

## 1 APPENDIX

**Table 1: Classical Symbolic Regression Benchmarks(SRB).**

FileNumber	FileName	Objective Function	Data Set
F1	Keijzer-5	$\log(x)$	U[0,2,20]
F2	Nguyen-8	$\sqrt{x}$	U[0,2,20]
F3	Korns-1	$1.57 + 24.3x$	U[-1,1,20]
F4	Korns-6	$6.87 + 11\cos(7.23x^3)$	U[-0.5,0.5,20]
F5	Nguyen-4	$x^6 + x^5 + x^4 + x^3 + x^2 + x$	U[-1,1,20]
F6	Nguyen-3	$x^5 + x^4 + x^3 + x^2 + x$	U[-1,1,20]
F7	Koza-1,Nguyen-2	$x^4 + x^3 + x^2 + x$	U[-1,1,20]
F8	Nguyen-1	$x^3 + x^2 + x$	U[-1,1,20]
F9	Koza-3	$x^6 - 2x^4 + x^2$	U[-1,1,20]
F10	Koza-2	$x^5 - 2x^3 + x$	U[-1,1,20]
F11	Nguyen-5	$\cos(x)\sin(x^2) - 1$	U[-1.6,1.6,20]
F12	Nguyen-6	$\sin(x) + \sin(x + x^2)$	U[-1,1,20]
F13	Nguyen-11	$x^y$	U[2,4,400]
F14	Keijzer-11	$xy + \sin((x - 1)(y - 1))$	U[-1,1,400]
F15	Nguyen-12	$x^4 - x^3 + y^2/2 - y$	U[-1,1,400]
F16	Keijzer-13	$6\sin(x)\cos(y)$	U[-1,1,400]
F17	Keijzer-15	$x^3/5 + y^3/2 - y - x$	U[-1,1,400]
F18	Nguyen-9	$\sin(x) + \sin(y^2)$	U[-1,1,400]
F19	Nguyen-10	$2\sin(x)\cos(y)$	U[-1,1,400]
F20	Vladislavleva-1	$\exp(-(x - 1)^2/(1.2 + (y - 2.5)^2))$	U[-1,1,400]
F21	Keijzer-3	$30xz/((x - 10)y^y)$	x,z:[-1,1,1000] y:U[1,3,1000]
F22	Korns-2	$0.23 + 14.2(x + y)/(3z)$	x,y:U[-1,1,1000] z:U[1,3,1000]
F23	Vladislavleva-5	$30((x - 1)(z - 1))/(y^2(x - 10))$	U[0,2,1000]

**Table 2: Feynman Symbolic Regression Benchmarks(FSRB).**

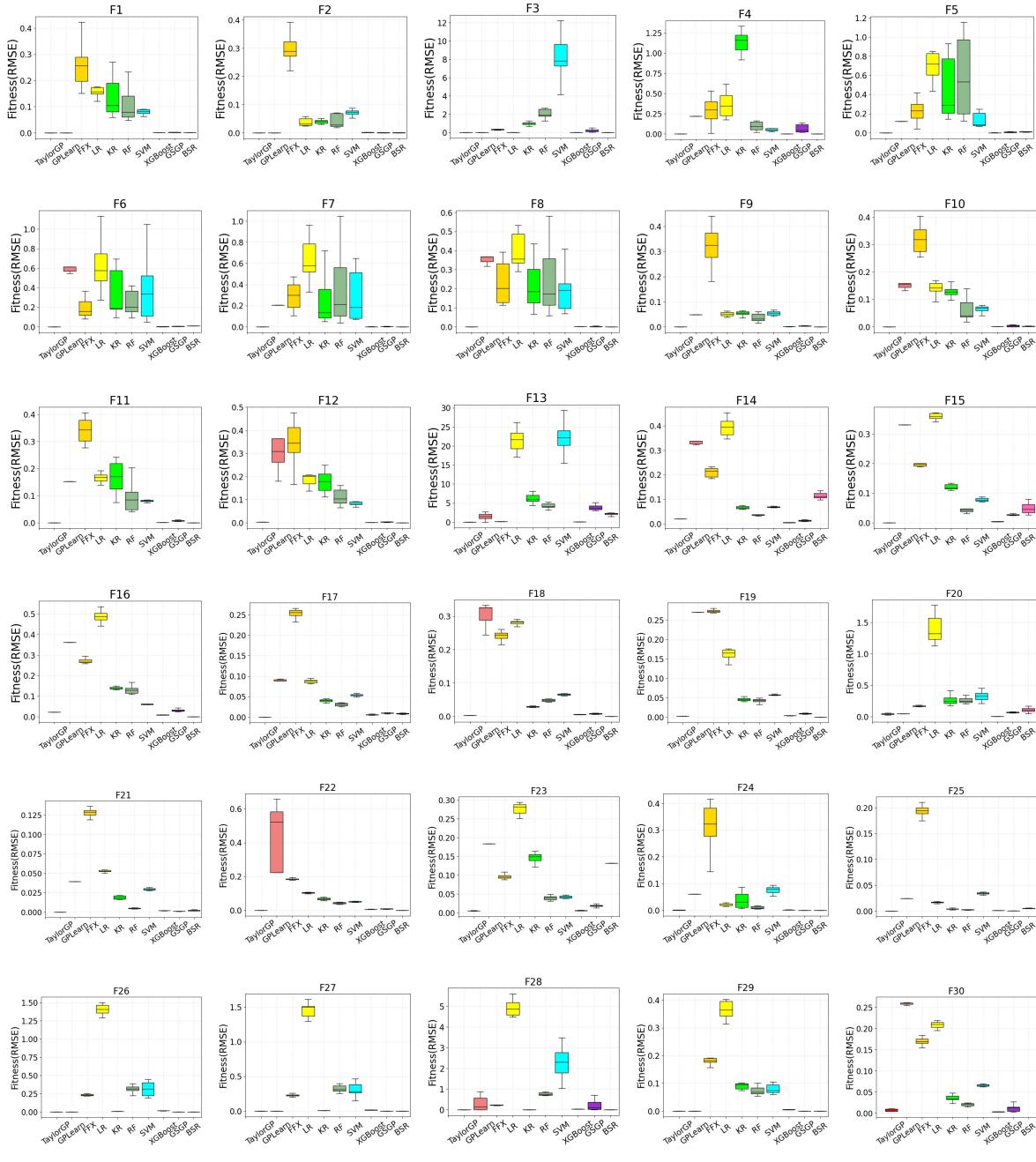
FileNumber	FileName	Output	Objective Function	Data Set
