UNIVERSITY OF BATH

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MSc. in Modern Applications of Mathematics

The Dynamics of Crowds

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Abstract

Crowds pose an interesting example of a complex system in which emergent behaviour is observed out of the interaction of many individual agents. This behaviour can be very important in the safe design of sports and other stadia, especially in the case of possible emergency situations. For this reason crowd dynamics is of great interest to architects. The purpose of this project is to review the current literature on crowd dynamics and to look at some simulations of different types of crowd behaviour in certain specific situations namely: the meeting of two crowds in a 'scramble crossing', the motion of a crowd though an exit and the response of a crowd to an emergency (such as a fire). The student will implement a differential equation model of the crowd crossing problem. In particular, they will study the emergent behaviour that arises in the scramble crossing problem and see how this depends on various parameters relating to the individual actions of the members of the crowd.

Declaration: I hereby certify that the work in this document is my own, unless otherwise referenced.

Signed .... Gareth Parry

Acknowledgements

I would like to thank Professor Budd in particular for his help and input whilst writing this thesis. The lively debate and insightful questions raised in our meetings was one of the particularly enjoyable aspects of the project. I would also like to thank Dr Chris Williams and Odysseas Kontovourkis for their help in providing an Architects insight into pedestrian dynamics.

Movies and Code

Submitted along with this thesis are copies of several movies created by the simulator and converted to *.avi files. Also included on the CD are the final MATLAB script and function files (hard copies are in Appendix A) along with several *.mat files which are saved data from different simulations. The movie and *.mat files are named with the number of pedestrians simulated followed by the geometry the flow was simulated in. So if 500 pedestrians were modelled in a Scramble Crossing then the associated files would be:

Movie: 500scramblecrossing.avi

Data: 500scramblecrossing.mat

Finally a *.PDF version of this thesis is also provided for submission to a plagiarism prevention website.

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Chapter 1

Introduction

The design and planning of large pedestrian areas has become increasingly important in recent times with the vast number of facilities having to cope with large volumes of pedestrian traffic. Notable examples of such buildings are airports, shopping centres, night clubs and sports stadia which have become larger and more commonplace in recent times. One of the key considerations in the architecture of these situations is the behaviour of the people that use them, in particular their reaction in panic situations. With the prevalence of these types of structures and tragic events which have resulted in the massive loss of life, the modelling, simulation and understanding of pedestrian movement in emergency egress situations is a necessity.

In many instances the fatalities and injuries in emergency evacuations were not caused by the hazard leading to the need for egress but the actions of the crowd itself. Stampedes are caused both by real hazards, such as fire, but also the behaviour of the crowd, perhaps in forcing its way out of a stadium. The most obvious example of such a situation is the Hillsborough disaster, when on April 15th, 1989; ninety-six Liverpool FC fans lost their lives resulting in the conversion of many football stadiums in the United Kingdom to all-seater and the removal of barriers at the front of stands.

The key aim of this thesis is to explore the current theories and models relating to pedestrian flows and implement two of these schemes. There is a massive breadth of literature relating to such models which have been developed over the last forty years. The model this thesis will focus on and implement is the *Social Force Model* which was developed D. Helbing and P. Molnár[1],[2].

The ultimate aim of this is to explore the emergent behaviour of pedestrian systems, particularly in the case of a scramble crossing and the expected striping effects. Other models will be discussed and outlined in some detail to highlight the variety of theories suggested to simulate pedestrian dynamics. The scheme proposed by V. J. Blue and J. L. Adler, their *Cellular Automata*[3] model, will be explored in some detail as initially it was intended that this model also be implemented in the MSc project.

Chapter 2

An Overview of Models for the Simulation of Pedestrian Dynamics

The simulation of pedestrian movement has being explored in a variety of ways. In this chapter the aim will be to provide an outline of the most prominent of these models along with an exposition of the governing equations of each scheme. In general these models describe the forces each pedestrian feels; treating each pedestrian as a particle in a larger system and using Newton's Second Law to evaluate position, velocity and acceleration. A numerical solver then can be used over a discrete timestep providing a velocity and position update. One of the key differences between pedestrian traffic models and other roadway-based traffic systems is that pedestrian locations are not restricted to a single dimension. Whilst many of the models are initially based on vehicle traffic systems, these are only one dimensional models and unlike pedestrian movement subject to a number of laws and restrictions governing traffic. Pedestrian movement is inherently more changeable than that of vehicles, as unlike vehicle flow which is controlled by well defined lane markings with lane change and passing opportunities restricted, there are no such restrictions on pedestrian walkways. Pedestrian interaction is also markedly different to that of cars since safety concerns are much less, clearly pedestrians can actually touch each other without incident, which is certainly not true of moving vehicles. Also pedestrians often move in pair or clusters, such as couples or family groups, whilst such attractive influences are rare in vehicle traffic. This leads to interactions between people that have to be considered in any model, examples of which are bumping into each other, exchange of places or bypass when pedestrian density is high as opposed to sidestepping which would be analogous to the behaviour of a car. The velocity and acceleration characteristics of pedestrians are also very different, with each pedestrian having their own desired and maximum speeds. They are also able to accelerate to full speed from standstill almost immediately and can change speed more rapidly allowing them to take advantage of gaps in traffic when they arise.

2.1 Pedestrian Modelling approaches

Pedestrian flow models can be classified in different ways depending upon how the scheme treats the pedestrians and the level of detail of the models. These classifications are:

1. Microscopic models, which consider individual pedestrian behaviour separately. The pedestrian behaviour in these models is often described by their interactions with other pedestrians in the system.
2. Mesoscopic models, which do not consider each pedestrian individually but the overriding characteristics, such as velocity distributions. The pedestrian behaviour is described microscopically though not specifically but rather in terms of velocity distributions.
3. Macroscopic models, which do not make distinctions between individual pedestrians nor describe their individual behaviour but consider the flow in terms of density, average velocity and flow patterns.

As previously discussed, the main focus of this thesis will be on the Social Force Model and Cellular Automata which are both examples of microscopic models. Whilst these are computationally demanding, both the individual and emergent group behaviour of pedestrians is of interest.

2.2 The Gas-Kinetic Model of Pedestrian Flows

This model was first suggested by L. F. Henderson[4], who treats large pedestrian crowds like molecules in a dilute gas. Whilst there are some seemingly random fluctuations in the movements of people in large crowds, the fact that each individual has a mass and velocity suggests that the classical Maxwell-Boltzmann statistics could be used to describe the motion of a crowd using a density function $f\left(\vec{x},\vec{v},t\right)$. Henderson only applies the Maxwell-Boltzmann equations to the so called gaseous phase. This is when the crowd is moving and has a low particle density, defined as the number of persons per unit area. If this is small then each individual is assumed to be able to move at their desired speed. However the model becomes problematic when boundary interactions occur and particle density increases, for example at a doorway. This is described as a phase transformation to a densely packed crowd liquid phase. To describe the crowd gas several assumptions are made, firstly that movement takes place on a continuous plane and that at time t each of the N pedestrians has a position (x,y) and velocity (V_x,V_y). Secondly the crowd is considered to be homogeneous, that is each particle will have the same mass and probability of velocity components. This homogeneity is analogous to chemical purity in molecular systems, although Henderson suggests that this homogeneity may not be a fair assumption due to what he describes as sexual inhomogeneity, the idea that men and women behave differently in crowds. This nonuniformity may extend beyond gender and also be attributed to other social and environmental factors such, as the age of each pedestrian. The next assumption is that the particles are independent of each other in position and velocity components with position and velocity of the individual pedestrian being uncorrelated. Finally, the crowd is assumed to be in equilibrium and that it can be treated as a statistical ensemble of any individual. The Maxwell-Boltzmann equations for the described assumptions are as follows, with the probability density function $P(V_x)$, for a single fluctuating velocity component, V_x, is

$$P\left(V_x\right) \equiv \frac{1}{N}\frac{dN_{v_x}}{dV_x} = \frac{1}{\sqrt{2\pi v_{r.m.s}}}\exp\left(-\frac{1}{2}\frac{V_x^2}{v_{r.m.s}^2}\right) \tag{2.2.1}$$

Where $v_{r.m.s}$ is the standard deviation of the speed $v \equiv |V|$. The expression for V_y is analogous and can be combined to get the resultant velocity, V

$$P\left(V\right) \equiv \frac{1}{N}\frac{dN_V}{dV} = \frac{1}{2\pi v_{r.m.s}^2}\exp\left(-\frac{1}{2}\frac{V^2}{v_{r.m.s}^2}\right) \tag{2.2.2}$$

With the probability density function for speed

$$P\left(|V|\right) \equiv \frac{1}{N}\frac{dN_v}{dv} = \frac{\pi}{4}\frac{v^2}{\overline{v}^2}\exp\left(-\frac{\pi}{4}\frac{v^2}{\overline{v}^2}\right) \text{ with } \overline{v} = \sqrt{\pi/2}v_{r.m.s} \tag{2.2.3}$$

These functions can be extended to situations where there may be a superimposed flux upon the system, for instance a crowd moving along a corridor. (2.2.1) Is shifted in this case with a new velocity component $V_x' \equiv V_x - \overline{V}_x$

$$P\left(V_x'\right) \equiv \frac{1}{N}\frac{dN_{V_x'}}{dV_x'} = \frac{1}{\sqrt{2\pi}v_{r.m.s}}\exp\left(-\frac{1}{2}\frac{V_x'^2}{v_{r.m.s}^2}\right) \tag{2.2.4}$$

With analogous treatment of (2.2.2) and (2.2.3).

Clearly this is a somewhat simplistic view of pedestrian dynamics, but it is the starting point for S. Hoogendoorn and P. H. L. Bovy's formulation [5]. In the simplest case where there is no distinction between pedestrian types, the gas-kinetic equations represent describe the dynamics of the generalised phase-space density $\rho = \rho(t, \underline{x}, \underline{v}, w)$ defined by $r(t, \underline{x}) f(\underline{v}, w \mid t, \underline{x})$. Here $r(t, \underline{x})$ is a multidimensional density which reflects the expected number of traffic entities per unit volume at $(t, \underline{x})$, with $f(\underline{v}, w \mid t, \underline{x})$ the joint probability density function of the velocity $\underline{v}$ and continuous attributes w which reflects characteristics of traffic flow and its constituent entities such as desired velocity. The gas-kinetic equation in n dimensions is

$$\frac{\partial \rho}{\partial t} + \overbrace{\nabla_{\underline{x}}.(\rho \underline{v})}^{(I)} + \overbrace{\nabla_{\underline{v}}.(\rho A)}^{(II)} + \overbrace{\nabla_{\underline{w}}.(\rho B)}^{(III)} = \overbrace{\left(\frac{\partial \rho}{\partial t}\right)_{event}}^{(IV)} + \overbrace{\left(\frac{\partial \rho}{\partial t}\right)_{cond}}^{(V)} \quad (2.2.5)$$

This equation shows how the phase-space density changes, with term (*I*) being convection, (*II*) acceleration, (*III*) the adaptation of continuous attributes, (*IV*) event based noncontinuum processes and (*V*) the condition-based noncontinuum processes. The so-called pedestrian phase-space density (P-PSD – this is consistent with Hoogendoorn's notation) $\rho(t, \underline{x}, \underline{v}, w)$ conforms to the setup laid out in the introduction to this chapter, that is it is a two dimensional system to describe the pedestrian flow where each pedestrian may have different velocity, desired velocity and acceleration considerations. With this in mind, pedestrian density $(r, \underline{x})$ is defined to be the expected number of pedestrians per unit area.

In (2.2.5) terms (*I*) – (*III*) are the continuum processes, that is they are smooth and describe the change in the spatial distribution of the pedestrians (in terms of density). They represent continuous changes in the independent variables $\underline{x}$, $\underline{v}$, w. Terms (*IV*) and

(V) describe the noncontinuum processes of pedestrian interaction – Hoogendoorn calls this a stimulus-response mechanism: "an event causes a remedial manoeuvre of the impeded pedestrian" and as such aren't continuous but occur when pedestrians engage with each other. The current research omits terms (III) and (V). The resulting equation to describe the P-PSD is

$$\frac{\partial\rho}{\partial t}+\overbrace{\frac{\partial}{\partial x_1}(\rho v_1)+\frac{\partial}{\partial x_2}(\rho v_2)}^{(I)}+\overbrace{\frac{\partial}{\partial v_1}(\rho A_1)+\frac{\partial}{\partial v_2}(\rho A_2)}^{(II)}=\overbrace{\left(\frac{\partial\rho}{\partial t}\right)^{+}_{event}+\left(\frac{\partial\rho}{\partial t}\right)^{-}_{event}}^{(IV)} \quad (2.2.6)$$

With A_1 and A_2 describing the acceleration laws of the system and the terms in (IV) describing noncontinuum events which increase or decrease the phase-space density respectively. Hoogendoorn and Bovy describe in detail the derivation of each term, however it is sufficient for this thesis to describe the model in general without detailing specifics.

One area of interest in the derivation however is the role of transition probabilities in the pedestrian interactions, as these probabilities describe how the pedestrian's direct environment affects their behaviour. Three types of pedestrian interaction are distinguished, which correspond to the stimulus-response mechanisms:

1. One-sided interaction – where pedestrian *p* catches up to a slower moving pedestrian *q*. Here *p* is held up by *q* but the converse isn't true.
2. Two-sided interaction – where two pedestrian travelling in opposite directions meet head on and hold each other up.

3. Passive interaction – this is the same as 1 except the pedestrian being considered is the one being caught up and so will not take any action as their progress is unimpeded.

Transition probabilities are then used to describe the expected behaviour of each pedestrian in the system in one of the three interactions described.

The model also touches on factors that are relevant in several models, particularly the pedestrian's spatial requirements and different classes of pedestrian. The Gas-Kinetic model traditionally assumes that the particles in the system are infinitesimally small, this however is not necessarily a valid assumption for modelling pedestrian flows since the amount of space each pedestrian occupies is of dominant importance. The pedestrian classification is the same problem Henderson touched on, in that gender, age or other demographic characteristics may affect the behaviour of individuals in the system.

Hoogendoorn and Bovy then solve the formulated problem using the Monte Carlo method in a variety of simple cases such as unidirectional, bidirectional and crossing pedestrian flows. Their results are reasonable and reproduce the expected velocity-density relations qualitatively.

2.3 The Magnetic Force Model

The next model was developed by S. Okazaki and S. Matsushita[6] and as the title suggests treats the pedestrians within the system as charged objects within the resulting magnetic field. Each pedestrian in the system is given a positive charge and destinations such as doorways or service counters a negative charge. Clearly the attractive nature of opposite magnetic charges results in the pedestrians moving towards these destination points. This magnetic effect also means that pedestrians exert a repulsive force upon each other which physically corresponds to them avoiding collisions with each other and objects such as boundaries.

Once the room has been setup, it is inputted as a series of vertexes described in the Cartesian coordinates. With the destination details configured the model then requires a detailed amount of input data to produce realistic results before the simulation can be started. The required data is the desired destination, initial position, initial velocity, the pedestrian orientation, time that the pedestrian starts walking and their method of walk. This final input is the most interesting as the simulation can implement a wayfinding technique where the pedestrians will follow a sequence of points (corners in the case of this simulator) until they have an unobstructed route, that is no boundaries in their path, to their desired destination. Another point of interest in this model is the velocity input, which is a maximum velocity. This is simply because if there were no upper bound on the velocity the pedestrians would accelerate without limit according to Coulomb's Law.

