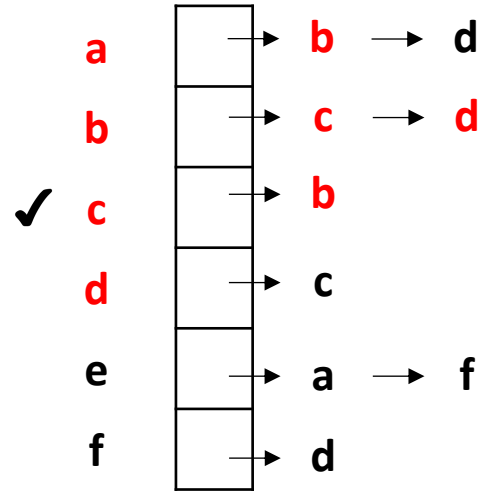
G	G	B	W	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

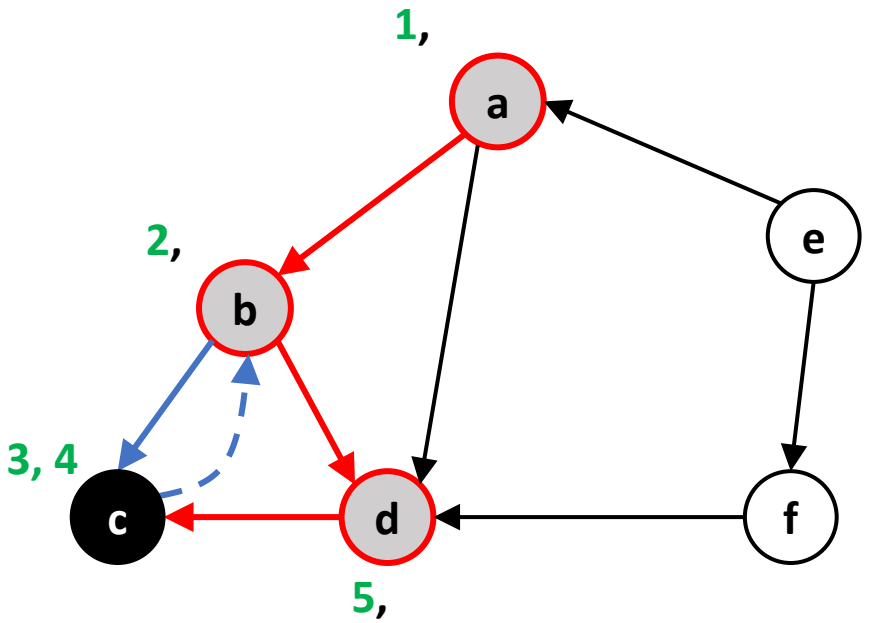


Colors of nodes:

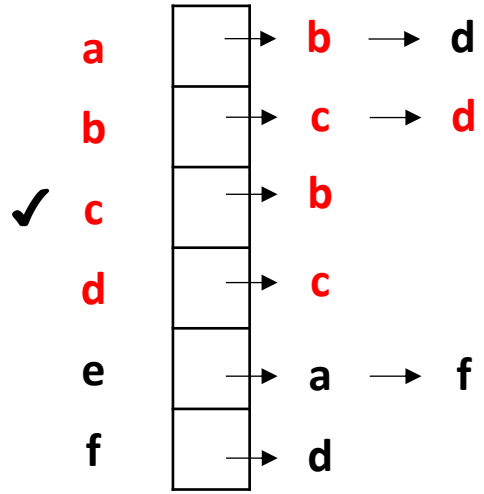
G	G	B	G	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

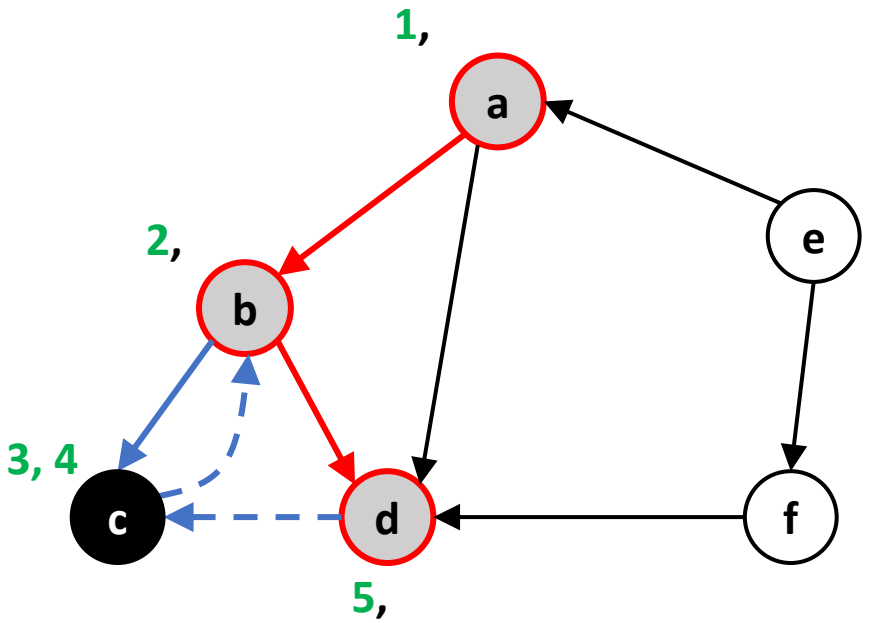


Colors of nodes:

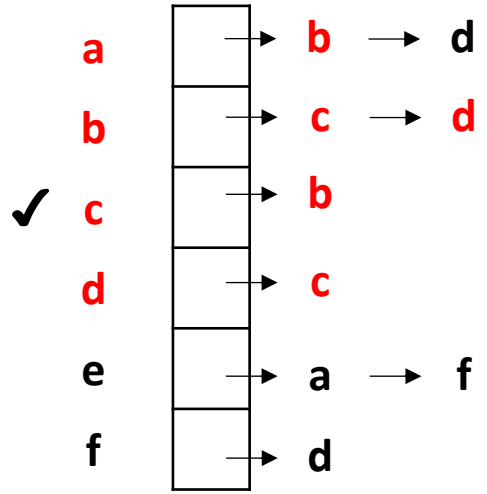
G	G	B	G	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

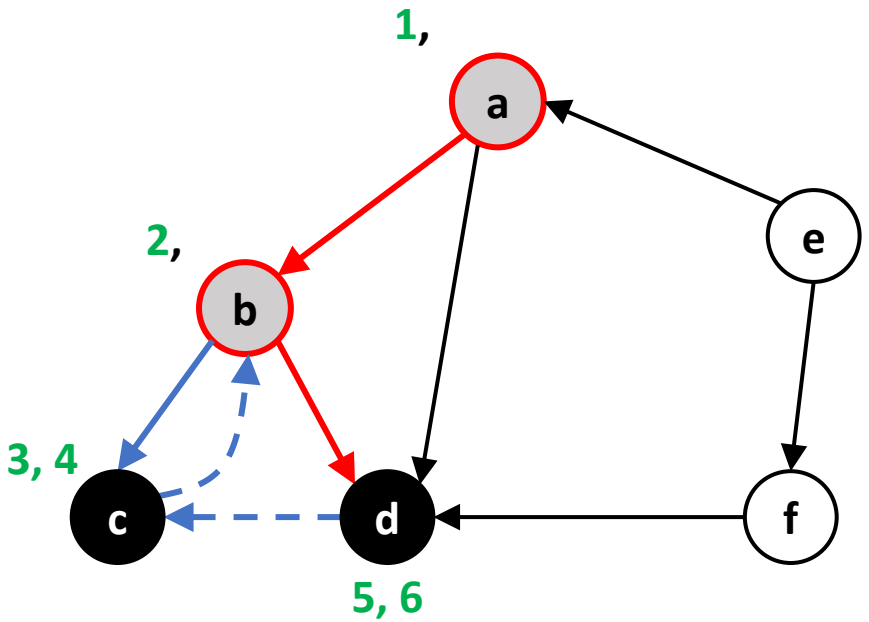


Colors of nodes:

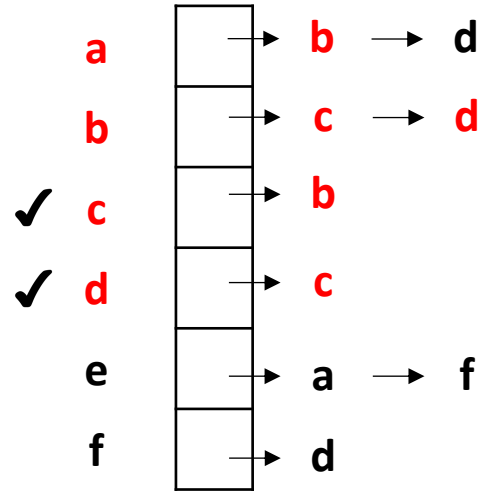
G	G	B	G	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

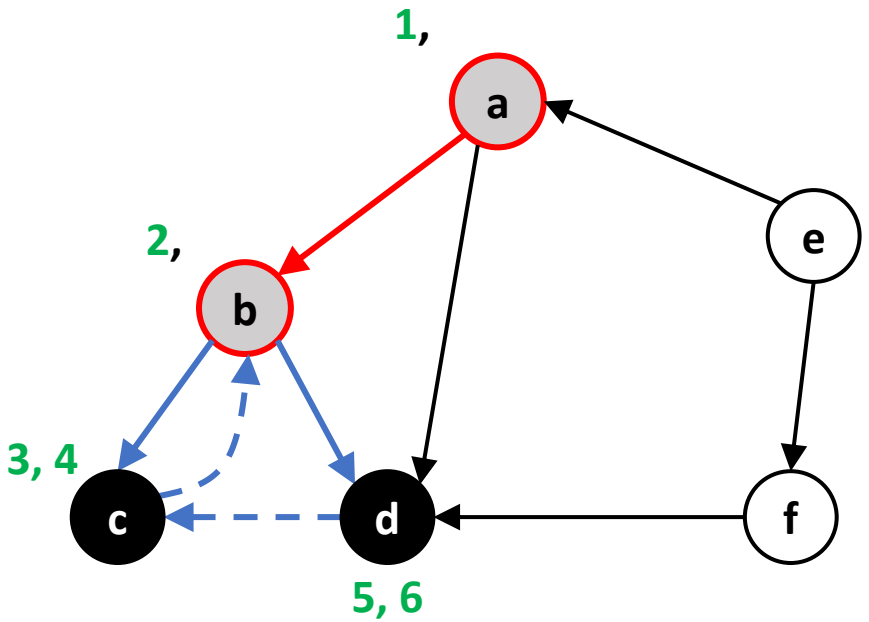


Colors of nodes:

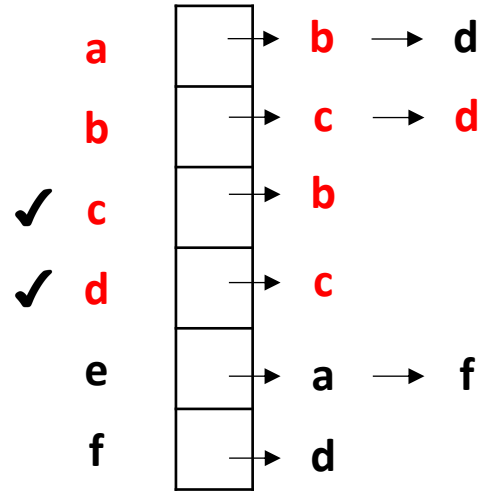
G	G	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

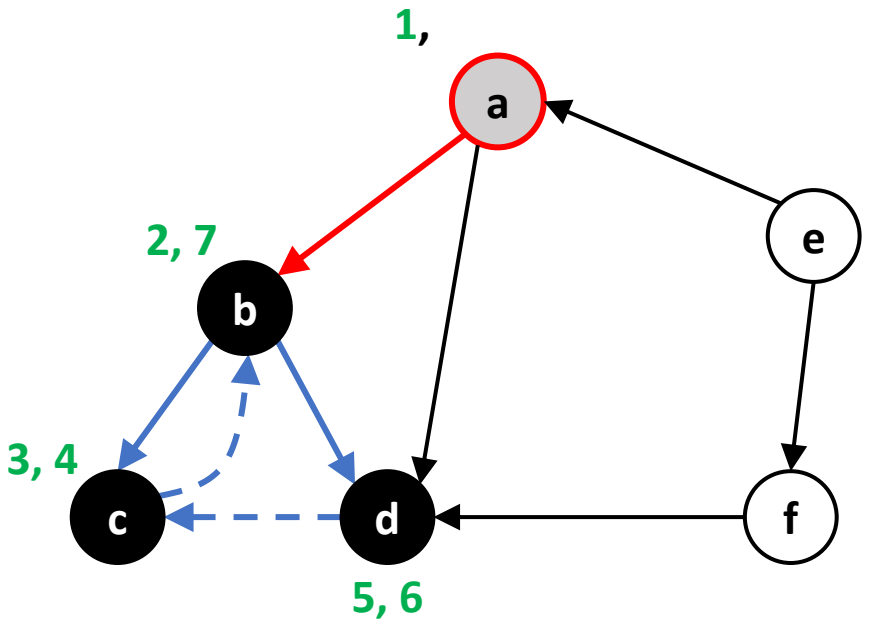


Colors of nodes:

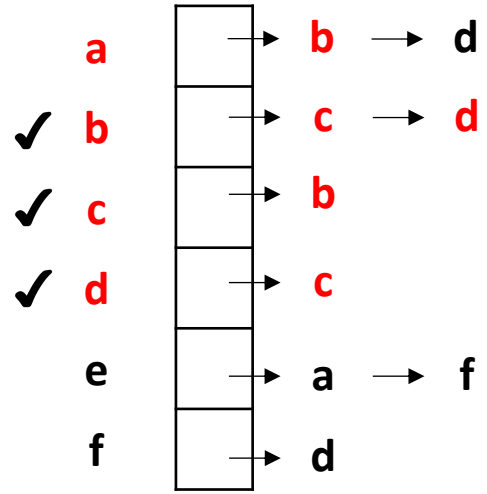
G	G	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

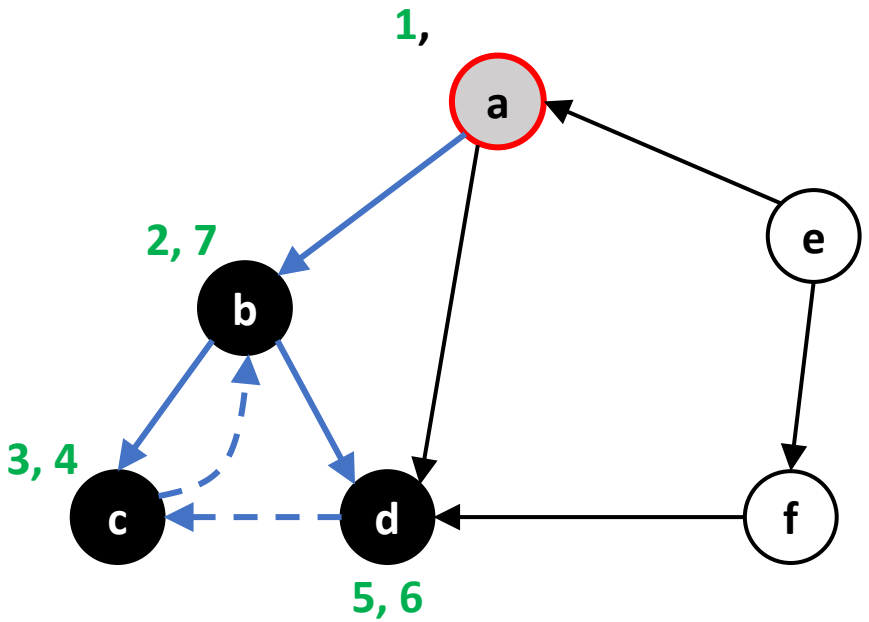


Colors of nodes:

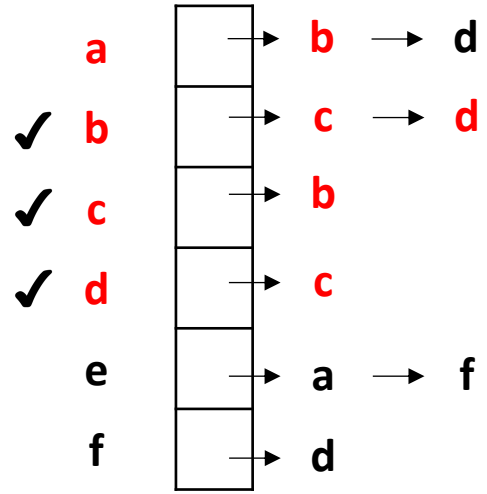
G	B	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

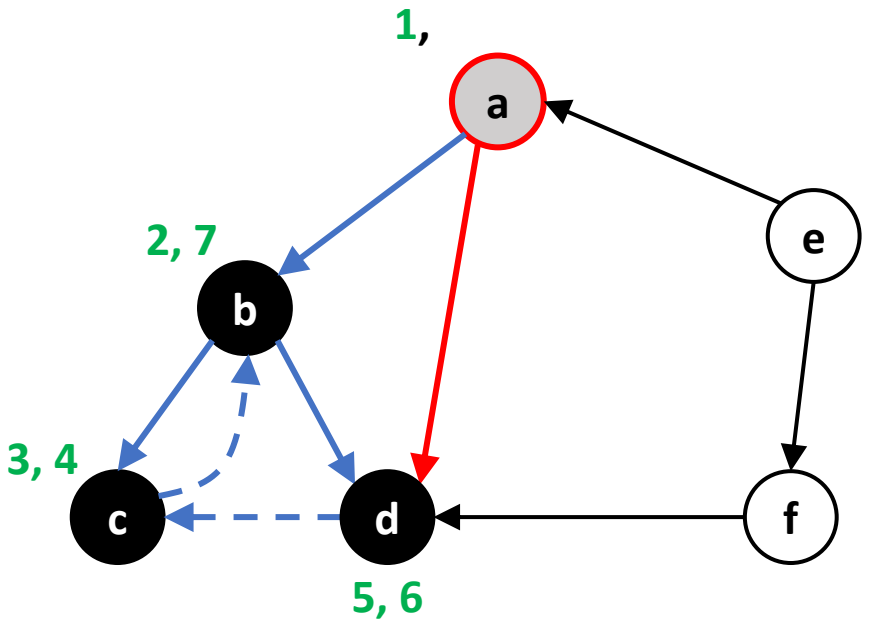


Colors of nodes:

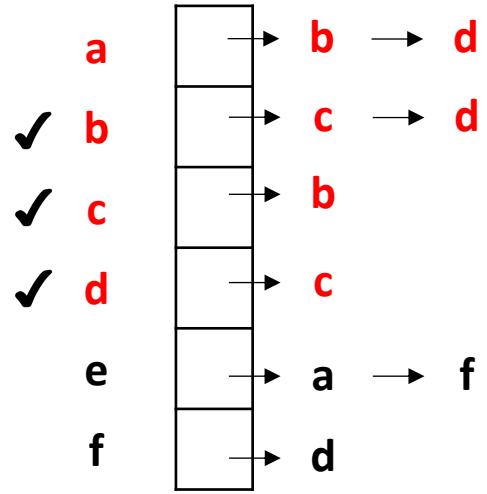
G	B	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

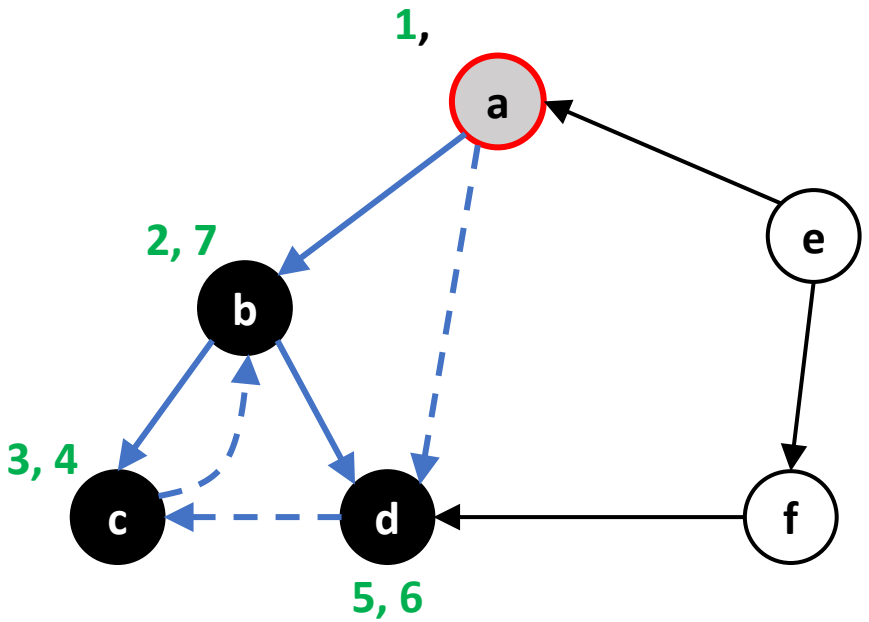


Colors of nodes:

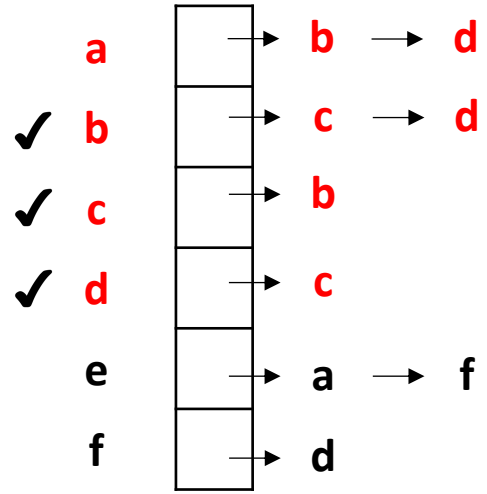
G	B	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

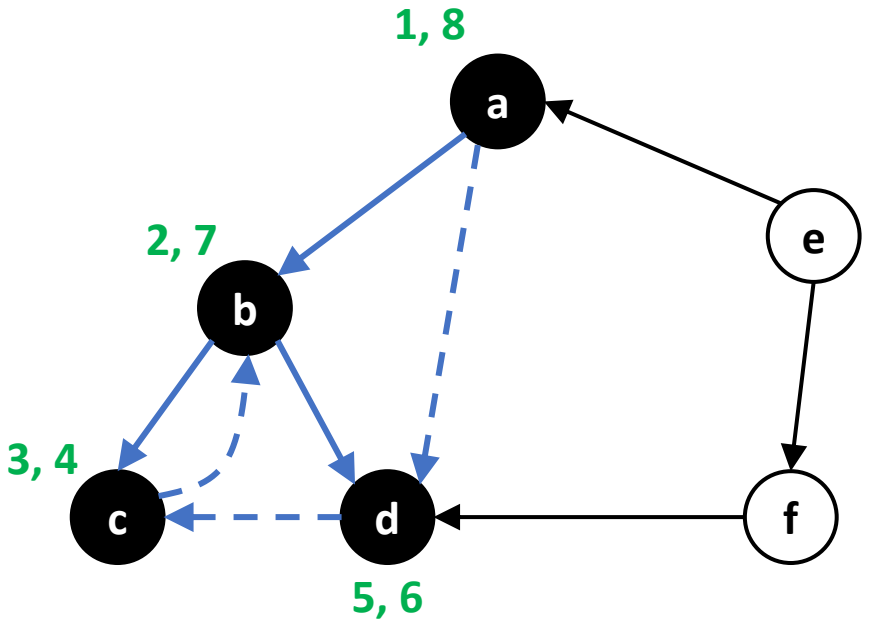


Colors of nodes:

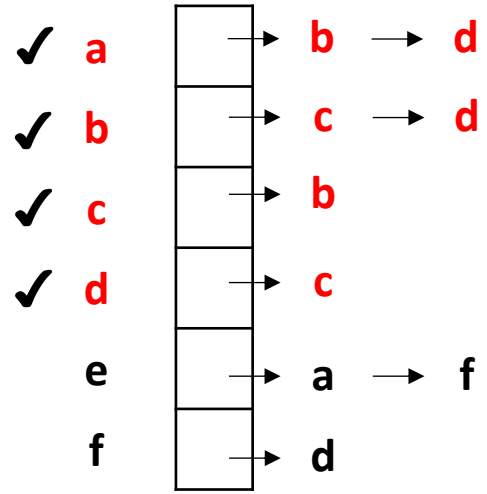
G	B	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **a)**

DFS(G)



Adj List of G :

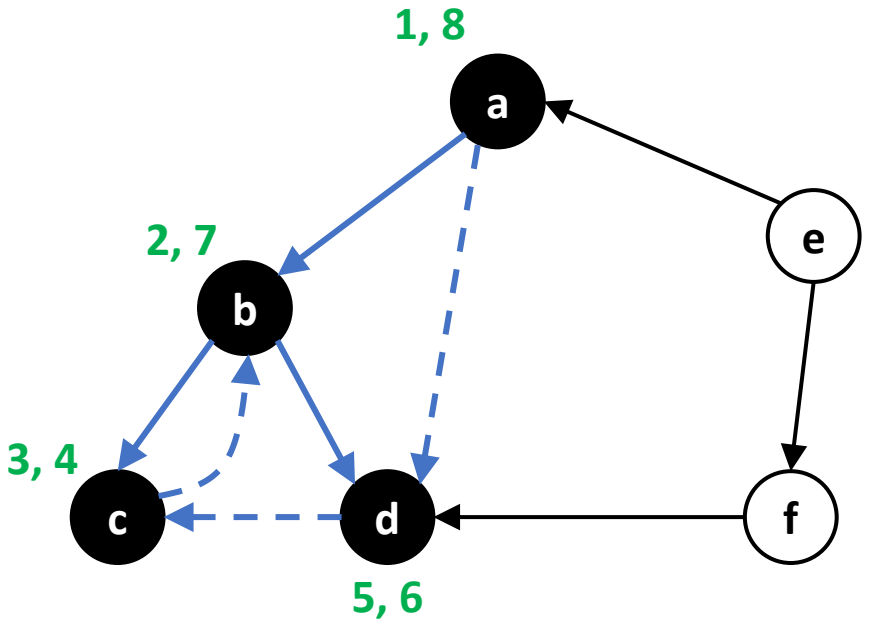


Colors of nodes:

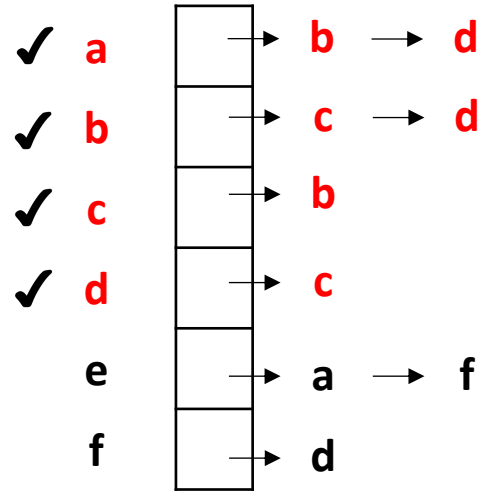
a	b	c	d	e	f
B	B	B	B	W	W

DFS-Explore(G, **a)**

DFS(G)



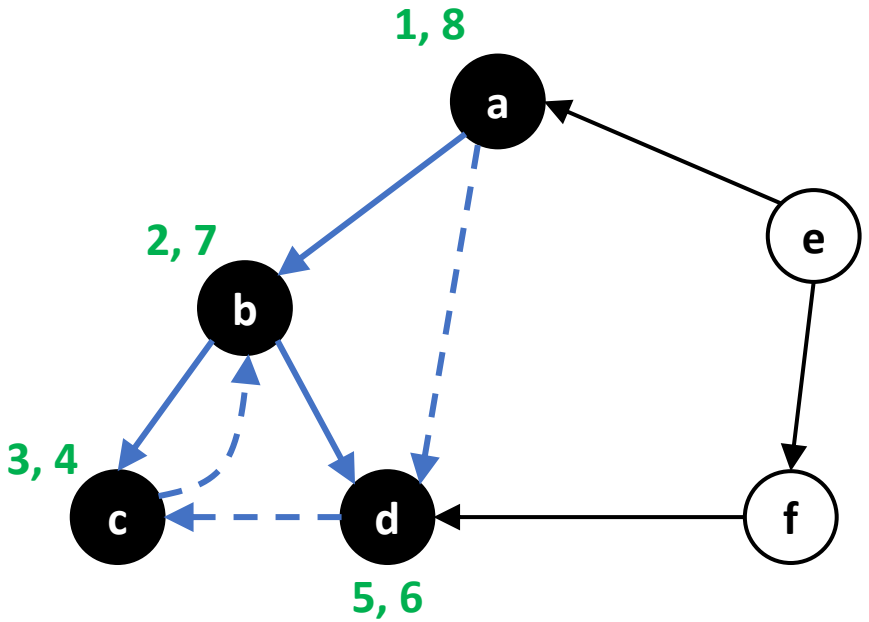
Adj List of G :



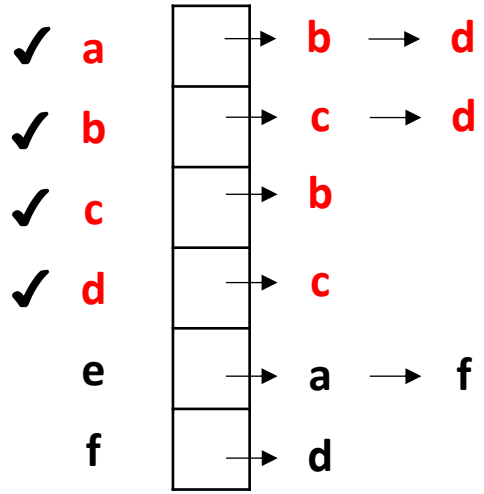
Colors of nodes:

a	b	c	d	e	f
B	B	B	B	W	W

DFS(G)



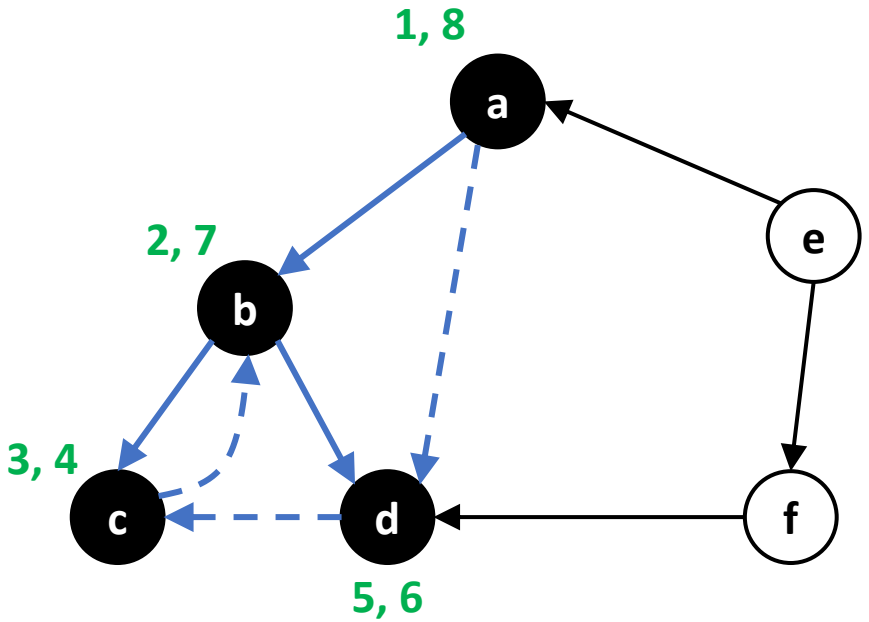
Adj List of G :



