20.06.04 #KUPT 07

< Hypothesis Testing >

Types errors

- $\alpha = P(\text{reject } H_0 \mid H_0 \text{ is true})$
- $\beta = P(\text{accept } H_0 \mid H_a \text{ is true})$

P-value

- probability of a result at least as extreme as the result we actually got, assuming H_0 to be true.
- smaller p-values are stronger evidence against H_0 in the favor of H_a

Power (θ)

probability that the test rejects H_0
when true parameter value is θ

$P(\text{reject } H_0 \mid \theta)$ 검정력

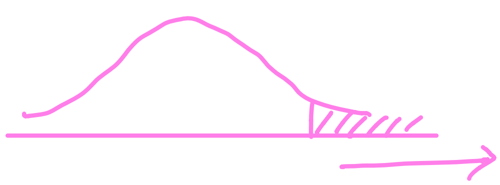
$\alpha = P(RR \mid H_0)$

$\beta = P(RR^c \mid H_a)$

power at H_a = $P(RR \mid H_a) = 1-\beta$

p-value = $P(H_a$에 가깝 $\mid H_0)$

after observation
sample에 따라서 다르게 나온다



p-value랑
H_a 방향 같다

ex) assume $Y \sim N(\mu, 9)$

$H_0 : \mu = 25$ vs $H_a : \mu < 25$

$\bar{Y} \sim N(\mu, \frac{9}{n})$

$N(\mu, \frac{9}{4})$

$Y_1, Y_2, Y_3, Y_4 \Rightarrow n = 4$

$RR : \{ \bar{Y} \leq 22.5 \}$

a) what is α ? $\alpha = P(RR \mid H_0)$

$P(\bar{Y} \leq 22.5 \mid \mu = 25) = P\left(\frac{\bar{Y} - 25}{\sqrt{9/4}} \leq \frac{22.5 - 25}{\sqrt{9/4}} \right)$

$= P(Z \leq \frac{22.5 - 25}{3/2})$

b) Power when $\mu = 23$? $P(RR \mid H_a) = 1 - \beta$

$P(\bar{Y} \leq 22.5 \mid \mu = 23)$

$P(Z \leq \frac{22.5 - 23}{3/2})$

$\bar{Y} = 24$

c) Suppose we observe $y_1 = 22,\ y_2 = 24.5,\ y_3 = 23,\ y_4 = 26.5$

p-value of this shape? $P(H_a$에 가깝 $\mid H_0)$

$P(\bar{Y} \leq 24 \mid \mu = 25)$

$= P(Z \leq \frac{24 - 25}{3/2})$

· have one observation on discrete y

$H_0 : f(y) = f_0(y)$ vs $H_a : f(y) = f_a(y)$

Y	H_0 $f_0(y)$	H_a $f_a(y)$	f_0/f_a
0	0.1	0.3	1/3
1	0.4	0.4	1
2	0.2	0.1	2
3	0.1	0.2	1/2
4	0.2	0	∞

$P(Y=3 \mid H_a)$

↳ 분포가 $f_a(y)$를 따를 때 관측값

$P(Y=1 \mid H_0)$

↳ 분포가 $f_0(y)$를 따를 때 관측값

$\boxed{\frac{f_0}{f_a}}$ 크면 H_0에 가까움
작으면 H_a에 가까움 → regret H_0

	RR	Y	$\alpha = P(RR \mid H_0)$	$\beta = P(RR^c \mid H_a)$	$= 1-\beta$ $= P(RR \mid H_a)$ Power at H_a
①	$f_0/f_a \le \frac{1}{3}$	{0} : RR	$P(Y=0 \mid H_0)$ 0.1	$P(Y=1,2,3,4 \mid H_a)$ 0.4+0.1+0.2+0	1−0.7
②	$f_0/f_a \le \frac{1}{2}$	{0, 3}	$P(Y=0, Y=3 \mid H_0)$ 0.1+0.1=0.2	$P(Y=1,2,4 \mid H_a)$	1−0.5
③	$f_0/f_a \le 1$	{0, 3, 1}	$P(Y=0,3,1 \mid H_0)$ 0.1+0.1+0.4	$P(Y=2,4 \mid H_a)$ 0.1+0	1−0.1
④	$f_0/f_a \le 2$	{0, 3, 1, 2}	$P(Y=0,3,1,2 \mid H_0)$ 0.1+0.1+0.4 +0.2	$P(Y=4 \mid H_a)$ 0	1−0