Since the pedestrians are treated as magnetic objects, the appropriate force law is Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 . Q_2}{s^2} \tag{2.3.1}$$

Q_1 and Q_2 are the signed values for the magnetic charge of the objects they represent, s is the distance between the two particles and k, the so called Coulomb constant, is the constant term $\frac{1}{4\pi\varepsilon}$. If Q_1 represents a positively charged pedestrian, a, say and Q_2 a negatively charged pole representing the desired destination. Since a two dimensional setup is being considered the force is a vector quantity resulting in the following form of Coulomb's Law

$$\underline{F} = k.\frac{Q_1.Q_2}{s^3}.\hat{\mathbf{s}} \tag{2.3.2}$$

Here $\hat{\mathbf{s}}$ is the unit vector pointing from Q_1 to Q_2. If the charges have the same sign, as in a pedestrian-pedestrian interaction, then the resulting force is positive which corresponds to a repulsive interaction. Conversely if the signs are different, in the case of a pedestrian-destination interaction, there is a negative force and so an attractive force is felt which leads to the particle accelerating towards the destination. When more than two charges are present in the system the forces are superimposed, that is the force between any pair is the sum of all the exerted forces from the component charges.

The model also incorporates another force which acts upon the pedestrians in order to simulate the collision avoidance characteristics of the crowd. If two pedestrians come within a certain distance of each other then the new force is exerted upon the pedestrians. That is, if *a*, intersects within a specific area of another pedestrian then *a* feels the following repulsive force and resulting acceleration to cause a change of direction and thus preventing a collision. This acceleration is represented by

$$\frac{dv_a}{dt} = \tan\beta.\cos\alpha.v_a \tag{2.3.3}$$

Where v_a is the velocity of pedestrian *a*, β is the angle between the relative velocity (**RV**) of *a* to pedestrian *b* and the contacting line from the position of pedestrian *a* to the circle around pedestrian *b*. This circle is the "Pedestrian Area Module" which is equivalent to the “Territorial Sphere” in Helbing’s Social Force Model [1]. Finally α is the angle between relative velocity of pedestrian *a* (**RV**) to the pedestrian *b* and the velocity of pedestrian *a*.

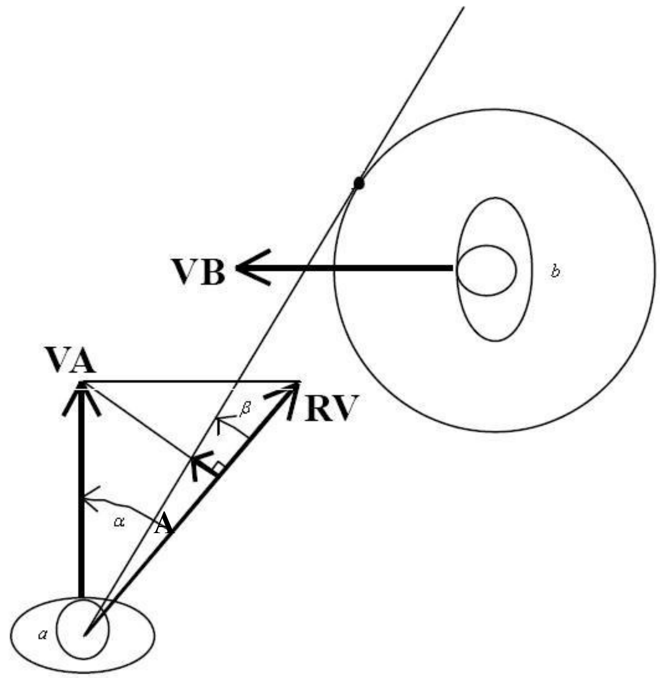


Figure 1: Acceleration force **A** acting on *a* to avoid collision with *b*

This model was used to simulate to simulate an escape from fire on one floor of an office building, to plot the movement of pedestrians in part of an underground railway station and pedestrian flows in a hotel lobby. The model can be used to evaluate how long an emergency escape might take, the behaviour of pedestrians in queuing situations – that is the number in each queue, the length of their wait and movement processes.

2.4 Queuing Systems

Queuing theory, which is generally considered to be a branch of operations research, describes pedestrian flows in terms of probability functions. The pedestrian will arrive at a given node, which represents a server, with a certain probability. They will then spend a certain amount of time being served, at a shop till for example, and then continue on to their next destination, leaving the queue. A queuing system is comprised of three elements, the pedestrian's arrival in the queue, the service mechanism and the service discipline. A queuing discipline determines the manner in which the exchange handles calls from customers. Examples are

- *First In, First Out* – This principle states that customers are served one at a time and that the customer that has been waiting longest is served first
- *Last In First Out* – This principle also serves customers one at a time, however the customer with the shortest waiting time will be served first
- *Processor Sharing* – Customers are served equally. Network capacity is shared between customers and they all effectively experience the same delay

Queuing is handled by control processes within exchanges, which can be modelled using state equations. Queuing systems use Markov Chains which model the system in each state where Incoming traffic to these systems is modelled via a Poisson distribution. The stochastic process in a queuing system is the population of a particular room.

The first model to be discussed was formulated by S. J. Yuhaski, Jr and J. Macgregor Smith[7] which develops a state dependent queuing model for the congestion effects of movement through circulation systems of a building. Circulation systems are, in this case, the pathways of movement such as corridors, stairways and ramps. The problem they describe is that of crowded pedestrian flows in confined spaces and the paper describes the following as "crucial aspects to movement Systems"

1. The service rate of the movements system decays with increasing traffic
2. The amount of available space within the movement system is finite

These characteristics are the basis for the construction of their model and their aim to "best capture the congestion effects". The first step is to generate a representation for the facility; this is its floor plan. This can be represented by a planar graph $G'(V',E')$ (this is consistent with their notation) and so the queuing network is the Dual Graph of G'. There are then two distinct types of spatial entity which must be distinguished, that is the activity network and the circulation network. So V is partitioned into two sets $V=\{A,S\}$. The set $A=\{A_1,A_2,...,A_N\}$ is the set of activity nodes which represent so called "activity areas", say a department of a shop. The set $S=\{S_1,S_2,...,S_M\}$ then is the circulation nodes which are the movement pathways joining activity areas.

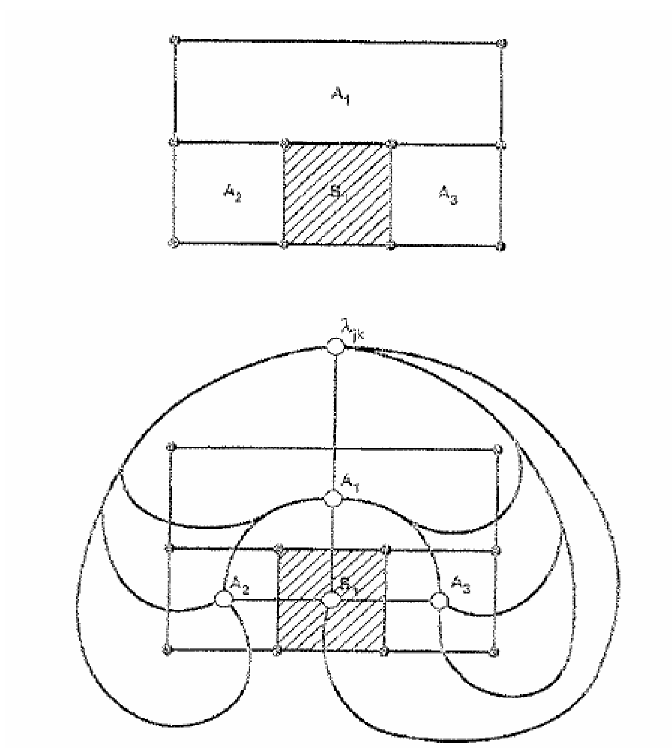


Figure 2: Planar Graph $G'(V',E')$ and dual graph $G(V,E)$

One feature of modelling pedestrian facilities is that transitions within a network are not virtually instantaneous as in other systems (telephone networks for example) and so the set *S* is required. Vertices from this set represent additional nodes within a facility which handle the flow of pedestrians from A_i to A_k without interrupting service.

The model makes the following assumptions; firstly that there are *J* customer classes, which wish to use the facility, drawn from an infinite population and each class has *K* generating sources. Then the average arrival rate of type *j* per unit time from source *k* is $\lambda_{jk}\left(j=1,2,\ldots J;k=1,2,\ldots K\right)$. The next assumption is that each pedestrian of type *jk* will follow a deterministic routing vector through the facility, called a "customer chain". This vector has elements $r_{jkl}\left(l=1,2,\ldots L_{jk}\right)$ where the *l*th element represents the destination of the pedestrian to the next resource after they have been served at their previous destination. So, customers of the type *jk* enter a system of queues in independent Poisson streams at a rate λ_{jk} and follow a sequence of queues before leaving the facility. A Poisson stream of arrivals corresponds to arrivals at random. In a Poisson stream successive customers arrive after intervals which are independently exponentially distributed.

The next stage is to model a single corridor as a queuing system. The corridor is given a maximum capacity of C = [5*LW*] where *L* is the corridor length and *W* the width. It is then assumed that pedestrians enter the corridor with the behaviour of a Poisson stream of rate λ, and the time each person occupies the corridor is exponentially distributed with rate μ_n. Thus, there is a state dependent service rate, meaning it is a function of the number of pedestrians in the corridor. The model now uses the Chapman-Kolmogorov steady-state difference equations for the state probabilities $p_1, p_2, ..., p_c$

$$p_n = \frac{\lambda_0 \lambda_1 ... \lambda_{n-1}}{\mu_1 \mu_2 ... \mu_n} p_0 \tag{2.4.1}$$

Such that

$$\frac{1}{p_0} = 1 + \sum_{n=1}^{C} \left[\frac{\lambda_0 \lambda_1 ... \lambda_{n-1}}{\mu_1 \mu_2 ... \mu_n} \right] \tag{2.4.2}$$

Here the arrival rates are not influenced by the number in the queue so $\lambda = \lambda_0 = \lambda_1 = ... = \lambda_C$. The paper suggests two congestion models with approximate overall walking-speed V_n, firstly the linear relation

$$V_n = \frac{1.5}{C}(C + 1 - n) \tag{2.4.3}$$

Or the exponential relation, which may be more accurate, is

$$V_n = A \exp\left[-\left(\frac{n-1}{\beta} \right)^{\gamma} \right] \tag{2.4.4}$$

A is the amplitude with parameters β and γ are called the scale and shape parameters. In both models the service rate r_n, is the average of the inverse of the time it takes for the pedestrians to travel the length of the corridor

$$r_n = \frac{V_n}{L} \tag{2.4.5}$$

With overall service rate

$$\mu_n = n r_n \tag{2.4.6}$$

The overall service rate for the linear model then becomes

$$\mu_n = \frac{1.5}{C}\left(C+1-n\right) \tag{2.4.7}$$

Then the Chapman-Kolmogorov equations become

$$p_n = \frac{\lambda_n}{\left(\frac{A}{LC}\right)^n \prod_{i=1}^{n}\left(C-i+1\right)i} p_0 \tag{2.4.8}$$

And

$$\frac{1}{p_0} = 1 + \sum_{n=1}^{C} \frac{k^n}{\prod_{i=1}^{n}\left(C-i+1\right)i} \tag{2.4.9}$$

With $k^n = \frac{\lambda LC}{A}$. Similarly the Chapman-Kolmogorov equations for the exponential model are

$$p_n = \frac{\lambda_n}{\prod_{i=1}^{n} i\left(\frac{A}{L}\right)\exp\left\{\left[-\left(\frac{i-1}{\beta}\right)^{\gamma}\right]\right\}} p_0 \tag{2.4.10}$$

And

$$\frac{1}{p_0} = 1 + \sum_{n=1}^{C} \left\{ \frac{\lambda_n}{\prod_{i=1}^{n} i\left(\frac{A}{L}\right)\exp\left[-\left(\frac{i-1}{\beta}\right)^{\gamma}\right]} \right\} \tag{2.4.11}$$

Another notable Queuing Model is the one developed by Løvas[8] introduced a similar stochastic model where pedestrians can be modeled in a queuing network. This model is setup similarly where Nodes in the network represent rooms and links the doors. Each pedestrian will then select a new node with a given probability. The model can evaluate several performance measures such as the mean number of persons in a node and the mean number of safe evacuees. Løvas has developed a tool called EVACSIM which has simulated egress in several different setups. The system visualizes the movement of the pedestrians and provides qualitative information about behaviour at bottlenecks in the queuing systems.

2.5 Cellular Automata

A **cellular automaton** is a discrete model studied in computability theory, mathematics, and theoretical biology. It consists of a regular grid of *cells*, each in one of a finite number of *states*. Time is discretised and the state of a cell at time t is a function of the states of a finite number of cells (called its *neighbourhood*) at time $t - 1$. These neighbours are a selection of cells relative to the specified cell, and do not change (though the cell itself may be in its neighbourhood, it is not usually considered a neighbour). Every cell has the same rule for updating, based on the values in this neighbourhood. Each time the rules are applied to the whole grid a new *generation* is created. The Cellular Automata method can be used to simulate pedestrian flow. It is fast and relatively simple, with the walkway represented by a cellular grid. In this representation each cell within the grid is occupied by one pedestrian. The pedestrian flow is modelled by a set of governing rules which differ according to the particular model being considered.