F24	I.6.2a	$f$	$\exp(-\theta^2/2)/\sqrt{2\pi}$	U[1,3,20]
F25	I.6.2	$f$	$\exp(-(\theta/\sigma)^2/2)/(\sqrt{2\pi}\sigma)$	U[1,3,400]
F26	I.12.1	$F$	$mUN_n$	U[2,4,400]
F27	I.12.5	$F$	$q_2Ef$	U[2,4,400]
F28	I.14.4	$U$	$1/2k_{spring}x^2$	U[2,4,400]
F29	I.25.13	$Volt$	$q/C$	U[2,4,400]
F30	I.26.2	$\theta_1$	$\arcsin(ns\sin(\theta_2))$	$n:U[0,1,400] \theta_2:U[2,4,400]$
F31	I.29.4	$k$	$\omega/c$	U[2,4,400]
F32	I.34.27	$E_n$	$(h/(2\pi))\omega$	U[2,4,400]
F33	I.39.1	$E_n$	$3/2prV$	U[2,4,400]
F34	II.3.24	$flux$	$Pwr/(4\pi r^2)$	U[2,4,400]
F35	II.8.31	$E_{den}$	$\epsilon Ef^2/2$	U[2,4,400]
F36	II.11.28	$\theta$	$1+n\alpha/(1-(n\alpha/3))$	U[0,1,400]
F37	II.27.18	$E_{den}$	$\epsilon Ef^2$	U[2,4,400]
F38	II.38.14	$mu_S$	$Y/(2(1+\sigma))$	U[2,4,400]
F39	III.12.43	$L$	$n(h/(2\pi))$	U[2,4,400]
F40	II.37.1	$E_n$	$mom(1+chi)B$	U[2,4,1000]
F41	I.18.12	$tau$	$rFsin(\theta)$	U[2,4,1000]
F42	I.6.2b	$f$	$\exp(-((\theta-\theta_1)/\sigma)^2/2)/(\sqrt{2\pi}\sigma)$	U[1,3,1000]
F43	I.10.7	$m$	$m_0/\sqrt{1-v^2/c^2}$	$m_0, c:U[3,5,1000] v:U[1,2,1000]$
F44	I.12.4	$Ef$	$q_1r/(4\pi\epsilon r^3)$	U[2,4,1000]
F45	I.14.3	$U$	$mgz$	U[2,4,1000]
F46	I.15.1	$p$	$m_0v/\sqrt{1-v^2/c^2}$	$m_0, c:U[3,5,1000] v:U[1,2,1000]$
F47	I.16.6	$v_1$	$(u+v)/(1+uv/c^2)$	U[2,4,1000]
F48	I.27.6	$foc$	$1/(1/d_1+n/d_2)$	U[2,4,1000]
F49	I.30.3	$Int$	$\text{Int}_0 \sin(n\theta/2)^2/\sin(\theta/2)^2$	U[2,4,1000]
F50	I.30.5	$\theta$	$\arcsin(\lambda/(nd))$	$\lambda :U[1,2,1000] d, n:U[2,4,1000]$
