2. (20pt)

Suppose that $n > d + 1$. Then, we cannot compute the inverse of A since A is not a square matrix. In this case, how can we solve the linear equation $A\mathbf{w} = \mathbf{y}$?

To solve this equation for $\vec{w}$, I will find the pseudo inverse of matrix A. The Moore-Penrose pseudo inverse is defined as

$$B = (A^T A)^{-1} A^T$$

where A^TA is the Gramian matrix, which has the property of being square positive semidefinate with the same eig. vectors and squared eig. values. Inversing this and multiplying with A^T preforms a least square fit, which is essential for our regresion.

$$A\vec{w} = \vec{y}$$

make it square →

$$A^TA\vec{w} = A^T\vec{y}$$

Solve for $\vec{w}$ →

$$\vec{w} = (A^TA)^{-1}A^T\vec{y}$$

Moore-Penrose →

$$\vec{w} = B\vec{y}$$