The most notable Cellular Automata model is the one developed by V. J. Blue and J. L. Adler ([3],[9]) which models the walkway as a circular lattice (closed loop) with width W, length G and class $L = WG$. Each cell within the lattice is assigned a label $L(i, j)$ with $1 \le i \le W$ and $1 \le j \le G$. The density of pedestrians on the walkway is determined at the start of the simulation and remains constant throughout. The

Microsimulation continues in discrete timesteps t_i for $i = 1, 2, ..., T$. The lane assignments and speed updates change the position of all the pedestrians within the lattice in four stages determined by the local rules which are applied to each individual on the walkway. The stages are

1. A set of lane change rules determining the lane for each pedestrian on the lattice
2. The pedestrians are moved in to the assigned lanes
3. A set of rules is applied to find the allowable speed of each pedestrian based on the available gap ahead and the pedestrians desired speed
4. Forward movements based on the allowed speeds are made

The rule sets for each stage are

Lane Change (parallel update 1 – stages 1 and 2)

1) Eliminate conflicts: If two walkers are adjacent then they cannot sidestep into each other
   a) If a cell is available between two walkers then assign it to one of then with a 50/50 split

2) Identify gaps: The lane (same or left/right adjacent) is chosen which best advances forward movement upto *v_max* according to the gap computation subprocedure that follows the step forward update

   a) For Dynamic Multiple Lanes (DML):
      i) Step out of lane if a walker in the opposite direction is within 8 cells by assigning *gap* = 0
      ii) Step behind a pedestrian moving in the same direction when avoiding a collision with an oncoming walker by choosing any available lane with *gap_same* = 1 when *gap* = 1

   b) Ties of equal maximum gaps ahead are resolved according to:
      i) If the 2-way tie is between adjacent lanes then a 50/50 split between which lane is chosen
      ii) If the 2-way tie is between the current and an adjacent lane then the walker stays in lane
      iii) If there is a 3-way tie between the current and adjacent lanes then the pedestrian stays in lane

3) Move (this is stage 2): Each pedestrian p_n is moved 0, +1 or -1 lateral sidesteps after 1) – 3) is completed

Step forward (parallel update 2 – stages 3 and 4)

1) Update velocity: Let $v\left(p_n\right) = gap$ where *gap* is from the subprocedure below

2) Exchanges: IF *gap* = 0 or 1 AND *gap* = *gap_opp* (cell occupied by an opposite moving pedestrian) THEN with probability *p_exchg,* $v\left(p_n\right) = gap + 1$ ELSE $v\left(p_n\right) = 0$. That is the walker and the opposite moving pedestrian either exchange cells with a predefined probability *p_exchg* or stop when they meet each other and sidestep at the next timestep

3) Move (this is stage 4): each pedestrian p_n is moved $v\left(p_n\right)$ cells forward

Gap Computation Subprocedure

1) Same direction: Look ahead a up to 8 cells (8 = 2* the largest *v_max*) IF an occupied cell is found with a pedestrian moving in the same direction THEN set *gap_same* to the number of cells between the two pedestrians ELSE set *gap_same* = 8

2) Opposite Direction: IF an occupied cell is found with an opposite moving pedestrian THEN set *gap_opp* to (0.5*number of cells between the pedestrians) ELSE *gap_opp* = 4

3) Assign *gap* = MIN(*gap_same, gap_opp, v_max*)

The lane switching procedure is illustrated in Figure 3. For pedestrian 1, the adjacent cells on both the left and right are available. The unoccupied distance on however is uniquely maximal in the present lane so no switch is required. Pedestrian 4 conflicts to the right with pedestrian 6 and in this instance pedestrian 6 is given access to this cell, hence 4 can only sidestep to the left. The unoccupied distance is largest in this lane and so 4 will switch. The change is represented by 4’. Pedestrian 7 cannot switch lanes since both adjacent lanes are occupied and thus the lane change procedure is terminated.

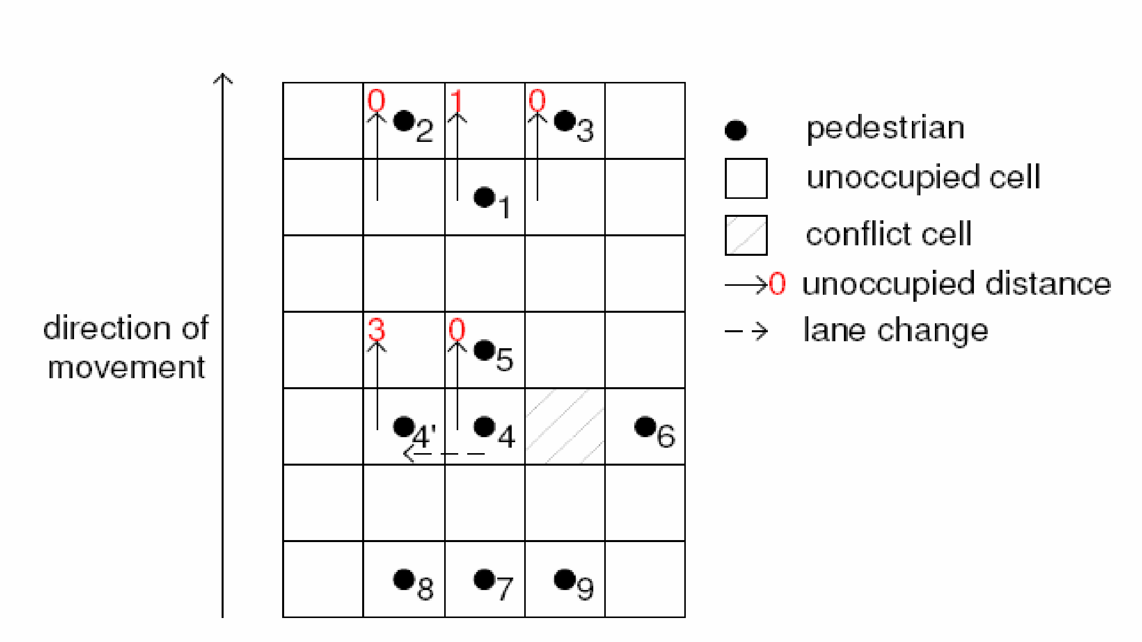


Figure 3: Lane switching behaviour in Blue/Adler model

This model was successfully applied to both uni and bi-directional flows. The results reproduce observed phenomena particularly the expected lane formation which is shown in figure 4 where grey cells are left moving pedestrians and the black right moving pedestrians

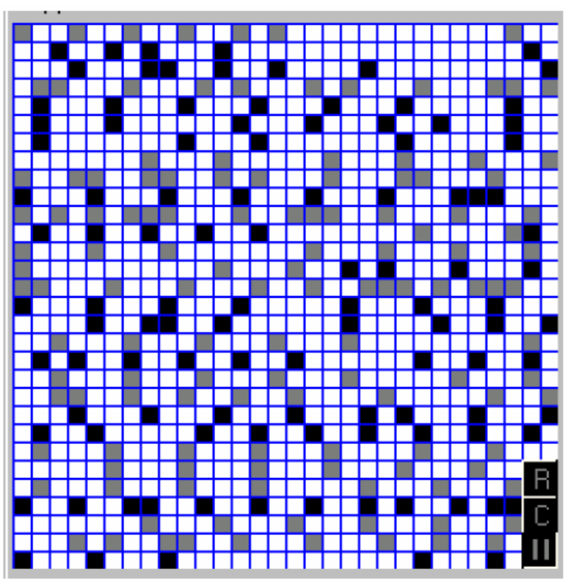


Figure 4: Emerging Lane Formation

Another interesting model was developed by J. Dijkstra, J. Jessurun and H. Timmermans [10] which looks at pedestrian movement with a shopping centre. The rule set for this model is

1) Check decision point: If a pedestrian has passed a decision point (the end of an activity or node in the network) then got 3)
2) Check the cell type – examine the behaviour of the pedestrian and the walkers desired direction followed by a change into that direction then a decision point will be passed
3) If the cell is free then the pedestrian can move into that cell, otherwise got 4)
4) If the cell to the left/right isn't occupied then move there

In this model the movement is directed only toward the destination and can only change at decision points with pedestrian interactions not being considered.

2.6 The Social Force Model

This model has been developed primarily by D. Helbing and P. Molnár ([1], [2], [11]). They describe the idea of social forces in the context of ordinary pedestrian behaviour. That is, in general a pedestrian will be used to the majority of situations that confront them and they have prescribed behavioural strategies based on previous experience of similar situations. As such they will react in the *best* way, that is the most efficient for them and as such pedestrian movement is automatic and thus predictable. The Social Force Model is a Microsimulation of each individuals behaviour and so each pedestrian, α, in the system can be represented by a point $\vec{r}_\alpha(t)$ in space, which changes continuously with speed being governed by the equation of motion

$$\frac{d\vec{r}_\alpha(t)}{dt} = \vec{v}_\alpha(t) \tag{2.6.1}$$

Similarly, the speed $\vec{v}_\alpha(t)$ is continuously changing and thus acceleration is governed by the social forces $\vec{f}_\alpha(t)$, which represent the sum of the different influences upon the individual pedestrian (that is environment and other pedestrians). There is also a consideration for random fluctuations within the system which account for random behavioural fluctuations which gives rise to $\vec{\xi}_\alpha(t)$. So, the acceleration obtained is

$$\frac{d\vec{v}_\alpha}{dt} = \vec{f}_\alpha(t) + \vec{\xi}_\alpha(t) \qquad (2.6.2)$$

The model being implemented in this thesis takes into account an acceleration force $\vec{f}_\alpha^0(\vec{v}_\alpha)$, repulsive effects of boundaries $\vec{f}_{\alpha B}(\vec{r}_\alpha)$, repulsive interactions with other pedestrians $\vec{f}_{\alpha\beta}(\vec{r}_\alpha, \vec{v}_\alpha, \vec{r}_\beta, \vec{v}_\beta)$ and attraction effects $\vec{f}_{\alpha i}(\vec{r}_\alpha, \vec{r}_i, t)$ leading to

$$\vec{f}_\alpha(t) = \vec{f}_\alpha^0(\vec{v}_\alpha) + \vec{f}_{\alpha B}(\vec{r}_\alpha) + \sum_{\beta(\neq\alpha)} \vec{f}_{\alpha\beta}(\vec{r}_\alpha, \vec{v}_\alpha, \vec{r}_\beta, \vec{v}_\beta) + \sum_i \vec{f}_{\alpha i}(\vec{r}_\alpha, \vec{r}_i, t) \qquad (2.6.3)$$

2.6.1 The Driving Force

As the name suggests, the driving force, is the component of the social forces which describes each individuals desire to move to their intended destination with some desired velocity v_α^0. The desired direction of motion is given by $\vec{e}_\alpha$ and deviations of the actual velocity $\vec{v}_\alpha$ from the desired velocity $\vec{v}_\alpha^0(t) = v_\alpha^0(t)\vec{e}_\alpha(t)$ are corrected within the so called relaxation time τ_α. The equation which describes this motivation is

$$\vec{f}_\alpha^0 = \frac{1}{\tau_\alpha}\left(v_\alpha^0(t)\vec{e}_\alpha(t) - \vec{v}_\alpha(t)\right) \qquad (2.6.4)$$

The desired direction of the pedestrian is described by

$$\vec{e}_\alpha(t) = \frac{\vec{p} - \vec{r}_\alpha}{\|\vec{p} - \vec{r}_\alpha\|} \qquad (2.6.5)$$

Where $\vec{p}$ is the desired destination and $\vec{r}_\alpha$ the current position.

It is often the case that pedestrians may be delayed, at bottlenecks or doorways for example, which leads to an increase in the desired speed over the course of time. One way to describe this effect is to implement the following

$$v_\alpha^0(t) = \left[1 - n_\alpha(t)\right] v_\alpha^0(0) + n_\alpha(t) v_\alpha^{\max} \tag{2.6.6}$$

With, $v_\alpha^{\max}$ the maximum desired velocity and $v_\alpha^0(0)$ the initial one. The parameter

$$n_\alpha(t) = 1 - \frac{\overline{v}_\alpha(t)}{v_\alpha^0(t)} \tag{2.6.7}$$

Describes the impatience of the pedestrian to reach their destination, with $\overline{v}(t)$ the average speed into the desired direction of motion. This particular effect may lead to a crowd developing pushy behaviour thus increasing the pressure within the crowd, which may lead to clogging effects which may have disastrous consequences.

2.6.2 Pedestrian Interactions

The repulsive force term $\vec{f}_{\alpha\beta}(\vec{r}_\alpha, \vec{v}_\alpha, \vec{r}_\beta, \vec{v}_\beta)$ describes the interactions between two pedestrians α and β, and the desire of pedestrian α to keep a certain distance from β. This term is described by

$$\vec{f}_{\alpha\beta}(t) = A_\alpha^1 \exp\left[\frac{\left(r_{\alpha\beta} - d_{\alpha\beta}\right)}{B_\alpha^1}\right] \vec{n}_{\alpha\beta} . F_{\alpha\beta} + A_\alpha^2 \exp\left[\frac{\left(r_{\alpha\beta} - d_{\alpha\beta}\right)}{B_\alpha^2}\right] \vec{n}_{\alpha\beta} \tag{2.6.8}$$

The first term of this force describes the tendency to respect the *private sphere* of each individual and also helps to avoid collisions if there are sudden changes within the system. The second term accounts for the behaviour of physical interaction in high densities and pushy crowds if so called frictional effects are ignored. A second formulation which includes frictional effects will be briefly introduced later. The parameters A_α^i and B_α^i denote the interaction strength and range respectively. These parameters are often dependent on cultural influences, for example the personal space expected may vary depending on the society being modelled. The parameter $d_{\alpha\beta}$ is the distance between the centres of mass of the pedestrians being considered, $r_{\alpha\beta}$ is the sum of the radii of pedestrians α and β and $\vec{n}_{\alpha\beta}$ is the normalised vector pointing from β to α

$$\vec{n}_{\alpha\beta} = \frac{\vec{x}_\alpha(t) - \vec{x}_\beta(t)}{d_{\alpha\beta}(t)} \tag{2.6.9}$$

Where $\vec{x}_\alpha(t)$ is the point of the centre of mass of α at time t, and similarly for β. Figure 5 shows visually these parameters.

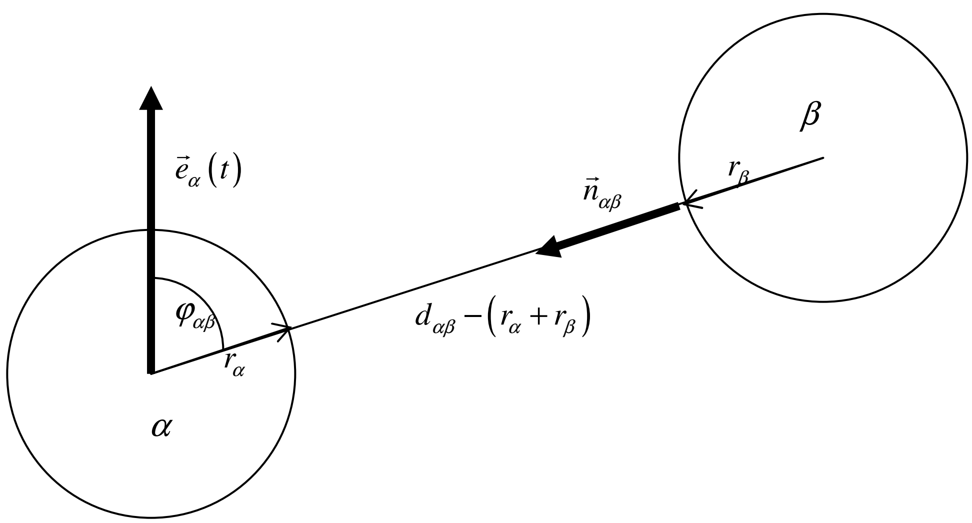


Figure 5: Distance between α and β

Each pedestrian in the system is modelled to have a solid core, rather than just be a point. However this does not have a great impact on the results of the simulation and in [2] the pedestrians were considered to be point particles without any problem. The final parameter in (2.6.8) is $F_{\alpha\beta}$ accounts for the anisotropic behaviour of the pedestrians. In this context it means that the actions of those in front of our pedestrian have a greater impact on their movement than those behind. Pedestrians can often predict what may occur infront of them and can react accordingly. Thus $F_{\alpha\beta}$ is a factor which gives the pedestrians within view greater influence than those out of view. This is done by having this factor depend on the angle, φ, between the walkers desired direction of movement and the direction of the pedestrian exerting the repulsive force. This is also shown in figure 5.

$$F_{\alpha\beta} = \lambda_\alpha + (1-\lambda_\alpha)\frac{1+\cos(\varphi_{\alpha\beta})}{2} \qquad (2.6.10)$$