↓

RR이 관대해진다

MP test at $\alpha = 0.1$

의미: 2번 RR 구하라는 뜻 RR {Y=0}

〈LRT〉

$Y_1, \dots, Y_n \quad exp(\theta)$

$H_0: \theta = 3 \quad H_a: \theta \neq 3$

LRT ⇒ 계산복잡
실수주의
Y, C_1, C_2

1. $\hat{\theta}^{MLE}$

$$L(\theta) = \left(\frac{1}{\theta^n}\right) e^{-\Sigma y_i/\theta}$$

UMP만 구하기 : NP lemma 문제

$\ln L(\theta)$

$$\frac{\partial}{\partial\theta} \ln L(\theta) \overset{set}{=} 0 \quad , \quad \frac{\partial^2}{\partial\theta^2} \ln L(\theta) < 0$$

※ 변수변환
$\ln Y, \ e^Y,$

$$\hat{\theta}^{MLE} = \frac{\Sigma Y_i}{n} = \bar{y}$$

범위바꾸기 $Y > -1$
$Y+1 > 0$

2.

$$\lambda = \frac{\overset{\text{귀무가설}}{L(3)}}{\underset{\text{MLE}}{L(\bar{y})}} = \frac{\frac{1}{3^n} e^{-\Sigma Y_i/3}}{\frac{1}{\bar{y}^n} e^{-\Sigma Y_i/\bar{y}}}$$

$$= \left(\frac{\bar{y}}{3}\right)^n e^{-\overset{n\bar{y}}{\Sigma y_i}\left(\frac{1}{3} - \frac{1}{\bar{y}}\right)} \le k$$

N-P lemma : increase/decrease 찾기
LRT : $g(x)$

$$g(\bar{y}) \to g(x) = \left(\frac{x}{3}\right)^n e^{-nx\left(\frac{x-3}{3x}\right)}$$

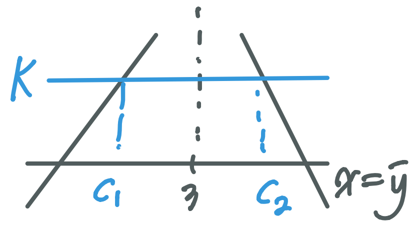
$$= \left(\frac{x}{3}\right)^n e^{-n\left(\frac{x-3}{3}\right)}$$

$g(x)$의 개형 실수주의

$$\ln(g(x)) = n\ln x - n\ln 3 - n\left(\frac{x-3}{3}\right)$$

$$\frac{\partial}{\partial x}\ln g(x) = \frac{n}{x} - \frac{n}{3} \overset{set}{=} 0$$

critical pt $x=3$



$g(x) \le k$ 일 때 reject H_0

$$\Leftrightarrow \{\bar{y} \le c_1 \text{ or } \bar{y} \ge c_2\}$$

$$Y \sim exp(\theta)$$

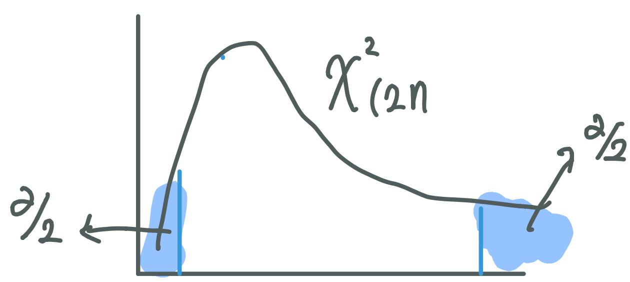
$\Leftrightarrow \{\Sigma y_i \le c_1^* \text{ or } \Sigma y_i \ge c_2^*\}$ 일 때 reject H_0

$$\Sigma Y_i \sim gamma(n, \theta)$$

$P(RR|H_0) = \alpha$ 이용 $H_0: \theta = 3$

$$\frac{2}{3}\Sigma Y_i \sim gamma(n, 2) \quad \chi^2(2n)$$

$$P\left(\frac{2}{3}\Sigma Y_i \le \frac{2}{3}c_1 \text{ or } \frac{2}{3}\Sigma Y_i \ge \frac{2}{3}c_2 \;\middle|\; H_0\right) = \alpha$$



$\frac{2}{3}c_1 = \chi^2(1-\alpha/2, 2n)$ $\qquad$ $\frac{2}{3}c_1 = \chi^2(\alpha/2, 2n)$

$$C_1 = \frac{3}{2}\chi^2(1-\alpha/2, 2n)$$

$$C_2 = \frac{3}{2}\chi^2(\alpha/2, 2n)$$