

PHY422/820: Classical Mechanics

FS 2020

Exam Preparation

December 1, 2020

Problem P4 – Hoop and Pulley

A mass M is attached to a massless hoop (of radius R) which lies in a vertical plane. The hoop is free to rotate about its fixed center. M is tied to a string which winds part way around the hoop, then rises vertically up and over a massless pulley. A mass m hangs on the other end of the string (see figure).

1. Show that the Lagrangian of the machine is given by

$$L = \frac{1}{2} (M + m) R^2 \dot{\theta}^2 + MgR \cos \theta + mgR\dot{\theta}. \quad (1)$$

2. Find the equation of motion for the angle of rotation of the hoop. What is the frequency of small oscillations around the equilibrium? Assume that m moves only vertically, and that $M > m$.