λ_α is the potential of the anisotropic character of the pedestrians, with

$$\cos(\varphi_{\alpha\beta}) = -\vec{n}_{\alpha\beta}(t).\vec{e}_\alpha(t) \text{ where } \vec{e}(t) = \frac{\vec{v}_\alpha(t)}{\|\vec{v}_\alpha(t)\|} \qquad (2.6.11)$$

Setting $\lambda_\alpha < 1$ creates the aforementioned anisotropic character, with a pedestrian directly infront having the greatest effect as $\varphi = 0$ so $F_{\alpha\beta} = 1$ and pedestrians in the range $0 < \varphi < \frac{\pi}{2} \bigcup \frac{3\pi}{2} < \varphi < 2\pi$ have the strongest influence. Figure 6 shows this for various λ

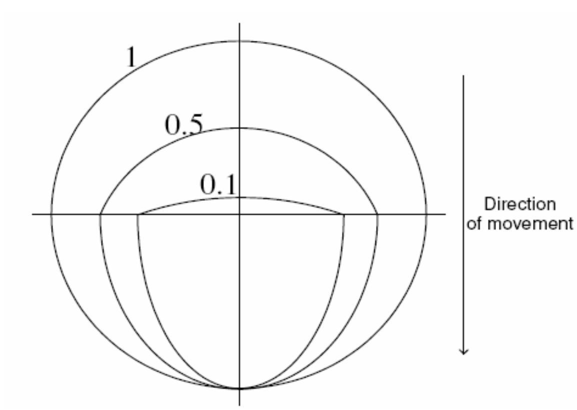


Figure 6: $F_{\alpha\beta}$ characterises anisotropic behaviour

2.6.3 Boundary Interactions

The treatment of boundaries within the model is analogous to that of other pedestrians, excluding the anisotropic effects of the pedestrian interactions. This leads to

$$\vec{f}_{\alpha B}\left(\vec{r}_{\alpha}\right) = A_{\alpha B} \exp\left(\frac{r_{\alpha} - d_{\alpha B}}{B_{\alpha B}}\right)\vec{n}_{\alpha B} \tag{2.6.12}$$

Where $d_{\alpha B}$ is the distance between the boundary and the pedestrian and $\vec{n}_{\alpha B}$ the normal vector pointing from the boundary to α. In most situations there is more than one boundary to consider leading to a question as to which boundaries influence to account for. There are three possible ways of considering the boundary interaction

1. Superposition: All boundaries influence the pedestrian so the forces are summed
2. Shortest distance: Only the closet boundary element is considered
3. Biggest impact: only the boundary with the largest impact is considered

In most geometries the biggest impact and shortest distance model are equivalent; this is often an appropriate choice too however this may not however be reasonable for angled passageways and superposition may be better. It was decided that in this project only the nearest boundary element would be considered, again this was the approach of [2] and produced realistic results in that case.

2.6.4 Attractive Interactions

Often pedestrians demonstrate certain joining characteristics, such as families or groups of tourists, and will wish to move through the walkway together. Other instances may be shops, window displays or performances in the street. These two cases are however separate, in the former the attractive force is constant and independent of time reflecting the desire of these groups to remain together over the whole time interval. In the latter case however the attraction is time dependent as the pedestrian will not wish to be late and clearly will ignore the attraction if this is the case. Such attractions have a similar modelling to pedestrian interactions however the attraction range $B_{\alpha i}$ is typically larger with a smaller, negative, time dependent interaction strength $A_{\alpha i}$.

2.6.5 Individuality and Random Behaviour Fluctuations

As previously mentioned, each pedestrian may display some random behaviour arising from accidental or deliberate changes to the expected and optimal actions. This force $\vec{\xi}_\alpha(t)$ is Gaussian distributed and perpendicular to the desired direction. One such formulation would be

$$\xi_\alpha = \left\langle \vec{e}_\alpha(t), \vec{f}_\alpha(t) \right\rangle X \vec{e}_\alpha^{\,\mathrm{perp}} \tag{2.6.13}$$

Here $X \propto N\left(0, \sigma^2\right)$ with probability density

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{2.6.14}$$

2.6.6 Another Formulation

As previously mentioned, there is another formulation of the Social Force model which takes into account the 'sliding friction force' in the pedestrian and boundary interaction terms. For the pedestrian interactions the model becomes

$$\vec{f}_{\alpha\beta} = \left\{ A_\alpha \exp\left(\frac{r_{\alpha\beta} - d_{\alpha\beta}}{B_\alpha}\right) . F_{\alpha\beta} + k.g\left(r_{\alpha\beta} - d_{\alpha\beta}\right) \right\} \vec{n}_{\alpha\beta} + \kappa.g\left(r_{\alpha\beta} - d_{\alpha\beta}\right) \Delta v_{\alpha\beta}^t \vec{t}_{\alpha\beta} \tag{2.6.15}$$

Here $\vec{t}_{\alpha\beta} = \left(-n_{\alpha\beta}^2, n_{\alpha\beta}^1\right)$ is the tangential direction and $\Delta v_{\alpha\beta}^t = \left(\vec{v}_\beta - \vec{v}_\alpha\right) . \vec{t}_{\alpha\beta}$ the relative tangential velocity. The values k, κ are given constants of the system and the function

$g(x)$ is zero when its argument is negative and equal to its argument otherwise. In this case

$$g\left(r_{\alpha\beta}-d_{\alpha\beta}\right)=\begin{cases}0 & \text{if } d_{\alpha\beta}>r_{\alpha\beta}\\ r_{\alpha\beta}-d_{\alpha\beta} & \text{else}\end{cases} \tag{2.6.16}$$

Physically this means that the ‘sliding friction force’ is only felt if the pedestrians are touching. Again boundary interactions are analogous, so

$$\vec{f}_{\alpha B}=\left\{A_{\alpha B}\exp\left(\frac{r_{\alpha}-d_{\alpha B}}{B_{\alpha B}}\right)+k.g\left(r_{\alpha}-d_{\alpha B}\right)\right\}\vec{n}_{\alpha B}-\kappa.g\left(r_{\alpha}-d_{\alpha B}\right)\left(\vec{v}_{\alpha}.\vec{t}_{\alpha B}\right)\vec{t}_{g\alpha B} \tag{2.6.17}$$

This alternative model will not be implemented (although in principle once the first model is coded the changes required are not difficult) and is only included for completeness. Similarly the attractive interactions and random fluctuations described earlier will not be implemented either.

Chapter 3

Self-Organisation Phenomena in Pedestrian Flows

One of the main objectives of this thesis is to explore and simulate emergent behaviours of pedestrian systems. If a pedestrian flow has certain conditions such phenomena are often observable. Of particular interest are

- Lane formation and striping effects
- Oscillatory flows and clogging effects at doorways
- Shock waves in dense crowds

These self-organisation effects are patterns of behaviour that are not externally planned or organised by an outside source such as traffic signals or behavioural conventions.

3.1 Intersecting Flows

Intersecting flows are commonplace in many pedestrian facilities and are nearly unavoidable as alternatives, such as bridges, are costly and generally impractical. Of particular interest are Scramble Crossings which have been observed to display striping effects when they intersect. A paper by Dzubiella and Löwen [12] exploring the pattern forming in such systems describe these stripes as density waves moving into the direction of the sum of the directional vectors of both flows.

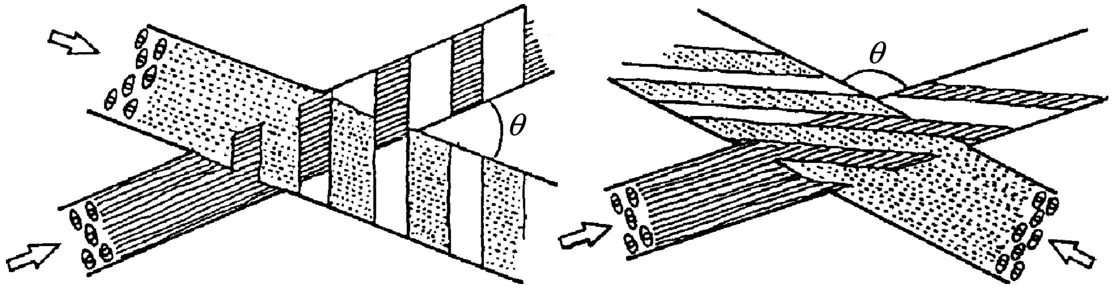


Figure 7: Striping phenomena in Intersecting flows

The stripe formation reduces the number of obstructing interactions and maximisation of average pedestrian speeds.

3.2 Bottlenecks

Bottlenecks such as doorways are also common in pedestrian areas and the way pedestrians interact at them is of interest. Often the flows through bottlenecks are irregular and inefficient. If there is an opposite flow into a bottleneck it is often the case that the flow of pedestrians is oscillating and unidirectional as opposed to the bidirectional flow in an ordinary corridor without a bottleneck. Similar to lane formation, which will be discussed in the next section, it is typical for groups to move through the bottle neck since it is easier to follow someone than to move against them. This leads to a pressure increase, due in part to impatience, on the side of the bottleneck where flow is halted and a decrease on the side that is flowing. When the difference between the two pressures is large enough the flow will change. Thus the process is reversed and the flows will oscillate.

3.3 Lane Formation

A bidirectional system, or counterflow, in everyday conditions has pedestrians moving in opposite directions which are unevenly distributed over the walkway. One of the most commonly observed phenomena is lane formation where pedestrians form into lanes of unidirectional flow. This effect leads to a reduction in the number of evasive manoeuvres each pedestrian need to make and so increases the efficiency of the walkway. In this case, improved efficiency means that the average pedestrian speed is maximal and necessary avoidance and braking actions minimal. The number of lanes formed is dependent on the geometry of the walkway. Initially the lanes are small channels, created when opposite moving pedestrians meet, which then merge with each other to form larger lanes.

3.4 Shockwaves

When the flow of pedestrians is slowed, the shockwave effect can be observed. The velocity of each pedestrian, as described by (2.6.6) for the Social Force model, is dependent upon their average speed. Physically this corresponds to the walker becoming impatient and the desire to increase their velocity becomes stronger than the desire to keep a certain distance from the pedestrian infront. This effect is cumulative and when one pedestrian moves forward those behind him also move and thus the shockwave effect occurs.

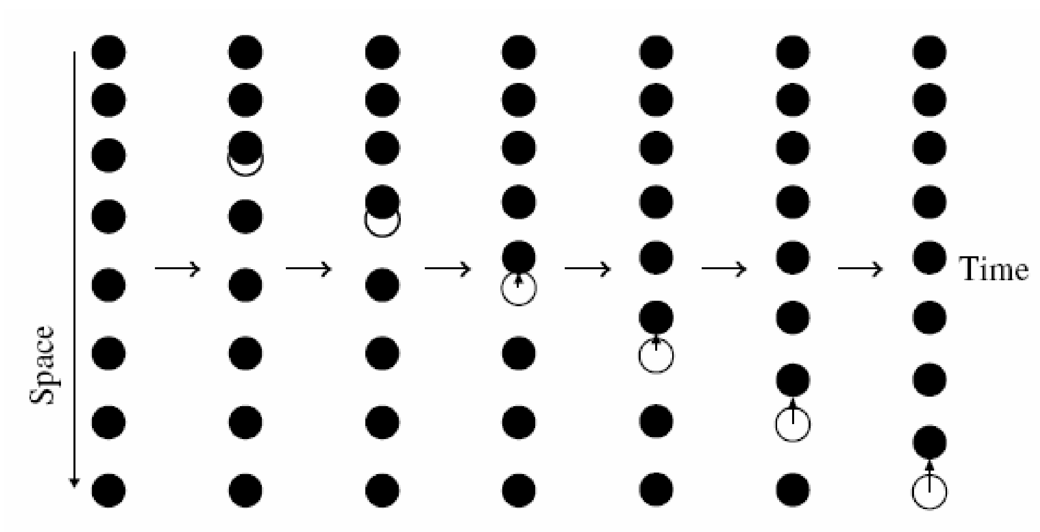


Figure 8: illustration of pedestrian behaviour causing shockwaves

Chapter 4

Implementation of the Social Force Model

One of the key aims of this project was to successfully implement the Social Force Model, as described by Helbing [1], and attempt to do some quantitative analysis of the results along with reproducing the qualitative behaviours of large pedestrian flows as described in chapter 3. In this chapter the implementation of this model will be explored with particular emphasis given to detailing some of the problems observed with the model and how they were overcome.

4.1 Numerical Solution of the Social Force Model

The Social Force Model described in chapter 2 is a first order Ordinary Differential Equation (ODE) which needs to be solved numerically to calculate the velocity and position of the pedestrians within the system. Importantly the model is an initial value problem, for both the velocity and displacement equations, which means that there are several excellent MATLAB solvers available for this type of problem. The ODE described by the Social Force model with initial conditions

$$\begin{aligned}\vec{r}_\alpha(0) &= \left(r_\alpha^1(0), r_\alpha^2(0)\right)^t \\ \vec{v}_\alpha(0) &= \left(v_\alpha^1(0), v_\alpha^2(0)\right)^t\end{aligned} \tag{4.1.1}$$

Is a *non-stiff* system, as all the elements evolve on a similar time-scale. As this is the case the suggested solver initially was `ode45` which is extremely efficient for these types of problem. This solver implements a Runge Kutta method of the fourth order and was developed by Dormand and Prince. However, an unspotted error in the initial code actually meant that it appeared that the system was *stiff* and so a different solver was chosen, namely `ode15s` which is a Gear solver and generally very efficient for *stiff* systems. This error was subsequently spotted in the de-bugging procedure and obviously raised the question as to whether `ode15s` was an appropriate solver for the system. To choose the best solver some tests were run to calculate the time each one was taking to simulate an intersecting pedestrian flow. Each solver was tested with systems of 10 to 100 pedestrians in increments of 10 and the `MATLAB` function `cputime` was used to calculate the time taken.

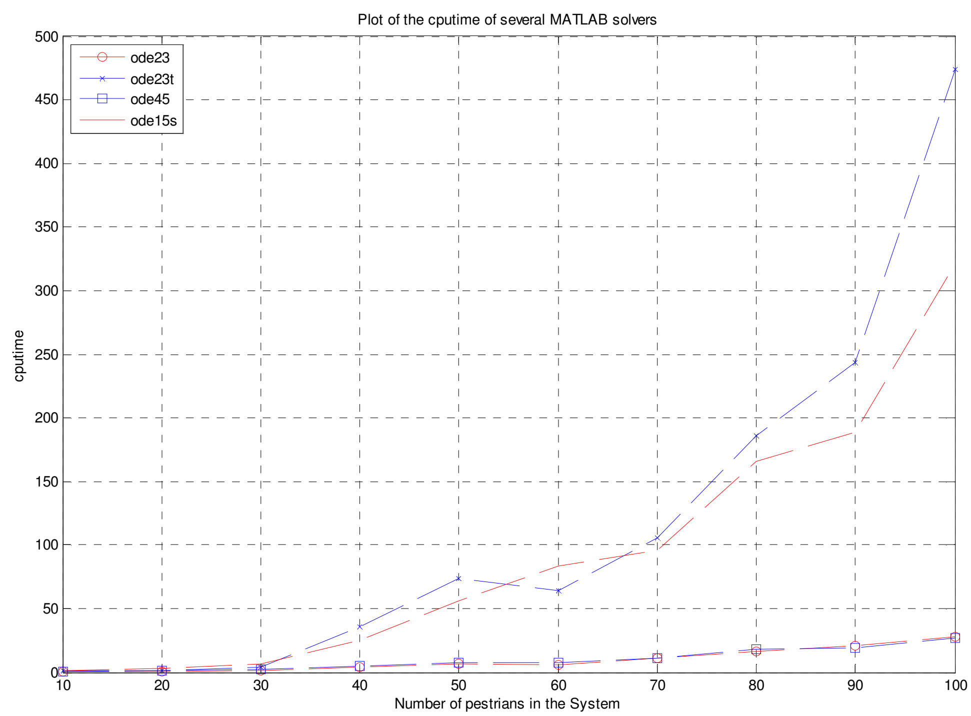


Figure 9: Cputime for several solvers – clearly ode23 and ode45 are the best solvers for the Social Force Model

Whilst the *stiff* solver produced accurate results, they were no more accurate than the *non-stiff* solvers. As such in the final implementation `ode45` was chosen. The efficiency of the system is of paramount importance as the Social Force model is of n^2 complexity.