F51	I.34.1	$\omega$	$\omega_0/(1-v/c)$	$c, \omega_0 :U[3,5,1000] v:U[1,2,1000]$
F52	I.34.14	$\omega$	$(1+v/c)/\sqrt{1-v^2/c^2}\omega_0$	$c, \omega_0 :U[3,5,1000] v:U[1,2,1000]$
F53	I.37.4	$Int$	$I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta)$	U[2,4,1000]
F54	I.39.11	$E_n$	$1/(\gamma-1)prV$	U[2,4,1000]
F55	I.43.31	$D$	$mob_k b_T$	U[2,4,1000]
F56	I.47.23	$c$	$\sqrt{\gamma pr/\rho}$	U[2,4,1000]
F57	II.4.23	$Volt$	$q/(4\pi\epsilon r)$	U[2,4,1000]
F58	II.8.7	$E_n$	$3/5q^2/(4\pi\epsilon d)$	U[2,4,1000]
F59	II.10.9	$Ef$	$\sigma_{den}/\epsilon/(1+chi)$	U[2,4,1000]
F60	II.13.23	$rhoc$	$rhoc_0/\sqrt{1-v^2/c^2}$	$rhoc_0, c:U[3,5,1000] v:U[1,2,1000]$
F61	II.13.34	$j$	$rhoc_0 v/\sqrt{1-v^2/c^2}$	$rhoc_0, c:U[3,5,1000] v:U[1,2,1000]$
F62	II.27.16	$flux$	$ecEf^2$	U[2,4,1000]
F63	II.34.2a	$I$	$qv/(2\pi r)$	U[2,4,1000]
F64	II.34.2	$mom$	$qvr/2$	U[2,4,1000]
F65	II.34.29a	$mom$	$qh/(4\pi m)$	U[2,4,1000]
F66	III.7.38	$\omega$	$2momB/(h/(2\pi))$	U[2,4,1000]
F67	III.8.54	$prob$	$\sin(E_n t/(h/(2\pi)))^2$	U[1,2,1000]
F68	III.15.12	$E_n$	$2U(1-\cos(kd))$	U[2,4,1000]
F69	II.15.4	$E_n$	$-momB\cos(\theta)$	U[2,4,1000]
F70	II.15.5	$E_n$	$-p_dEf\cos(\theta)$	U[2,4,1000]
F71	I.18.14	$L$	$mr\sin(\theta)$	U[2,4,4000]

**Table 3: Penn Machine Learning Benchmarks(PMLB).**

FileNumber	FileName	Samples	Variables	Task
F72	210_cloud	108	5	regression
F73	519_vinnie	380	2	regression
F74	573_cpu_act	1000	21	regression