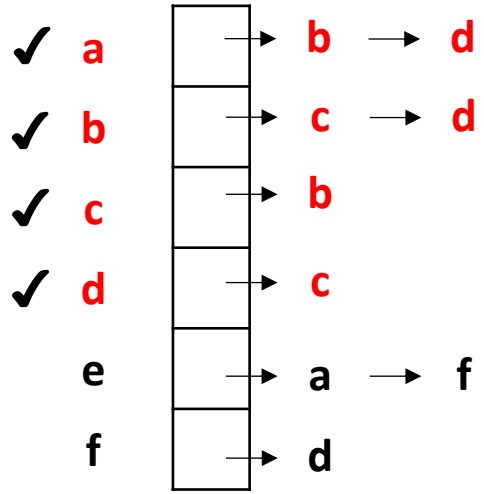
Colors of nodes:

a	b	c	d	e	f
B	B	B	B	W	W

DFS(G)



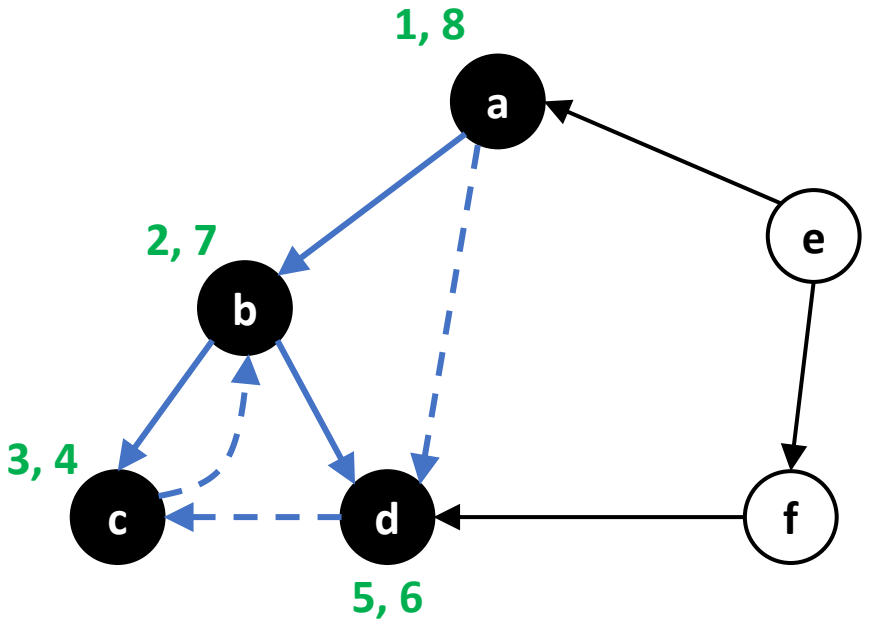
Adj List of G :



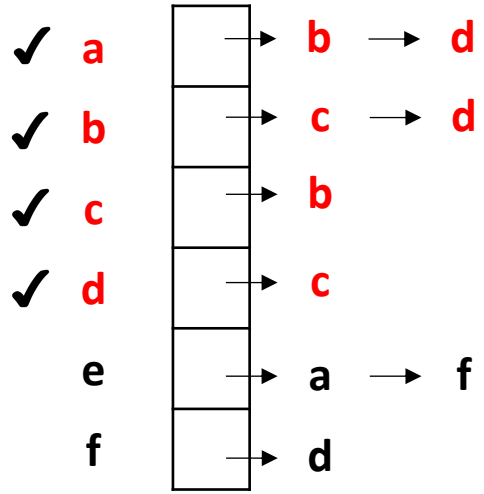
Colors of nodes:

B	B	B	B	W	W
a	b	c	d	e	f

DFS(G)



Adj List of G :

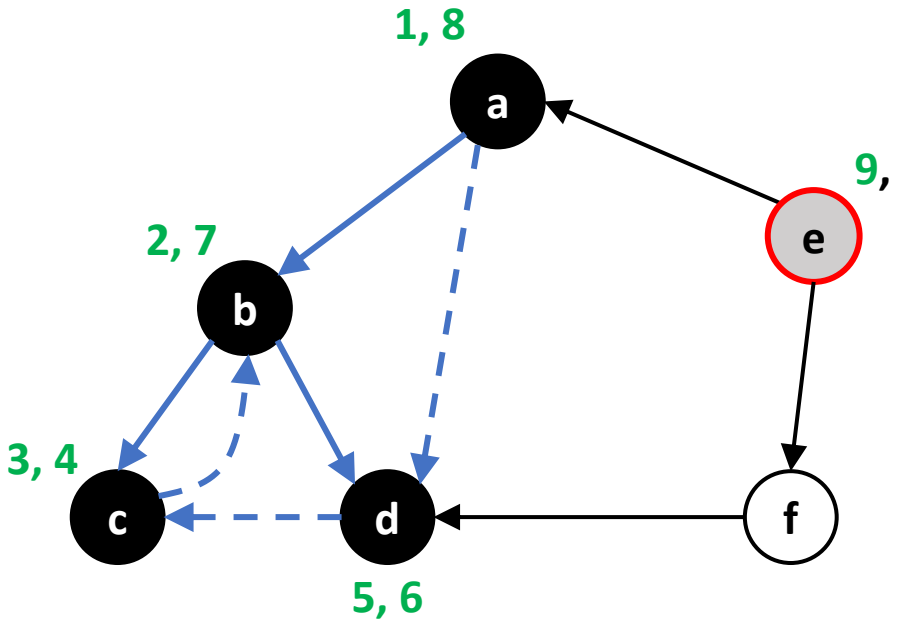


Colors of nodes:

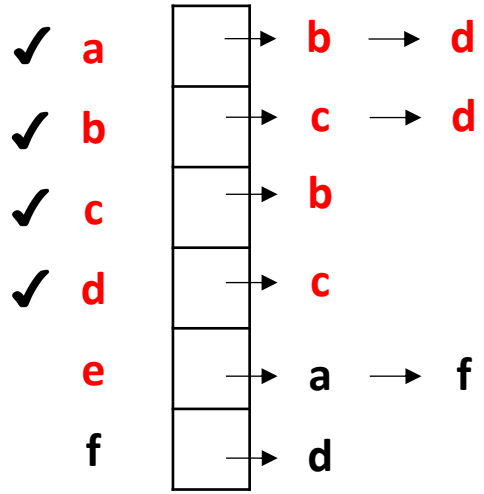
B	B	B	B	W	W
a	b	c	d	e	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

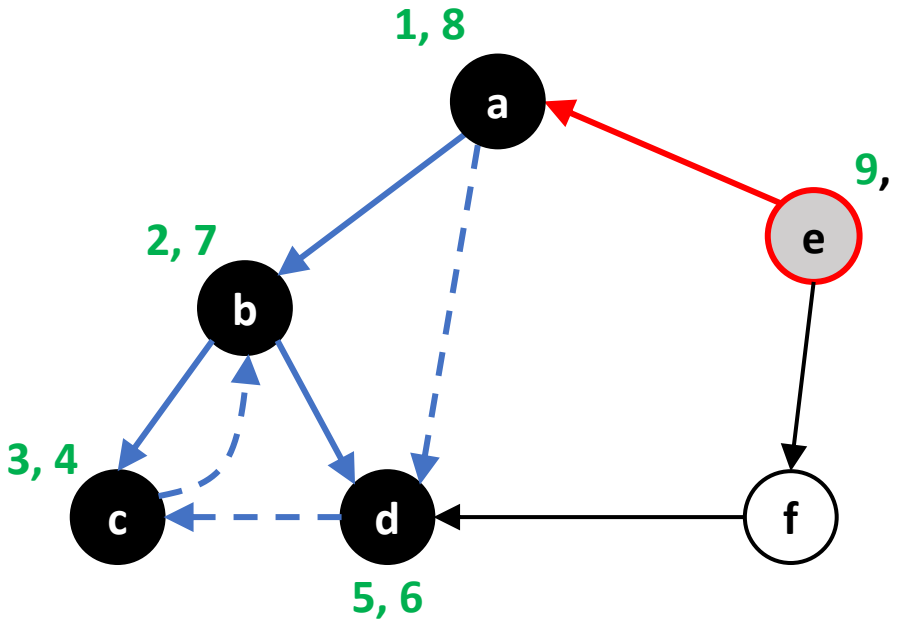


Colors of nodes:

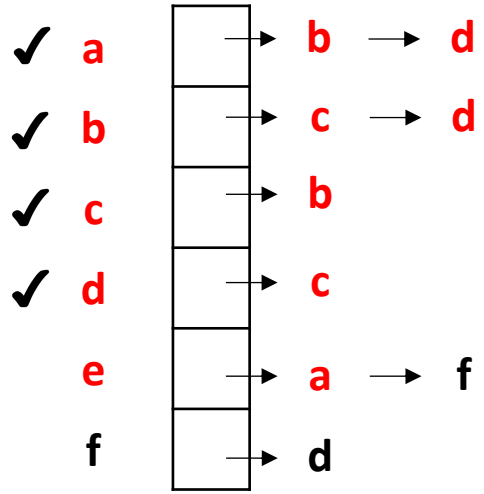
B	B	B	B	G	W
a	b	c	d	e	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

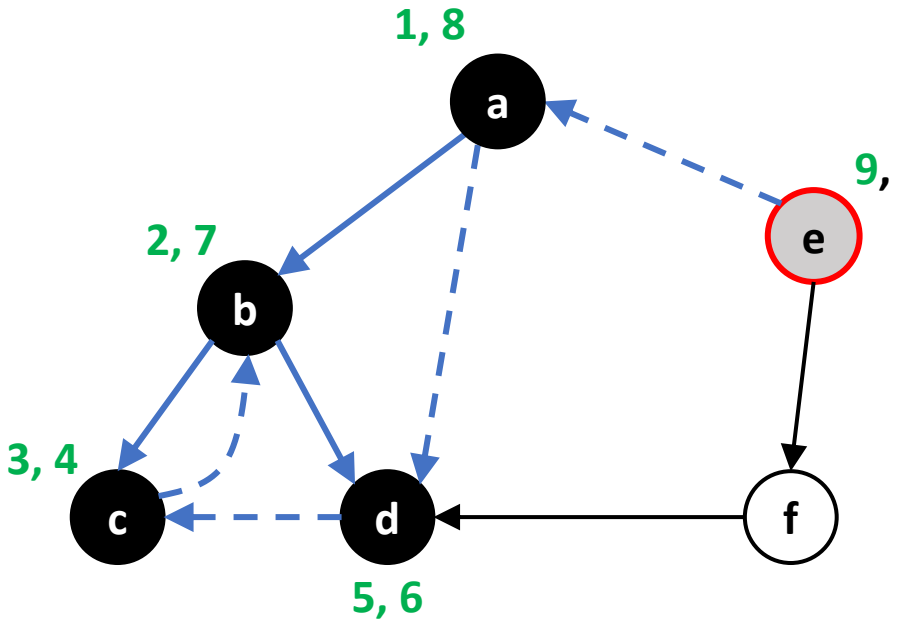


Colors of nodes:

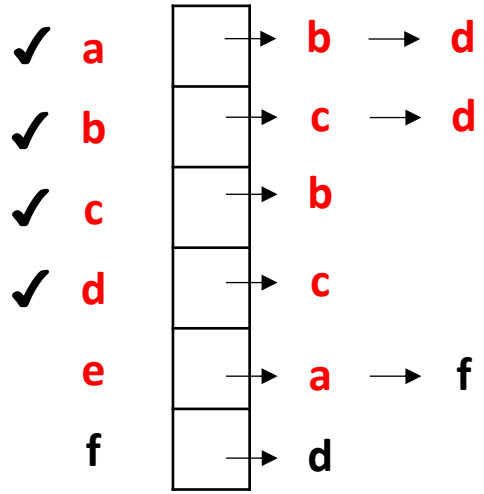
B	B	B	B	G	W
a	b	c	d	e	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

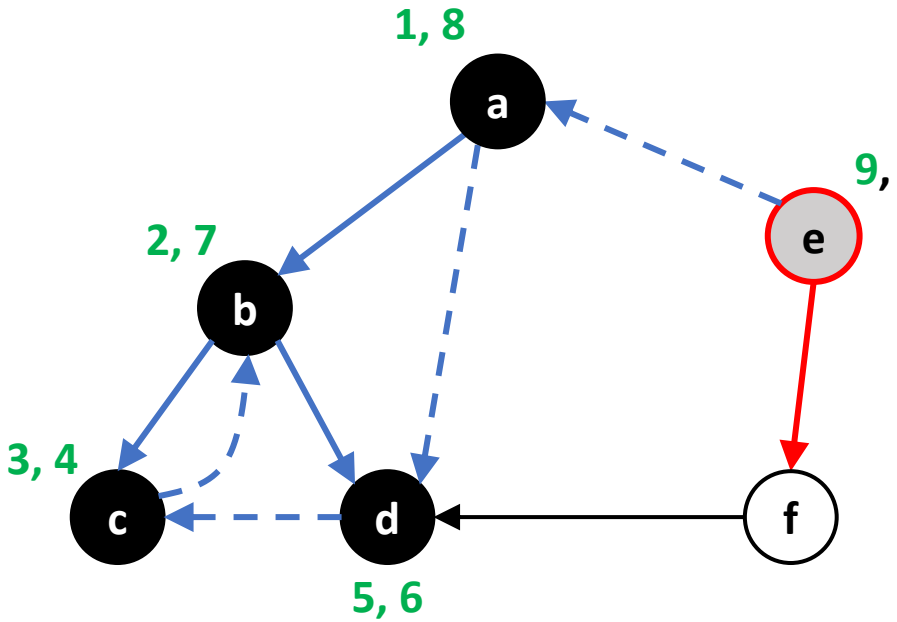


Colors of nodes:

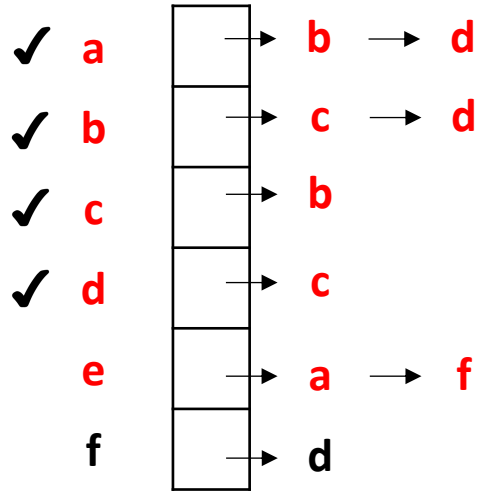
a	b	c	d	e	f
B	B	B	B	G	W

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

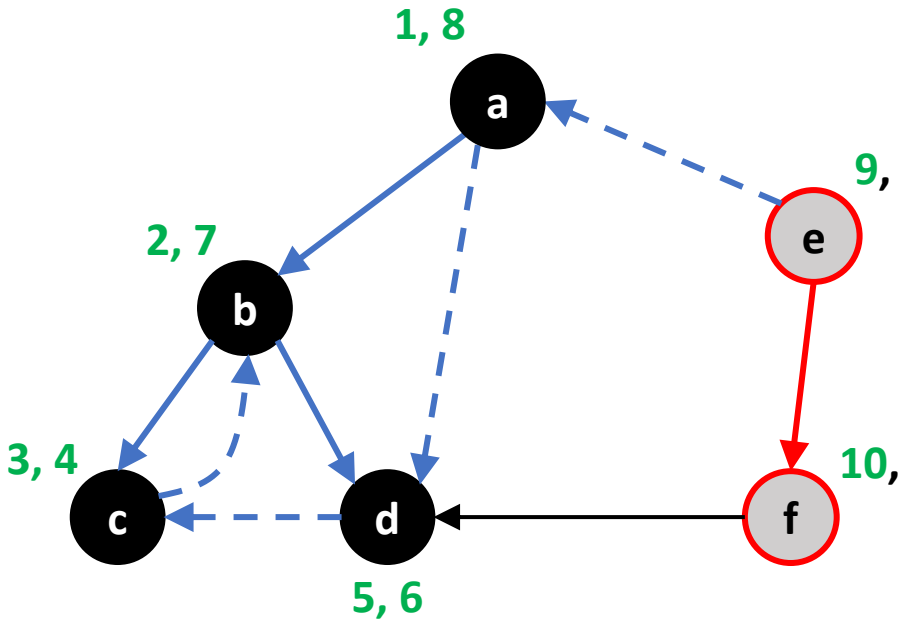


Colors of nodes:

a	b	c	d	e	f
B	B	B	B	G	W

DFS-Explore(G, **e)**

DFS(G)



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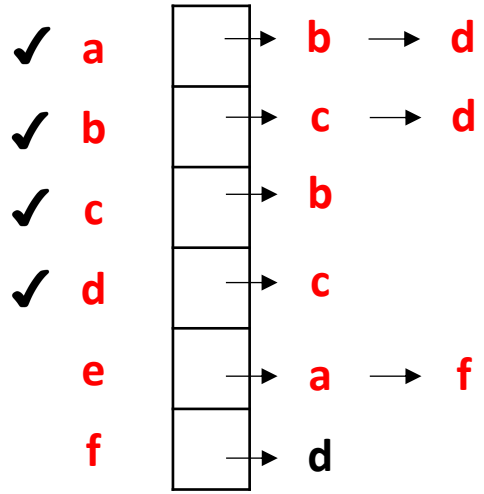

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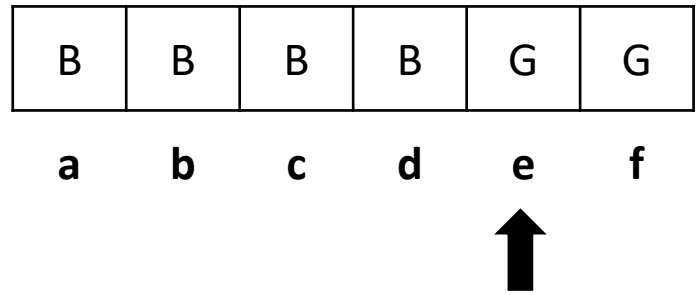
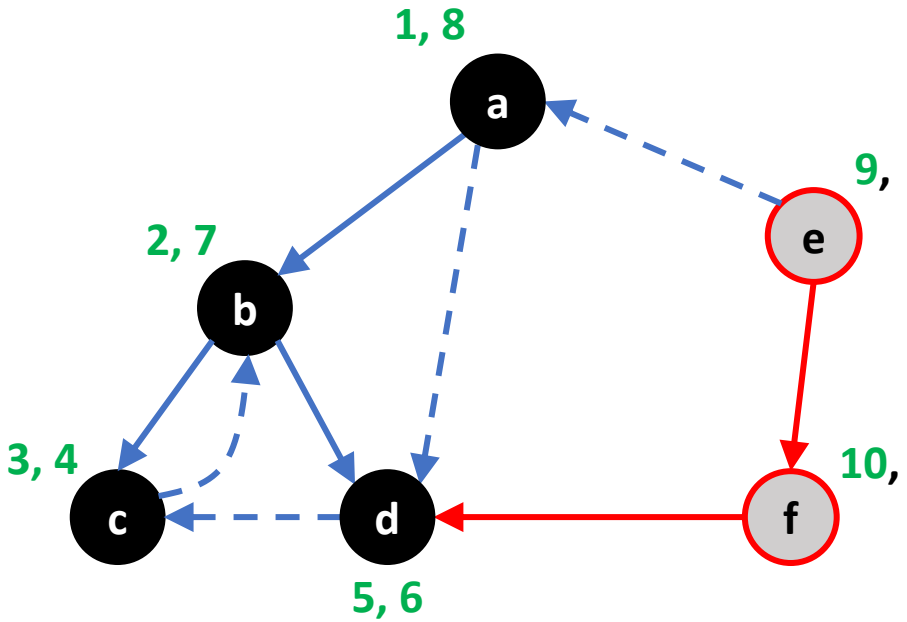


DFS(G)



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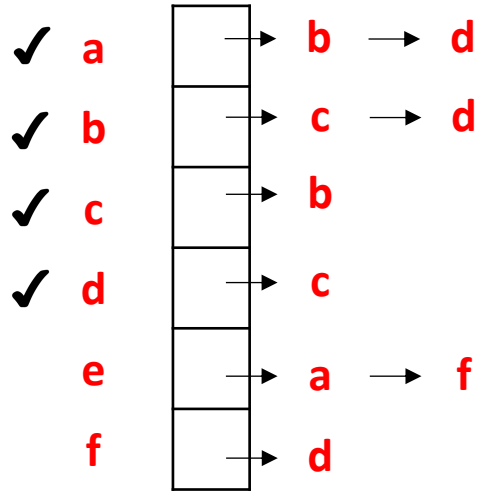
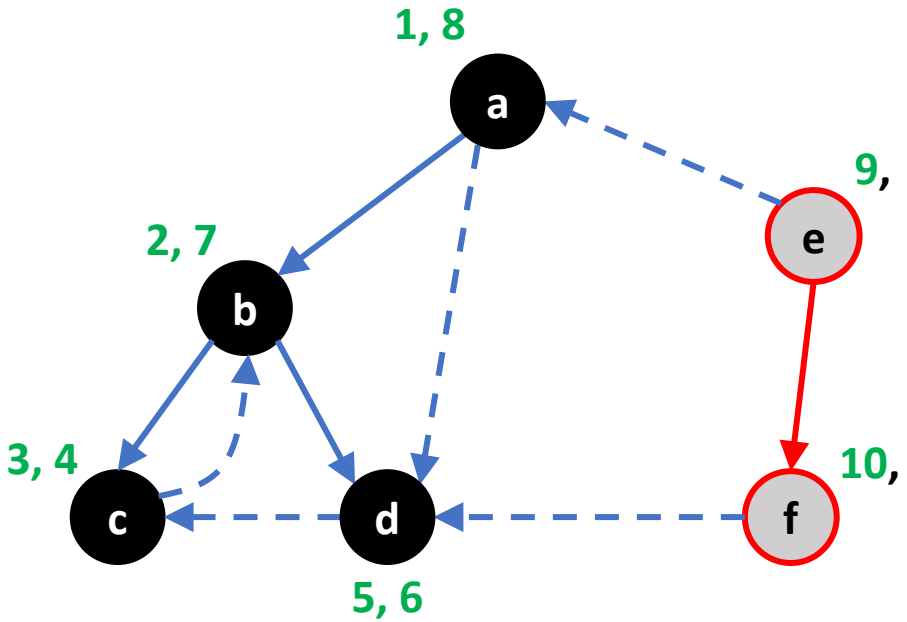


Colors of nodes:

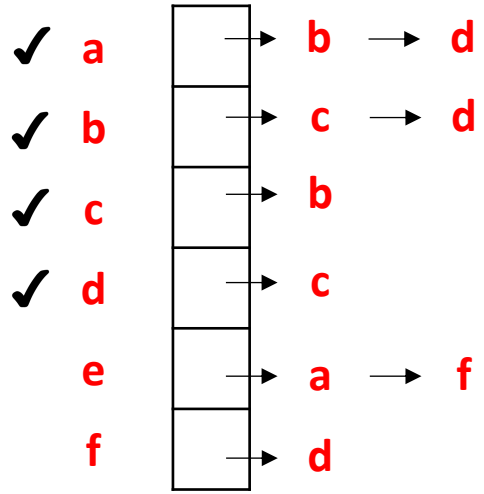
B	B	B	B	G	G
a	b	c	d	e	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

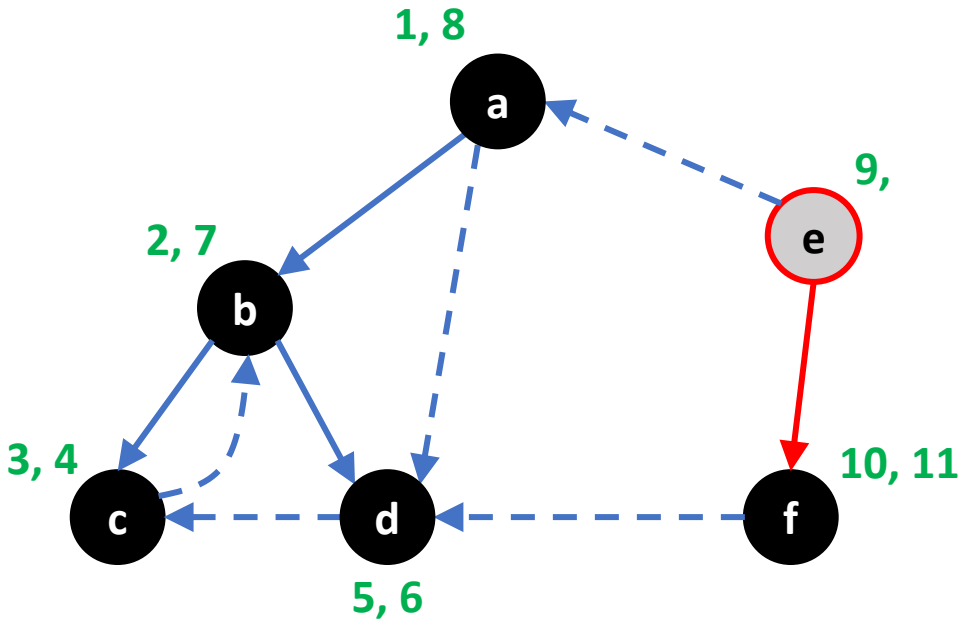


Colors of nodes:

B	B	B	B	G	G
a	b	c	d	e	f

DFS-Explore(G, **e)**

DFS(G)



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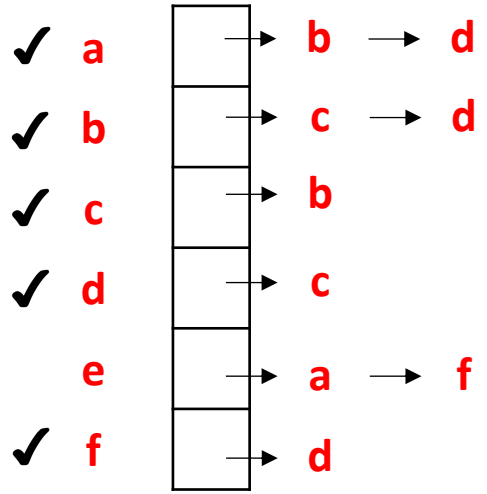
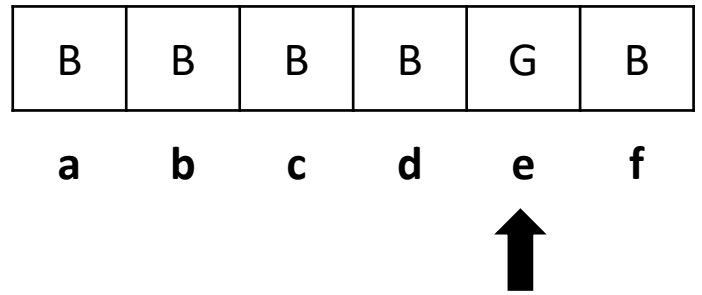
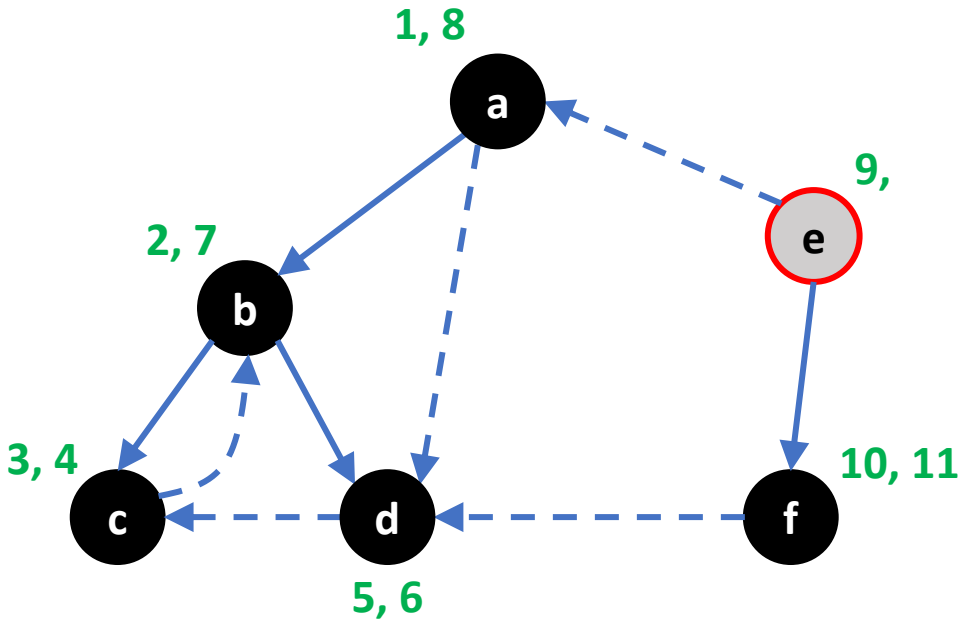


Colors of nodes:



DFS(G)



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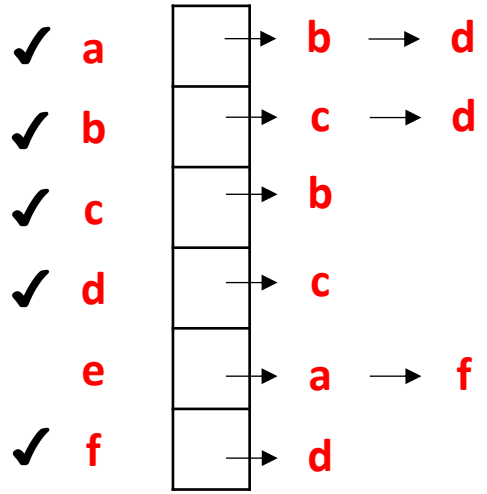
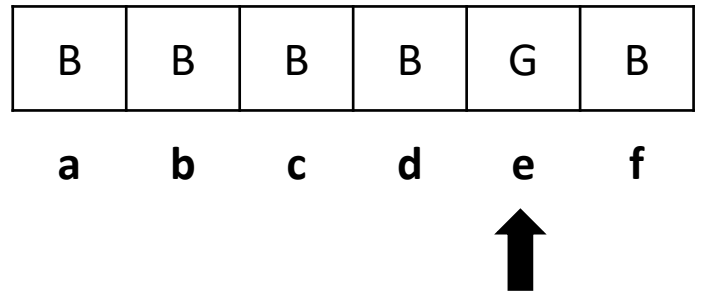
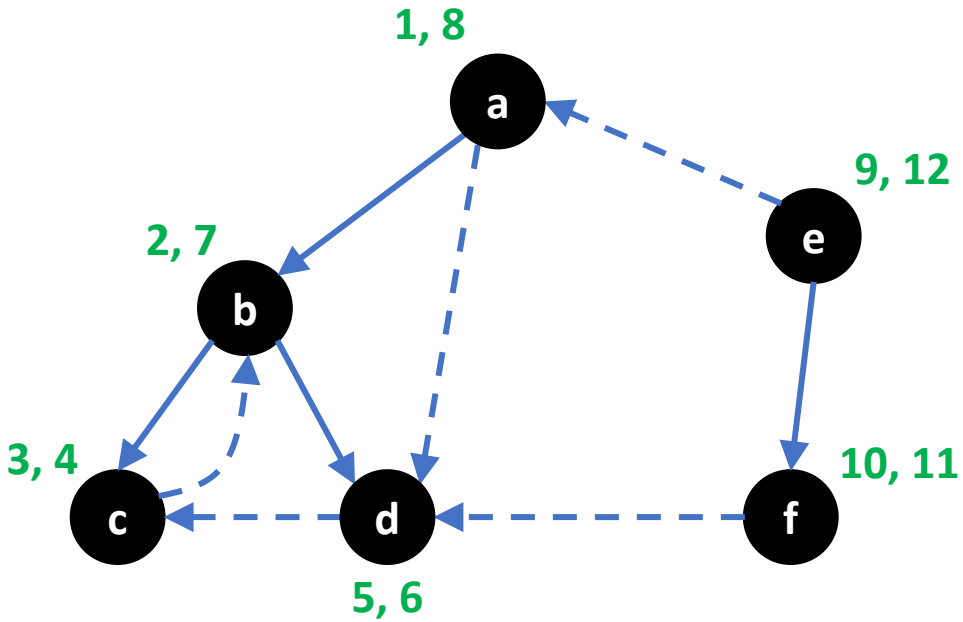