This complexity comes from the pedestrian interaction equation (2.6.8), where the effect of every other individual in the system upon α must be calculated and then summed. This then has to be done for each pedestrian leading to $O\left(n^2\right)$ calculations, this corresponds to a double `for` loop in the function file. This algorithm can infact be reduced to an order *n* system, however this is not straightforward and time constraints prevent its implementation. The first step of this reduction however was included in the final code, which is to use a Verlet-Sphere [13]. For each pedestrian a circle, of radius *r*, surrounding them is introduced with *r* being the radius of the interaction length of the social force model. This radius is the maximum distance between any two pedestrian for which their social force interaction, as defined by (2.6.8), will be calculated. This represents physically the idea that pedestrians beyond a certain distance away will not effect the decision making of the individual α, which is both intuitively and mathematically reasonable, as (2.6.8) becomes negligible for $r = r_{\alpha\beta} - d_{\alpha\beta} \sim O(10)$.

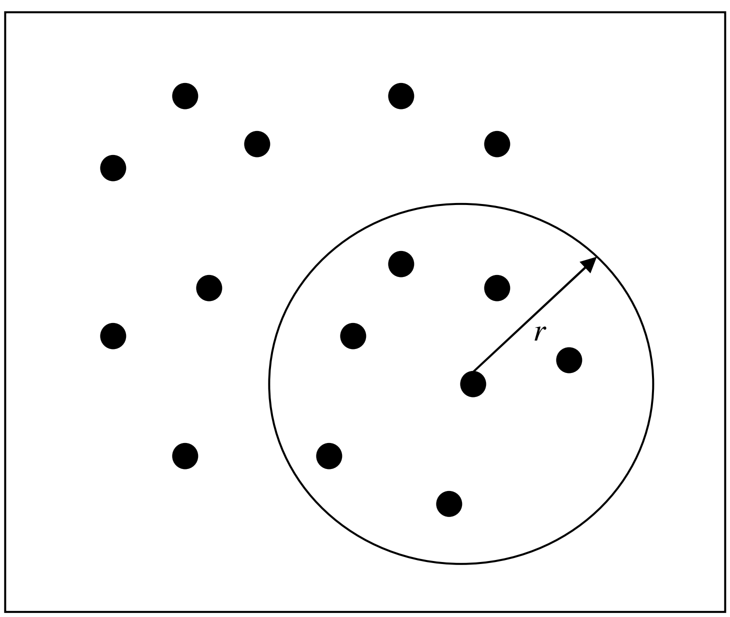


Figure 10: Verlet - Sphere

The final code for the simulation solves 4*n* differential equations, where *n* is the number of pedestrians in the system. To solve, `ode15s` requires a vector input with each component corresponding to a differential equation. The first 2*n* differential equations are the *x* and *y* velocity components for pedestrians 1 to *n* and the second 2*n* differential equations the *x* and *y* displacement values for pedestrians 1 to *n*. That is

$$\frac{d}{dt}\begin{bmatrix} v_\alpha^1(t) \\ v_\alpha^2(t) \end{bmatrix} = \begin{bmatrix} f_\alpha^1(t) \\ f_\alpha^2(t) \end{bmatrix} \text{ for } \alpha = 1,\ldots,n \tag{4.1.2}$$

and

$$\frac{d}{dt}\begin{bmatrix} r_\alpha^1(t) \\ r_\alpha^2(t) \end{bmatrix} = \begin{bmatrix} v_\alpha^1(t) \\ v_\alpha^2(t) \end{bmatrix} \text{ for } \alpha = 1,\ldots,n \tag{4.1.3}$$

The following *pseudo-code* describes how the differential equations are formulated in the function file that `ode15s` solves.

```
FOR every pedestrian in the system do
    Calculate desired velocity according to (2.6.6)
    Calculate desired destination
    Calculate desired direction according to (2.6.5)
    Calculate driving force (2.6.4)
    FOR every other pedestrian do
        Check distance between pedestrians
        IF this distance < radius of Verlet-Sphere
            Calculate Social Force (2.6.8)
            Add Social Force to driving force
    FOR each boundary element do
        Check distance to element
        Calculate the influence of the closest element from (2.6.12)
        Add to the Social and driving force
END

Set first 2n vector components as (4.1.2)
Set second 2n vector components as (4.1.3)
```

The simulation parameters are set outside of the function file in the associated script file. These parameters include the constants of each term, the desired destination, initial position and velocities and the boundary array which describes the geometry of the room being considered.

4.2 The Desired Destination and Waypoints

In the earlier version of the code, the desired destination was set as a single point which the pedestrian would head for. Whilst setting the desired destination in this way did not prevent the simulations from displaying the emergent characteristics expected, it leads to simulated behaviour which was qualitatively unreasonable. A good example to illuminate this would be two pedestrians moving in opposite directions meeting in a corridor.

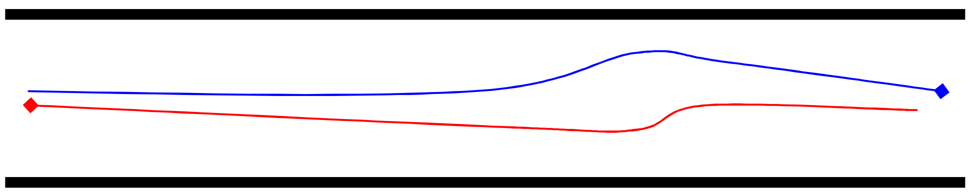


Figure 11: Pedestrian paths with a single point for $\vec{p}$, desired destination

In figure 11, the behaviour of pedestrians with fixed desired destinations is show. However, when walking down a corridor the destination is only the opposite end and not a fixed point. Thus the expected behaviour would be for the pedestrian to take the shortest path to the end of the corridor and as such they would re-evaluate their desired destination in the event of interaction with another walker. To implement this behaviour the desired destination was re-evaluated with each iteration. This was achieved setting a desired destination array and then calculating the closest point of this and using that as $\vec{p}$.

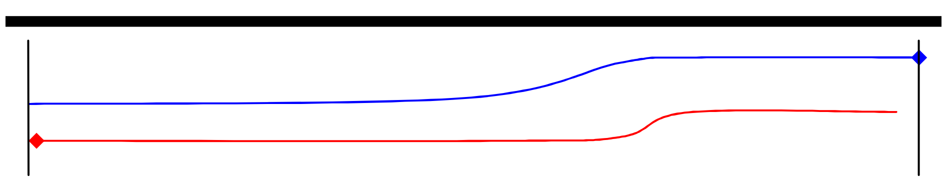


Figure 12: Pedestrian paths using a desired destination array

In Figure 12 the desired destination arrays are shown at each end of the corridor and the new pedestrian paths. In this small test case the difference may seem to be unimportant but its value in the modelling come as the number of pedestrians in the system increase. If the desired destination remained as a single point then individuals maybe simulated trying to force their way through a dense crowd to reach the specified point instead of taking the more efficient route. The principle at the core of all pedestrian modelling is that individuals will try to use the most efficient route for themselves and that the model must reflect the decision making abilities of humans and their capability to change and adapt to the system around them.

Another problem found in earlier version of the code was so called Hunting or Chattering effects. This phenomenon occurs when the direct line between the pedestrian and their desired destination is blocked by the boundary of the setup being simulated. If the model isn't modified to counter this effect the walker will head directly towards the desired destination until they reach the boundary. The repulsive interaction with the boundary is (obviously) stronger than the driving force and the pedestrian will become *stuck* on the wall. Clearly this behaviour is not consistent with observed pedestrian movement and so a solution must be found.

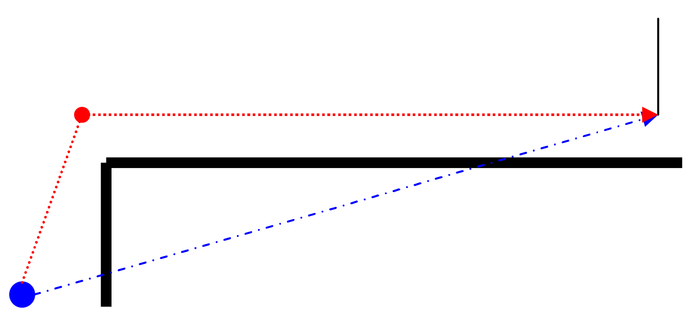


Figure 13: Pedestrians path with and without a waypoint

The solution implement in here is to use a waypoint system. In Figure 13 the blue dashed line illustrates the desired path of the pedestrian in the original code and the red line the desired path after a waypoint is included. The process of implementing waypoints is relatively straight forward, the *pseudo-code* is as follows:

```
FOR each pedestrian do
    Check the distance from current position to waypoint (i)
    IF the distance to waypoint (i) < specified distance
        Set desired destination as the waypoint (i + 1)
    ELSE
        Set desired destination as waypoint (i)
    END
END
```

Again it is important to note that waypoints are intuitively consistent with pedestrian behaviour, in particular a pedestrians immediate destination is always within their line of sight. That is, an individual will always travel to a place they can see until their ultimate destination is in view. In the case of angled passage ways, corners will be appropriate waypoints since moving directly toward them will provide the shortest path to the overall goal.

4.3 The Desired Velocity

One of the self-organisation phenomena of interest are shockwaves in dense crowds. This effect comes from an impatience factor of the modelled pedestrians. This is described in the formulation of the desired velocity, (2.6.6). The only extra calculation required is the average velocity, which is

$$\overline{v}_\alpha = \frac{\vec{r}_\alpha(t) - \vec{r}_\alpha(0)}{t} \tag{4.3.1}$$

In earlier versions of the code the desired velocity was fixed, the following graph shows the acceleration of an unobstructed pedestrian with both fixed and variable desired velocities.

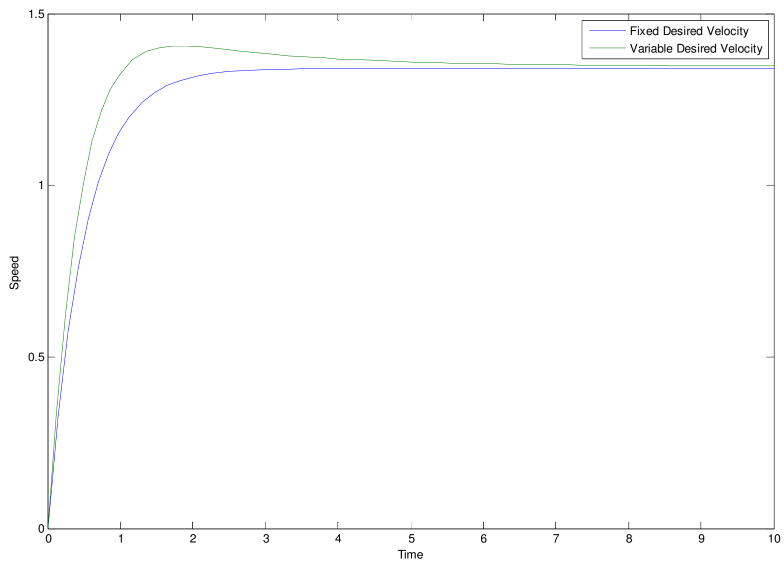


Figure 14: The difference between a fixed desired velocity and a variable one

As Figure 14 shows, the variable desired velocity means that the pedestrian will want to travel faster if they are impeded. The graph shows that this effect means the pedestrian will move slightly faster than the initial desired velocity before settling into a comfortable walking speed. In the case of the fixed desired velocity the walker reaches the desired velocity smoothly and without going faster than this value. Certainly the variable desired velocity describes shockwave effects well but this effect in the acceleration phase is also reasonable.

Chapter 5

Simulation and Analysis of the Social Force Model

In this chapter the code created to implement the Social Force Model will be run for a variety of different pedestrian flows. These will be a counterflow within a corridor, with and without a bottleneck, a ninety degree corner and finally a Scramble Crossing. The aim will be to use the data from the simulation to do some quantitative analysis of pedestrian flows to supplement the qualitative observations laid out in chapter 3. Obviously it is expected that the model will recreate self organisation phenomena and where this is the case it will be highlighted.

5.1 The Model Constants

The Social Force Model has several parameters in each of the driving force (2.6.4), pedestrian interaction (2.6.8), and boundary interaction terms (2.6.12). Thus, several parameters must be set in the startup phase of the simulation, in this instance in the script file for the model.

The driving force parameters are consistent with those of Helbing [1] where the desired velocity is set to 1.34 ms^{-1} with a relaxation time, τ_α, of 0.5. The relaxation time is the time taken to correct disturbance in movement (e.g. obstacles or avoidance manoeuvres). Helbing also gives values for the parameters in the pedestrian and boundary interaction terms. To verify that these values were reasonable the simulation was run for a variety of parameters in a test room and the resulting pedestrian flows evaluated.

