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| Woodbury Matrix Identity | $(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}$ |
| Sherman Morrison Formula | Given <i>invertible</i> $A \in R^{n \times n}$ and $u, v \in R^n$ vectors. Then $A + uv^T$ is invertible iff $1 + v^T A^{-1}u \neq 0$. Then $(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^T A^{-1}}{1 + v^T A^{-1}u}$ |
| Cauchy Schwarz Inequality | $ \langle U, V \rangle \leq \langle U, U \rangle \cdot \langle V, V \rangle$ |
| Markov Inequality | X is a non-negative rv and $a > 0$ then $P(X \geq a) \leq \frac{E(X)}{a}$ |
| Minkowski inequality | $\ f + g\ _p \leq \ f\ _p + \ g\ _p$ |
| Holder Inequality | Let $p, q \in [1, \infty], 1/p + 1/q = 1$. Then for all real/complex functions $f, g, \ fg\ _1 \leq \ f\ _p \ g\ _q$ |
| Jensen Inequality | Given X is a rv and ϕ is convex. Then $\phi(E(X)) \leq E(\phi(X))$ |
| Johnson Lindenstrauss Property | A distribution on matrices $S \in R^{k \times n}$ has (ϵ, δ, l) -JL moment property if $\forall x \in R^n, x _2 = 1, E_S(Sx _2^2 - 1 ^l) \leq \epsilon^l \delta$ |
| Hoeffding Inequality | Let X_1, \dots, X_n be independent rv in $[0, 1]$. Let $\bar{X} = \frac{1}{n} \sum_i X_i$. Then $P(\bar{X} - E(\bar{X}) \geq t) \leq \exp(-2nt^2)$ |
| Chebyshev Inequality | $P(X - E(X) \geq a) \leq \frac{Var(X)}{a^2}$ |
| Golden Thompson | For Hermitian matrices A, B $tr \exp(A + B) \leq tr(\exp(A) \exp(B))$ |
| Oppenheimer Inequality | For PSD matrices A, B , $\det(A \circ B) \geq (\prod a_{ii}) \det(B)$ |
| Ky Fan Matrix Inequality | Let A, B be $n \times n$ Hermitian matrices. Then $\lambda(A) + \lambda(B) \succ \lambda(A + B)$, where \succ denotes the majorization relation. |
| McDiarmid Inequality | Multidimensional Hoeffding. Let X_1, \dots, X_m be independent rv taking values in χ . Further, let $f : \chi^m \mapsto \mathbb{R}$ be a function of X_1, \dots, X_m that satisfies coordinate-wise bounded differences, $\forall i \forall x_1, \dots, x_m, x_i^* \in \chi, f(x_1, \dots, x_i, \dots, x_m) - f(x_1, \dots, x_i^*, \dots, x_m) \leq c_i$. Then $\mathbb{P}(f - \mathbb{E}(f) \geq \epsilon) \leq \exp\left(\frac{-2\epsilon^2}{\sum_{i=1}^m c_i^2}\right)$. |
| Cortes Sampling Concentration Inequality | Let $X_i, i \in [m]$ be a set of drawn <i>without replacement</i> from an underlying finite set of $m + u$ elements. Let $f : \chi^m \mapsto \mathbb{R}$ be a function of X_1, \dots, X_m be symmetric (up to permutation of parameters) and obey coordinate-wise bounded differences, then $\mathbb{P}(f - \mathbb{E}(f) \geq \epsilon) \leq \exp\left(\frac{-2\epsilon^2}{\alpha(m, u)c^2}\right)$, where $\alpha(m, u) = \frac{mu}{m+u-0.5} \times \frac{1}{1-1/(2\max(m, u))}$. |
| Simple Gaussian Concentration Inequality | Let X_1, \dots, X_n be iid $(0, 1)$ Gaussian rv. Let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be 1-Lipschitz. Then for any $\lambda > 0$, $\mathbb{P}(f(X) - \mathbb{E}(f(X)) \geq \lambda) \leq C \exp(-c\lambda^2)$ for constants c, C . |
| Bounds for central term in binomial coefficient | $\frac{4^n}{2n+1} \leq \binom{2N}{N} \leq 4^n$ |

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| Generating function of Catalan Numbers | $G(z) = \sum_{i=0}^{\infty} \frac{1}{n+1} \binom{2n}{n} z^i = \frac{1-\sqrt{1-4z}}{2z} = \frac{2}{1+\sqrt{1-4z}}$ |
| Weyl Matrix Perturbation Inequality | If $M = H + P$ are $n \times n$ matrices with eigenvalues μ_i, ν_i, ρ_i (ordered in descending order) respectively, then $\nu_i + \rho_n \leq \mu_i \leq \nu_i + \rho_1$ |
| Walds Identity | Let X_1, \dots be a sequence of (potentially infinite) independent random variables with identical means. Let N be any <i>stopping time</i> (or independent of X s). Let $S_N = \sum_{i=1}^N X_i$. Then $\mathbb{E}(S_N) = \mathbb{E}(N)\mathbb{E}(X)$. Note N can be weakly dependent on X . |
| Variance of Random Sum | If N and X_i are independent and X s have equal variance, $\text{Var}(\sum_{i=1}^N X) = \mathbb{E}(N)\text{Var}(X_i) + \mathbb{E}(X)^2\text{Var}(N)$ |
| Eckart-Young-Mirsky Theorem | The k-truncated SVD of a matrix A gives the best rank-k approximation for A in the spectral and frobenius norm. $A_k = U\Sigma_k V^T = \argmin_{B \text{ rank-k}} \ A - B\ _2 = \sigma_{k+1}(A)$ |
| Wolfe Conditions | (i) (Armijo rule) $f(\mathbf{x}_k + \alpha_k \mathbf{p}_k) \leq f(\mathbf{x}_k) + c_1 \alpha_k \mathbf{p}_k^T \nabla f(\mathbf{x}_k)$ (ii) (Curvature) $-\mathbf{p}_k^T \nabla f(\mathbf{x}_k + \alpha_k \mathbf{p}_k) \leq -c_2 \mathbf{p}_k^T \nabla f(\mathbf{x}_k)$, where α_k is step size, \mathbf{p}_k is descent direction and $0 < c_1 < c_2 < 1$ |