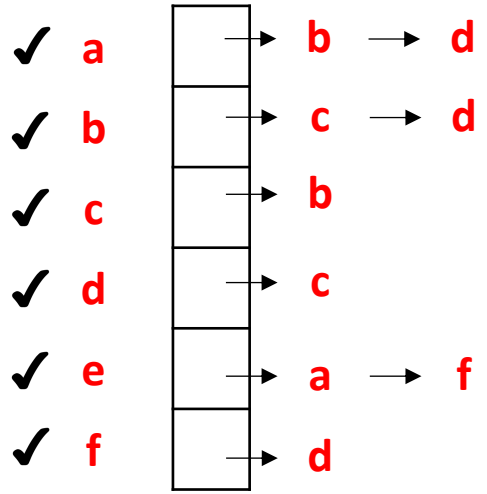
Colors of nodes:



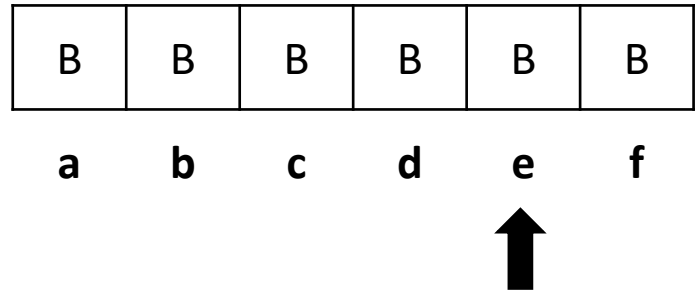
DFS(G)



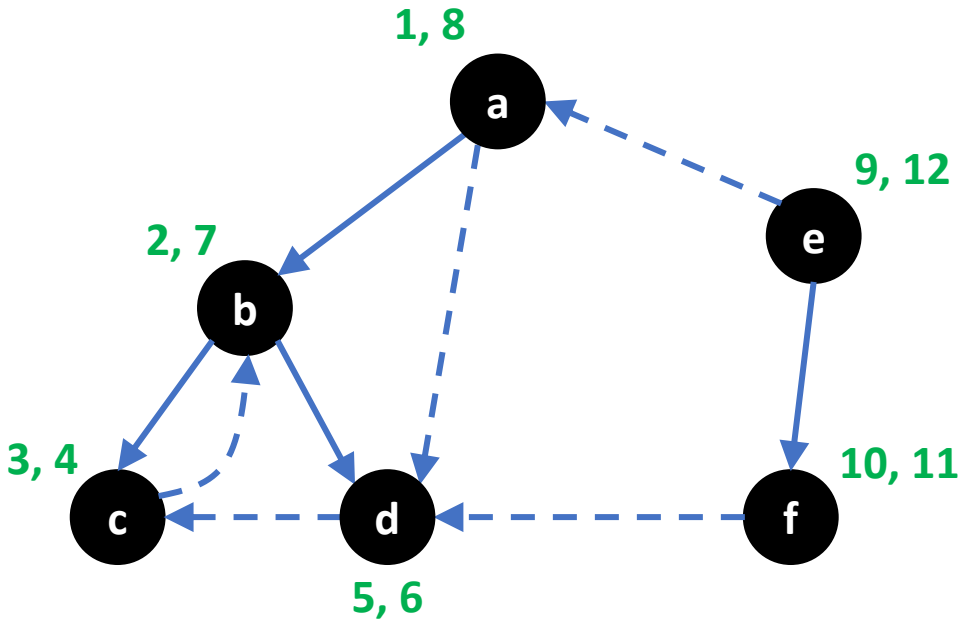
Adj List of G :



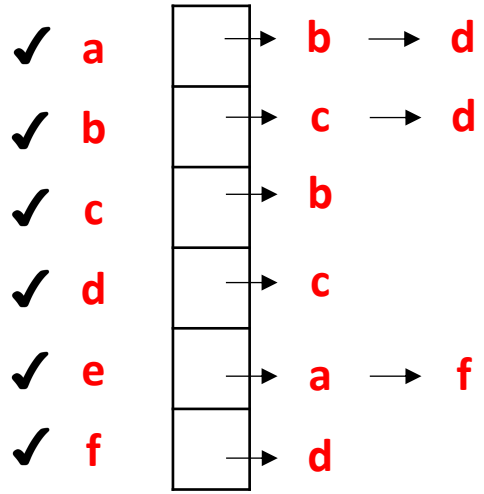
Colors of nodes:



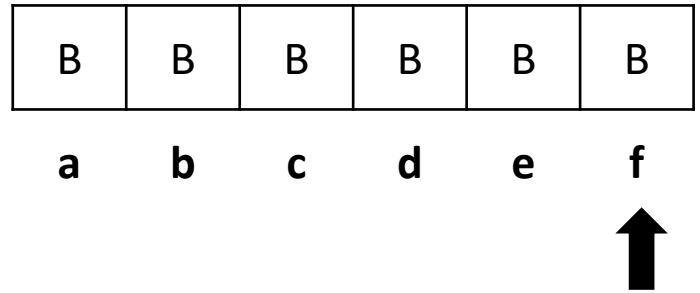
DFS(G)



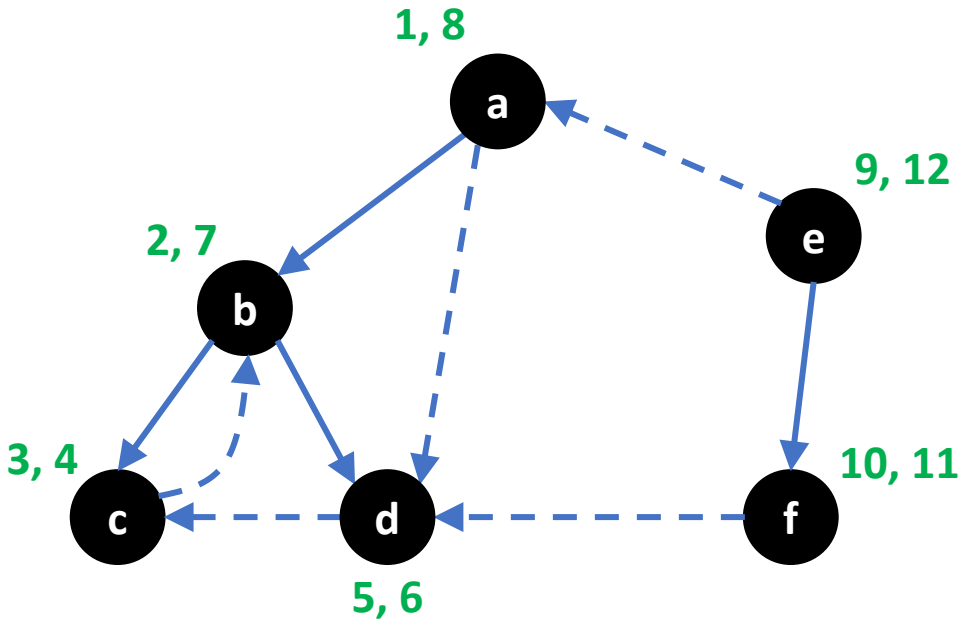
Adj List of G :



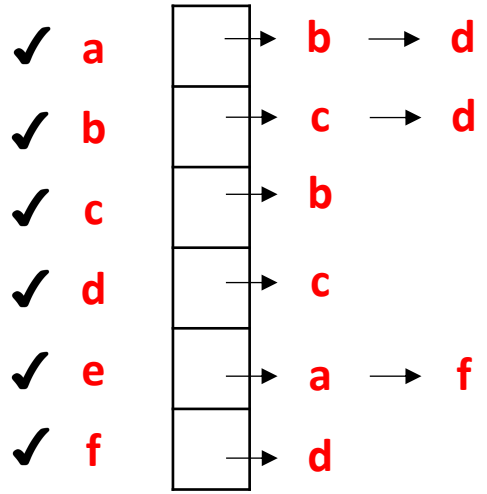
Colors of nodes:



DFS(G)



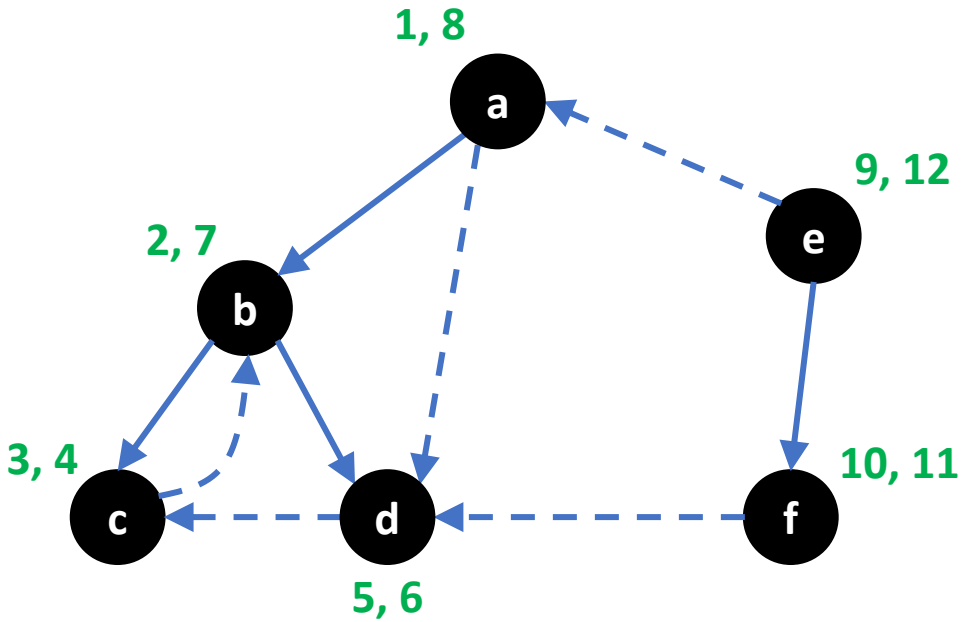
Adj List of G :



Colors of nodes:

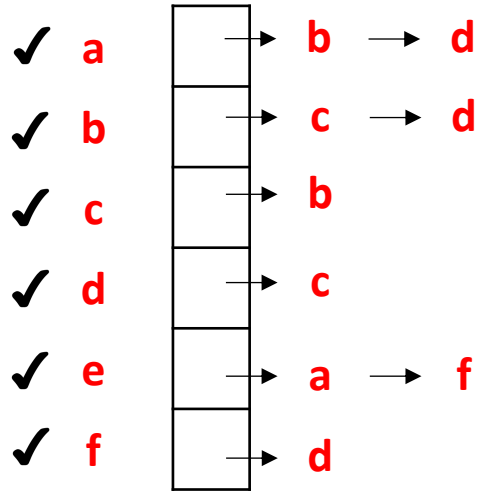
B	B	B	B	B	B
a	b	c	d	e	f

DFS(G)



Worst-case time complexity:

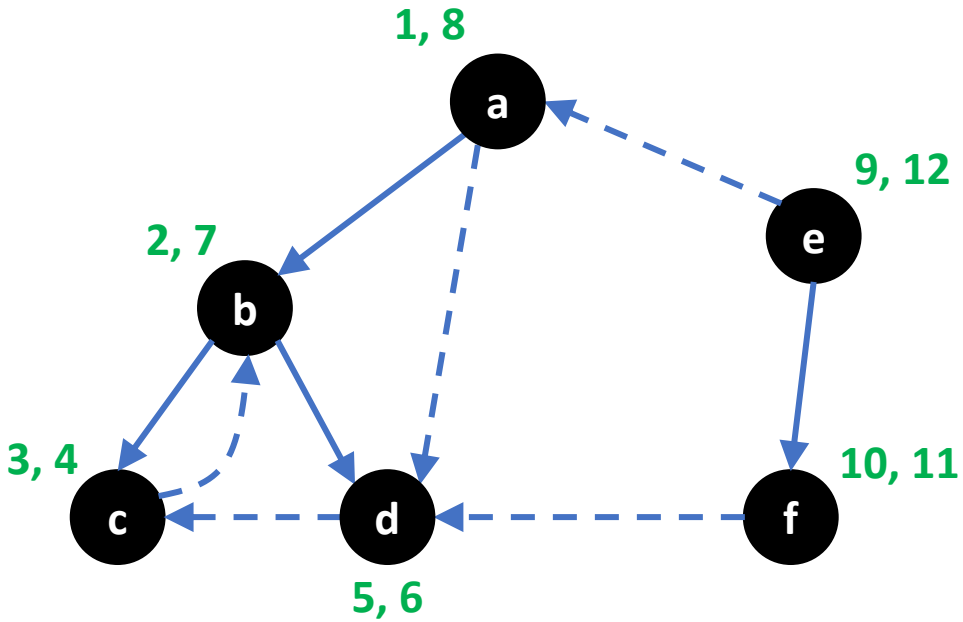
Adj List of G :



Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

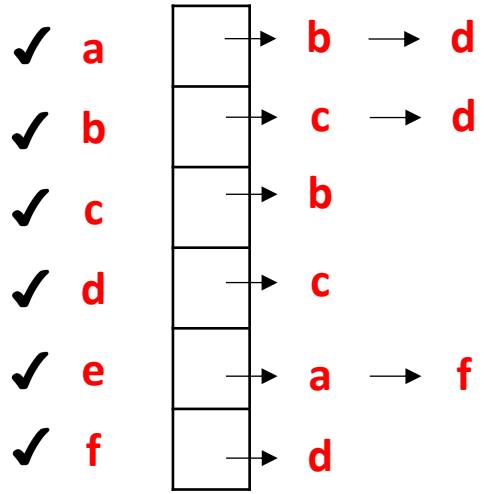
DFS(G)



Worst-case time complexity: O(|V| + |E|)

O(n + m)

Adj List of G :

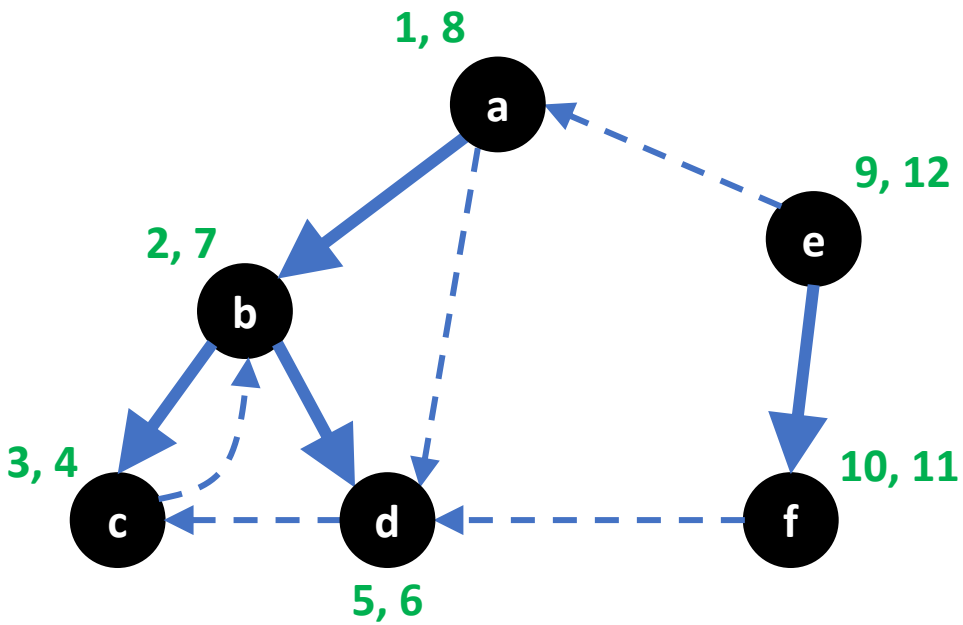


Colors of nodes:

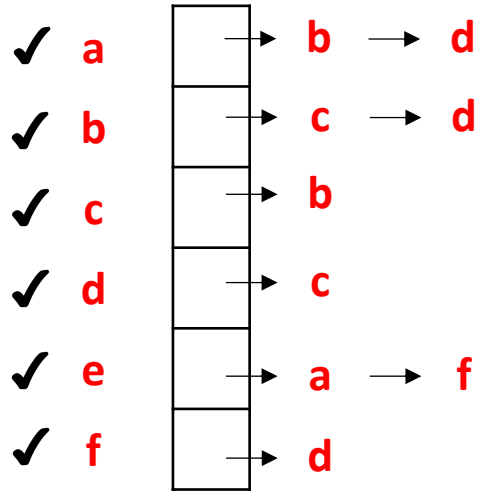
a	b	c	d	e	f
B	B	B	B	B	B

DFS(G)

DFS Forest : **solid** edges



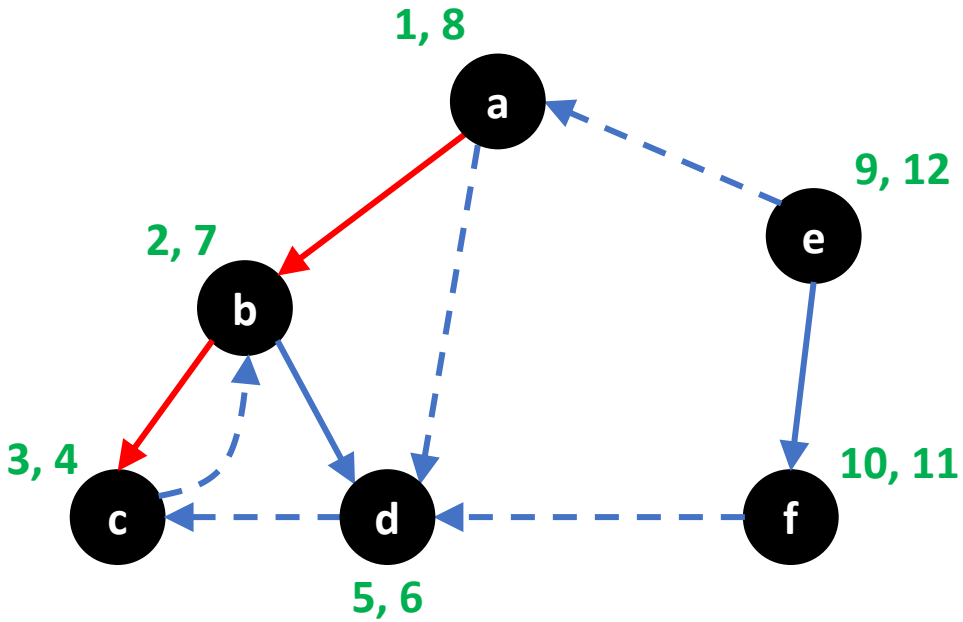
Adj List of G :



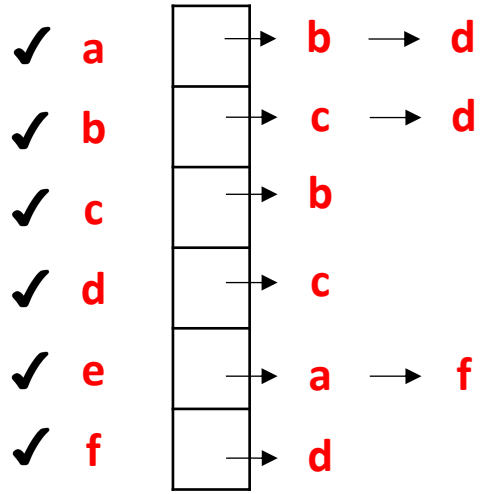
Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

DFS(G)



Adj List of G :

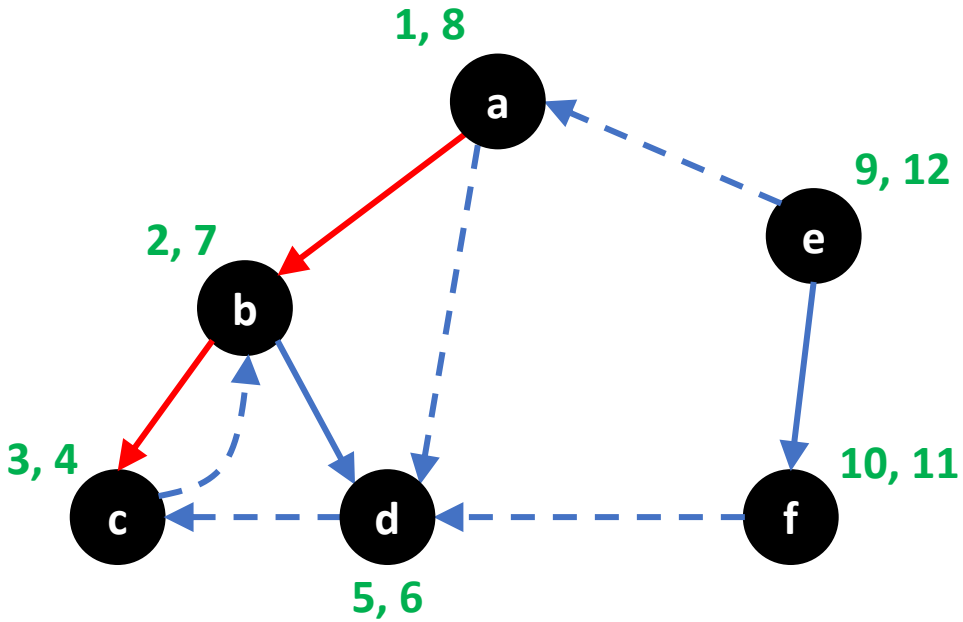


Colors of nodes:

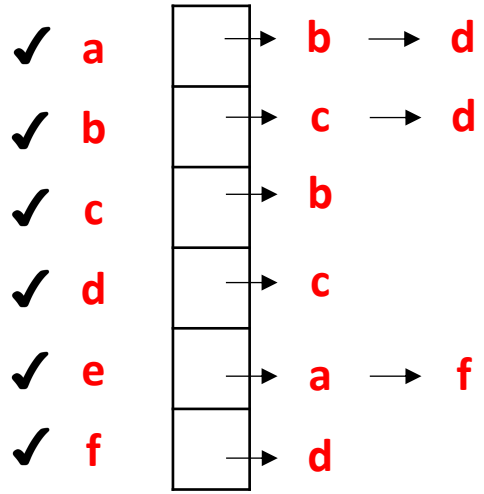
B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

a is **ancestor** of c

DFS(G)



Adj List of G :



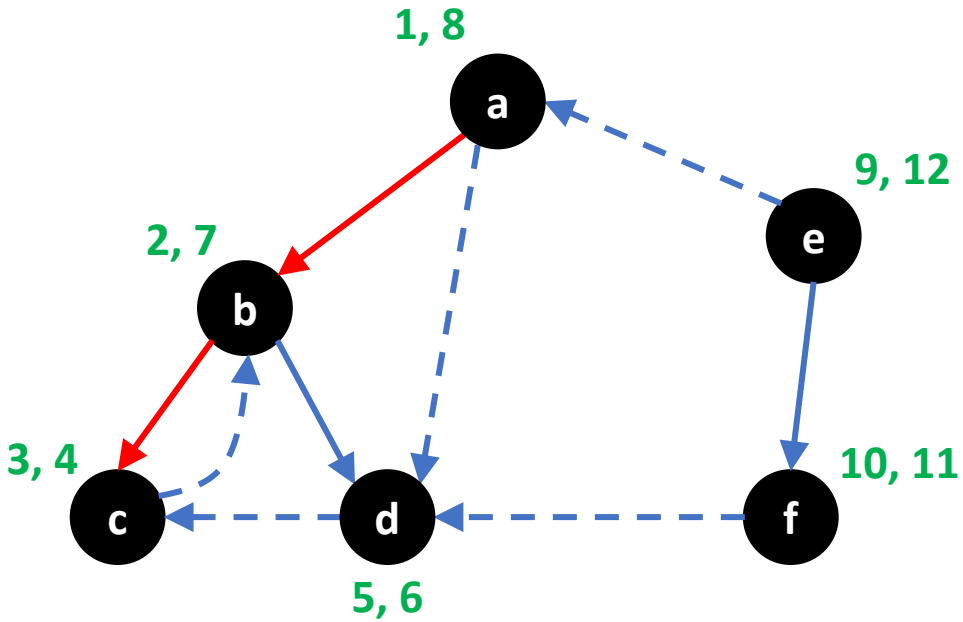
Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

a is **ancestor** of c

c is **descendant** of a

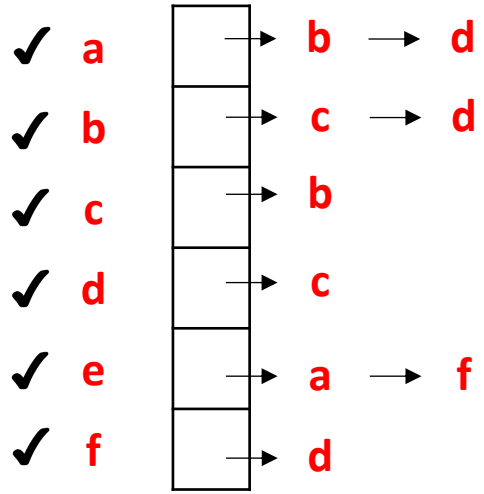
DFS(G)



1

d[a]

Adj List of G :



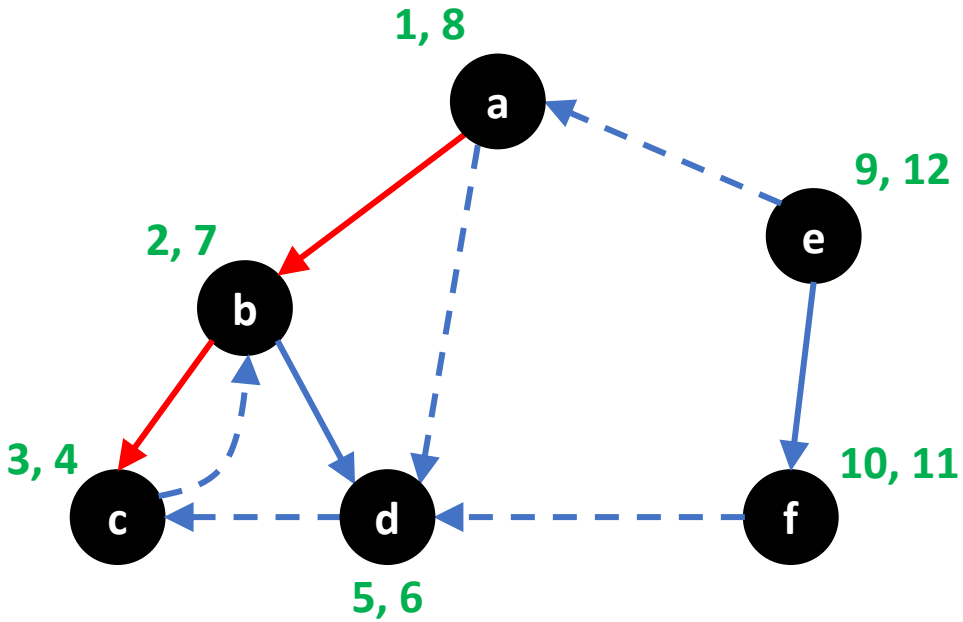
Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

a is **ancestor** of c

c is **descendant** of a

DFS(G)



1

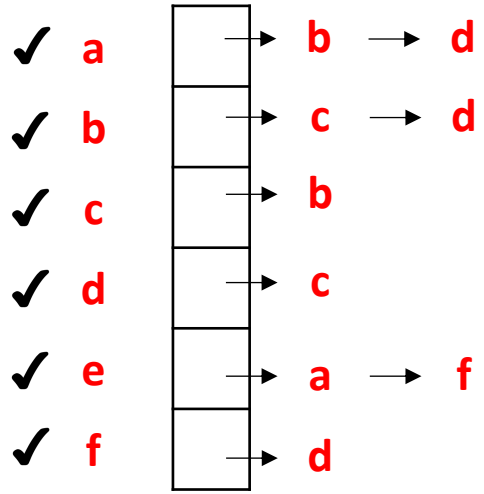
d[a] < d[c]

3

a is **ancestor** of c

c is **descendant** of a

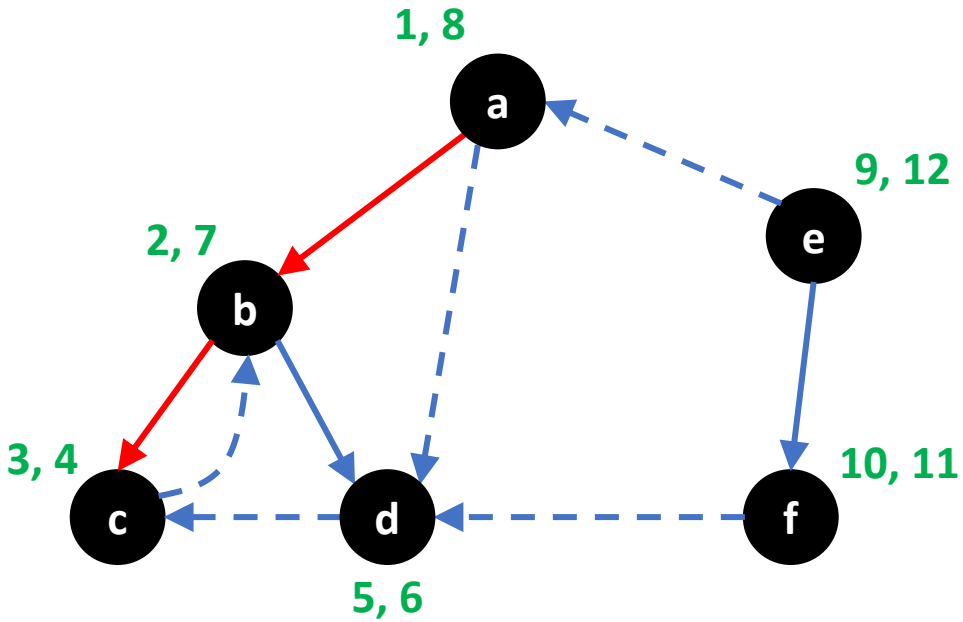
Adj List of G :



Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

DFS(G)



1

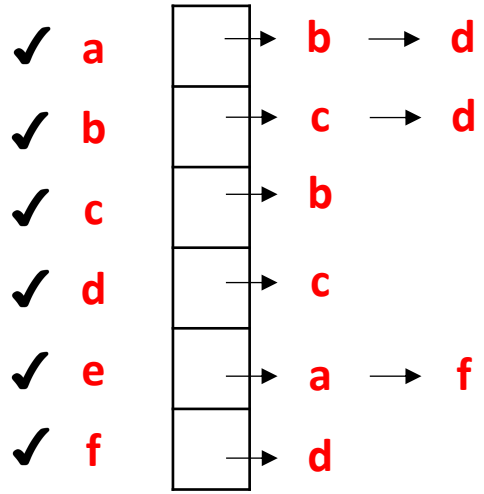
d[a] < d[c] < f[c]

3 4

a is **ancestor** of c

c is **descendant** of a

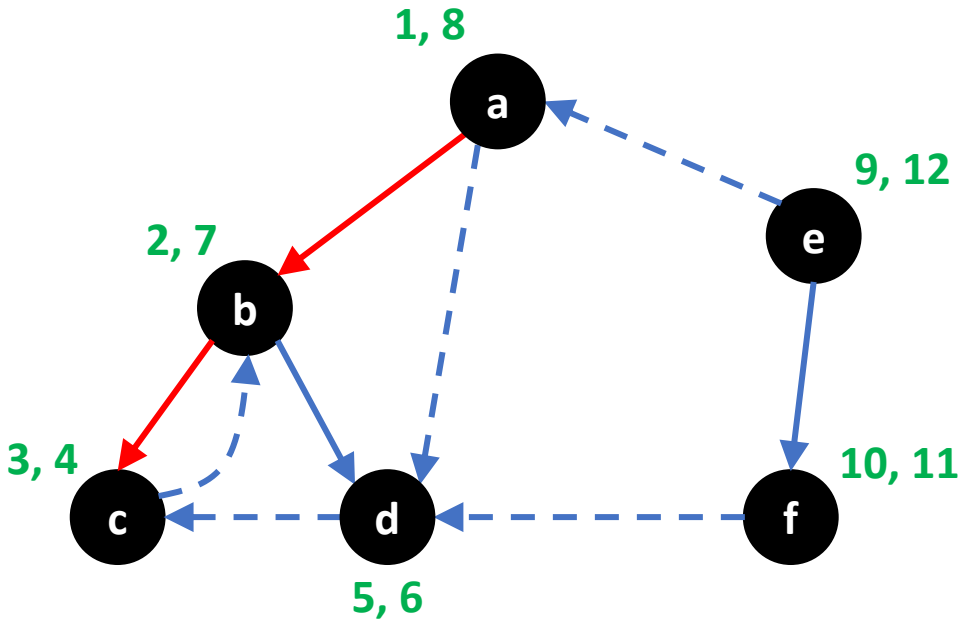
Adj List of G :



Colors of nodes:

B	B	B	B	B	B
a	b	c	d	e	f

DFS(G)



1 8

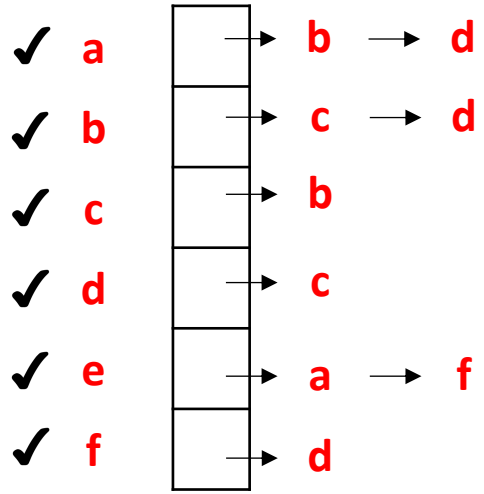
d[a] < d[c] < f[c] < f[a]

3 4

a is **ancestor** of c

c is **descendant** of a

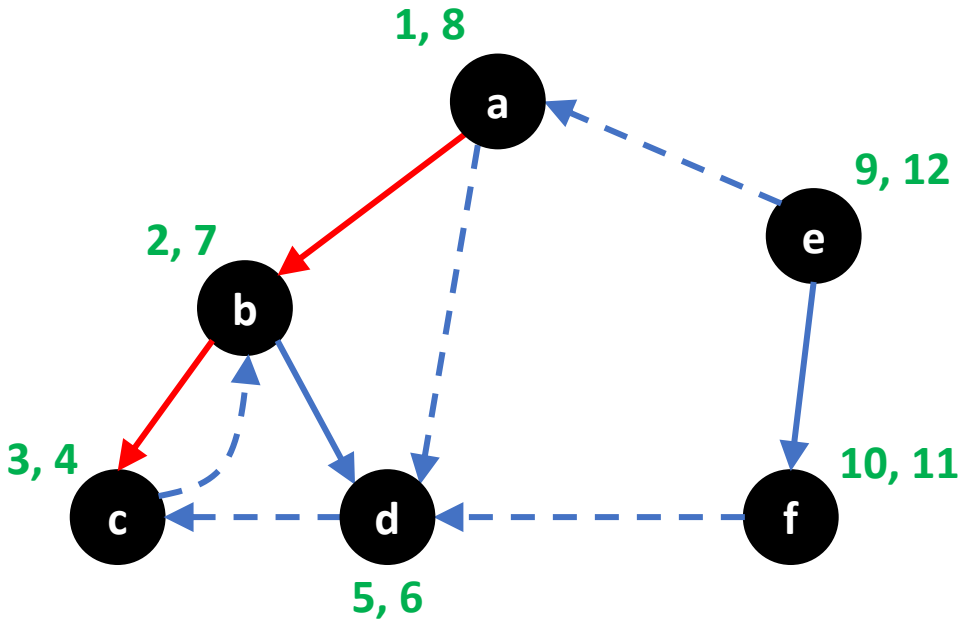
Adj List of G :



Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

DFS(G)



1 8

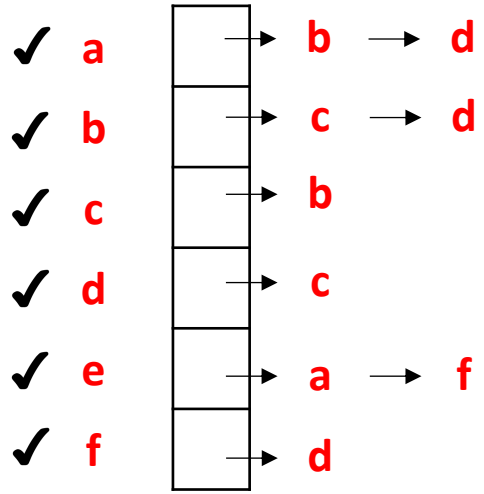
d[a] < d[c] < f[c] < f[a]

3 4

a is **ancestor** of c

c is **descendant** of a

Adj List of G :

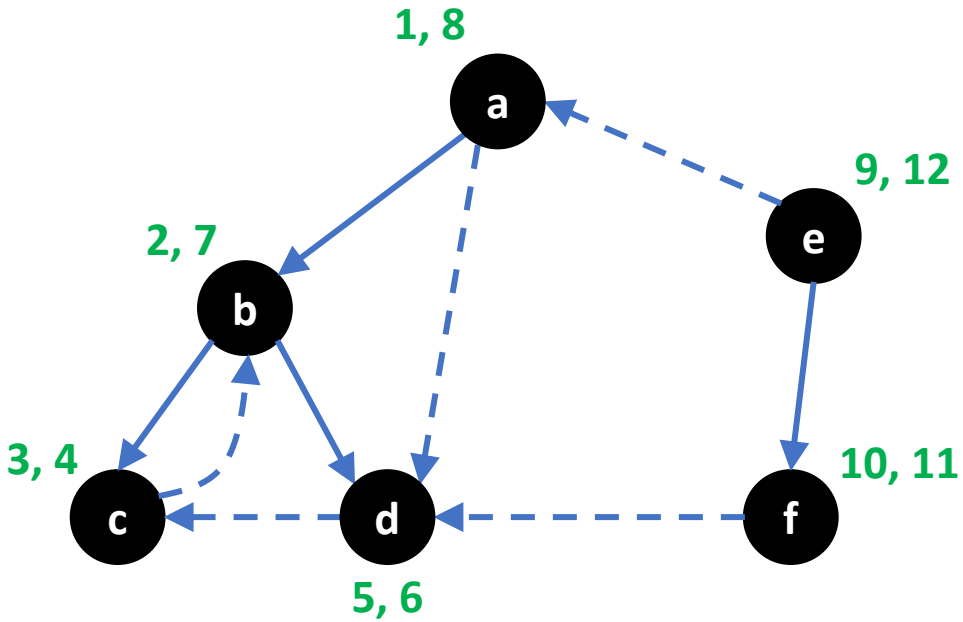


Colors of nodes:

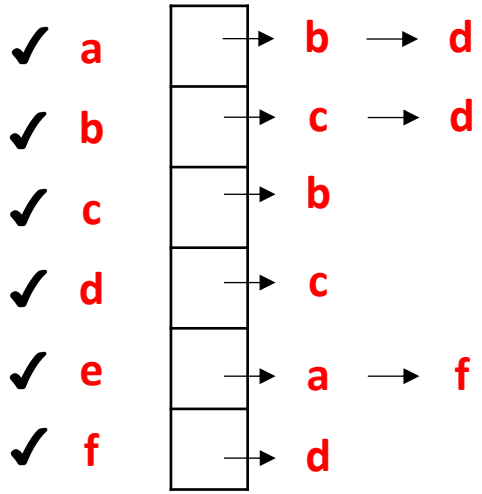
B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

DFS(G)

Edge Classification by DFS



Adj List of G :

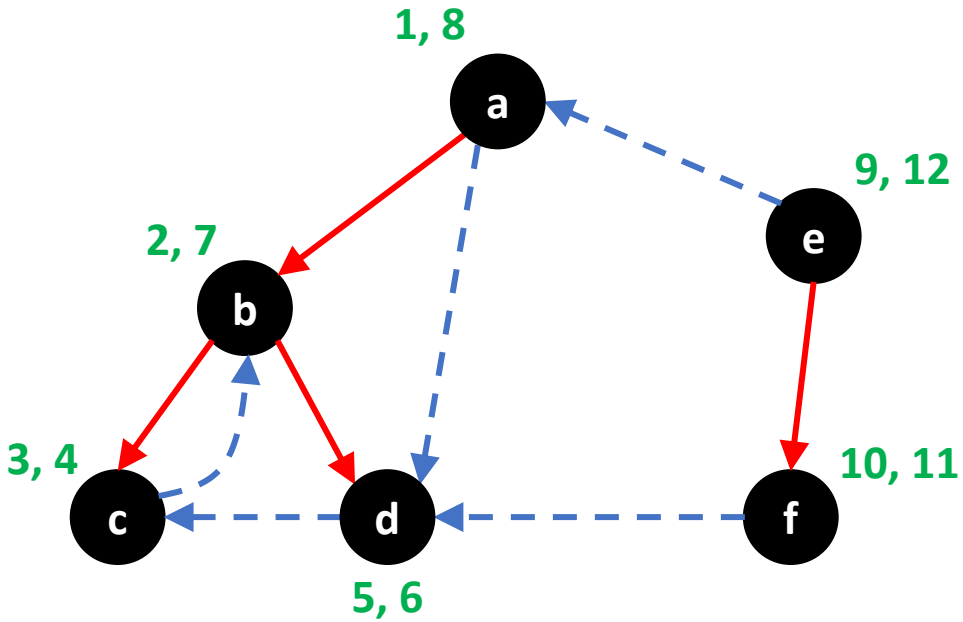


Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

DFS(G)

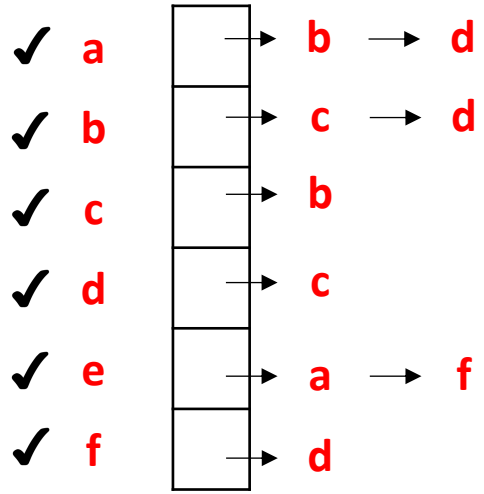
Edge Classification by DFS



An edge (u, v) of G is a

Tree edge ⇔ u is the **parent** of v in the DFS forest

Adj List of G :

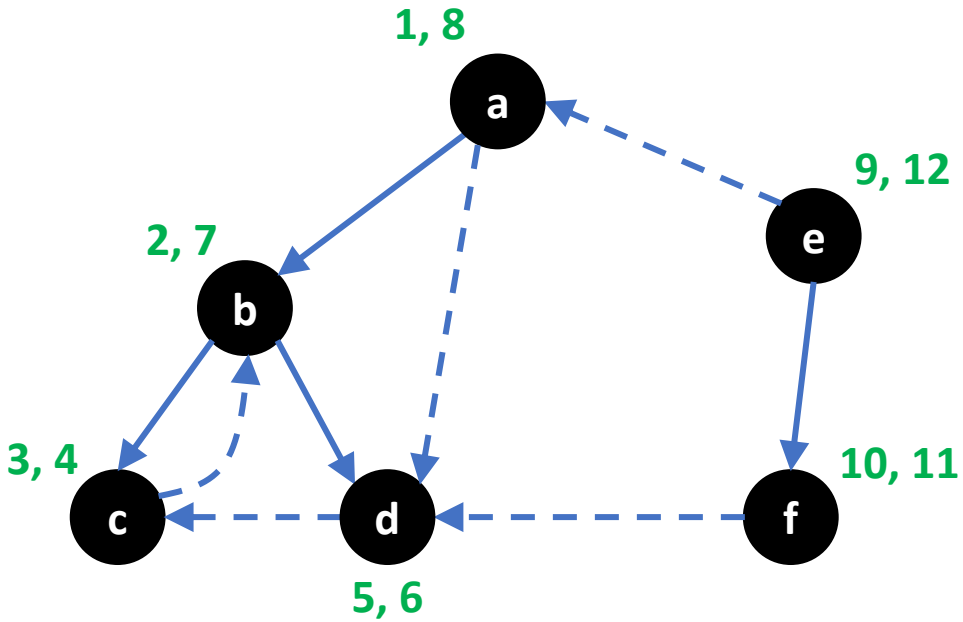


Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

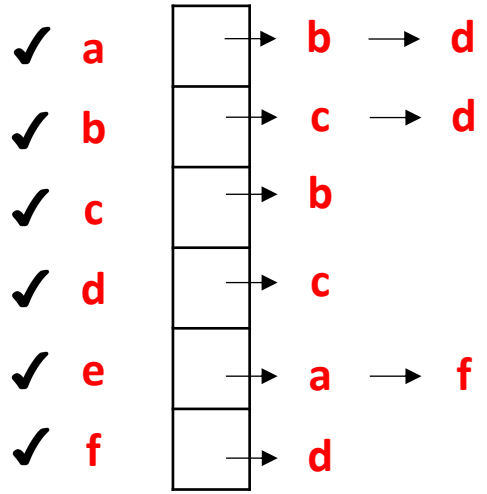
DFS(G)

Edge Classification by DFS



A **non-tree** edge (u, v) of G is a

Adj List of G :

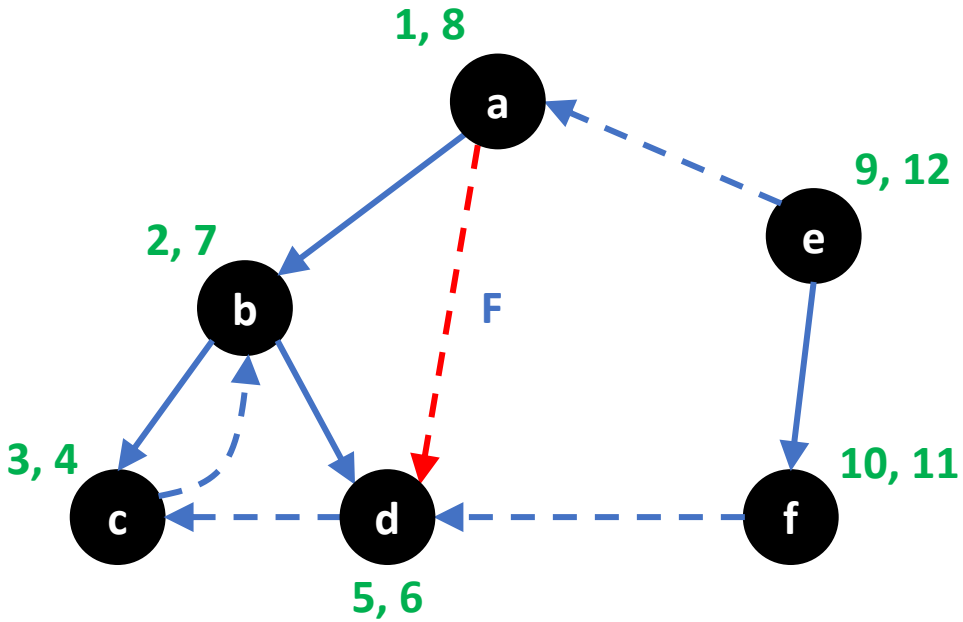


Colors of nodes:

B	B	B	B	B	B
a	**b**	**c**	**d**	**e**	**f**

DFS(G)

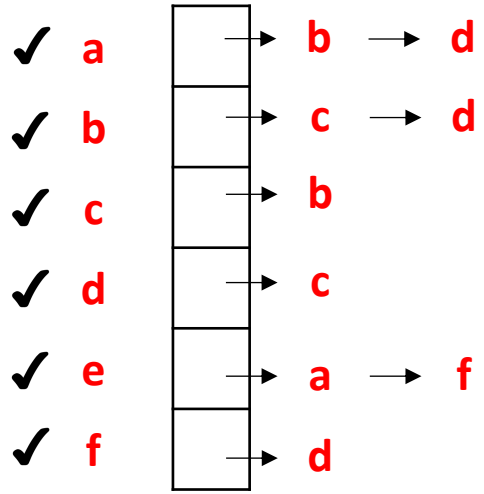
Edge Classification by DFS



A **non-tree** edge (u, v) of G is a

Forward edge ⇔ u is an **ancestor** of v in the DFS forest

Adj List of G :

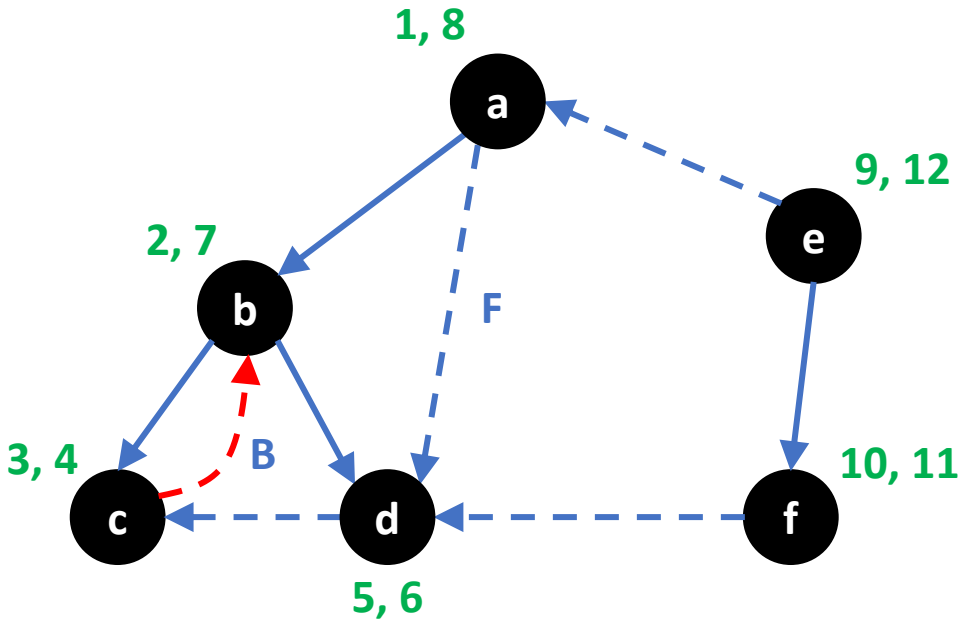


Colors of nodes:

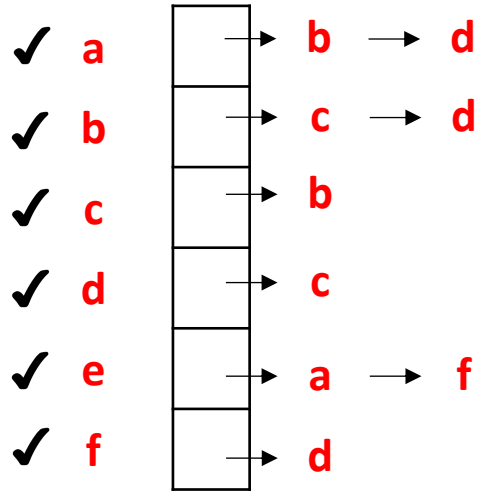
a	b	c	d	e	f
B	B	B	B	B	B

DFS(G)

Edge Classification by DFS



Adj List of G :



A **non-tree** edge (u, v) of G is a

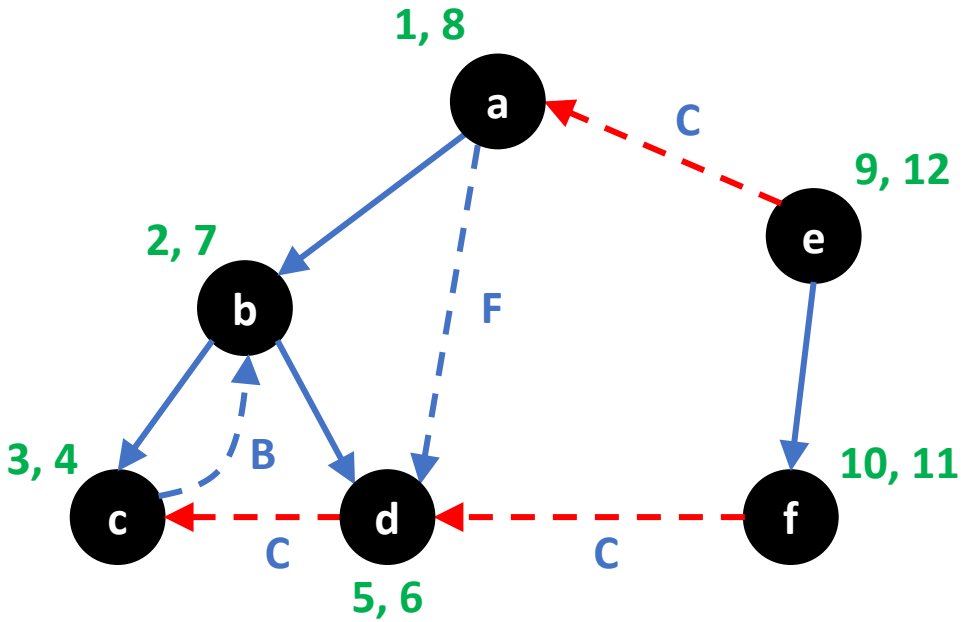
Back edge ⇔ u is a **descendent** of v in the DFS forest

Colors of nodes:

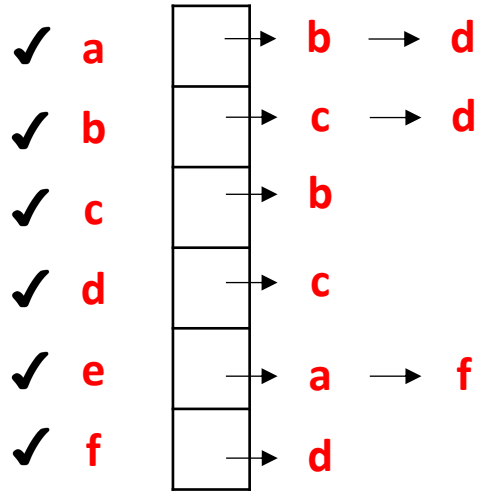
a	b	c	d	e	f
B	B	B	B	B	B

DFS(G)

Edge Classification by DFS



Adj List of G :



Colors of nodes:

a	b	c	d	e	f
B	B	B	B	B	B

A **non-tree** edge (u, v) of G is a

Cross edge ⇔ u is **neither ancestor nor descendent** of v in the DFS forest
⇔ not a forward or back edge

Edge Classification by DFS

A DFS of a directed graph G = (V, E) classifies the edges of G as follows:

Edge Classification by DFS

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(u,v) ∈ E is a :

1. **Tree edge**

Edge Classification by DFS

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(u,v) ∈ E is a :

1. **Tree edge** ⇔ u is the **parent** of v in the DFS forest

Edge Classification by DFS

A DFS of a directed graph G = (V, E) classifies the edges of G as follows:

(u,v) ∈ E is a :

1. **Tree edge** ⇔ u is the **parent** of v in the DFS forest

Non-tree edges

Edge Classification by DFS

A DFS of a directed graph G = (V, E) classifies the edges of G as follows:

(u,v) ∈ E is a :

1. **Tree edge** ⇔ u is the **parent** of v in the DFS forest
2. **Forward edge** ⇔ u is an **ancestor** of v in the DFS forest

Non-tree edges

Edge Classification by DFS

A DFS of a directed graph G = (V, E) classifies the edges of G as follows:

(u,v) ∈ E is a :

1. **Tree edge** ⇔ u is the **parent** of v in the DFS forest
2. **Forward edge** ⇔ u is an **ancestor** of v in the DFS forest
3. **Back edge** ⇔ u is a **descendant** of v in the DFS forest

Non-tree edges

Edge Classification by DFS

A DFS of a directed graph G = (V, E) classifies the edges of G as follows:

(u,v) ∈ E is a :

1. **Tree edge** ⇔ u is the **parent** of v in the DFS forest
2. **Forward edge** ⇔ u is an **ancestor** of v in the DFS forest
3. **Back edge** ⇔ u is a **descendant** of v in the DFS forest
4. **Cross edge** ⇔ u is **neither ancestor nor descendent** of v in the forest

Non-tree edges

Claim 1 :

u is an ancestor of v in a DFS of G

Claim 1 :

u is an ancestor of v in a DFS of G $\Leftrightarrow$

Claim 1 :

u is an ancestor of v in a DFS of G $\Leftrightarrow$ d[u]

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u is an ancestor of v in a DFS of G $\Leftrightarrow$ d[u] < d[v]

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Exploration of u

Exploration of v

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Exploration of u

Exploration of v

Claim 2 :

For any 2 nodes u and v,

Claim 1 :

u is an ancestor of v in a DFS of G $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

Exploration of u

Exploration of v

Claim 2 :

For any 2 nodes u and v, we CANNOT have d[u]

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Claim 3 :

If $(u,v) \in E$

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Exploration of u

Exploration of v

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Exploration of v

Exploration of u

Claim 3 :

If $(u,v) \in E$ then

v is surely discovered **before** we finish exploring u.

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Exploration of v

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Exploration of v

Exploration of u

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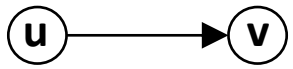
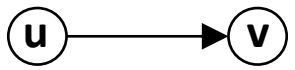
If $(u,v) \in E$ then $d[v] < f[u]$

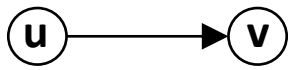
Claim: After a DFS of a directed graph, 

$\in E$ is of type:

Claim: After a DFS of a directed graph, 

$\in E$ is of type:

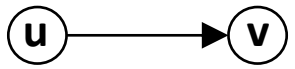
Tree or Forward edge $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

Claim: After a DFS of a directed graph, 

$\in E$ is of type:

Tree or Forward edge $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

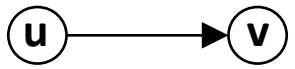
Back edge

Claim: After a DFS of a directed graph, 

$\in E$ is of type:

Tree or Forward edge $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

Back edge $\Leftrightarrow$ $d[v] < d[u] < f[u] < f[v]$

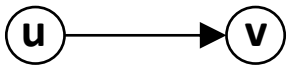
Claim: After a DFS of a directed graph, 

$\in E$ is of type:

Tree or Forward edge $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

Back edge $\Leftrightarrow$ $d[v] < d[u] < f[u] < f[v]$

Cross edge

Claim: After a DFS of a directed graph, 

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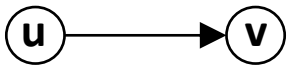
Cross edge $\Leftrightarrow$ $d[v] < f[v] < d[u] < f[u]$

Claim: After a DFS of a directed graph, 

$\in E$ is of type:

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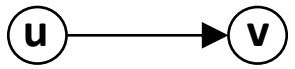
Back edge $\Leftrightarrow$ $d[v] < d[u] < f[u] < f[v]$

Cross edge $\Leftrightarrow$ $d[v] < f[v] < d[u] < f[u]$

OR

$d[u] < f[u] < d[v] < f[v]$?

Claim: After a DFS of a directed graph, 

$\in E$ is of type:

Tree or Forward edge $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

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Cross edge $\Leftrightarrow$ $d[v] < f[v] < d[u] < f[u]$

OR

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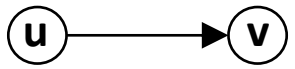
Claim: After a DFS of a directed graph, 

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Tree or Forward edge $\Leftrightarrow$ $d[u] < d[v] < f[v] < f[u]$

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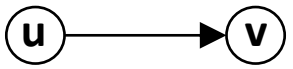
OR

$d[u] < f[u] < d[v] < f[v]$

Impossible because of Claim 3 !