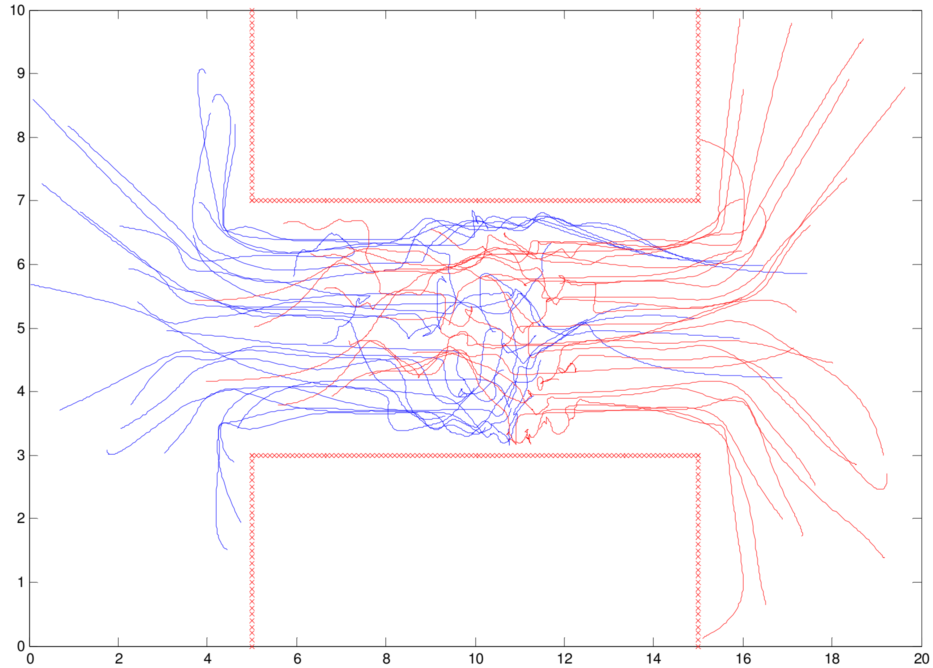


Figure 15: An example of pedestrian flows in the test room (Not to Scale)

Figure 15 shows one of the test runs for a given set of parameters, each line in the figure represents the movement of a pedestrian over time. The test room was simulated with 50 pedestrians and qualitative observations were made. An example of such an observation is that for large values of $B_{\alpha B}$ in (2.6.12) the pedestrians appeared to be too far from the boundaries given the amount of traffic in the room. To test the interaction strengths a simple test was to set the desired velocity of a pedestrian to 0 and place them next to a boundary. The repulsive force of the boundary for different parameter values could then be more accurately observed providing a better reference for deciding on realistic parameters. A similar process was done for pedestrian interactions. Whilst there was only a qualitative analysis at this stage it allowed for a set of realistic parameters to be chosen, where the conventions for pedestrian behaviour come from the authors' personal experience and the experiments done by Helbing [1]. The following table gives the chosen parameters

Parameter	Formula	Value
Initial Desired Velocity - $v_\alpha^0(0)$	(2.6.6)	1.34 ms^{-1}
Relaxation Time - τ_α	(2.6.4)	0.5 s
Maximum Speed - $v_\alpha^{\max}$	(2.6.6)	1.3* $v_\alpha^0(0)$
Territorial Sphere Pedestrian Interaction Strength - A_α^1	(2.6.8)	0
Territorial Sphere Pedestrian Interaction Range - B_α^1	(2.6.8)	0.3 m
Anisotropic Character - λ_α	(2.6.10)	0.75
Physical Pedestrian Interaction Strength - A_α^2	(2.6.8)	2
Physical Pedestrian Interaction Range - B_α^2	(2.6.8)	0.2 m
Boundary Interaction Strength - $A_{\alpha B}$	(2.6.12)	5
Boundary Interaction Range - $B_{\alpha B}$	(2.6.12)	0.1 m
Radius of Pedestrians – r_α	(2.6.8), (2.6.12)	0.3 m

Table 1: Simulation Parameters

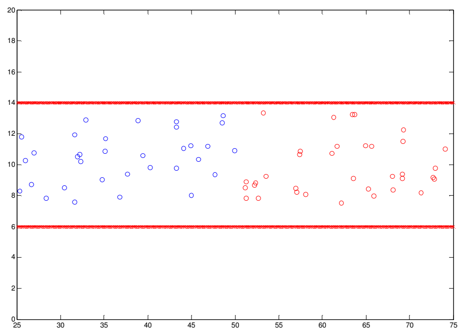
In the simulation it was chosen to take $A_\alpha^1 = 0$ as suggested by Helbing [1]. The purpose of this is to speed up the simulation in dense crowds, which is appropriate for the situations being considered. In all the cases that will be discussed in this thesis (in particular the Scramble Crossing) the emergent behaviour is most evident in densely populated situations. Since this term describes the tendency to respect each pedestrians territorial sphere these considerations can be ignored in dense crowds and emergency situations.

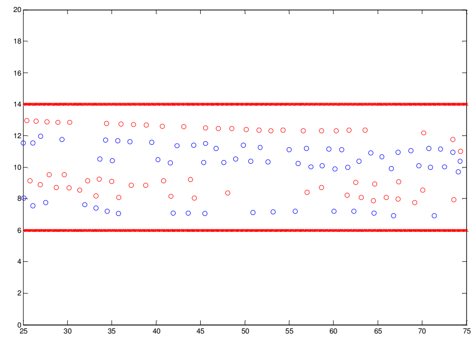
One final consideration in the startup phase is the size of the so-called Verlet-Sphere implemented in (2.6.8) discussed in chapter 4. The radius was chosen to be 10 m, anything further away than this has an overall contribution to the social force felt by α to be $\geq O\left(10^{-15}\right)$ which is negligible since the cumulative social force of all pedestrians upon α is of $O(1)$.

5.2 Pedestrian Counterflows

In this section the code developed will be used to simulate pedestrian counterflows both with and without a bottleneck in the walkway. The purpose of modelling these situations is to assess the validity of the simulation in a qualitative sense. In chapter 3 certain emergent phenomena were discussed; in particular lane formation in a counterflow and oscillatory flow at bottleneck. For these particular geometries a more detailed analysis of the pedestrian movements will not be considered, rather this section aims to confirm that the model simulates well experimentally observed phenomena ([1], [2]).

In a counterflow expected emergent behaviour is lane formation as discussed in chapter 3. The simulation was run for 250 pedestrians on a 200 meter walkway which is 6 meters wide. Half the pedestrians were randomly placed at each end of the walkway with a bivariate uniform distribution using the `rand` function in `MATLAB`. The wavefront of each the pedestrian bodies (in this case the group at either end) meet at the middle of the walkway. The simulation results were used to produce a movie (submitted on cd with the thesis – the file '250conterflow.avi') of the pedestrian movement.



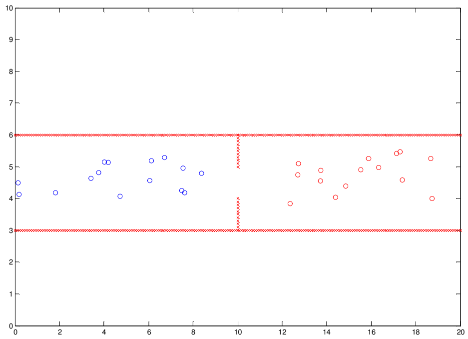


(a) The initial pedestrian distribution (b) The final pedestrian distribution

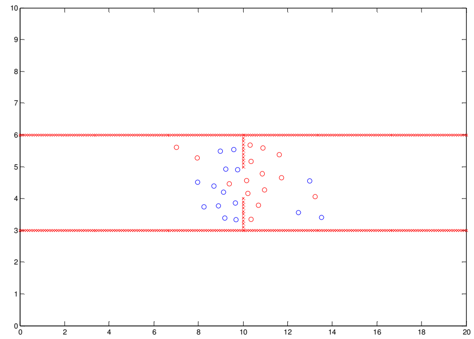
Figure 16: The initial and final pedestrian positions created for a counterflow

The results of the simulation are better than expected, reproducing the self organisation phenomena perfectly. However these results underline the idealised nature of the implemented model. In a real counterflow the pedestrians would not be homogeneous, that is the desired speed of the walkers would not be uniform and the pedestrians radii would not be the same. These inhomogeneities would lead to behaviour that isn't observed in this simulation, a particular example of this would be overtaking manoeuvres performed by faster moving pedestrians. In walkways with large pedestrian densities these manoeuvres can lead to a breakdown in the lane formation; this suggests that these idealisations may not always be valid assumptions.

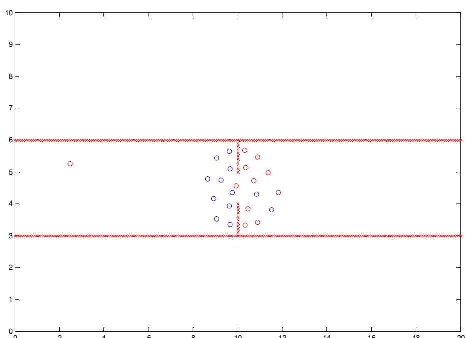
The next geometry to consider is a counterflow with a bottleneck. In this case the bottleneck refers to a doorway although in practice it could be a longer bottleneck (c.f. the test room geometry). The bottleneck, a 1 meter wide doorway, is positioned in the middle of the walkway which is 3 meters wide. 30 pedestrians were placed randomly, again with a bivariate uniform distribution, with half at each end of the corridor. Again a movie (on the cd – '100corridor.avi') of the simulation was produce – although it must be stressed it is not in real time. This is due to the fact that `ode45` is a variable timestep solver, which means that the time between each frame varies. However the movie creating tool in `MATLAB` has a fixed frame rate leading to the movies not being in real time.



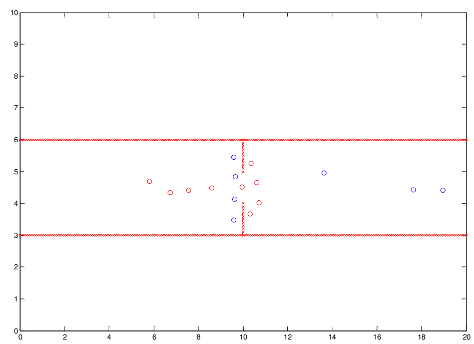
(a) Initial Pedestrian Positions



(b) Pedestrians meet at Bottleneck with right moving (red) pedestrians flowing through



(c) Switch in flow – now the left moving (blue) pedestrians are flowing through



(d) The flow has switched several times, now the right moving pedestrians are passing bottleneck

Figure 17: These figures show how flow oscillates at a bottleneck

The figure illustrates the oscillatory flow through a bottleneck is reproduced by the simulation. This oscillatory behaviour is much clearer in the associated movie. Again the results of the simulation are consistent with the experimental data of Helbing [1].

5.3 A Ninety Degree Corner

Having shown in two separate cases that the model is qualitatively accurate, the next stage is to do some quantitative analysis of the results for different geometries. The first situation to be modelled is a crowded corner in a corridor. A particular case where this geometry is of specific interest would be a modern sports stadium. There are often very large crowds which may need to evacuate the facility quickly and ninety degree corners are commonly found along escape routes. Thus the effect that they have upon pedestrian flow is of great interest.

The first thing to explore is how the corner effects the movement and speed of a single pedestrian. Obviously if a ninety degree corner has an effect on the large scale behaviour of pedestrian flows then it is reasonable to expect that there will be an observable change in the individual walkers movement at the corner.

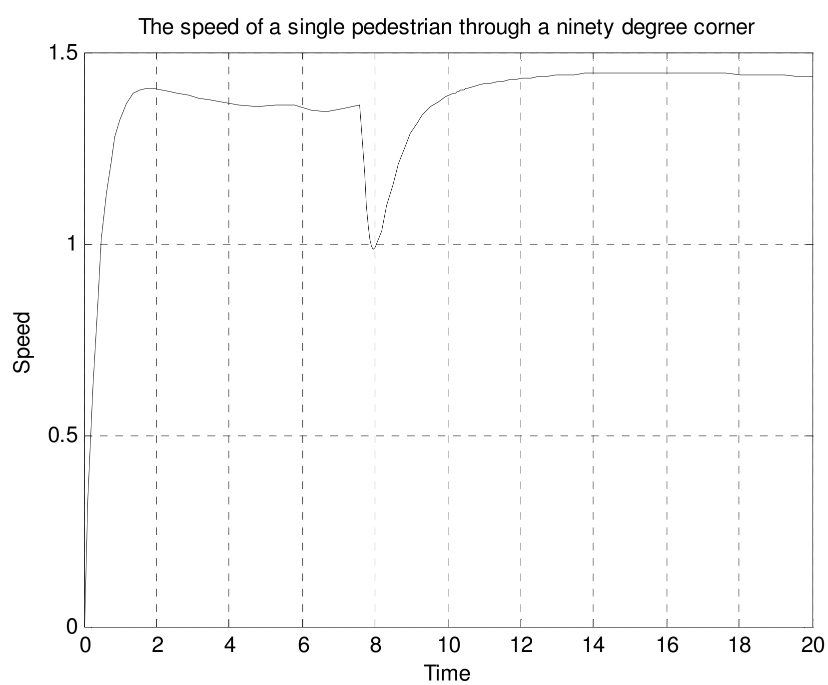


Figure 18: The Speed of a single Pedestrian around a ninety degree corner

Figure 18 clearly shows that the change of direction associated with moving around a corner leads to a sudden drop in speed as the walker reaches it. This highlights the problem with ninety degree corners in pedestrian facilities, that is that rather than allowing for a smooth change of direction and thus maintaining a steady speed it forces the pedestrians to change direction abruptly which has concertina effects on the following traffic. The figure also shows how the pedestrian speeds up after passing the corner to compensate for the obstruction. This effect results in the pedestrian density (pedestrians per unit area) being lower after the corner if the crowd is dense enough.

The next step is to model a large flow and analyse these concertina effects. As previously mentioned, the average pedestrian density is reduced after the corner and is greatest just before the corner, corresponding to the shockwave phenomena described in section 3.4.

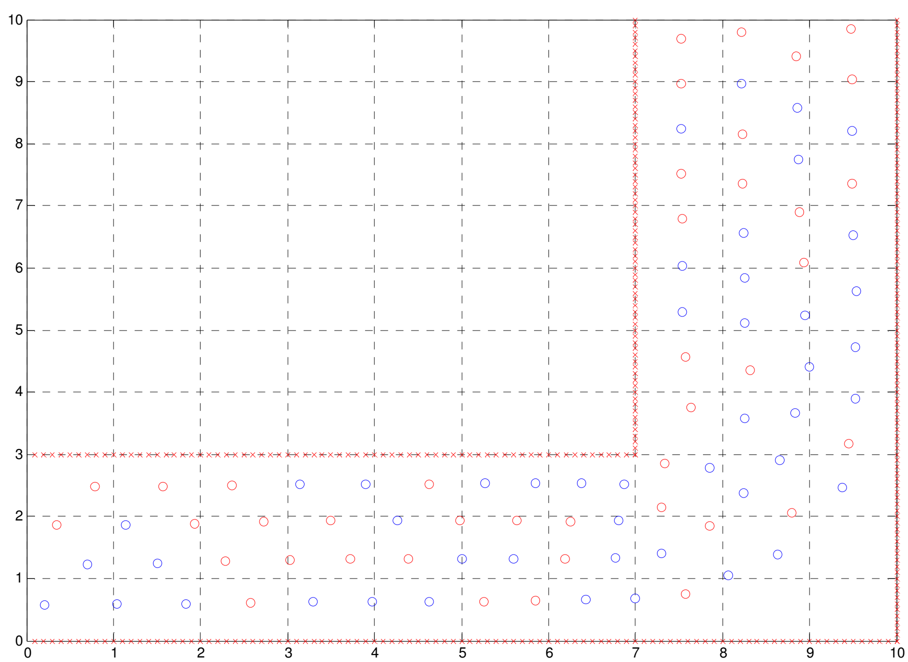


Figure 19: The simulation of a ninety degree corner

Figure 19 shows the simulation for only 200 pedestrians moving around a 3 meter wide corner, not a particularly large number when compared to what may be expected in a large sports stadium and already the corner is slowing traffic. This is highlighted in Figure 20, which shows that the overall mean speed of the pedestrians is reduced as the first walkers come to the corner. However, as more of the pedestrians pass the corner the overall mean speed starts to increase, reflecting the compensatory increase in speed of each individual as they pass the corner due to the change in their desired velocity described by (2.6.6).