F75	1027_ESL	488	4	regression
F76	1028_SWD	1000	10	regression
F77	1029_LEV	1000	4	regression
F78	analcatdata_boxing1	120	3	classification
F79	car-evaluation	1000	21	classification
F80	wine_quality_white	1000	11	classification
F81	towerData	1000	25	regression

Table 4: Fitness Metrics

Kinds	Function	TaylorGP		GPLEarn		FFX		GSGP		BSR		LR		KR		RF		SVM		XGBoost	
		CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE	CR	RMSE		
SRB	F1	100.0%	.0000	100.0%	.0000	0.0%	.2650	0.0%	.0023	0.0%	.0011	0.0%	.1839	0.0%	.1963	0.0%	.1226	0.0%	.0962	0.0%	
	F2	100.0%	.0000	100.0%	.0000	0.0%	.2730	0.0%	.0004	0.0%	.0004	0.0%	.0371	0.0%	.0407	0.0%	.0515	0.0%	.0751	0.0%	
	F3	100.0%	.0000	0.0%	.0285	0.0%	.2973	0.0%	.1966	100.0%	.0000	100.0%	.0000	0.0%	1.0412	0.0%	.20636	0.0%	.87351	0.0%	
	F4	0.0%	.0000	0.0%	.2189	0.0%	.2620	0.0%	.1641	0.0%	.0006	0.0%	.3308	0.0%	1.1220	0.0%	.1975	0.0%	.1207	0.0%	
	F5	100.0%	.0000	0.0%	.1267	0.0%	.2233	0.0%	.0066	0.0%	.0106	0.0%	.8329	0.0%	.4767	0.0%	.6471	0.0%	.5646	0.0%	
	F6	100.0%	.0000	0.0%	.5840	0.0%	.1887	0.0%	.0051	3.3%	.0089	0.0%	.6777	0.0%	.4043	0.0%	.3615	0.0%	.5594	0.0%	
	F7	100.0%	.0000	0.0%	.2741	0.0%	.2518	26.7%	.0023	93.3%	.0002	0.0%	.6353	0.0%	.2515	0.0%	.3047	0.0%	.2207	0.0%	
	F8	100.0%	.0000	0.0%	.3495	0.0%	.2223	60.0%	.0008	100.0%	.0000	0.0%	.3693	0.0%	.1798	0.0%	.2191	0.0%	.2142	0.0%	
	F9	100.0%	.0000	0.0%	.0487	0.0%	.5376	0.0%	.0033	0.0%	.0004	0.0%	.0529	0.0%	.0549	0.0%	.0346	0.0%	.0537	0.0%	