Claim: After a DFS of a directed graph, 

$\in E$ is of type:

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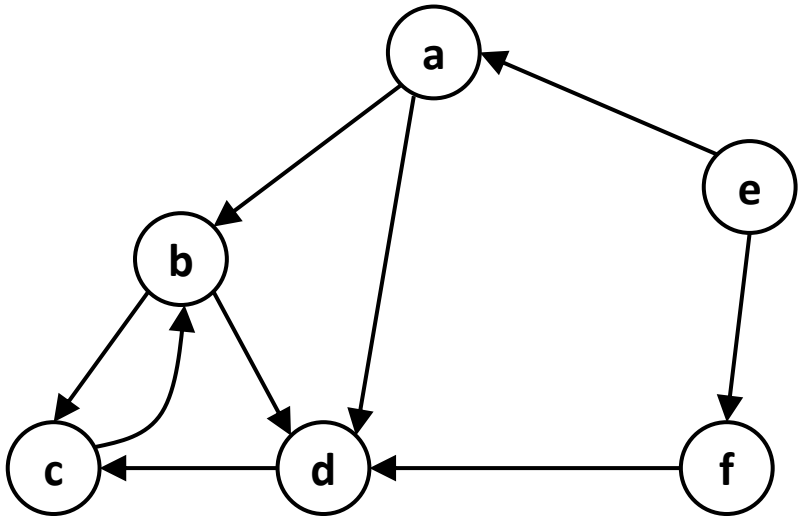
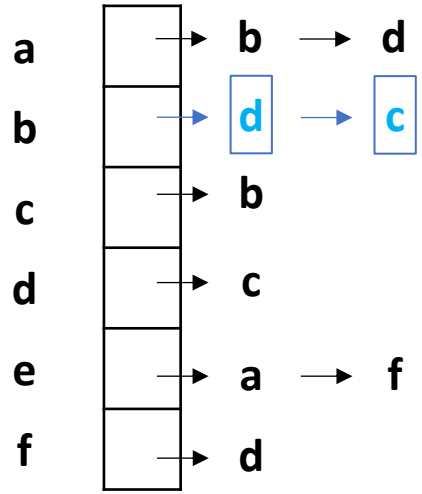
Back edge $\Leftrightarrow$ $d[v] < d[u] < f[u] < f[v]$

Cross edge $\Leftrightarrow$ $d[v] < f[v] < d[u] < f[u]$

DFS(G)



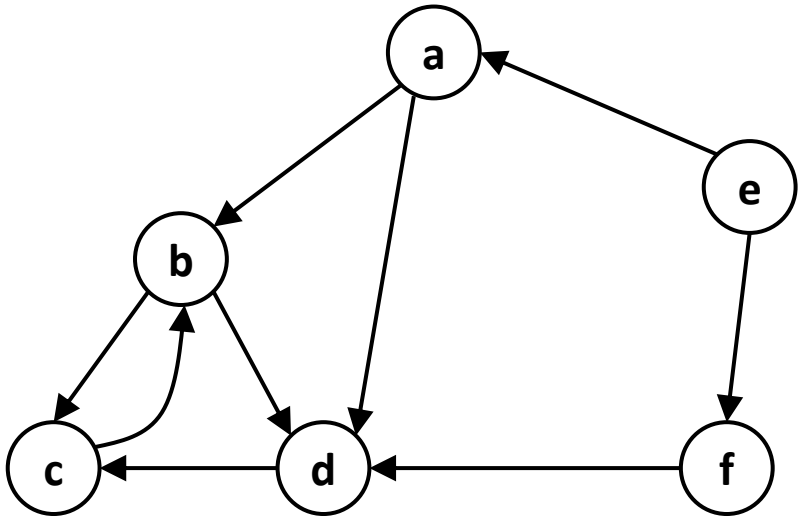
Adj List of G :



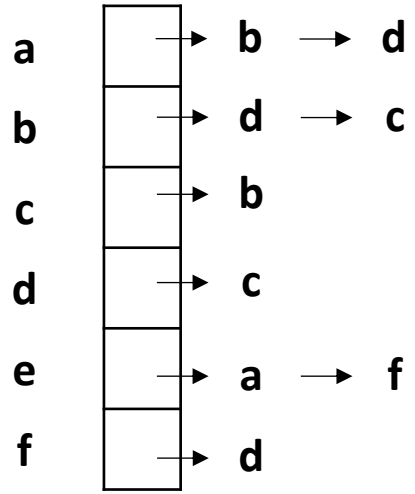
Colors of nodes:

e	a	b	c	d	f
W	W	W	W	W	W

DFS(G)



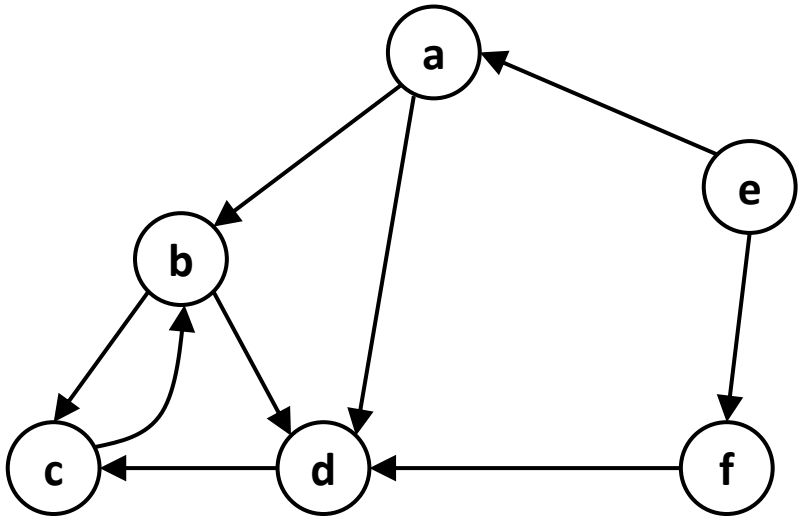
Adj List of G :



Colors of nodes:

e	a	b	c	d	f
W	W	W	W	W	W

DFS(G)



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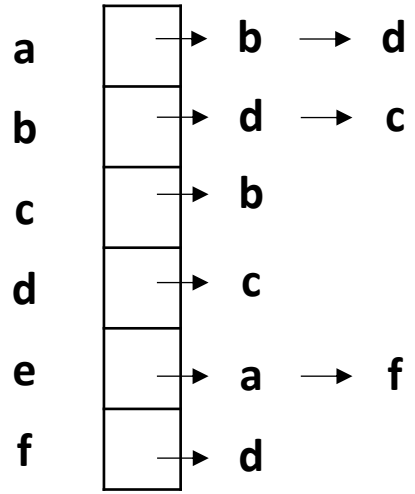
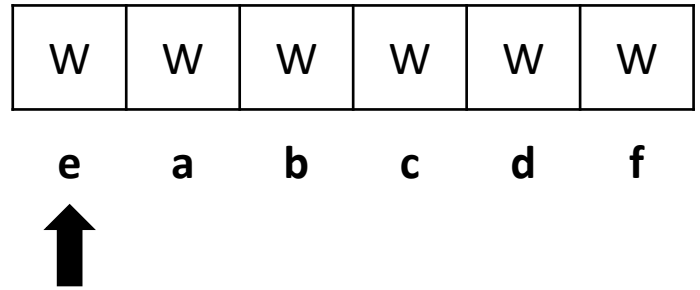
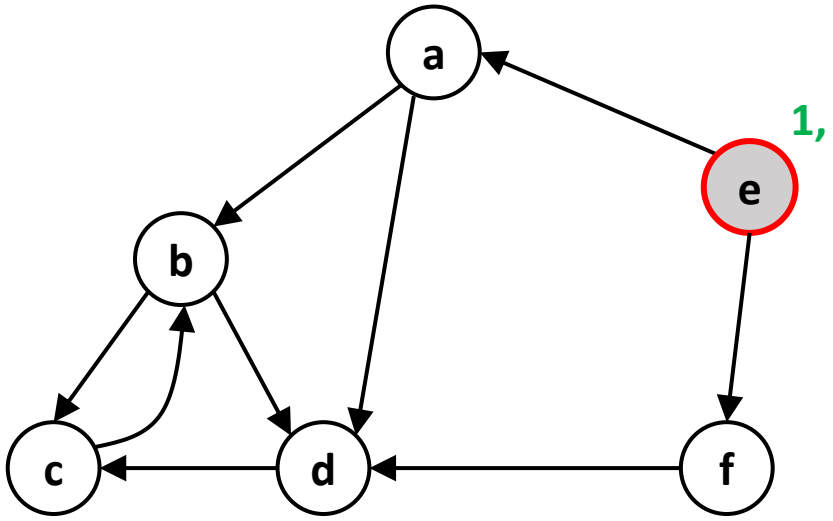


Colors of nodes:

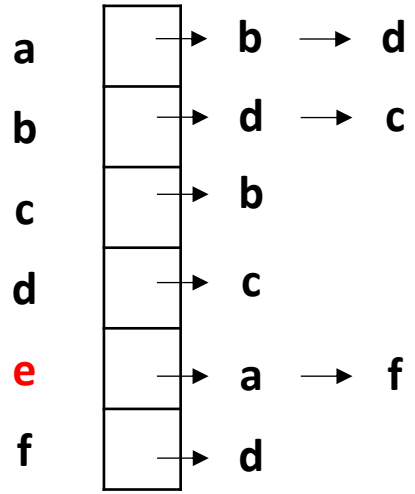


DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

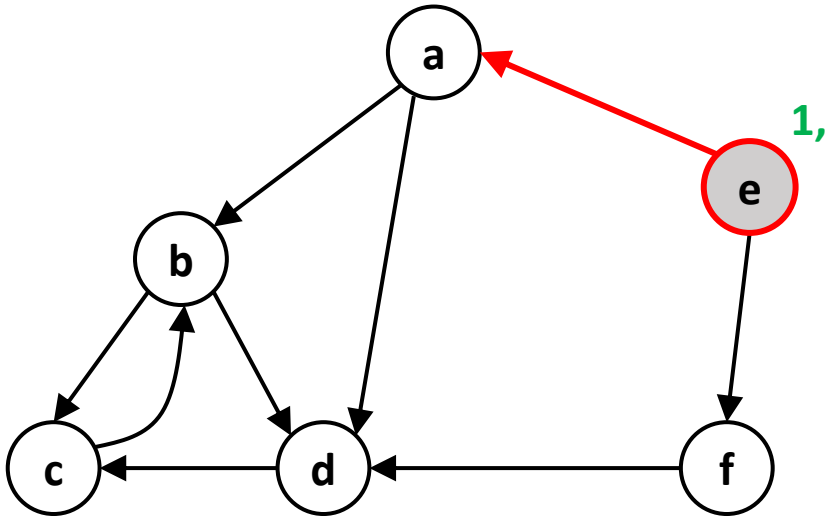


Colors of nodes:

G	W	W	W	W	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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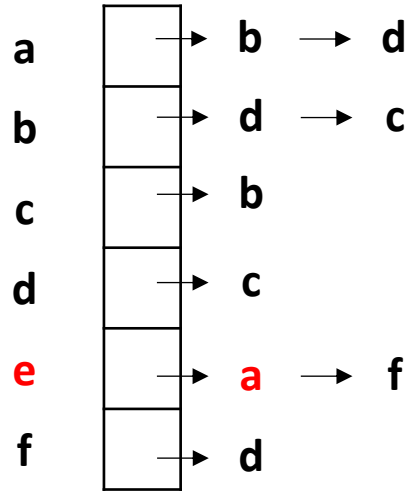
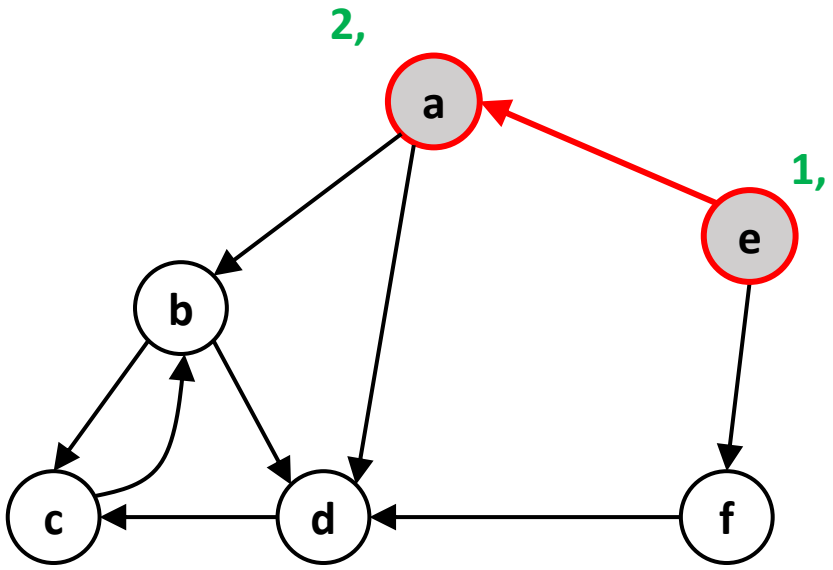


Colors of nodes:

G	W	W	W	W	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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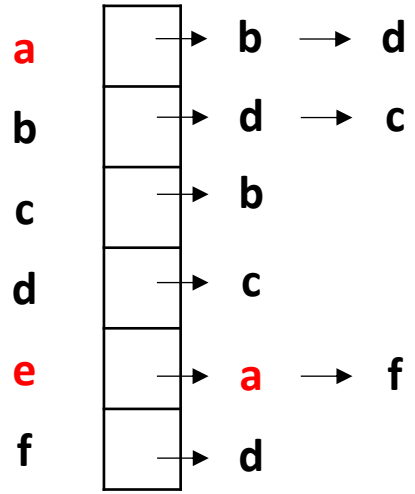
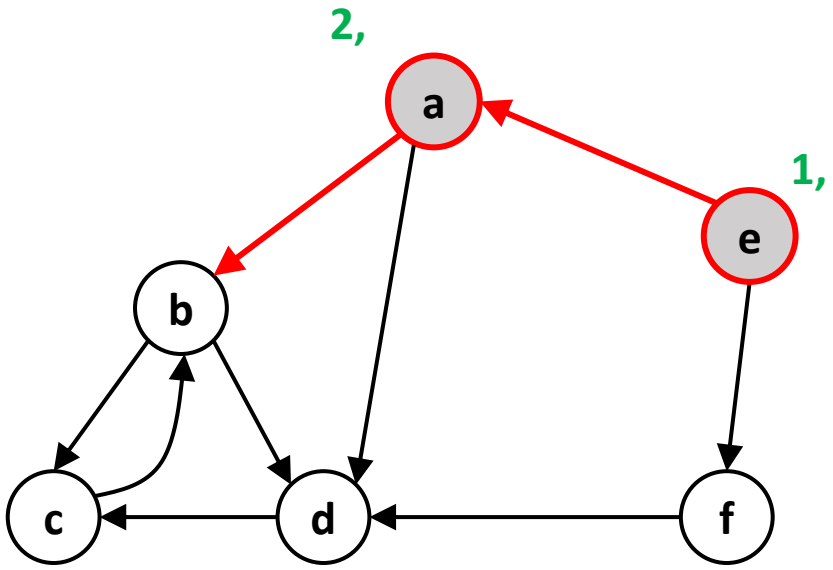


Colors of nodes:

G	G	W	W	W	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



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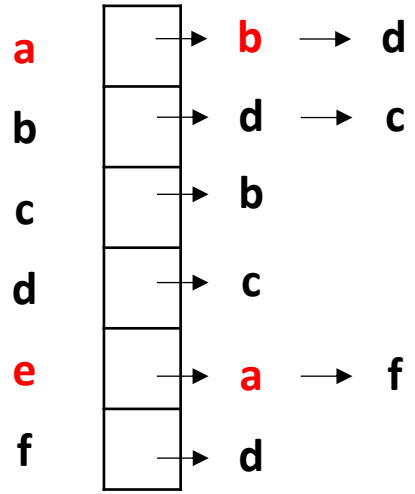
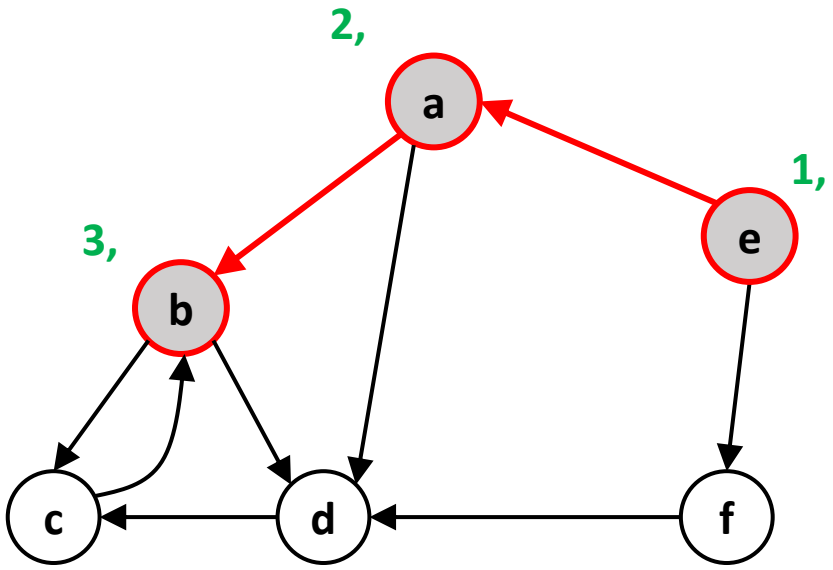


Colors of nodes:

G	G	W	W	W	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



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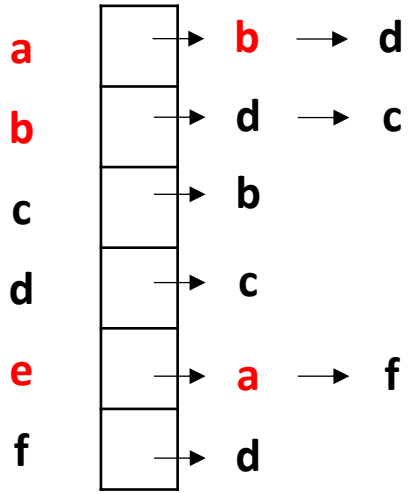
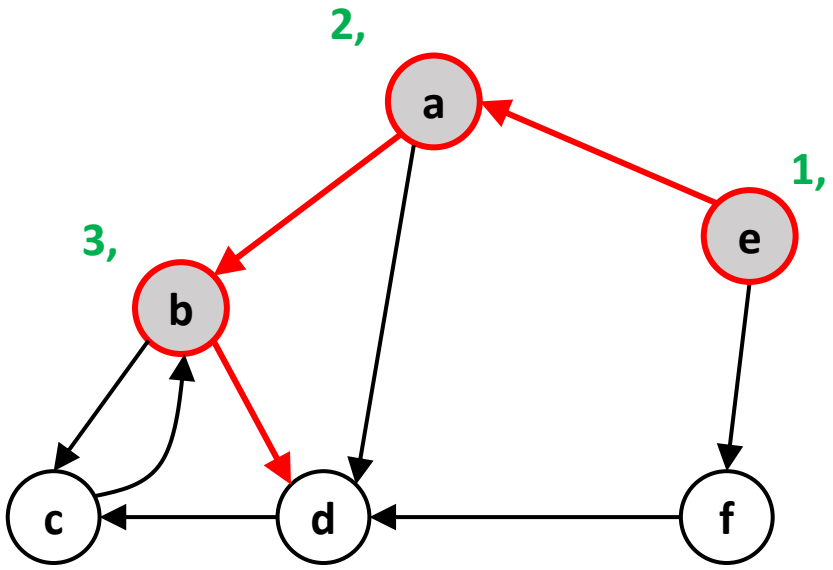


Colors of nodes:

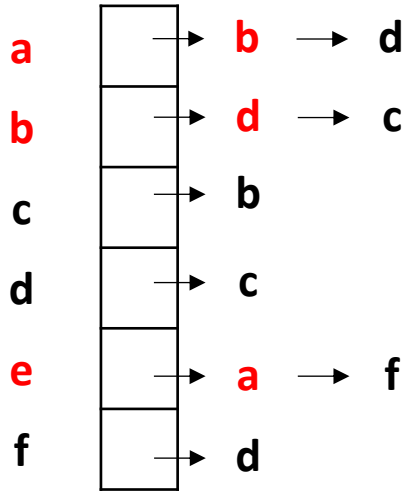
G	G	G	W	W	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

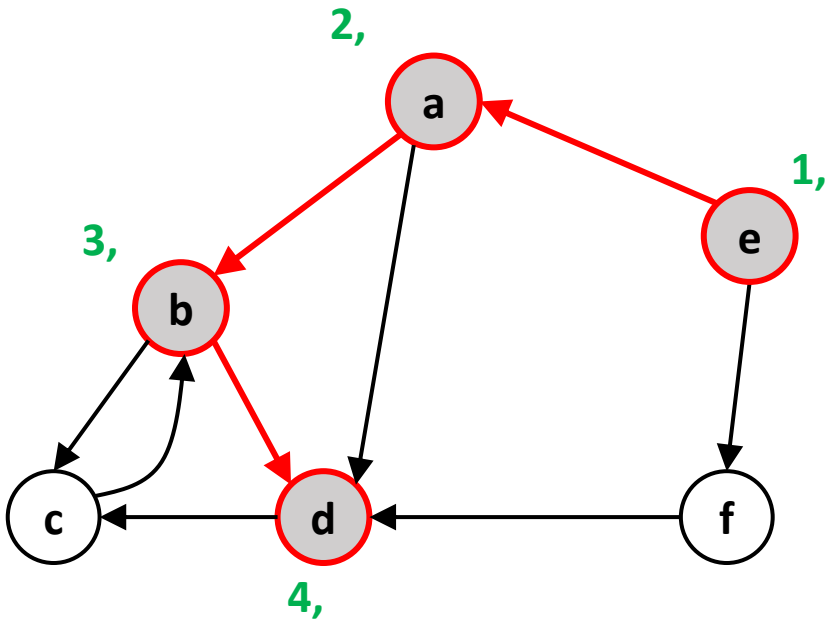


Colors of nodes:

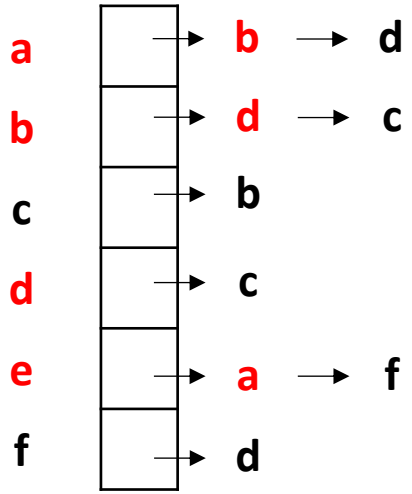
G	G	G	W	W	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

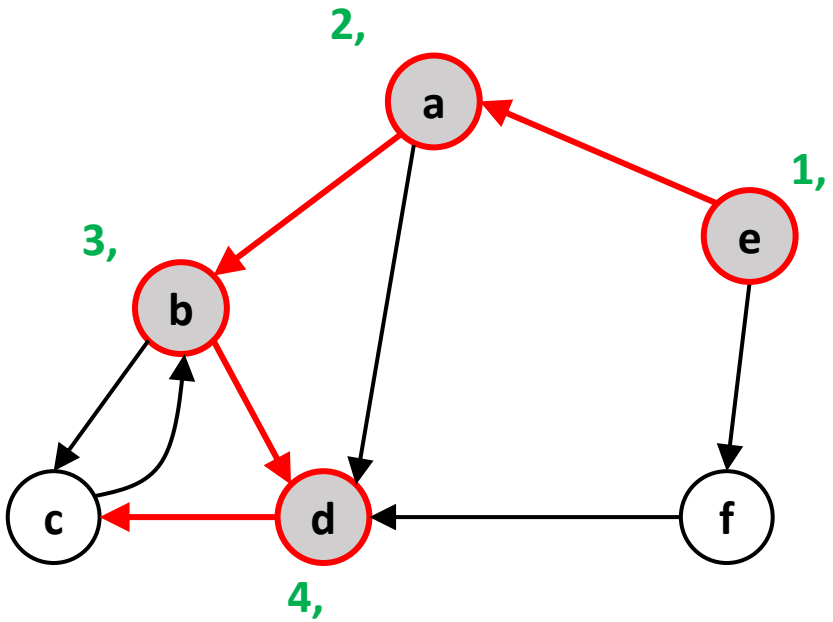


Colors of nodes:

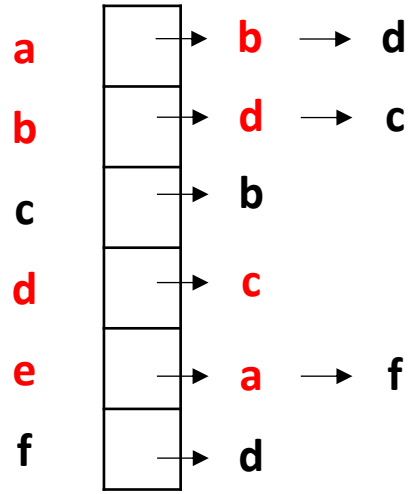
G	G	G	W	G	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

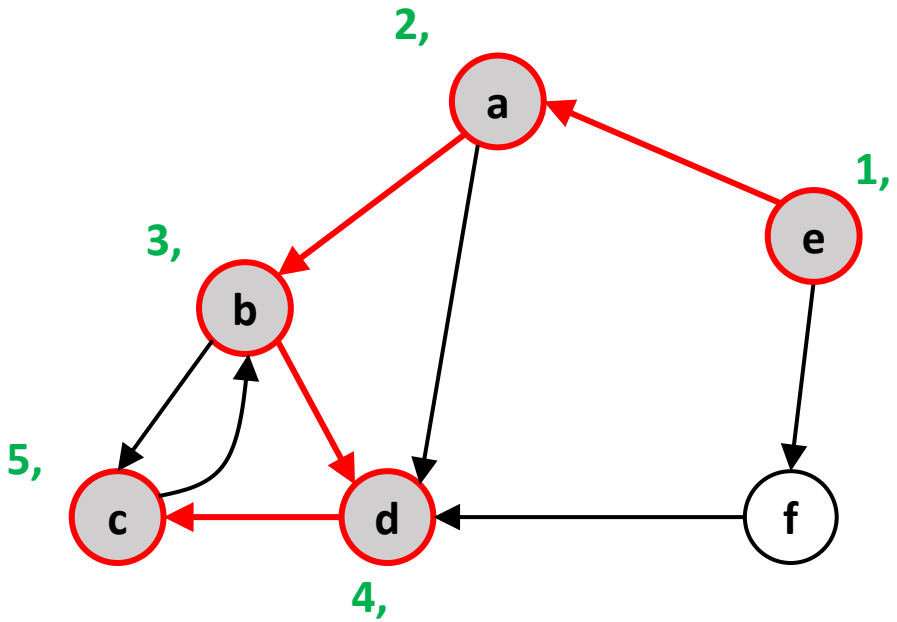


Colors of nodes:

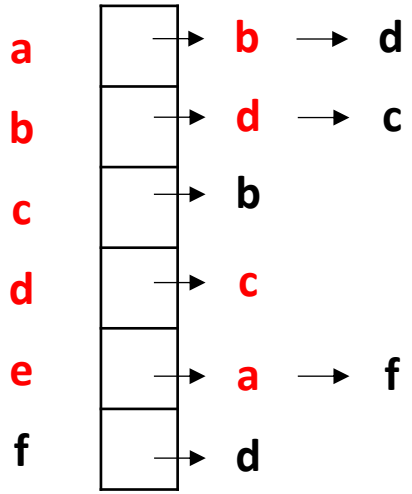
G	G	G	W	G	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

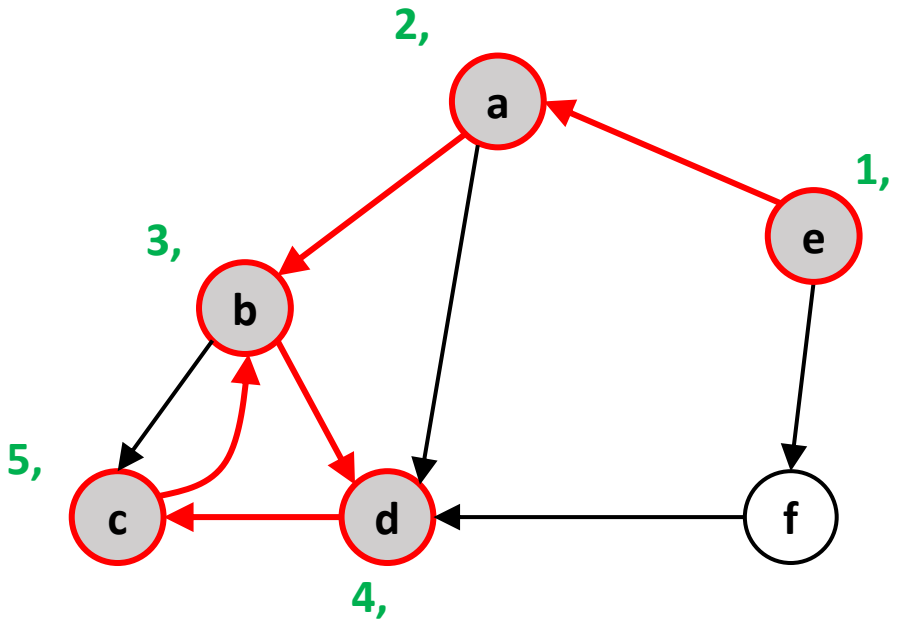


Colors of nodes:

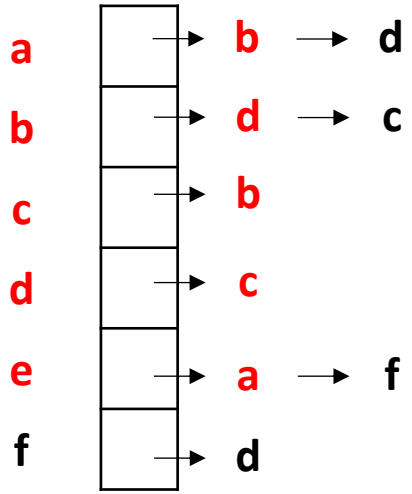
G	G	G	G	G	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

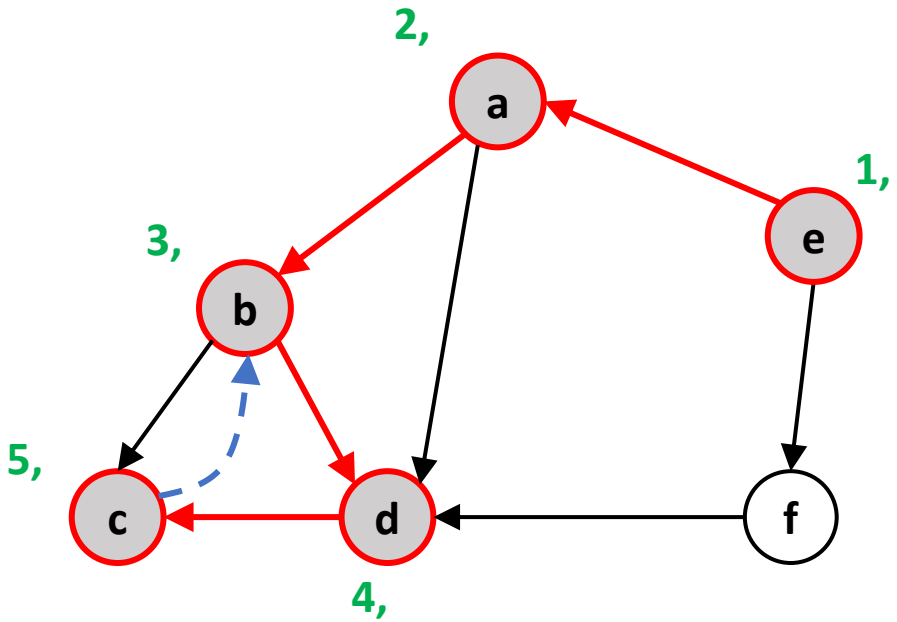


Colors of nodes:

G	G	G	G	G	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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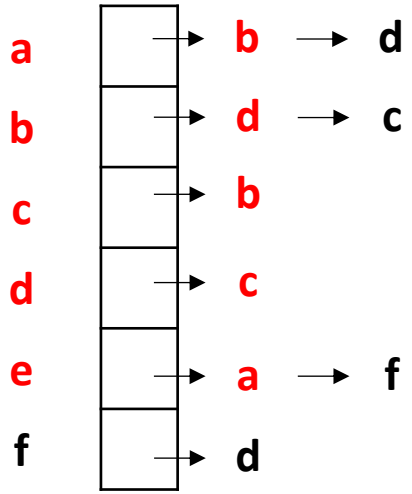
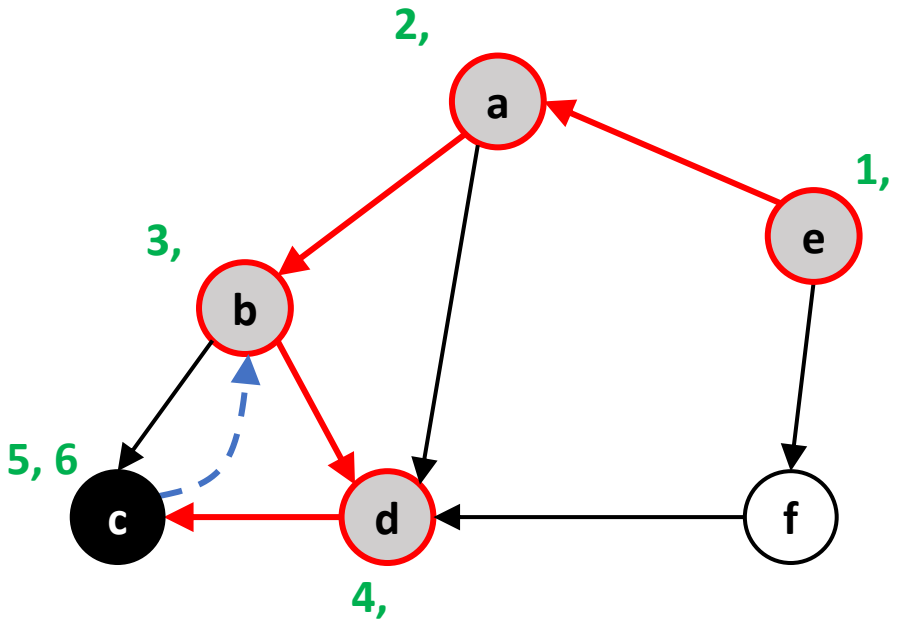


Colors of nodes:

G	G	G	G	G	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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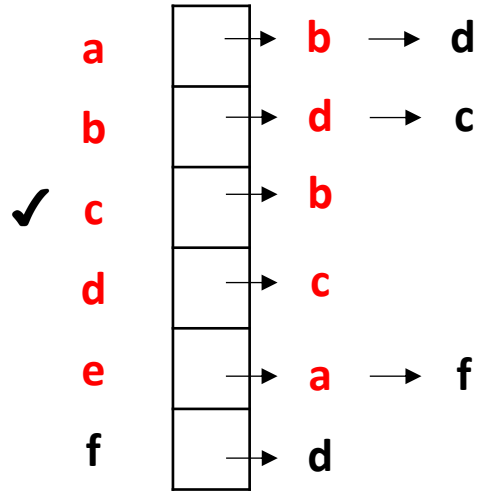
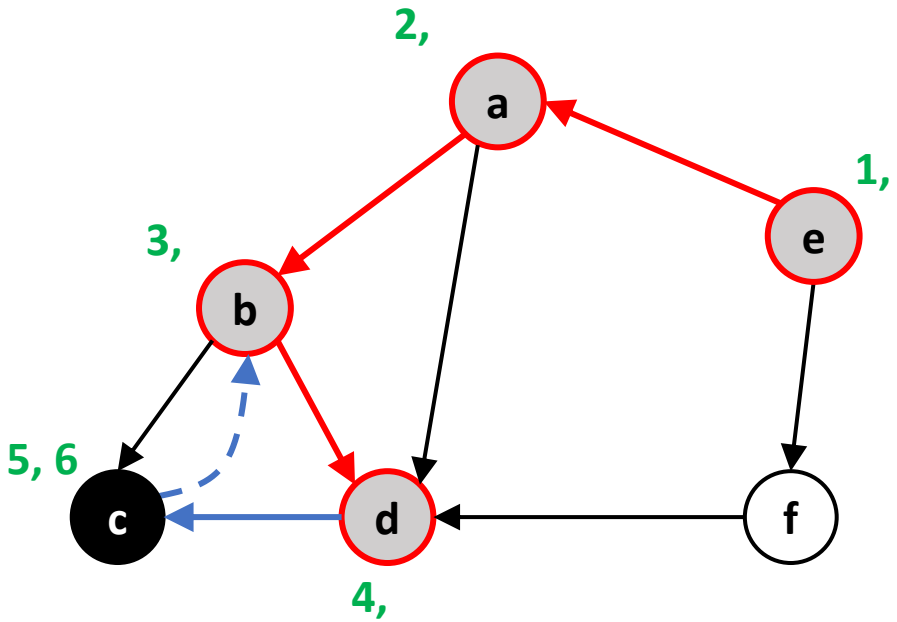


Colors of nodes:

G	G	G	B	G	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



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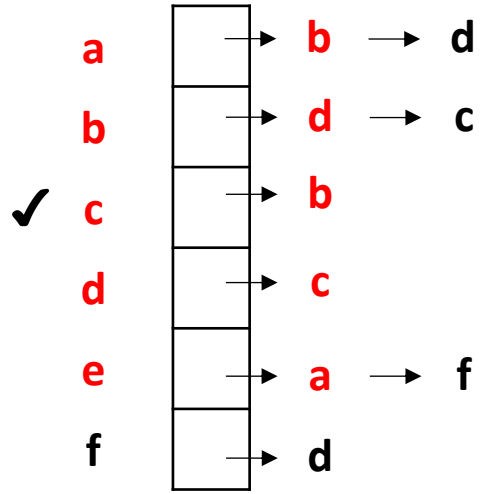
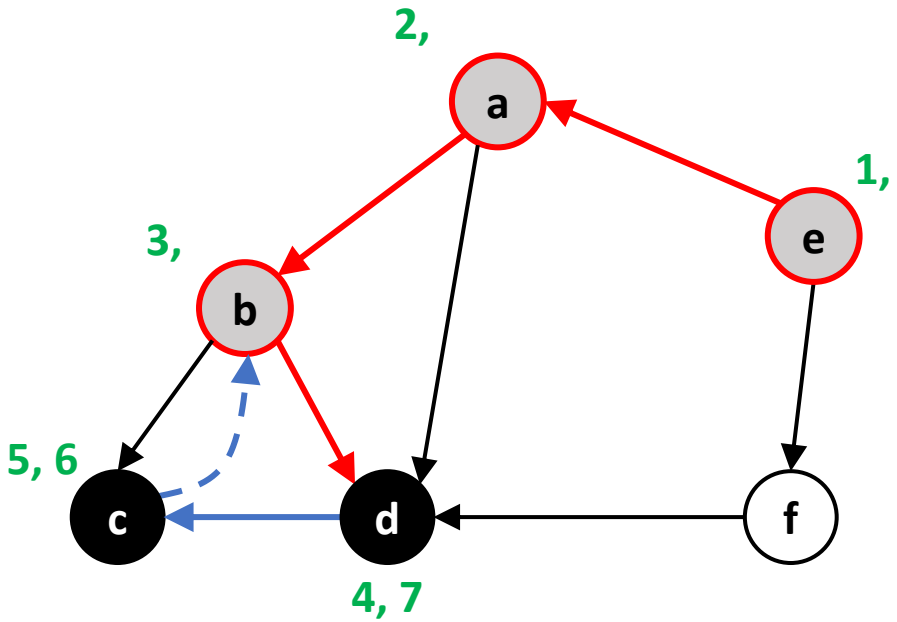


Colors of nodes:

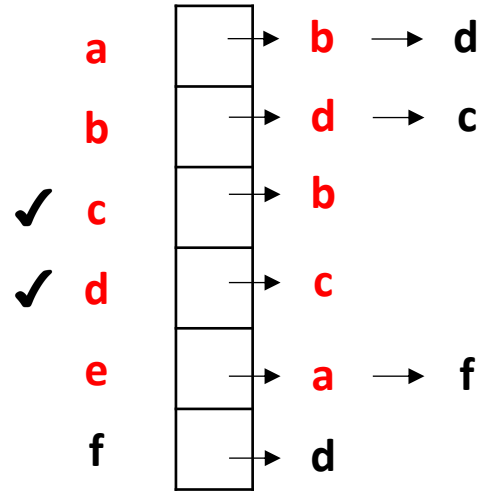
G	G	G	B	G	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

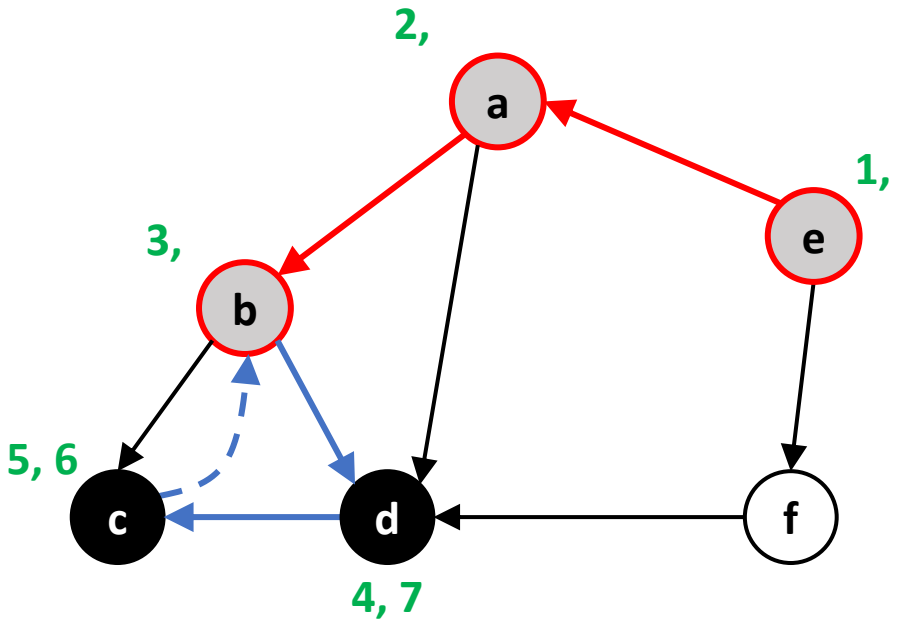


Colors of nodes:

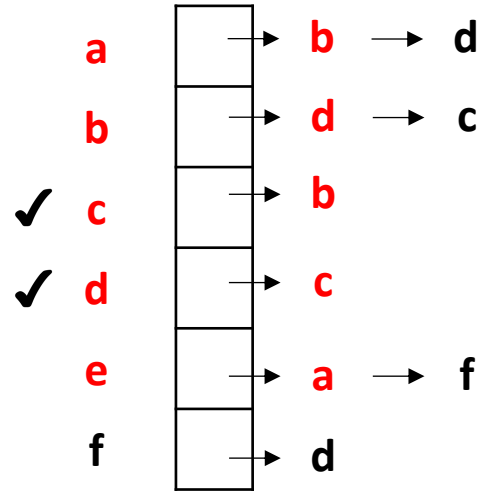
G	G	G	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

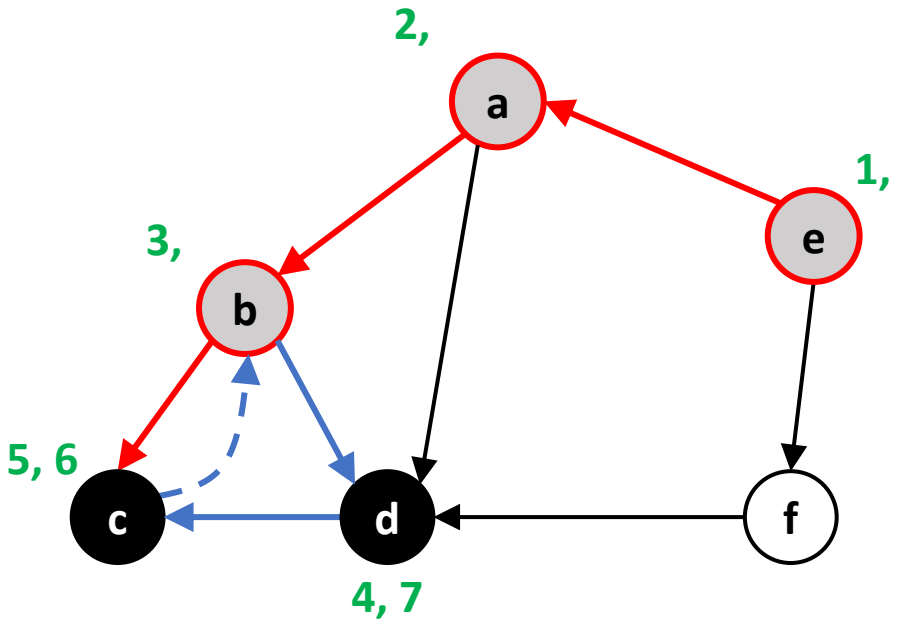


Colors of nodes:

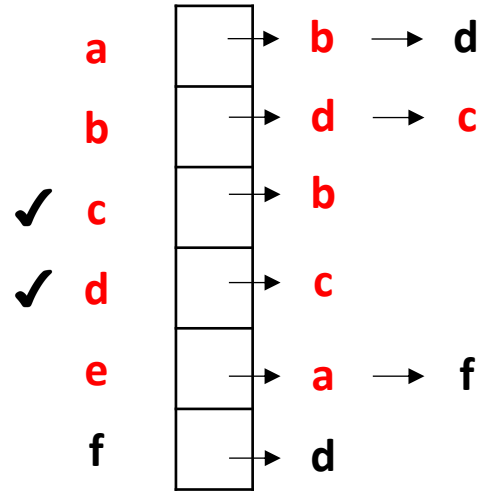
G	G	G	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

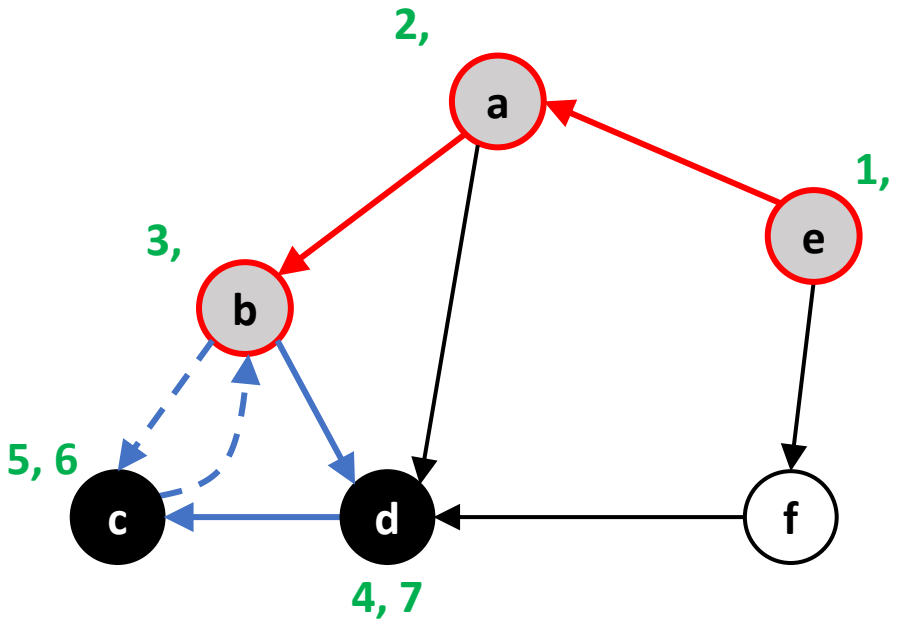


Colors of nodes:

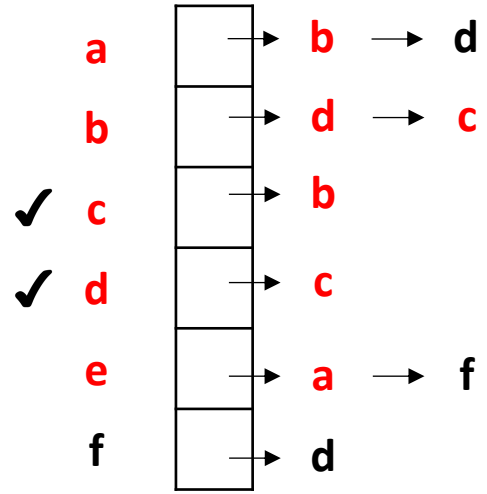
G	G	G	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

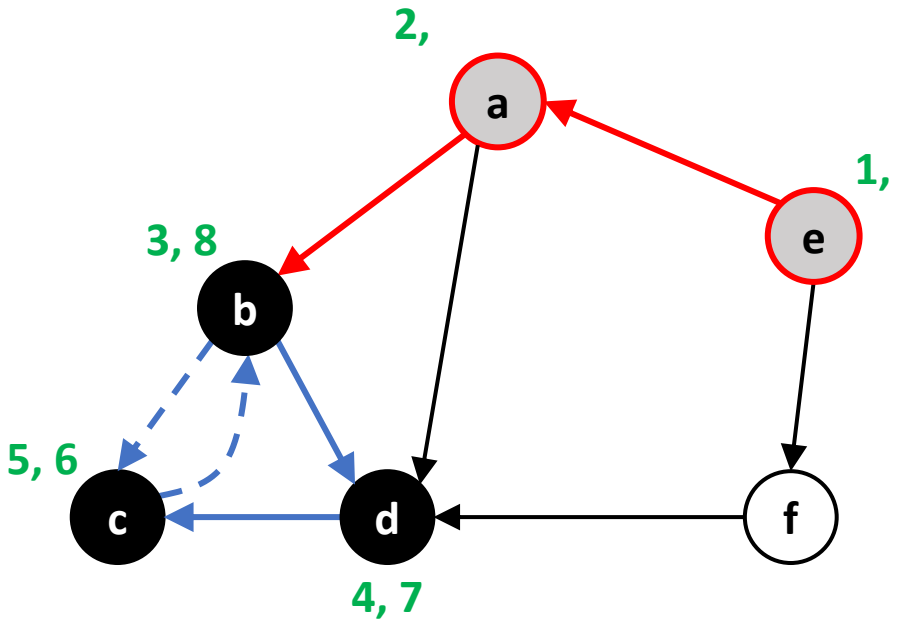


Colors of nodes:

G	G	G	B	B	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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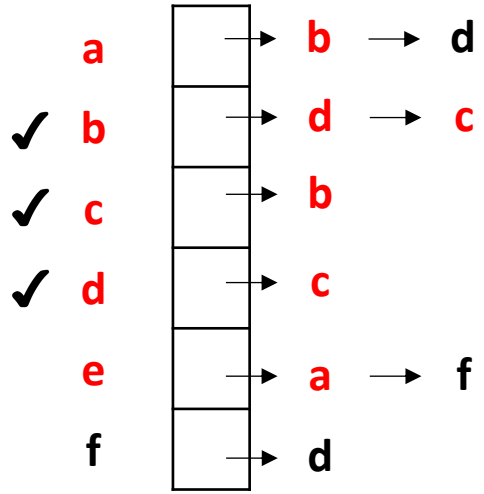
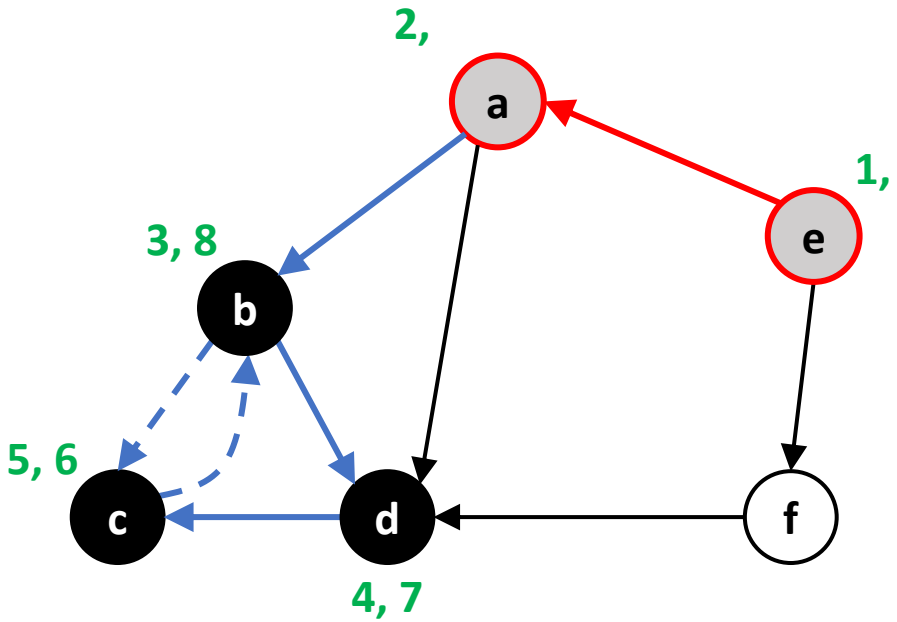


Colors of nodes:

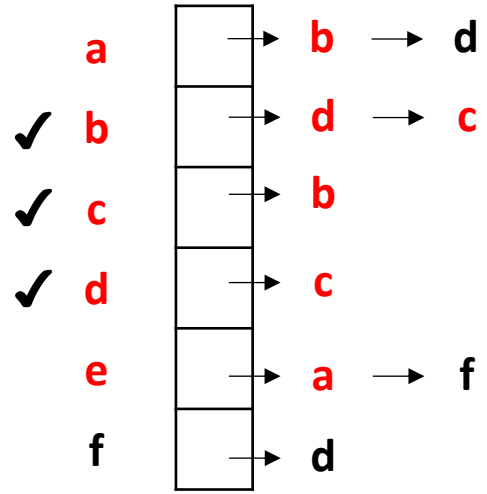
G	G	B	B	B	W
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

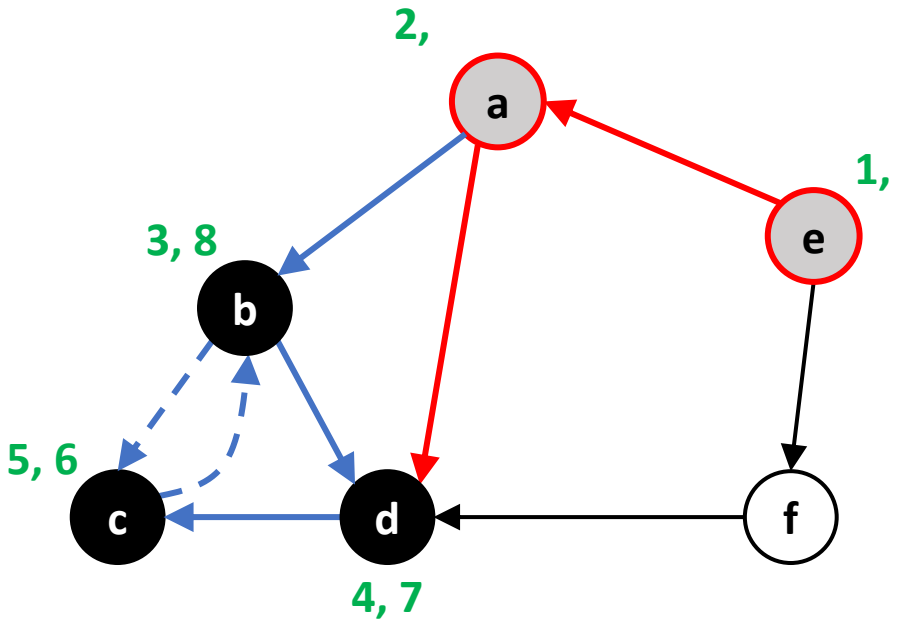


Colors of nodes:

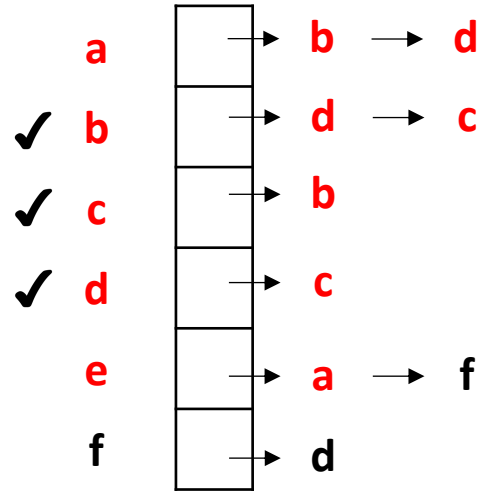
G	G	B	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

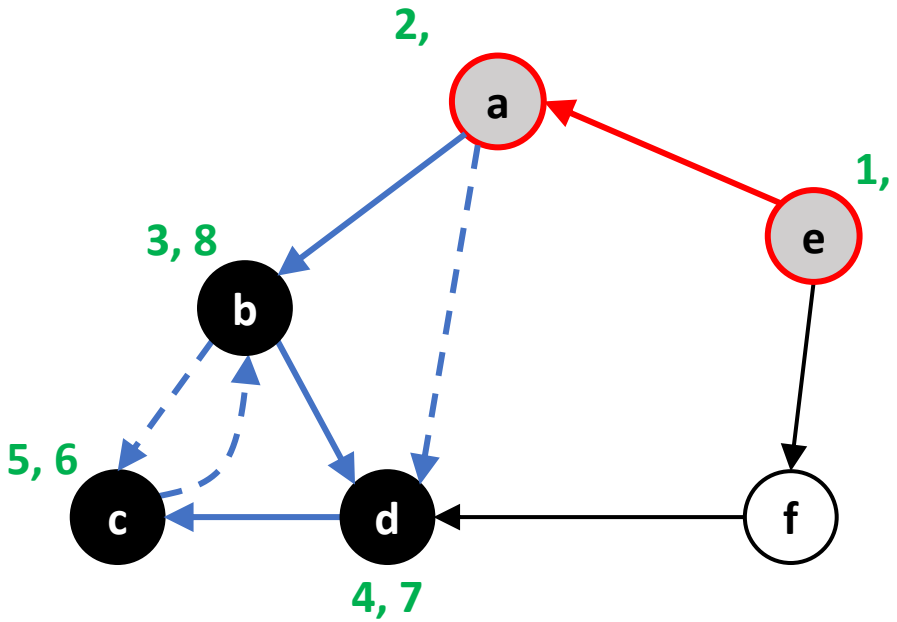


Colors of nodes:

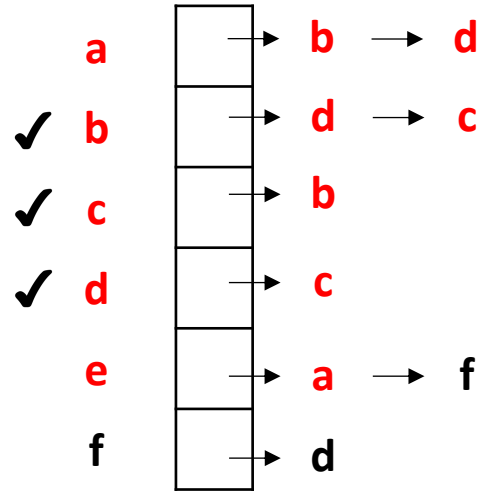
G	G	B	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

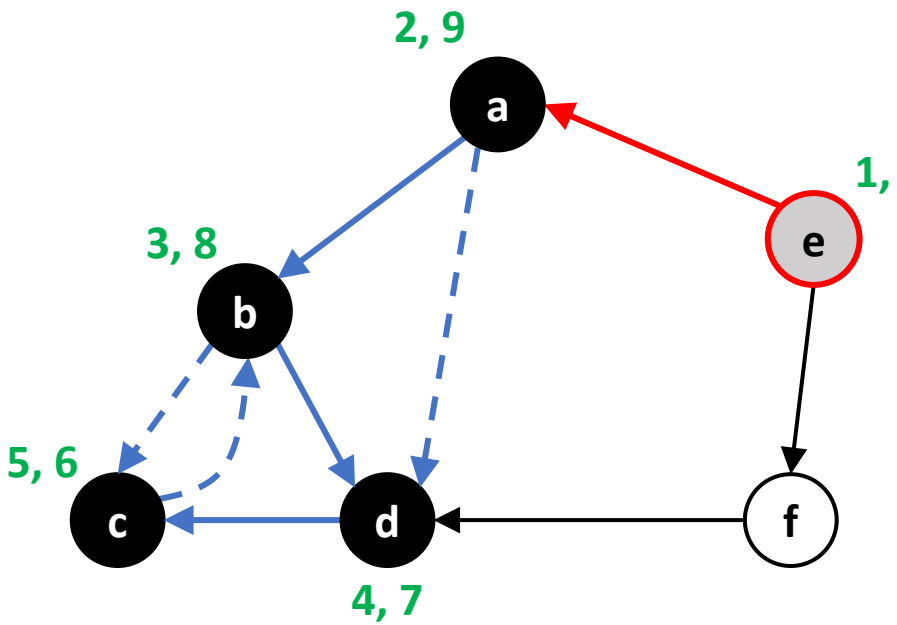


Colors of nodes:

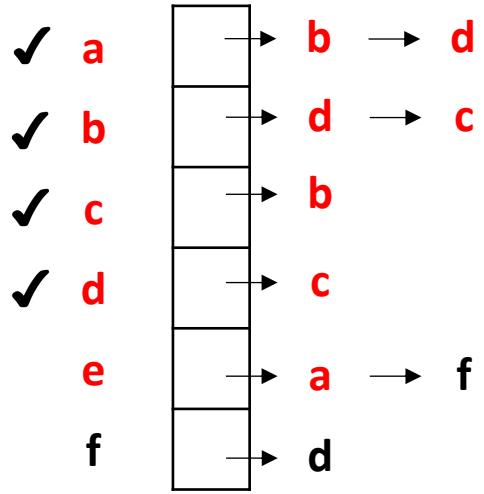
G	G	B	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

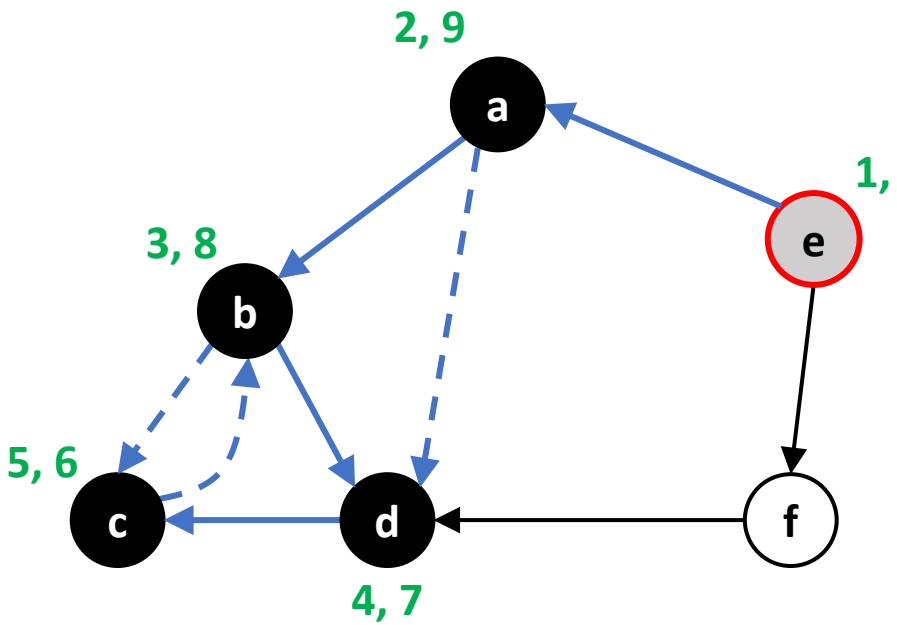


Colors of nodes:

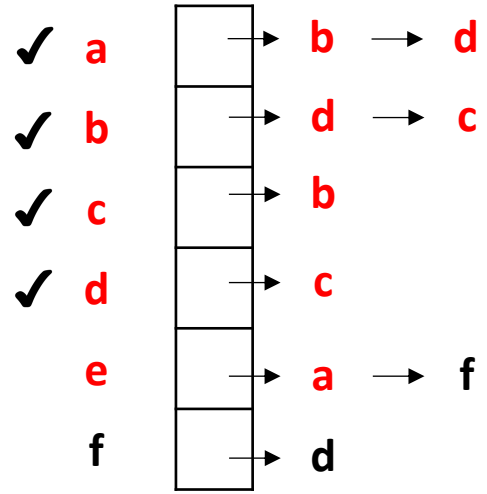
G	B	B	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

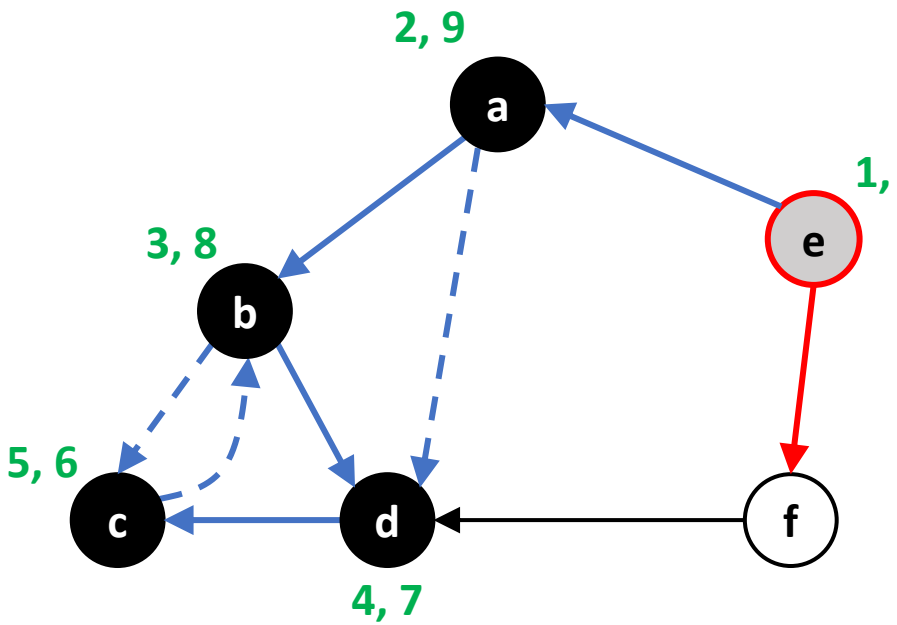


Colors of nodes:

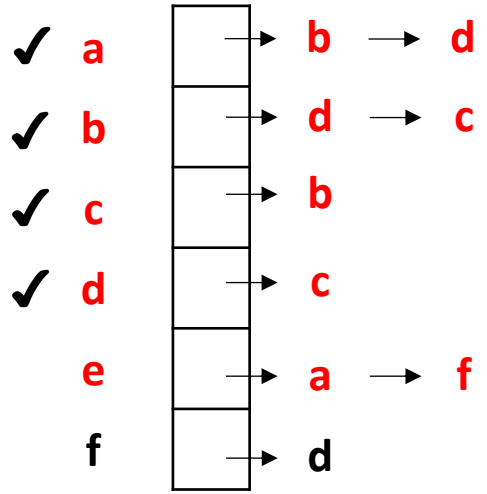
G	B	B	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

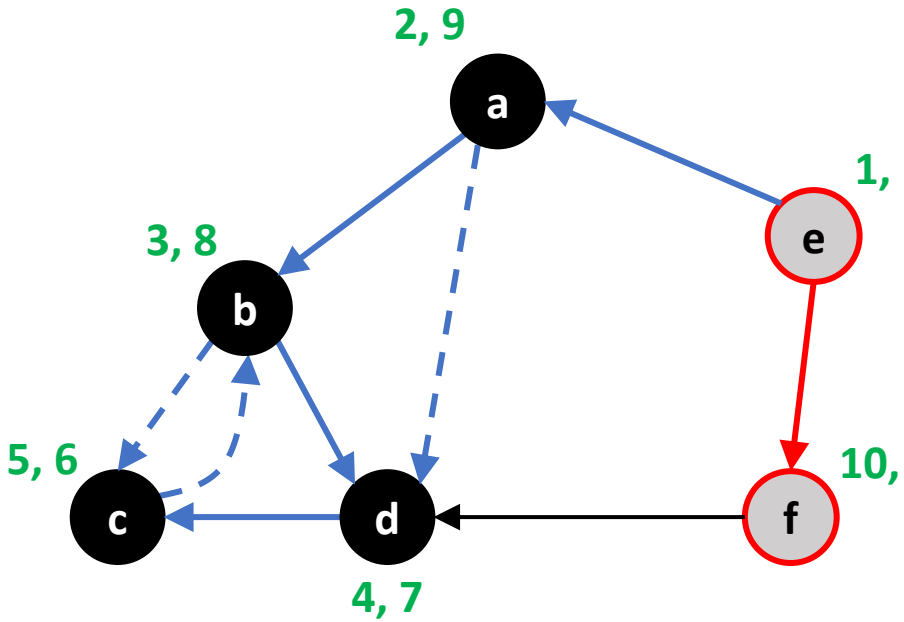


Colors of nodes:

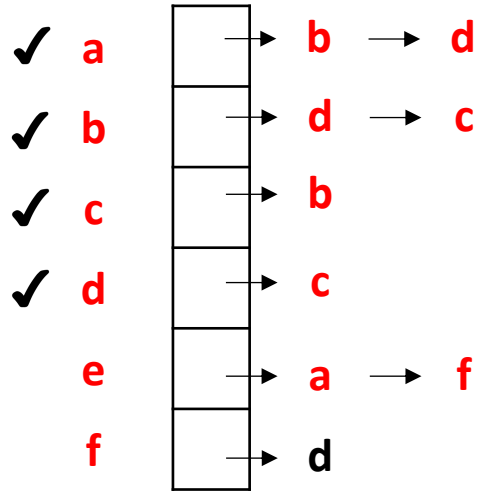
G	B	B	B	B	W
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

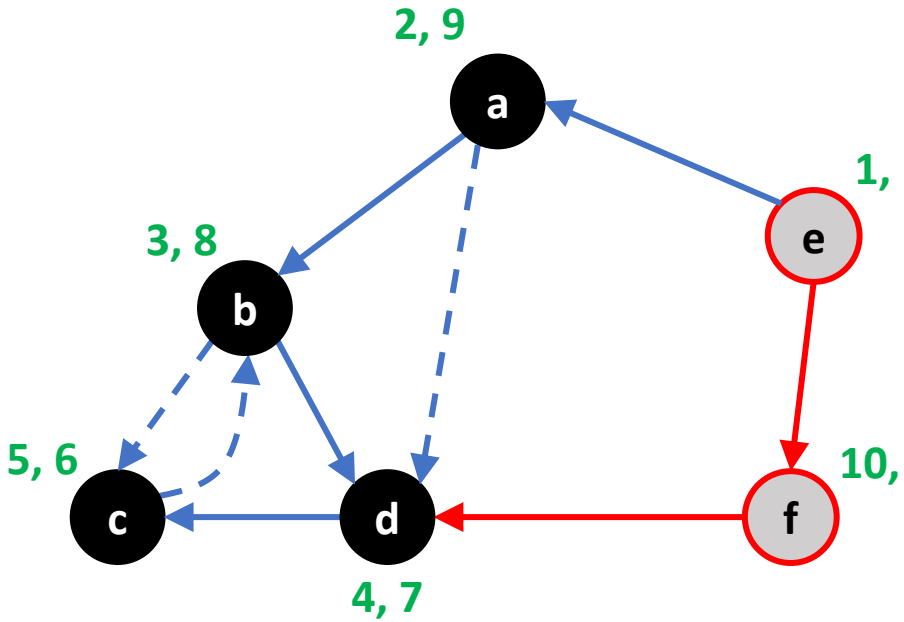


Colors of nodes:

G	B	B	B	B	G
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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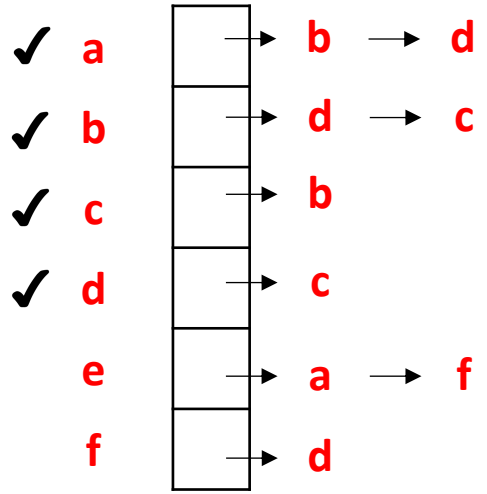
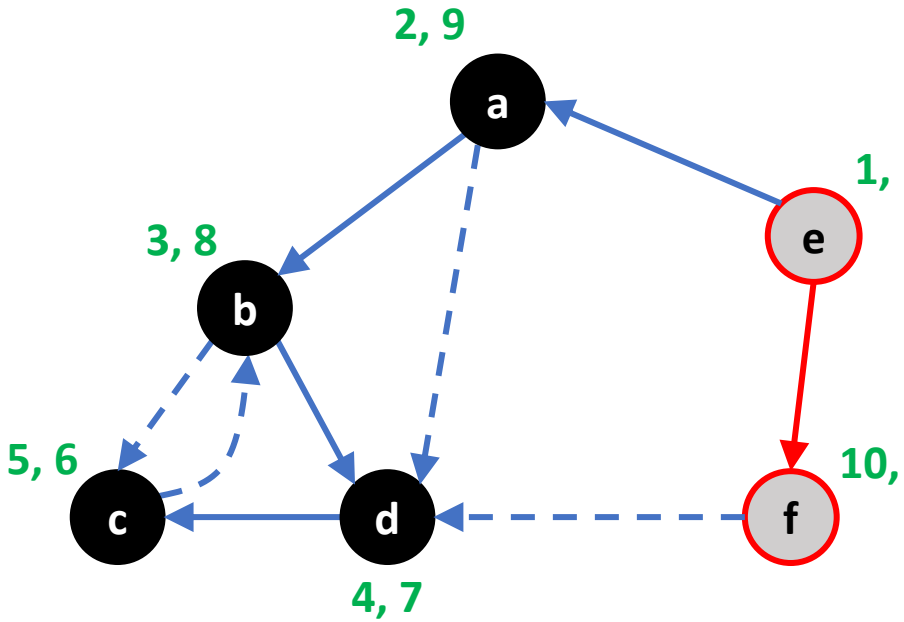


Colors of nodes:

G	B	B	B	B	G
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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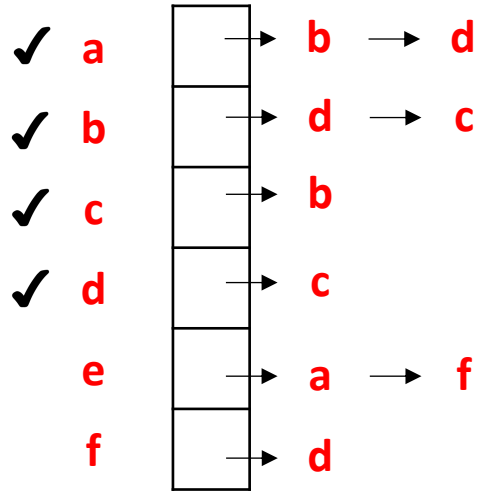
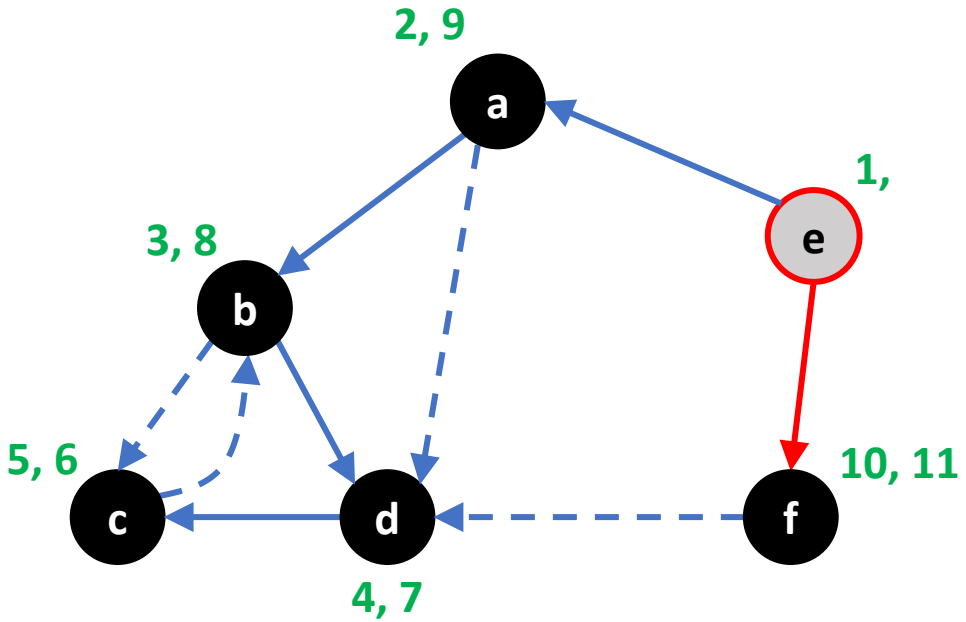


Colors of nodes:

G	B	B	B	B	G
e	a	b	c	d	f

DFS-Explore(G, **e)**

DFS(G)



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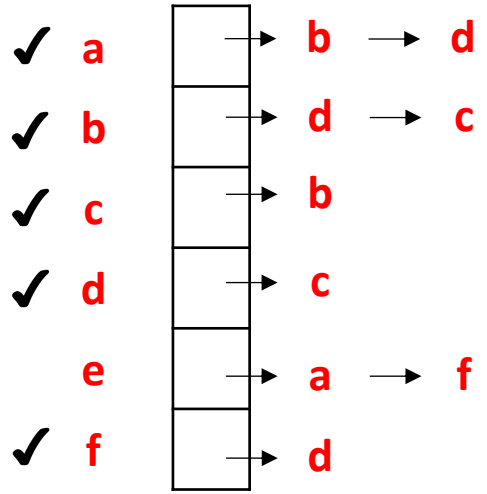
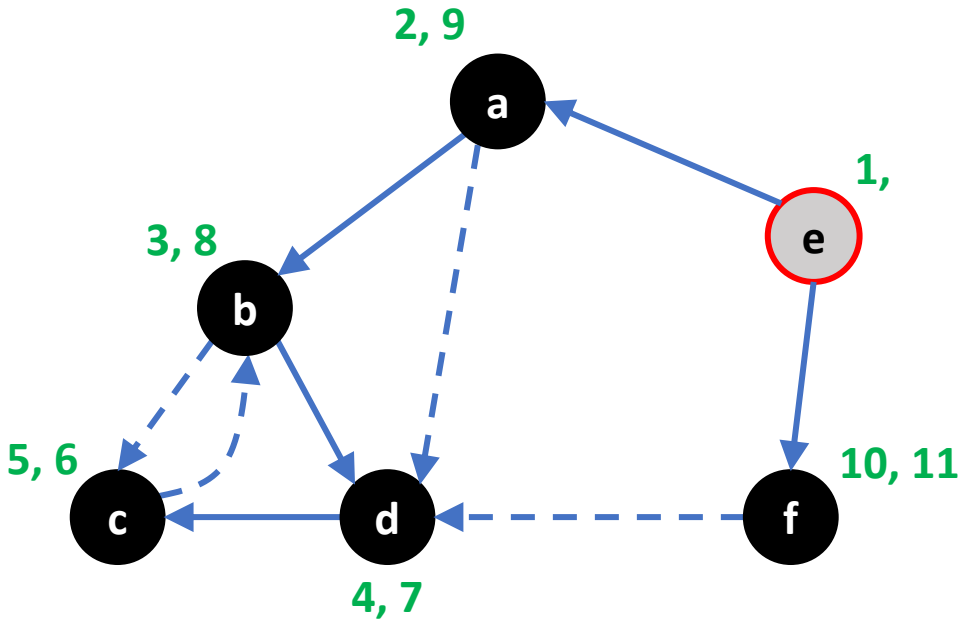


Colors of nodes:

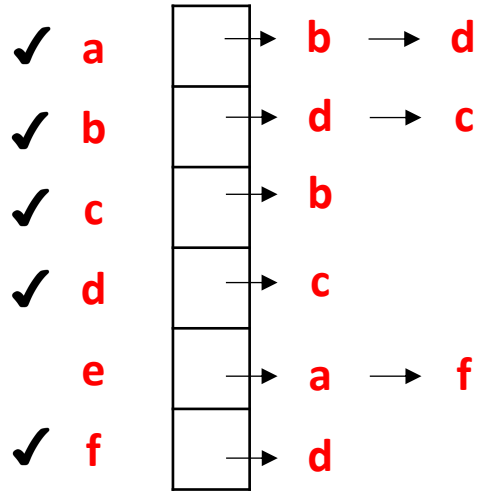
G	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

DFS(G)



Adj List of G :

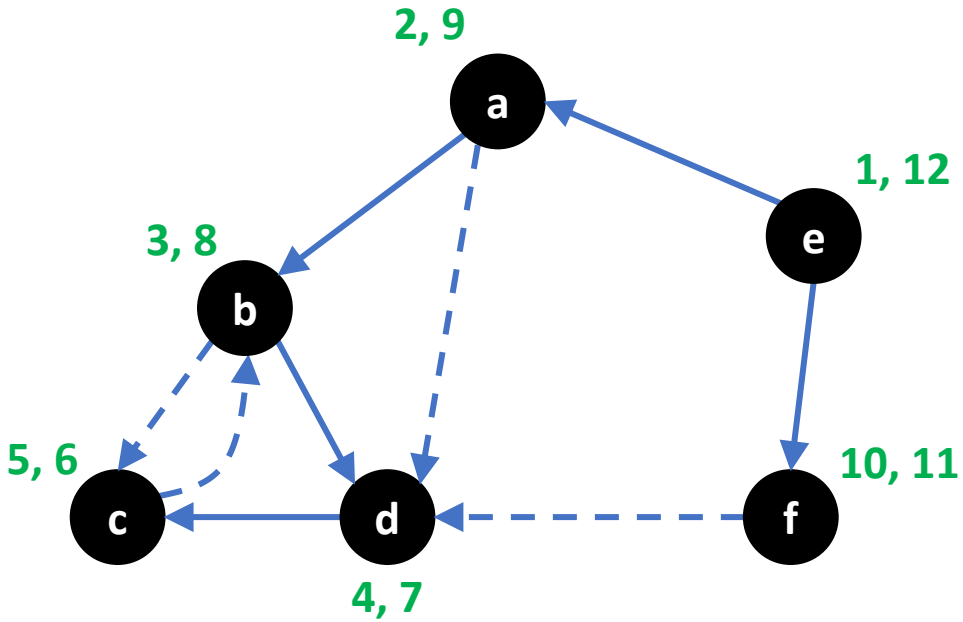


Colors of nodes:

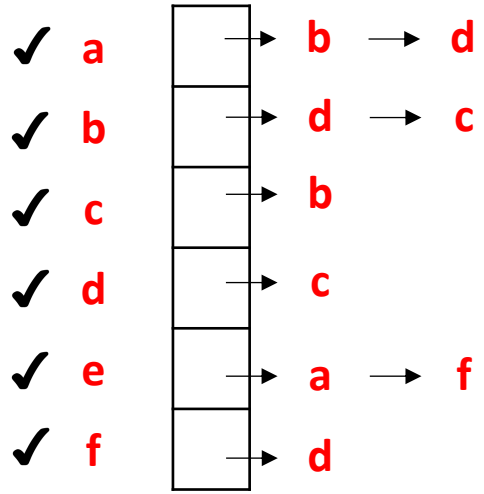
G	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS-Explore(G, **e)**

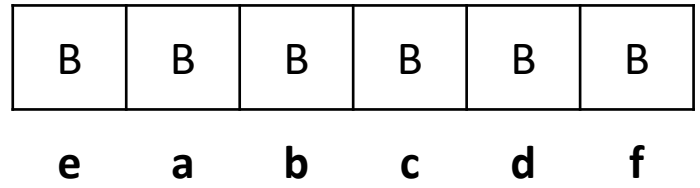
DFS(G)



Adj List of G :

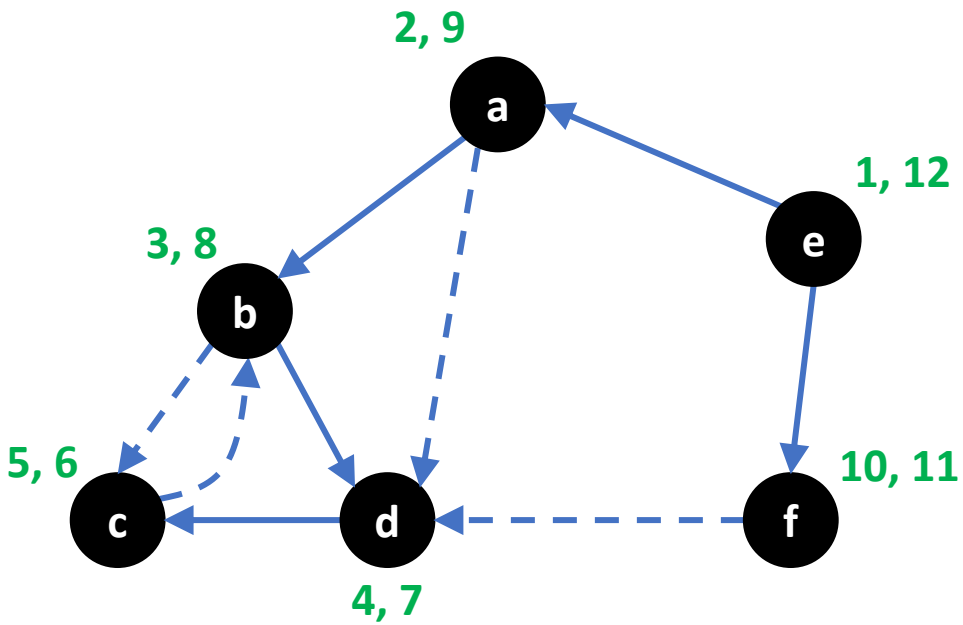


Colors of nodes:



DFS-Explore(G, **e)**

DFS(G)



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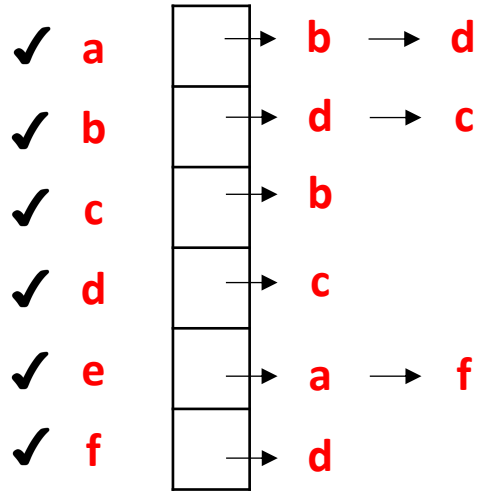
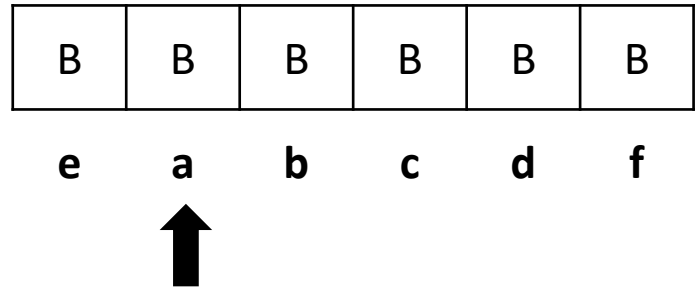
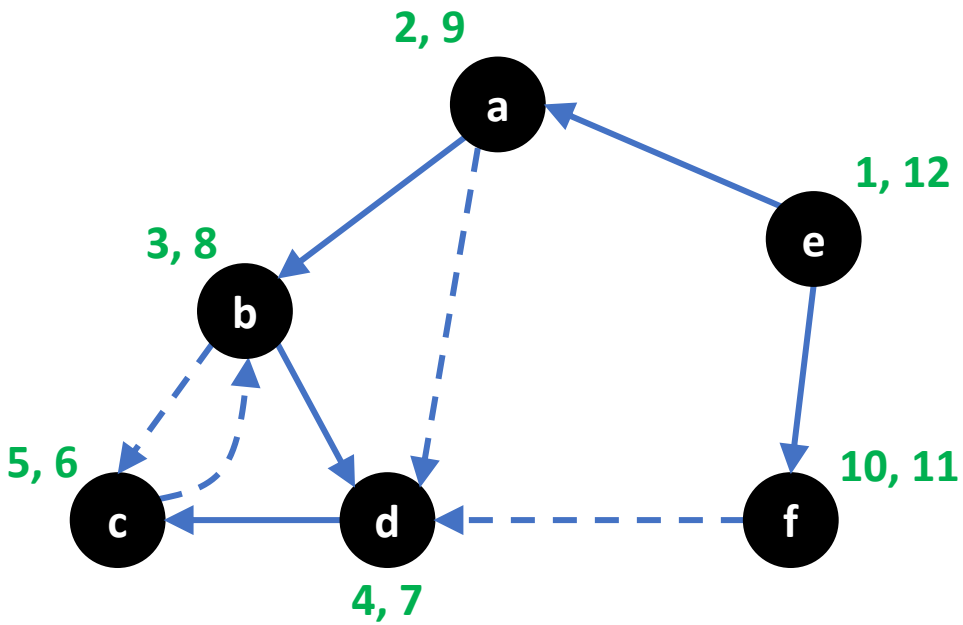


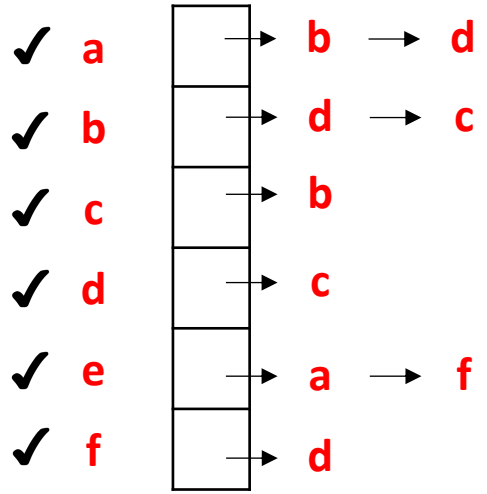
Colors of nodes:



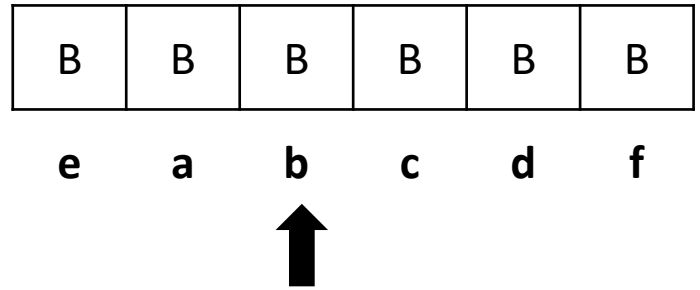
DFS(G)



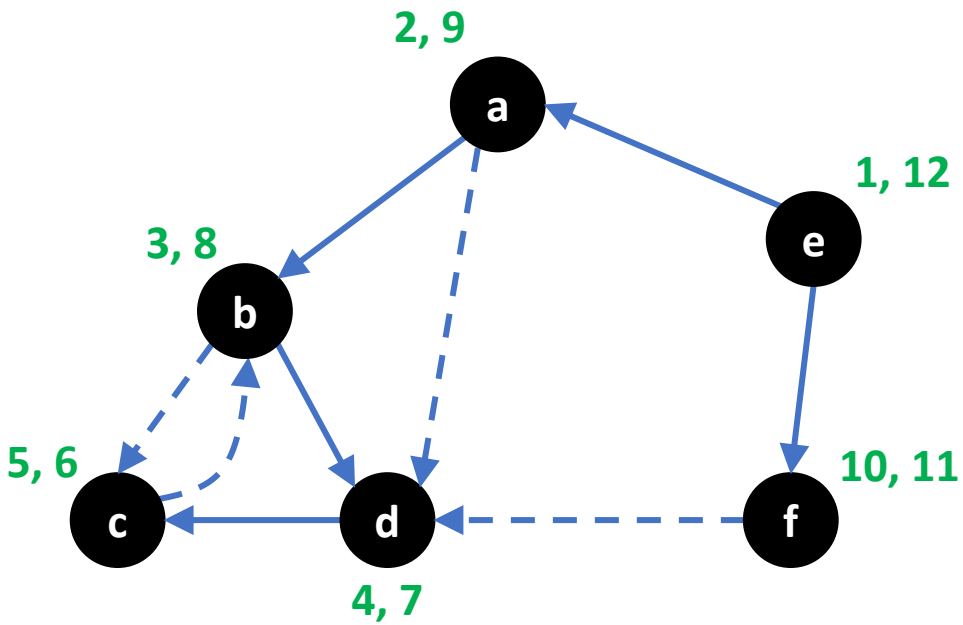
Adj List of G :



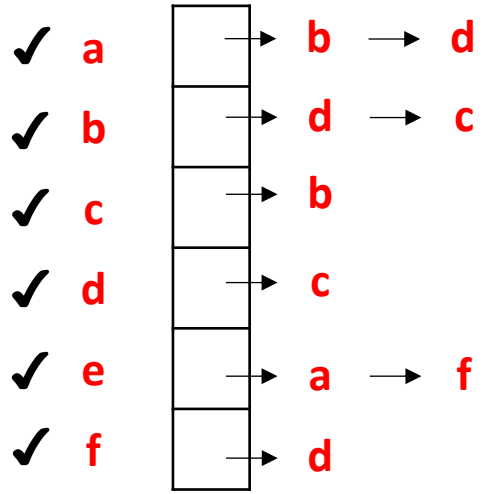
Colors of nodes:



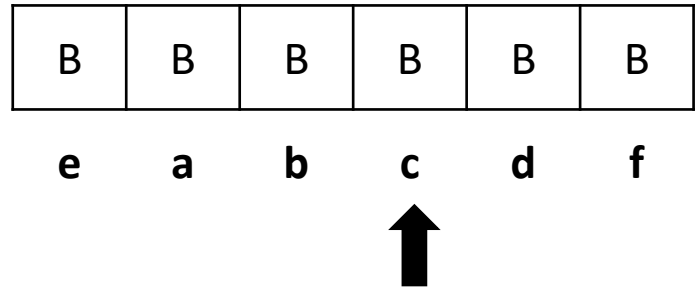
DFS(G)