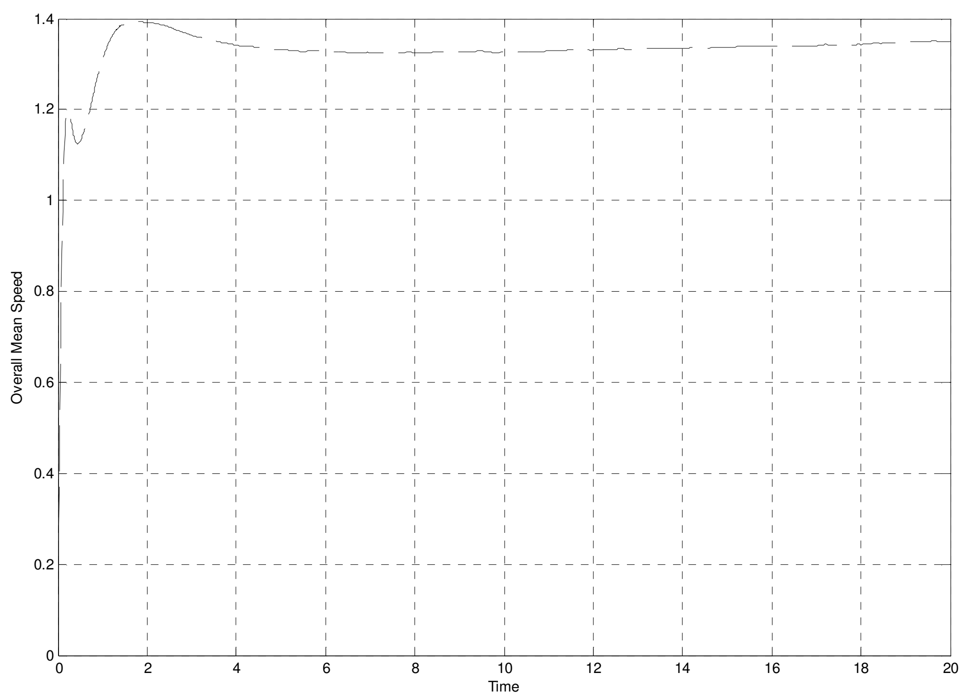


Figure 20: The overall mean speed of pedestrians around a ninety degree corner

The average densities were calculated for the 3 distinct phases of the corridor – the unidirectional flow before the corner, the corner itself and the corridor after it. The average pedestrian density before and after the corner are similar here, at approximately

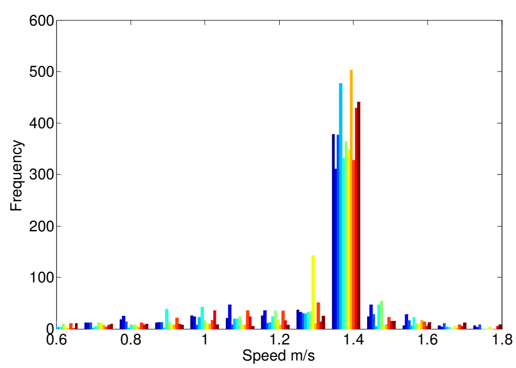
1.5 pedestrians per square meter, however on the corner this increases to approximately 2.7 pedestrians per square meter, if the 3 empty cells on thee outside of the corner are excluded. It is not unexpected that these 3 cells are empty; this is due to the fact that the natural instinct of a walker will be to head for the apex of the corner, as this will provide the shortest route to their desired destination. This reasoning also goes some way toward explaining why the corner itself is such a hindrance to traffic flow. Clearly all the pedestrians are seeking the shortest route and so all will want to pass as close to the apex of the corner as they can. This in turn will lead to increased densities in the cells next to the apex and explain the zero pedestrian density at the far edge of the corner.

In this case the densities before and after the corner is essentially the same; but, as figures 18 and 20 show, the speed of the walkers has increased after the corner. This quantitative effect is a good foundation for explaining why corners can cause clogging in dense crowds. Unfortunately, due to the n^2 complexity of the code, it wasn't possible to run the simulation for higher numbers of pedestrians. As discussed, for larger crowds, it would be expected that the corner would clog more noticeably leading to a significant reduction in pedestrian density after it. It should also be noted that although some shockwaving has occurred at the corner, this effect is not as prominent as the experimental data suggests it should be and again this may simply be because the current crowd isn't large enough. At this point it is worth mentioning again the homogeneity of the pedestrians in the current simulations, in particular the uniform desired speed. This uniformity is both unrealistic and means that no overtaking manoeuvres will occur. The lack of certain expected behaviours in the simulation therefore may, in part, be explained by this.

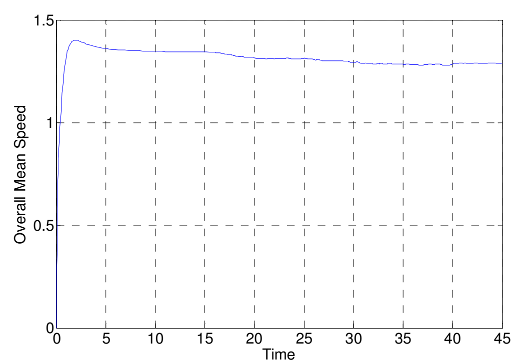
5.4 The Scramble Crossing

The final pedestrian flow to consider is the Scramble Crossing. Scramble Crossing is the term used to describe the meeting of two unidirectional flows with a non-zero angle of intersection (if the angle of intersection is zero this simply describes a bidirectional counterflow). Such intersecting flows are commonplace in pedestrian walkways and so the dynamics of them is of great interest. Only the simplest case, where there are no boundaries for the pedestrians to interact with, will be simulated in this project as the effect of particular interest is the stripping phenomena described in section 3.1.

A Scramble Crossing with 500 individuals was simulated with a ninety degree angle of intersection. The first quantitative data to consider is the velocities of the pedestrians in the simulation. It is expected that the pedestrians will interact in a manner that leads to the least deviation from the desired velocities.



(a) A Histogram of the speed of 10 random walkers



(b) The overall mean speed of pedestrians in a Scramble Crossing

Figure 21: Plots of the speed characteristics of a Scramble crossing

The plots in figure 21 illustrate well that the simulated interactions within a scramble crossing are optimal. In Figure 21(b) the drop in overall mean speed at about 15 seconds is when the two pedestrian wavefronts meet. This drop in the overall mean speed is approximately 0.1 ms^{-1} which would is clearly not particularly significant. Obviously the overall mean speed may hide the fact that movement in the area where the 2 flows meet is very slow. Clearly only a small proportion of the total number of pedestrians will be in the intersection at any one time and so the speed of the pedestrians outside of the point of intersection could mask the fact that movement where the flows meet is slow. Figure 21(a) however dispels this; it is a histogram of the speeds at each timestep of a random sample of 10 pedestrians where each pedestrian is represented by a different colour bar. It clearly shows that the pedestrian speeds are tightly distributed around the initial desired speed of 1.34 ms^{-1} indicating that even when a pedestrian is moving through the area of intersection their actual speed does not deviate too far from their desired speed.

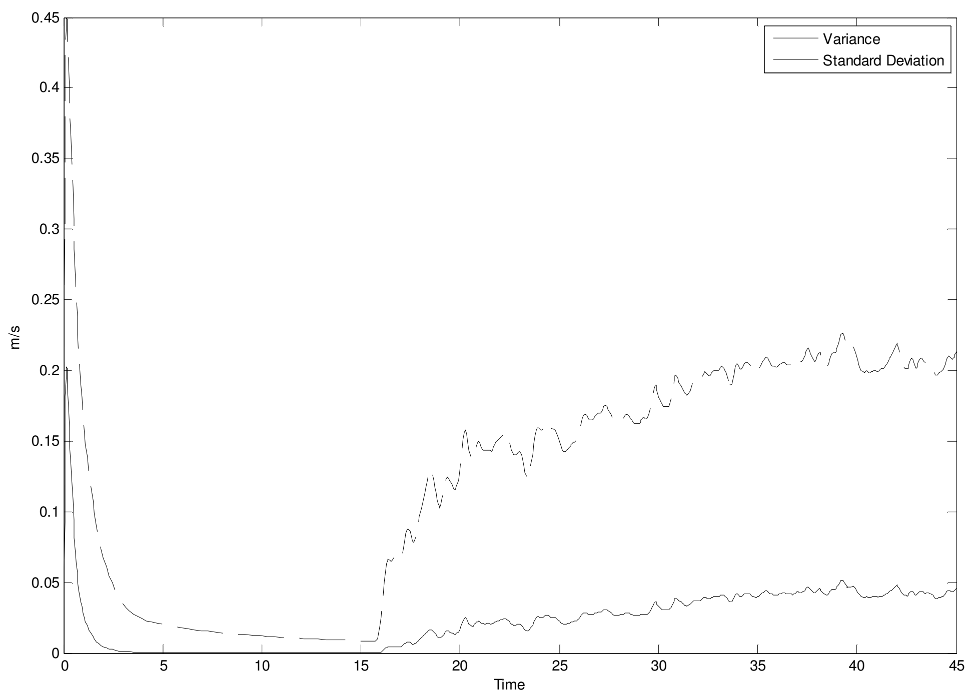


Figure 22: Variance and Standard Deviation of Pedestrian in a Scramble Crossing

Figure 22 shows both the variance and standard deviation of the pedestrian speeds in the simulated Scramble Crossing. These are perhaps better statistical measures of the pedestrian behaviour and are consistent with the previous conclusions. The mean standard deviation, after the two flows meet (at 16 seconds into the simulation), is 0.1888 which shows that the pedestrian speeds do not vary wildly when the flows meet supporting the conclusions drawn from the plots in Figure 21.

The most interesting aspect of the Scramble Crossing is the observed stripping phenomena. The simulation does reproduce this effect although it is not as obvious the lane formation of a counterflow shown in Figure 16 or the illustration of the effect in Figure 7. As this is the case there is little point showing still frames of the simulation. The effect is much more evident from the movies (submitted on CD with the thesis – '500scramblecrossing.avi') and certainly the movie shows that the simulation recreates the expect phenomena qualitatively. One statistical measure to show this stripping phenomena is occurring is the computation of Lattice Constants, l_1 and l_2. These are described by Ripley [14] in chapter 2. In the simplest case, where the standard basis is used, l_1 is the average distance between pedestrians in the x direction and l_2 the y direction. These Lattice Constants were calculated for 5 different regions, the 2 flows before the intersection (A and B) and the 2 flows after intersection (C and D) and the area of intersection itself (E). The Lattice constants were taken at $t = 40$ so a sufficient number

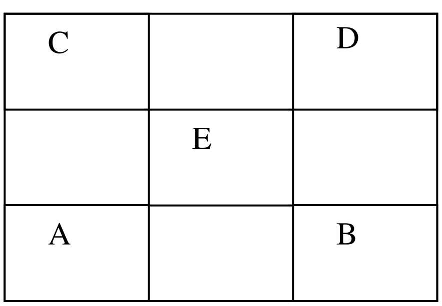


Figure 23: The regions the Lattice Constants are calculated in

of pedestrian had managed to pass the intersection of the flows. The Lattice constants are summarised in Table 2:

Region	l_1	l_2
A	0.5486	0.5491
B	0.5516	0.5521
C	0.6132	1.0236
D	0.5962	0.9742
E	0.3212	0.3195

Table 2: Lattice Constants

Initially the pedestrians were uniformly distributed and given the homogeneity of the walkers in this simulation it is to be expected that the lattice constants in regions A and B are very close to each other. In region E both the Lattice Constants are close to 0.3 which is the pedestrian radius defined for this simulation (See Table 1) describing the high density in the area of intersection. The interesting values are produced in regions C and D. In these area, l_1, the average separation in the x direction, is close to the initial values but l_2, the average separation in the y direction has doubled. This change, whilst visually obvious, provides a quantitative value highlighting the stripping effect.

5.5 Model Shortcomings

Having simulated a variety of pedestrian flows, the results obtained have highlighted a number of areas in which the model could be improved. The most obvious of these, which has already being touched upon, is the homogeneity of the system. Currently the pedestrians are modelled as uniform, which may not be a reasonable assumption and may lead to important effects not being simulated. The solution would be to obtain further information on the makeup of pedestrian crowds from experimental data. In the case of the desired velocity it would be a case of setting it a randomly distributed value, probably a Normal distribution with a mean value the same as the current constant desired velocity and a variance of 0.3 say, that is $v_{\alpha}^{0}(0) \sim N(1.34, 0.3)$. A similar approach could be adopted with the pedestrian radii.

Another problem that arises in the current implementation are shadowing effects. These occur when there is a solid wall between two pedestrian but the distance between them is less than the radius of the Verlet-Sphere. Even though the pedestrians can't see each other the current code still calculates the social force between them and adds it to the acceleration. Clearly this is unrealistic given the fact that there is no line of sight between them and thus no way either can know the other is there. One way to solve this problem would be to adjust the Verlet-Sphere for each pedestrian. This re-evaluation is shown below

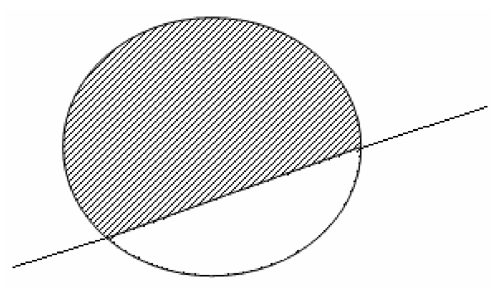


Figure 24: The New Verlet-Sphere

Only Pedestrians in the shaded area would be considered in the model, eliminating the shadowing effects.

Chapter 6

Summary and Future Directions

The aim of this project was to implement a version of the Social Force Model and use it to simulate various pedestrian flows. The results produced by the simulation with this implementation provided some excellent qualitative results, reproducing the expected self-organisation phenomena very well. These included oscillating flows at bottle necks, lane formation in a counter flow and stripping in a Scramble Crossing. However in the final chapter several flaws in the current implementation were identified though solution to each of them was suggested.

If this project were to be continued in future there are several areas that should be looked at. The first of these would be to include the additional pedestrian behaviours set out in section 2.6 such as the random fluctuations described by (2.6.13) and the joining behaviour discussed in section 2.6.4.

Currently the simulation requires order n^2 calculations at each iteration of the ODE solver. There are techniques to reduce this to an order n algorithm which would allow the simulation to be run for much larger systems on a single processor. Even if an order n algorithm is achieved it would be useful to parallelise the implementation so that the simulation could be extended to huge scale problems such as airport terminals or world sporting events, where there may be as many as 150,000 pedestrians present.

With the order *n* algorithm in place it would then be possible to explore the effect of vastly increased density upon the systems already investigated. Certainly in the case of the ninety degree corner it would be valuable to find a quantitative value for the critical density of the geometry. The effect of the flow reaching its critical density is of paramount importance in emergency situations as this is when such a density may be reached.

Further investigation should focus primarily on the effect of boundaries on pedestrian flows and try to develop solutions to flaws in current building design. This is simply because the most interesting pedestrian flows occur when boundaries are present and that in emergency situations it is the boundaries that become the biggest hazards when they restrict pedestrian flow.