	F10	100.0%	.0000	0.0%	.1503	0.0%	.3168	30.0%	.0037	33.3%	.0015	0.0%	.1440	0.0%	.1190	0.0%	.0575	0.0%	.0676	0.0%	
	F11	100.0%	.0000	0.0%	.1525	0.0%	.3478	0.0%	.0078	100.0%	.0000	0.0%	.1650	0.0%	.1718	0.0%	.0918	0.0%	.0752	0.0%	
	F12	3.3%	.0022	0.0%	.3177	0.0%	.3543	0.0%	.0044	100.0%	.0000	0.0%	.2112	0.0%	.1833	0.0%	.1118	0.0%	.0815	0.0%	
	F13	100.0%	.0000	73.3%	.4512	0.0%	.1846	0.0%	.36371	33.3%	.1589	0.0%	.210820	0.0%	.60769	0.0%	.40784	0.0%	.227293	0.0%	
	F14	0.0%	.0206	0.0%	.3290	0.0%	.2093	0.0%	.0136	0.0%	.1126	0.0%	.4062	0.0%	.0672	0.0%	.0402	0.0%	.0695	0.0%	
	F15	100.0%	.0000	0.0%	.3309	0.0%	.2028	0.0%	.0248	0.0%	.0493	0.0%	.3579	0.0%	.1188	0.0%	.0493	0.0%	.0778	0.0%	
	F16	0.0%	.0219	0.0%	.3614	0.0%	.2713	0.0%	.0303	100.0%	.0000	0.0%	.4800	0.0%	.1365	0.0%	.129	0.0%	.0613	0.0%	
	F17	100.0%	.0000	0.0%	.1279	0.0%	.2473	0.0%	.0096	0.0%	.0085	0.0%	.0879	0.0%	.0414	0.0%	.0320	0.0%	.0532	0.0%	
	F18	0.0%	.0028	0.0%	.3101	0.0%	.2454	0.0%	.0071	100.0%	.0000	0.0%	.2770	0.0%	.0289	0.0%	.0496	0.0%	.0635	0.0%	
	F19	0.0%	.0024	13.3%	.2261	0.0%	.2721	3.3%	.0086	100.0%	.0000	0.0%	.1611	0.0%	.0454	0.0%	.0421	0.0%	.0571	0.0%	
	F20	0.0%	.0396	0.0%	.0642	0.0%	.1687	3.3%	.0715	0.0%	.1058	0.0%	.13504	0.0%	.2706	0.0%	.2484	0.0%	.3687	0.0%	
	F21	0.0%	.0000	0.0%	.0394	0.0%	.1256	3.3%	.0012	0.0%	.0022	0.0%	.0533	0.0%	.0190	0.0%	.0048	0.0%	.0287	0.0%	