Adj List of G :



Colors of nodes:



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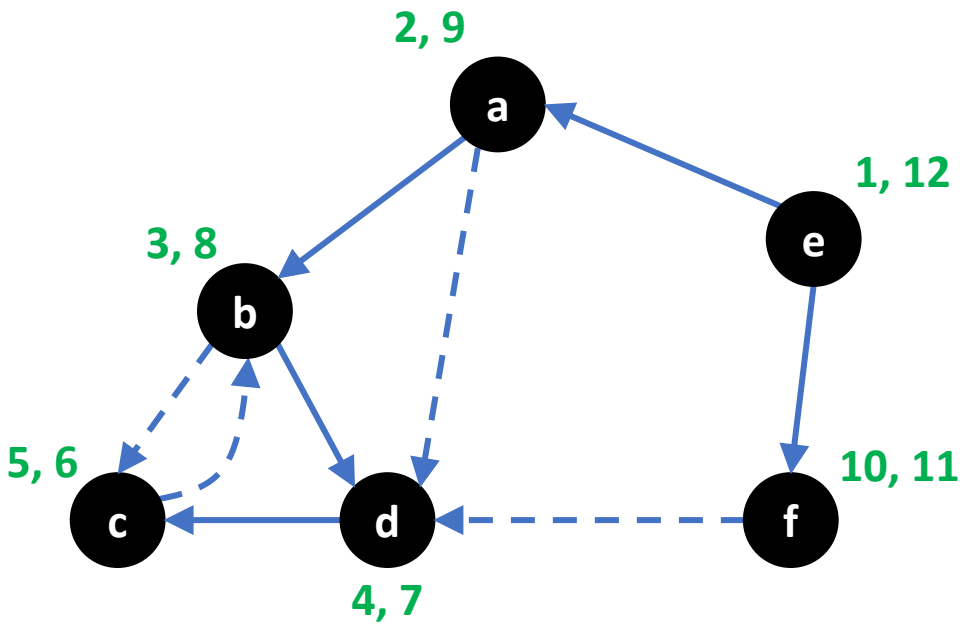
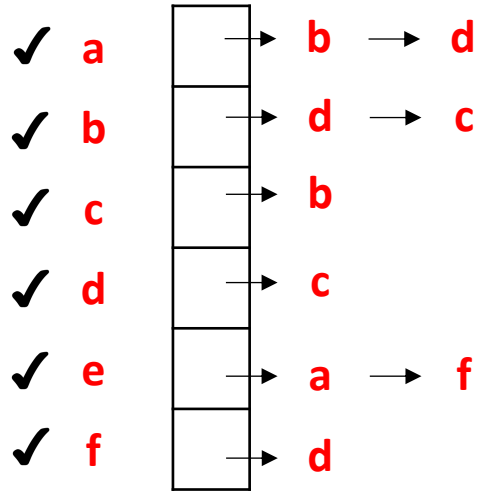
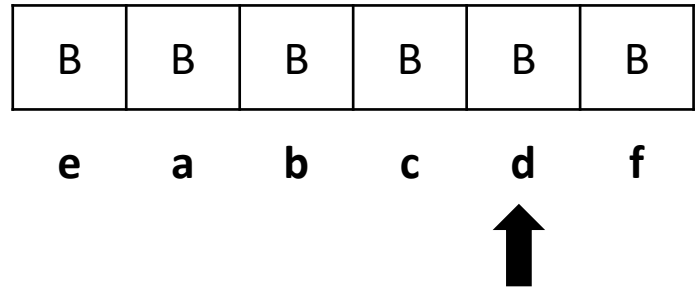


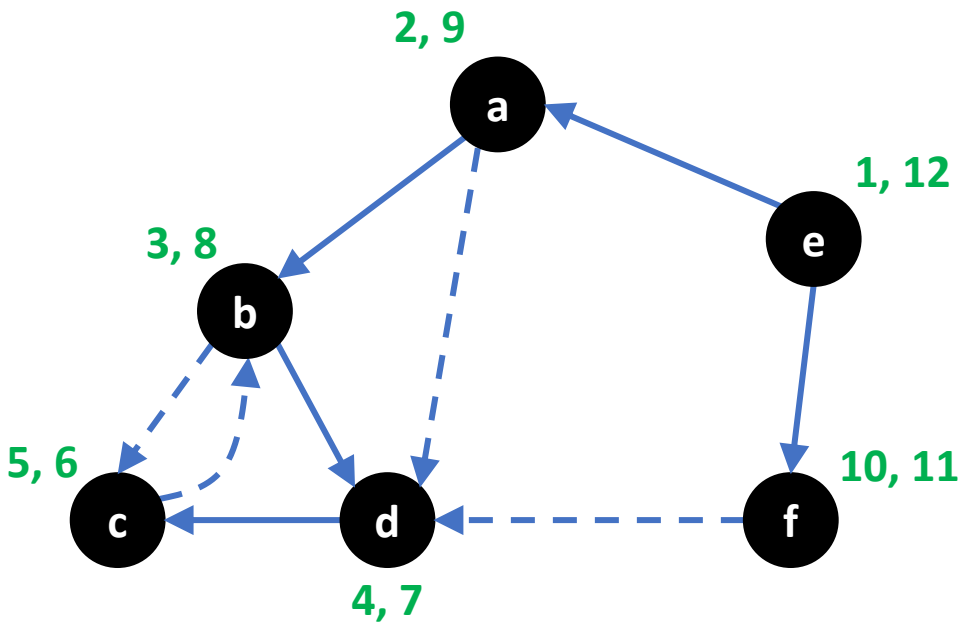
Adj List of G :



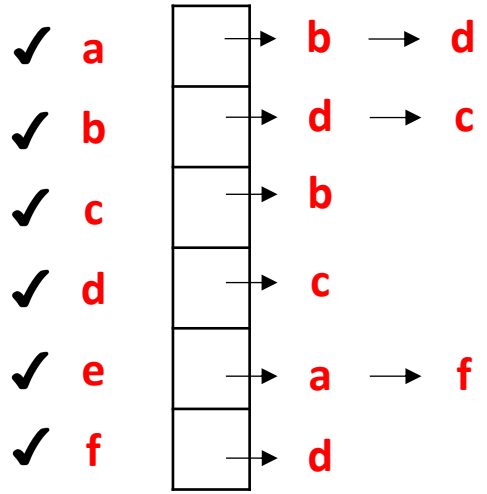
Colors of nodes:



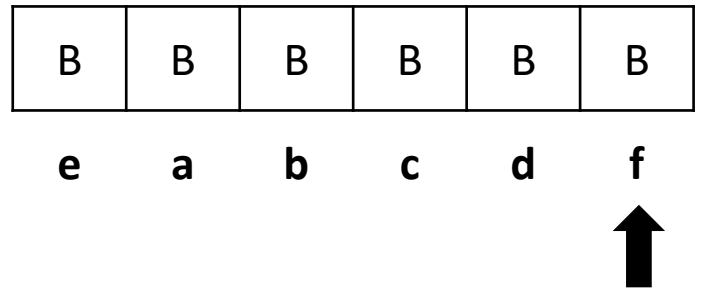
DFS(G)



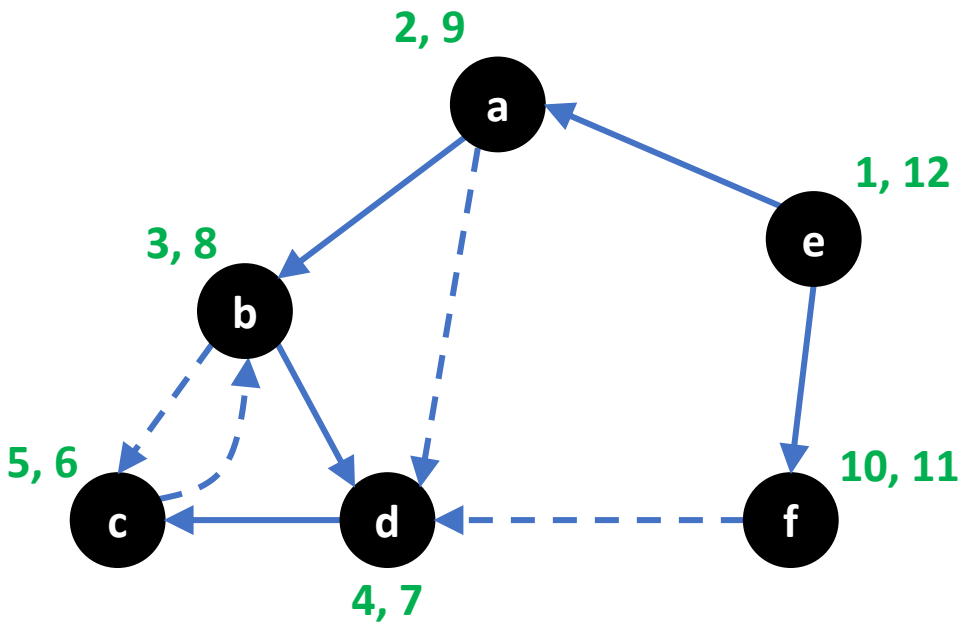
Adj List of G :



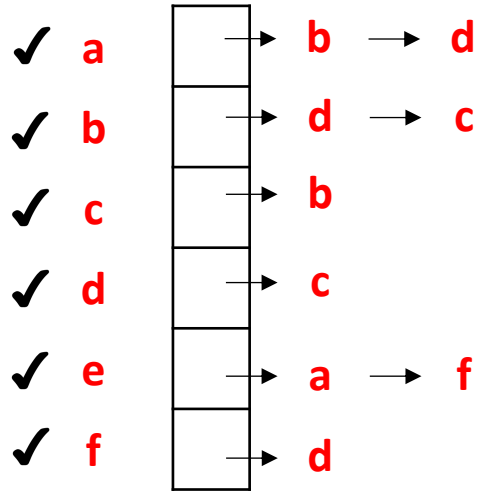
Colors of nodes:



DFS(G)



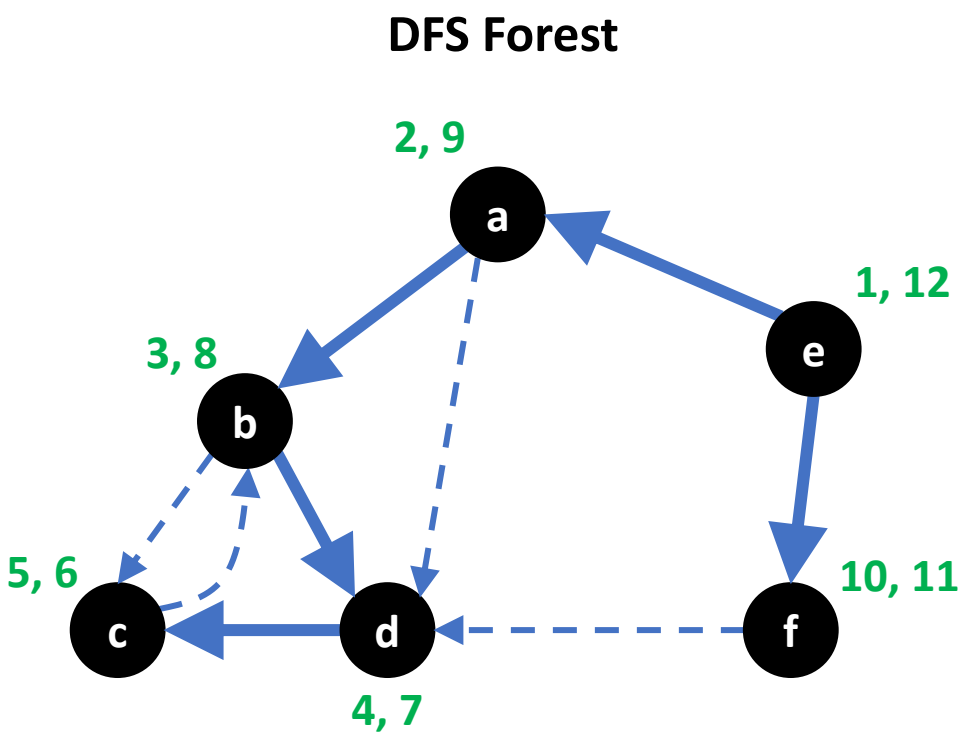
Adj List of G :



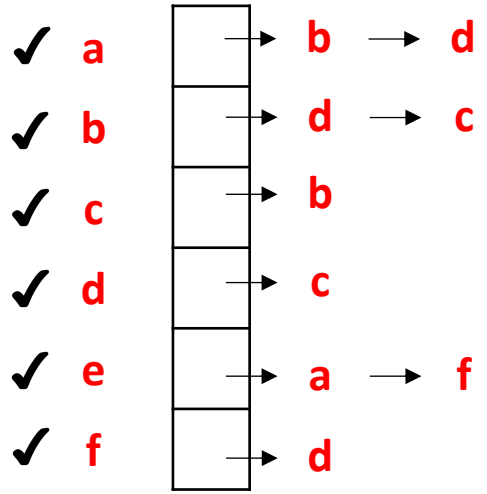
Colors of nodes:

B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS(G)



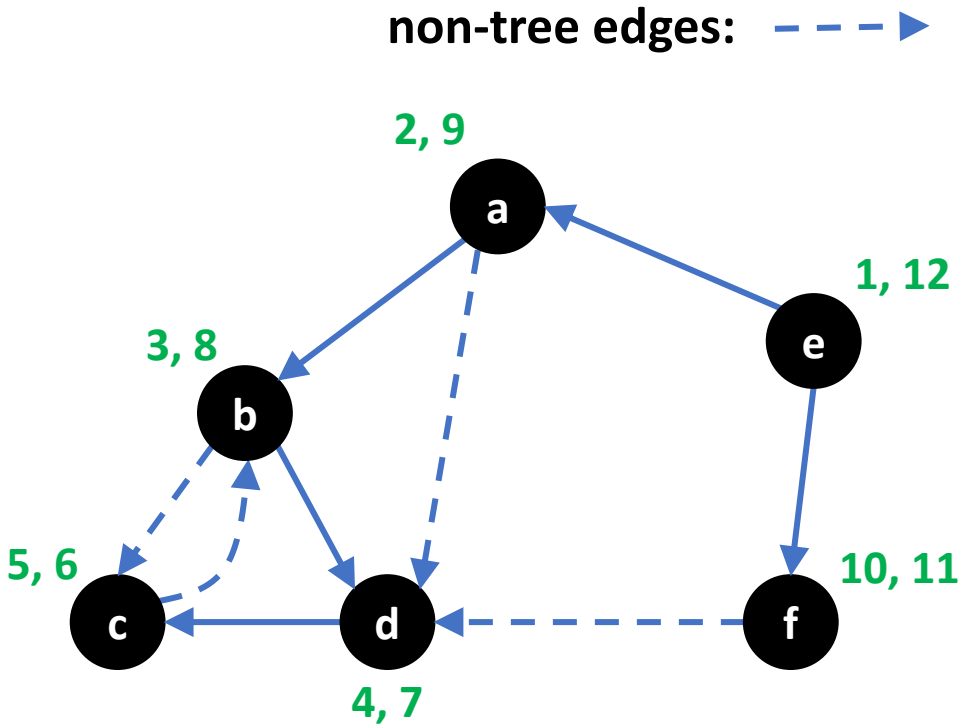
Adj List of G :



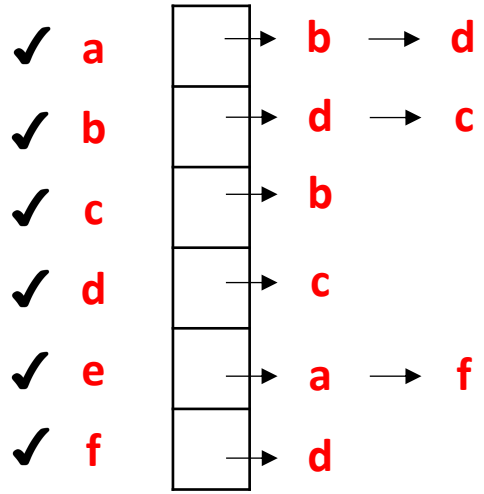
Colors of nodes:

B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS(G)



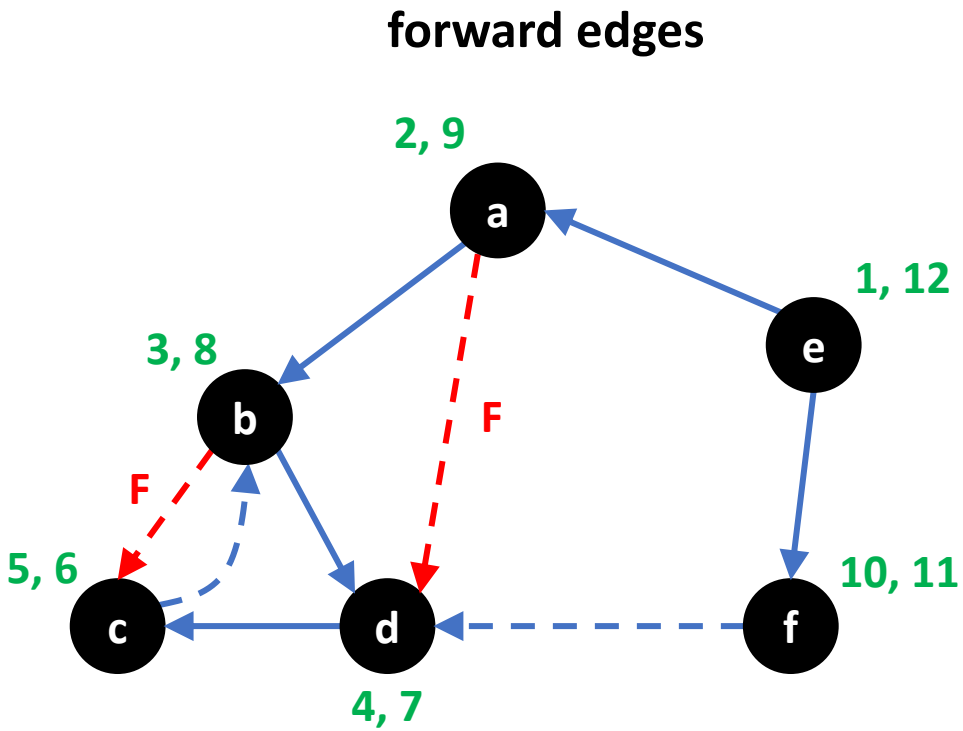
Adj List of G :



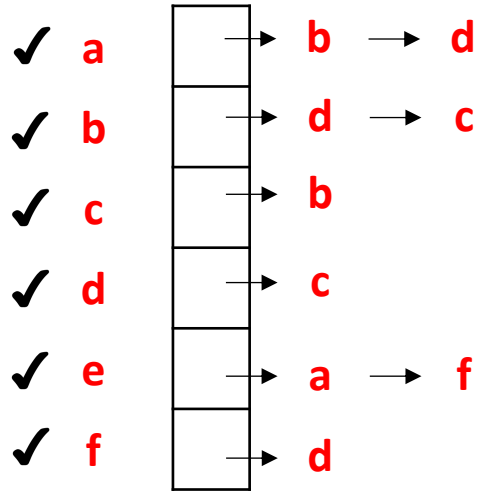
Colors of nodes:

B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS(G)



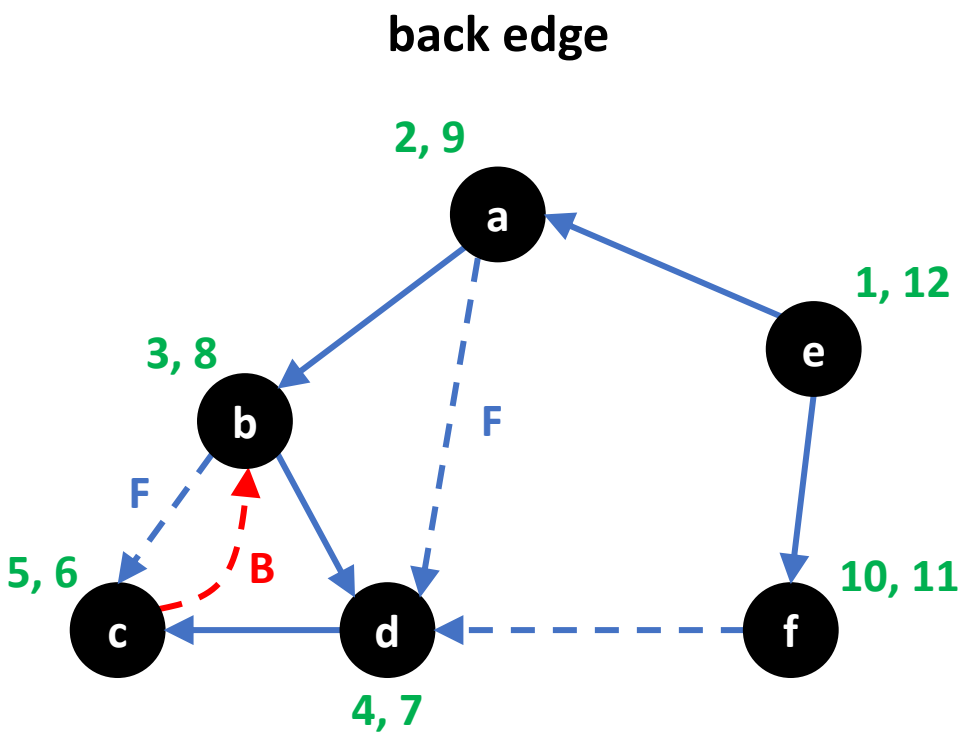
Adj List of G :



Colors of nodes:

B	B	B	B	B	B
e	a	b	c	d	f

DFS(G)



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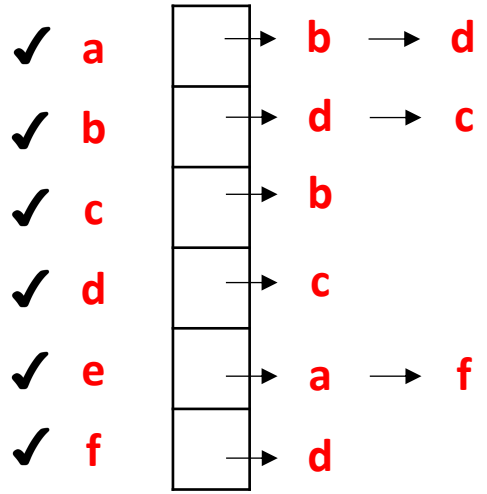
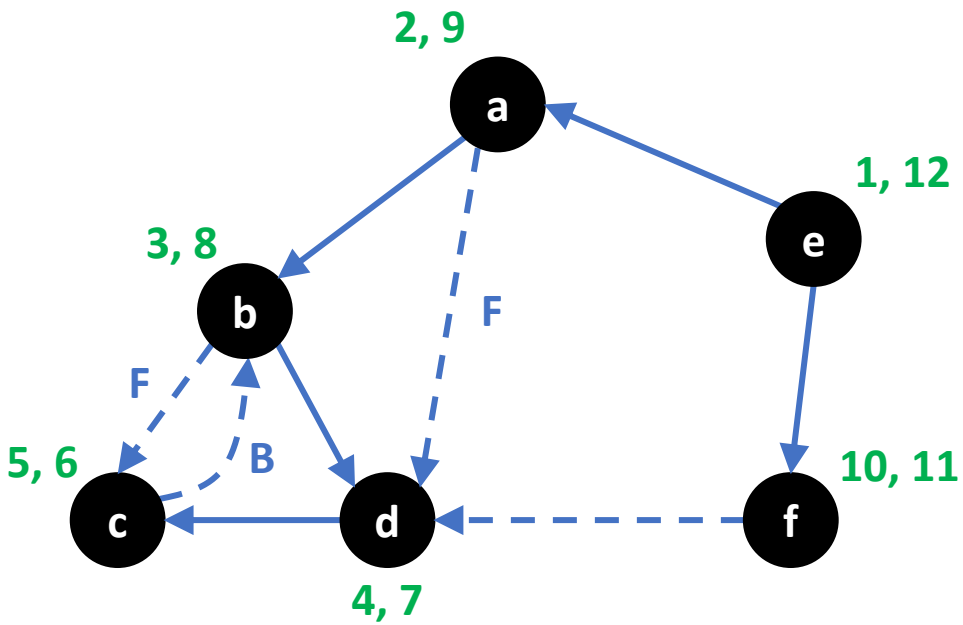


Colors of nodes:

B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS(G)



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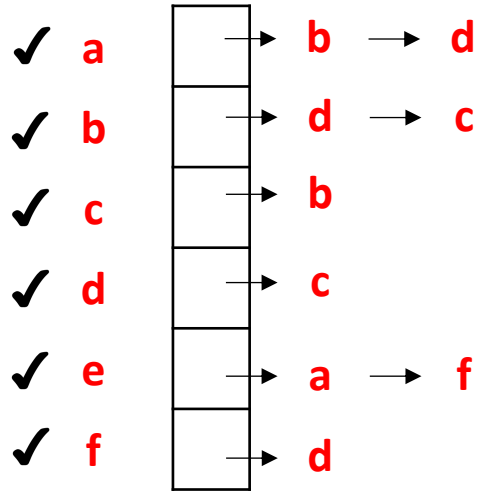
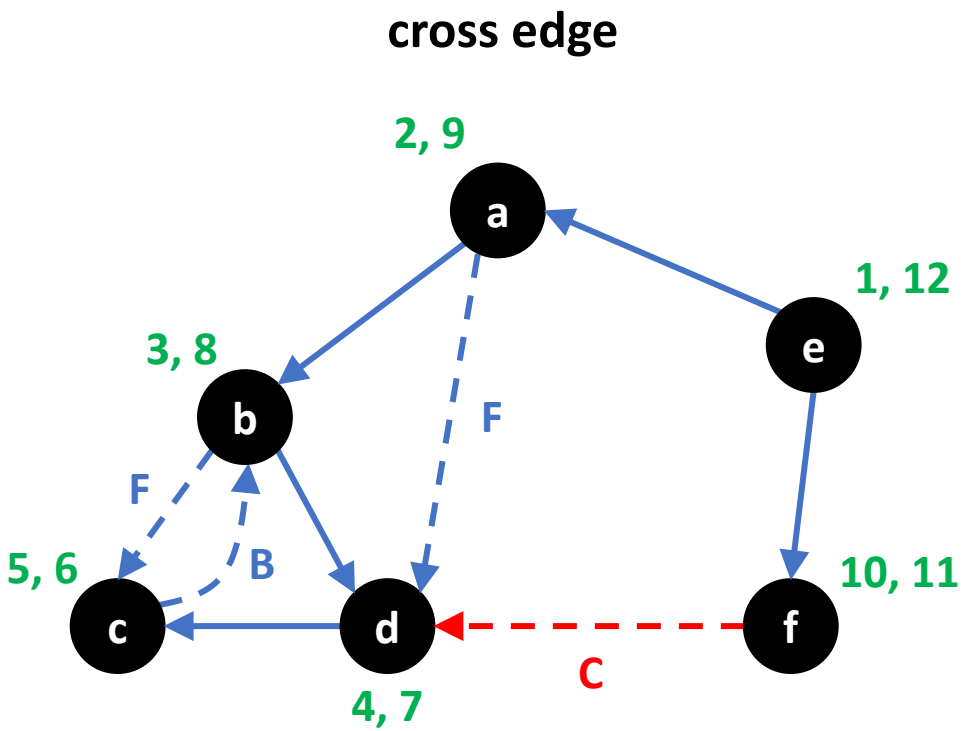


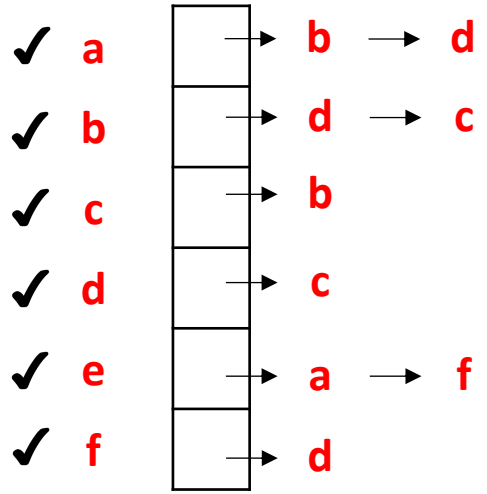
Colors of nodes:

B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS(G)



Adj List of G :

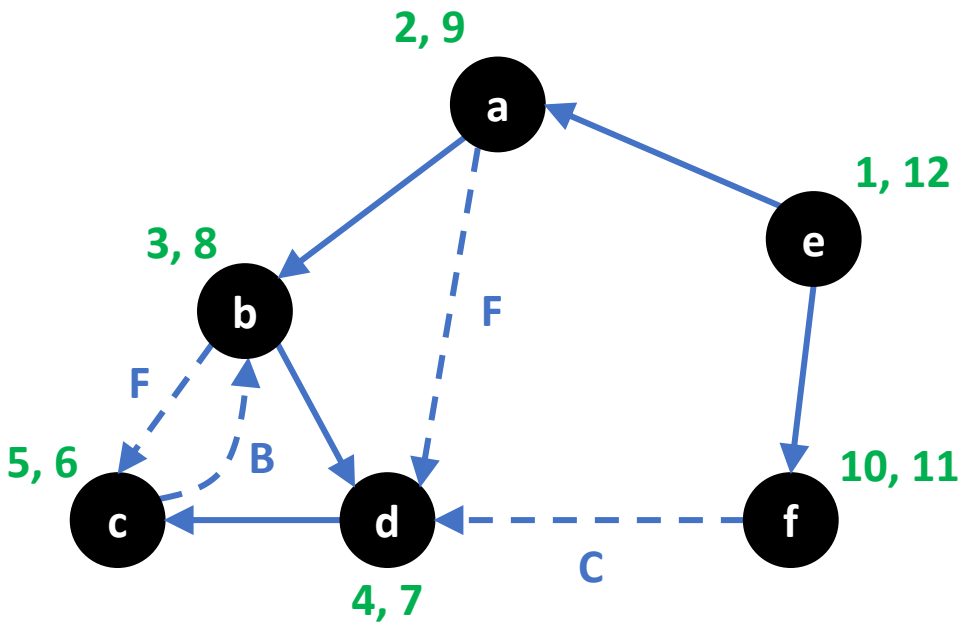


Colors of nodes:

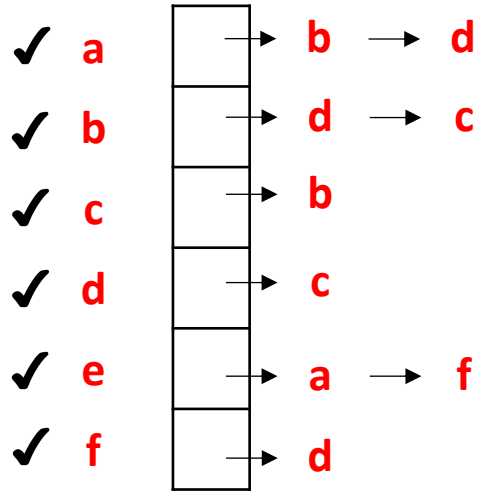
B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**

DFS(G)

DFS Forest and non-tree edges



Adj List of G :



Colors of nodes:

B	B	B	B	B	B
e	**a**	**b**	**c**	**d**	**f**