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Appendix A

MATLAB Codes

A.1 SFMDFPIBI.m

```
%   Driving Force, Pedestrian interactions and boundary interactions     %
%                    dv/dt = f0a + fab + faB                             %
%------------------------------------------------------------------------%

clear all
global m bdry p p1 p2 D way BDRY
 global tau v0a0 vmax r A1a A2a B1a B2a lambda A1 B1 VS dist

% Number of Pedestrians%
m = input('Enter the number of Pedestrians in the system (must be even):');
% Timespan of simulation %
T = input('Enter the desired length of the simulation (in seconds): ');
tspan = [0 T];
% Select the geometry to be simulated %
disp('The code has 5 geometries to choose from, they are:')
disp('1. A counterflow')
disp('2. A Counterflow with a doorway (bottleneck)')
disp('3. The test room')
disp('4. A Scramble Crossing')
disp('5. A ninety degree corner')
K = input('Enter the number of the room you wish to simulate: ');
%-------------------------------------------------------%
% Vector used in waypoint calculation - their value tells  %
% pedestrian alpha which waypoint to head for              %
%-------------------------------------------------------%
D = zeros(1,m);
%-----------------%
% model constants %
%-----------------%

% Relaxation time %
tau    = 0.5;
% Initial desired velocity %
v0a0   = 1.34;
% Maximum desired velocity %
vmax   = 1.3*v0a0;
% Pedestrian radii %
r      = 0.3*ones(1,m);
% Pedestrian Interaction constants from (2.6.8) %
A1a    = 0;
A2a    = 2;
B1a    = 0.3;
B2a    = 0.2;
lambda = 0.75;
% Boundary Interaction constants from (2.6.12) %
A1     = 5;
```

```
B1     = 0.1;
% Radius of Verlet Sphere %
VS     = 10;
% Distance to waypoint at which it deem to have been reached %
dist   = 1.2;
%-------------------------------------%
% Initial position and velocity vector %
%-------------------------------------%

if (K == 1)
    way = 1;
    Vx  = [1.34*rand(1,m/2),-1.34*rand(1,m/2)];
    Vy  = zeros(1,m);
    X   = [9*rand(1,m/2),9*rand(1,m/2)+11];
    Y   = [2*rand(1,m)+3.5];
elseif (K == 2)
    way = 0;
    Vx  = zeros(1,m);
    Vy  = zeros(1,m);
    X   = [100*rand(1,m/2)-50,100*rand(1,m/2)+50];
    Y   = [6*rand(1,m)+7.5];
elseif (K == 3)
    way = 1;
    Vx  = zeros(1,m);
    Vy  = zeros(1,m);
    X   = [5*rand(1,m/2),5*rand(1,m/2)+15];
    Y   = [10*rand(1,m)];
elseif (K == 4)
    Vx  = zeros(1,m);
    Vy  = zeros(1,m);
    X   = [45*rand(1,m/2)-40,45*rand(1,m/2)+35];
    for i = 1:m
        if (i <= m/2)
            Y(i) = X(i) - 10*rand(1,1) + 5;
        elseif (i > m/2)
            Y(i) = (35 - X(i)) + 10*rand(1,1);
        end
    end
elseif (K == 5)
    way = 1;
    Vx  = zeros(1,m);
    Vy  = zeros(1,m);
    X   = [35*rand(1,m)-30];
    Y   = [2*rand(1,m)+0.5];
end
% Setting u - the initial position and velocity vector %
for i = 1:m
    u((2*i)-1) = Vx(i);
    u((2*i))   = Vy(i);
end
for i = 1:m
    u((2*m)+(2*i)-1) = X(i);
    u((2*m)+(2*i))   = Y(i);
end
start = u;
%-----------------------------------------------------------------------%
% Desired destination - p is initially set as a waypoint (if required) %
%-----------------------------------------------------------------------%

if (K == 1)
```

```
    p    = [[10.5*ones(1,m/2),9.5*ones(1,m/2)]',[4.5*ones(1,m)]'];
    p1   = [[50*ones(1,(2/0.1)+1)]',[3.5:0.1:5.5]'];
    p2   = [[-30*ones(1,(2/0.1)+1)]',[3.5:0.1:5.5]'];
elseif (K == 2)
    p  = zeros(m,2);
    p1 = [[100*ones(1,(6/0.1)+1)]',[7:0.1:13]'];
    p2 = [[zeros(1,(6/0.1)+1)]',[7:0.1:13]'];
elseif (K == 3)
    p    = [[5*ones(1,m/2),15*ones(1,m/2)]',[5*ones(1,m)]'];
    p1   = [[20*ones(1,(4/0.1)+1)]',[3:0.1:7]'];
    p2   = [[zeros(1,(4/0.1)+1)]',[3:0.1:7]'];
elseif (K == 4)
    p    = zeros(m,2);
    p1   = [[50:0.1:55]',[55:-0.1:50]'];
    p2   = [[-15:0.1:-10]',[50:0.1:55]'];
elseif (K == 5)
    p    = [[8.5*ones(1,m)]',[2*ones(1,m)]'];
    p1   = [[7.5:0.1:9.5]',[40*ones(1,(2/0.1)+1)]'];
    p2   = [[7.5:0.1:9.5]',[40*ones(1,(2/0.1)+1)]'];
end
%----------------%
% Boundary Array %
%----------------%

if (K == 1)
    bdry = [[0:0.1:20,0:0.1:20,10*ones(1,(1/0.1)+1),10*ones(1,(1/0.1)+1)...
        ]',[3*ones(1,(20/0.1)+1),6*ones(1,(20/0.1)+1),3:0.1:4,5:0.1:6]',];
    BDRY = 1;
elseif (K == 2)
    bdry = [[-50:0.1:150,-50:0.1:150]'...
        ,[6*ones(1,(200/0.1)+1),14*ones(1,(200/0.1)+1)]'];
    BDRY = 1;
elseif (K == 3)
    bdry = [[5:0.1:15,5:0.1:15,5*ones(1,(3/0.1)+1),5*ones(1,(3/0.1)+1)...
        ,15*ones(1,(3/0.1)+1),15*ones(1,(3/0.1)+1)]'...
        ,[3*ones(1,(10/0.1)+1),7*ones(1,(10/0.1)+1),0:0.1:3,7:0.1:10,...
        0:0.1:3,7:0.1:10]'];
    BDRY = 1;
elseif (K == 4)
    BDRY = 0;
elseif (K == 5)
    bdry = [[-30:0.1:10,-30:0.1:7,10*ones(1,(40/0.1)+1),...
        7*ones(1,(37/0.1)+1)]',[zeros(1,(40/0.1)+1),3*ones(1,(37/0.1)+1)...
        ,0:0.1:40,3:0.1:40]'];
    BDRY = 1;
end
%-----------------------------------------%
% The solver for the system - here ode45 %
%-----------------------------------------%

options1 = odeset('AbsTol',1d-3,'RelTol',1d-4);
[t1,u1] = ode45(@fun5,tspan,start,options1);
pause
%----------------------------------------%
% Plotting the paths of the pedestrians %
%----------------------------------------%

% Plotting the boundaries %
if (K == 4)
    hold on
```

```
else
    plot(bdry(:,1),bdry(:,2),'rx')
    if (K == 1)||(K==3)
        axis([0,20,0,10]);
    elseif (K == 2)
        axis([25,75,0,20]);
    elseif (K == 5)
        axis([0,10,0,10]);
    end
    hold on
end
% Plotting the pedestrian paths %
for i = (2*m)+1:2:(4*m)
    if (i<=3*m)
        plot(u1(:,i),u1(:,i+1),'b-')
        if (K == 1)||(K == 3)
            axis([0,20,0,10]);
        elseif (K == 2)
            axis([25,75,0,20])
        elseif K == 4
            axis([0,40,0,40]);
        elseif K == 5
            axis([0,10,0,10]);
        end
        hold on
    else
        plot(u1(:,i),u1(:,i+1),'r-')
        if (K == 3)||(K ==1)
            axis([0,20,0,10]);
        elseif (K == 2)
            axis([25,75,0,20])
        elseif K == 4
            axis([0,40,0,40]);
        elseif K == 5
            axis([0,10,0,10]);
        end
        hold on
    end
end
pause
%-------------------------------------------------------------%
% Making the Movie - plots the pedestrian positions at each %
% timestep and then uses each plot as a movie frame         %
%-------------------------------------------------------------%

for j = 1:length(t1)
    % Plotting the boundaries %
    if (K == 4)
        hold off
    else
        hold off
        plot(bdry(:,1),bdry(:,2),'rx')
        if (K == 1)||(K == 3)
            axis([0,20,0,10]);
        elseif (K == 2)
            axis([25,75,0,20]);
        elseif (K == 5)
            axis([0,10,0,10]);
        end
        hold on
```

```
    end
    % Plotting the pedestrians %
    for i = (2*m)+1:2:(4*m)
        if (i<=3*m)
            plot(u1(j,i),u1(j,i+1),'bo')
            if (K == 1)||(K == 3)
                axis([0,20,0,10]);
            elseif (K == 2)
                axis([25,75,0,20])
            elseif K == 4
                axis([0,40,0,40]);
            elseif K == 5
                axis([0,10,0,10]);
            end
            hold on
        else
            plot(u1(j,i),u1(j,i+1),'ro')
            if (K == 3)||(K ==1)
                axis([0,20,0,10]);
            elseif (K == 2)
                axis([25,75,20])
            elseif K == 4
                axis([0,40,0,40]);
            elseif K == 5
                axis([0,10,0,10]);
            end
            hold on
        end
    end
    F(j) = getframe;
end

clf
```

A.2 fun5.m

```
% Driving force, pedestrian interaction and boundary interaction function %
%-------------------------------------------------------------------------%

function f0a = fun5(t,u)
global m bdry p p1 p2 D way BDRY x0
global tau v0a0 vmax r A1a A2a B1a B2a lambda A1 B1 VS dist

%--------------------------------------------------%
% Putting pedestrian positions in a separate vector %
%--------------------------------------------------%

for i = 1:m
    x(i,1) = u((2*i)-1+(2*m));
    x(i,2) = u((2*i)+(2*m));
end
%------------------------------------------------------%
% Setting inital position for average speed calculation %
%------------------------------------------------------%

if (t == 0)
    x0 = x;
end
```

```
%-------------------------------%
% Desired Velocity from (2.6.6) %
%-------------------------------%
for i = 1:m
    if t == 0
        v0a(i) = v0a0;
    else
        V (i,:)    = x(i,:) - x0(i,:);
        Vbar       = norm(V(i,:)/t);
        na         = 1 - (Vbar/v0a0);
        v0a(i)     = (1 - na)*v0a0 + na*vmax;
    end
    %----------------------%
    % Desired destination p %
    %----------------------%
    if (way == 1)
        % Pedestrians in area A (L-Hand corner for Scramble Crossing) %
        if (i <= m/2)
            nrmpx(i) = norm(p(i,:) - x(i,:));
            if (nrmpx(i) < dist)
                D(i) = 1;
            end
            if (D(i) == 1)
                p(i,:) = p1(1,:);
                nrmpx(i) = norm(p(i,:) - x(i,:));
                for j = 1:length(p1)
                    if (norm(p1(j,:) - x(i,:)) < nrmpx(i))
                        p(i,:) = p1(j,:);
                        nrmpx(i) = norm(p(i,:) - x(i,:));
                    end
                end
            end
            % Pedestrians in area B (R-Hand corner for Scramble Crossing) %
        elseif (i > m/2)
            nrmpx(i) = norm(p(i,:) - x(i,:));
            if (nrmpx(i) < dist)
                D(i) = 1;
            end
            if (D(i) == 1)
                p(i,:) = p2(1,:);
                nrmpx(i) = norm(p(i,:) - x(i,:));
                for j = 1:length(p2)
                    if (norm(p2(j,:) - x(i,:)) < nrmpx(i))
                        p(i,:) = p2(j,:);
                        nrmpx(i) = norm(p(i,:) - x(i,:));
                    end
                end
            end
        end
    elseif (way == 0)
        % Pedestrians in area A (L-Hand corner for Scramble Crossing) %
        if (i <= m/2)
            p(i,:) = p1(1,:);
            nrmpx(i) = norm(p(i,:) - x(i,:));
            for j = 1:length(p1)
                if (norm(p1(j,:) - x(i,:)) < nrmpx(i))
                    p(i,:) = p1(j,:);
                    nrmpx(i) = norm(p(i,:) - x(i,:));
                end
```

```
        end
        % Pedestrians in area B (R-Hand corner for Scramble Crossing) %
    elseif (i > m/2)
        p(i,:) = p2(1,:);
        nrmpx(i) = norm(p(i,:) - x(i,:));
        for j = 1:length(p2)
            if (norm(p2(j,:) - x(i,:)) < nrmpx(i))
                p(i,:) = p2(j,:);
                nrmpx(i) = norm(p(i,:) - x(i,:));
            end
        end
    end
end
%----------------------------------------%
% Desired Direction vector from (2.6.5) %
%----------------------------------------%
if (nrmpx(i) == 0)
    e(i,:) = [0,0];
else
    e(i,:) = (p(i,:) - x(i,:))/(nrmpx(i));
end
%---------------------------------------%
% Driving Force component from (2.6.4) %
%---------------------------------------%
f((2*i)-1) = ((v0a(i)/tau) * e(i,1)) - (u((2*i)-1)/tau);
f((2*i))   = ((v0a(i)/tau) * e(i,2)) - (u((2*i))/tau);
%-------------------------------------------------------------------------%
% Pedestrian Interactions from (2.6.8) - Summing all the interactions %
%-------------------------------------------------------------------------%
fab = [0,0];
for j = 1:m
    if (i ~= j)
        dab = norm(x(i,:) - x(j,:));
        if (dab < VS)
            rab = r(i) + r(j);
            nab = ((x(i,:) - x(j,:))/dab);
            fab = fab + ((A1a*exp((rab - dab)/B1a))*nab)...
                *(lambda + (1 - lambda)*((1+(-nab*e(i,:)'))/2))...
                + ((A2a*exp((rab - dab)/B2a))*nab);
        end
    end
end
f((2*i)-1) = f((2*i)-1) + fab(1,1);
f((2*i))   = f((2*i)) + fab(1,2);
%-------------------------------------------------------------------------%
% Boundary Interaction component, this is analogous to the pedestrian %
% interactions - infact the formula only needs different constants    %
%-------------------------------------------------------------------------%

%-----------------------------------------------------%
% Summing all the boundary interactions from (2.6.12) %
%-----------------------------------------------------%
if BDRY == 1
    for j = 1:length(bdry)
        if (j == 1)
            daB = norm(x(i,:) - bdry(j,:));
            BDY = bdry(j,:);
            rd  = r(i) - daB;
        elseif ((norm(x(i,:) - bdry(j,:))) < daB)
            daB = norm(x(i,:) - bdry(j,:));
```

```
                BDY = bdry(j,:);
                rd  = r(i) - daB;
            end
        end
        naB = (x(i,:) - BDY)/daB;
        faB = ((A1*exp(rd/B1))*naB);
        f((2*i)-1) = f((2*i)-1) + faB(1,1);
        f((2*i))   = f((2*i)) + faB(1,2);
    end
    %----------------------------------%
    % Displacement update from (2.6.1) %
    %----------------------------------%
    X((2*i)-1)= u((2*i)-1);
    X(2*i)    = u(2*i);
end

f0a = [f,X]';
```