	F22	0.0%	.0001	0.0%	.4750	0.0%	.1861	3.3%	.0078	100.0%	.0000	0.0%	.1052	0.0%	.0666	0.0%	.0401	0.0%	.0502	0.0%	
	F23	0.0%	.0050	0.0%	.1827	0.0%	.0958	0.0%	.0189	0.0%	.1298	0.0%	.2734	0.0%	.1461	0.0%	.0388	0.0%	.0406	0.0%	
best	16	16	5	2	3	0	5	0	11	8	4	1	3	0	3	0	3	0	2		
FSRB	F24	0.0%	.0008	0.0%	.0560	0.0%	.3044	0.0%	.0001	13.3%	.0001	0.0%	.0212	0.0%	.0283	0.0%	.0113	0.0%	.0771	0.0%	
	F25	0.0%	.0001	0.0%	.0246	0.0%	.1937	0.0%	.0006	0.0%	.0054	0.0%	.0167	0.0%	.0038	0.0%	.0026	0.0%	.0343	0.0%	
	F26	100.0%	.0000	100.0%	.0000	0.0%	.2363	100.0%	.0000	100.0%	.0000	0.0%	1.4143	0.0%	.0086	0.0%	.3157	0.0%	.3371	0.0%	
	F27	100.0%	.0000	100.0%	.0000	0.0%	.2265	100.0%	.0000	100.0%	.0000	0.0%	1.4301	0.0%	.0090	0.0%	.3248	0.0%	.3430	0.0%	
	F28	100.0%	.0000	53.3%	.1140	0.0%	.2257	6.7%	.2939	100.0%	.0000	0.0%	.48804	0.0%	.0205	0.0%	.8526	0.0%	.21924	0.0%	
	F29	100.0%	.0000	100.0%	.0000	0.0%	.1817	100.0%	.0000	100.0%	.0000	0.0%	.3584	0.0%	.0906	0.0%	.0690	0.0%	.0799	0.0%	
	F30	0.0%	.0070	0.0%	.2547	0.0%	.1709	0.0%	.0127	3.3%	.0125	0.0%	.2095	0.0%	.0374	0.0%	.0218	0.0%	.0666	0.0%	
	F31	100.0%	.0000	100.0%	.0000	0.0%	.1810	100.0%	.0000	100.0%	.0000	0.0%	.3685	0.0%	.0058	0.0%	.0685	0.0%	.0856	0.0%	
	F32	100.0%	.0000	0.0%	.0080	0.0%	.2283	0.0%	.0182	100.0%	.0000	0.0%	.2293	0.0%	.0014	0.0%	.0514	0.0%	.0660	0.0%	
	F33	100.0%	.0000	16.7%	.0017	0.0%	.2307	13.3%	.0163	100.0%	.0000	0.0%	.21527	0.0%	.0136	0.0%	.4842	0.0%	.6682	0.0%	
