

# PHY422/820: Classical Mechanics

FS 2021

Exam Preparation

October 6, 2021

## Problem P3 – Atwood Machines II

[cf. **problem G6**] Consider the Atwood machine shown in the figure, consisting of two masses  $m_1$ ,  $m_2$ , an ideal pulley and a string of fixed length  $l$ .

1. Show that the *unconstrained* Lagrangian of the machine is given by

$$L = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + m_1gx_1 + m_2gx_2, \quad (1)$$

where  $x_1$  and  $x_2$  are the lengths indicated in the figure, increasing in downward direction.

2. Starting from the *holonomic* form of the constraint, use the Lagrange formalism of the first kind to show that the tension in the string is

$$T = \frac{2m_1m_2}{m_1 + m_2}g. \quad (2)$$

3. Repeat the analysis based on the nonholonomic form of the constraint, and show that you obtain the same result for  $T$ .

