Homework 8

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1. 8.1 (a)
   For each x_i, there is 10 possibilities, so $|C| = \mathit{10}^n$.

   (c1)
   Pseudocode:

   Construction of configuration space.
   0 **for** each configuration a_i in C_m.

2. 8.2 transitivity of karp reduction
   It is easy to showthat if $L_1 \preccurlyeq L_2 \preccurlyeq L_3$ then $L_1 \preccurlyeq L_3$.
   proof: Knowing $(\forall x \in \Sigma_1^*)(x \in L_1 \iff f(x) \in L_2)$
   $(\forall x \in \Sigma_2^*)(x \in L_2 \iff g(x) \in L_3)$

   NTW $(\forall x \in \Sigma_1^*)(x \in L_1 \iff f(x) \in L_3)$

   we have
   $$g(f(x)) : \Sigma_1^* \to \Sigma_3^*$$
   for all $x \in \Sigma_1^*$, we have
   $$x \in L_1 \iff f(x) \in L_2 \iff g(f(x)) \in L_3$$
   The above two statements define the karp reduction from L_1 to L_3.

   The exponent of the reduction of L_1 to L_3 is cd. In other words, the Karp reduction f can be computed in time $O(n^{c+d})$.

3. 8.4
   3-colorability of graphs and RSA decryption. Breaking RSA is not a decision problem as 3-colorability is. However, the question that does integer n have a factor smaller than k is a decision problem. We can use binary search to find the factor of n in polynomial time by asking the decision question mentioned

4. 8.7 Clay mathematics institue carries the prize the correctio solution for the P versus NP problem.

5. 8.1 (b) extra credit
   This problem can be thought of unordered sampling with replacement since the monotonicity removes the ordering from part (a). After sampling with replacement from 0 9, we remove the sampling and fill each x_i using the monoticity rule. The total number of ways to do this is $\binom{n+k-1}{k}$ (from probability thoery knowledge). In this problem $k = n$ and $n = 10$. So the unordered sampling has $\binom{n+9}{n}$ ways to do the unordered sampling.