	F34	0.0%	.0004	0.0%	.0348	0.0%	.1672	0.0%	.0006	100.0%	.0000	0.0%	.0363	0.0%	.0128	0.0%	.0055	0.0%	.0888	0.0%	
	F35	100.0%	.0000	46.7%	.1284	0.0%	.2278	10.0%	.1321	100.0%	.0000	0.0%	.48270	0.0%	.0025	0.0%	.9328	0.0%	.24628	0.0%	
	F36	0.0%	.0000	0.0%	.2302	0.0%	.2178	0.0%	.0030	3.3%	.0002	0.0%	.1287	0.0%	.0302	0.0%	.0233	0.0%	.0521	0.0%	
	F37	100.0%	.0000	100.0%	.0000	0.0%	.2238	100.0%	.0000	100.0%	.0000	0.0%	.95658	0.0%	.0051	0.0%	.17310	0.0%	.84967	0.0%	
	F38	0.0%	.0001	0.0%	.1666	0.0%	.1995	6.7%	.0015	36.7%	.0006	0.0%	.0687	0.0%	.0108	0.0%	.0161	0.0%	.0602	0.0%	
	F39	100.0%	.0000	0.0%	.0081	0.0%	.2307	0.0%	.0218	100.0%	.0000	0.0%	.2295	0.0%	.0014	0.0%	.0519	0.0%	.0662	0.0%	
	F40	90.0%	.0002	90.0%	.0276	0.0%	.1917	6.7%	.0086	100.0%	.0000	0.0%	.2616	0.0%	.0201	0.0%	.2134	0.0%	.0815	0.0%	
	F41	100.0%	.0000	100.0%	.0000	0.0%	.1923	0.0%	.0195	100.0%	.0000	0.0%	.1400	0.0%	.0564	0.0%	.0597	0.0%	.0511	0.0%	
	F42	0.0%	.0020	0.0%	.0215	0.0%	.2736	0.0%	.0004	0.0%	.0018	0.0%	.0051	0.0%	.0050	0.0%	.0014	0.0%	.0256	0.0%	
	F43	0.0%	.0083	0.0%	.2578	0.0%	.2149	0.0%	.0033	0.0%	.0167	0.0%	.0895	0.0%	.0138	0.0%	.0190	0.0%	.0638	0.0%	
	F44	0.0%	.0000	0.0%	.0070	0.0%	.1796	0.0%	.0002	6.7%	.0009	0.0%	.0024	0.0%	.0014	0.0%	.0008	0.0%	.0143	0.0%	
	F45	100.0%	.0000	100.0%	.0000	0.0%	.1932	80.0%	.0028	100.0%	.0000	0.0%	.2215	0.0%	.0172	0.0%	.1632	0.0%	.0665	0.0%	
	F46	0.0%	.0091	0.0%	.2578	0.0%	.2183	0.0%	.0033	0.0%	.0168	0.0%	.0887	0.0%	.0143	0.0%	.0192	0.0%	.0641	0.0%	
	F47	0.0%	.0010	0.0%	.0220	0.0%	.2776	0.0%	.0017	0.0%	.0147	0.0%	.0310	0.0%	.0106	0.0%	.0052	0.0%	.0504	0.0%	
	F48	3.3%	.0002	3.3%	.0834	0.0%	.1882	3.3%	.0012	5.3%	.0078	0.0%	.0142	0.0%	.0066	0.0%	.0086	0.0%	.0458	0.0%	
	F49	0.0%	.0518	0.0%	.8525	0.0%	.2279	0.0%	.0404	0.0%	.1000	0.0%	.2555	0.0%	.1114	0.0%	.0605	0.0%	.0559	0.0%	
	F50	0.0%	.0008	0.0%	.0396	0.0%	.1870	0.0%	.0003	100.0%	.0000	0.0%	.0004	0.0%	.0075	0.0%	.0044	0.0%	.0039	0.0%	
	F51	100.0%	.0000	73.3%	.0748	0.0%	.1902	10.0%	.0049	20.0%	.0210	0.0%	.0758	0.0%	.0231	0.0%	.0596	0.0%	.0039	0.0%	
	F52	0.0%	.0003	0.0%	.2010	0.0%	.2007	0.0%	.0033	0.0%	.0141	0.0%	.0440	0.0%	.0159	0.0%	.0274	0.0%	.0665	0.0%	
	F53	0.0%	.0005	0.0%	.3903	0.0%	.2456	0.0%	.0186	0.0%	.0105	0.0%	.0946	0.0%	.0607	0.0%	.0359	0.0%	.0606	0.0%	
	F54	13.3%	.0001	0.0%	.2937	0.0%	.1806	0.0%	.0048	0.0%	.0037	0.0%	.0647	0.0%	.0272	0.0%	.0417	0.0%	.0448	0.0%	
	F55	100.0%	.0000	100.0%	.0000	0.0%	.1909	86.7%	.0016	100.0%	.0000	0.0%	.2201	0.0%	.0178	0.0%	.1605	0.0%	.0635	0.0%	
	F56	100.0%	.0000	93.3%	.0067	0.0%	.1829	0.0%	.0010	0.0%	.0070	0.0%	.0133	0.0%	.0108	0.0%	.0142</				



**Figure 1: Fitness comparison. The RMSE results on the benchmarks F1-F30.**

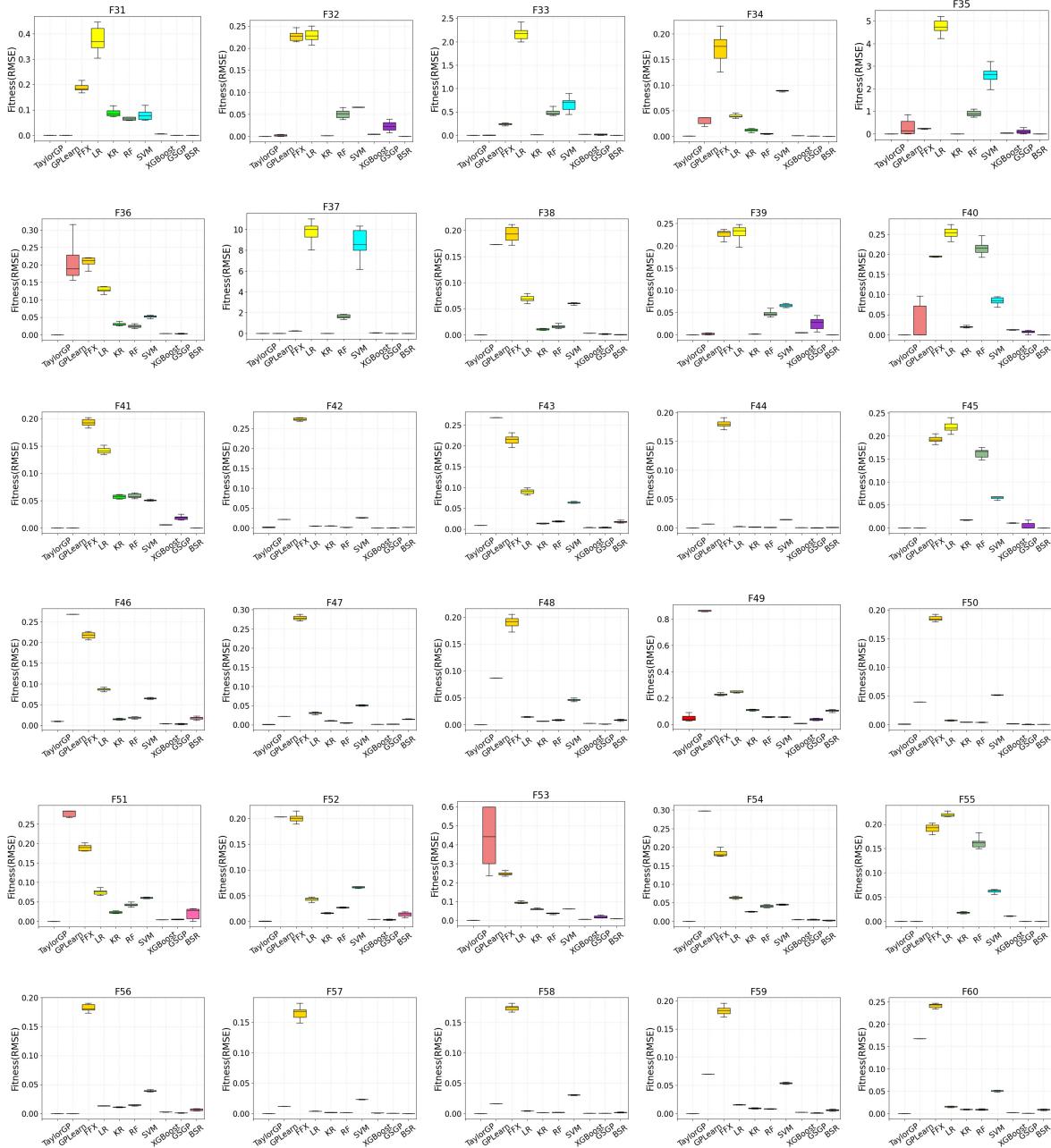
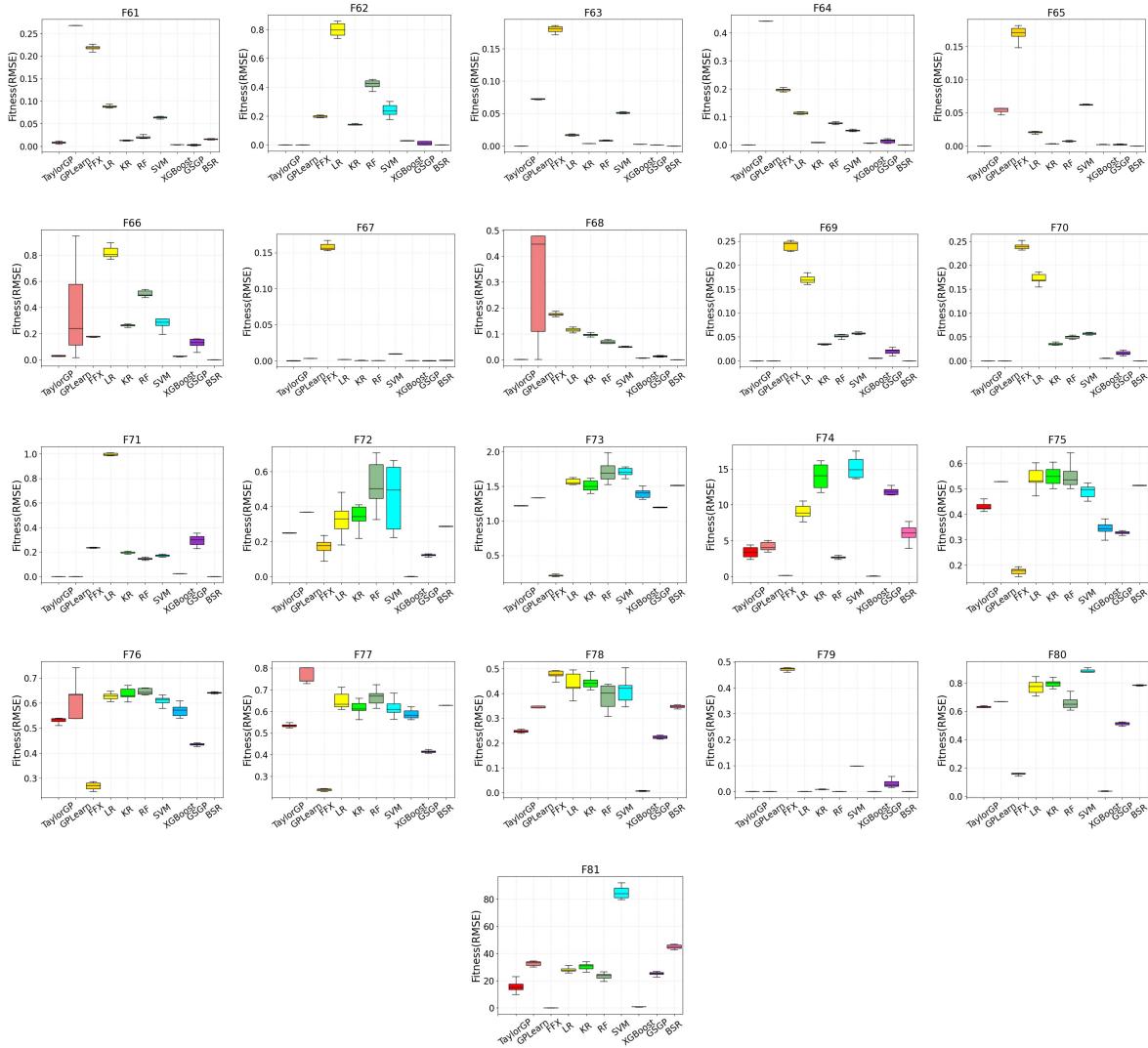


Figure 2: Fitness comparison. The RMSE results on the benchmarks F31–F60.



**Figure 3: Fitness comparison. The RMSE results on the benchmarks F61-F81.**

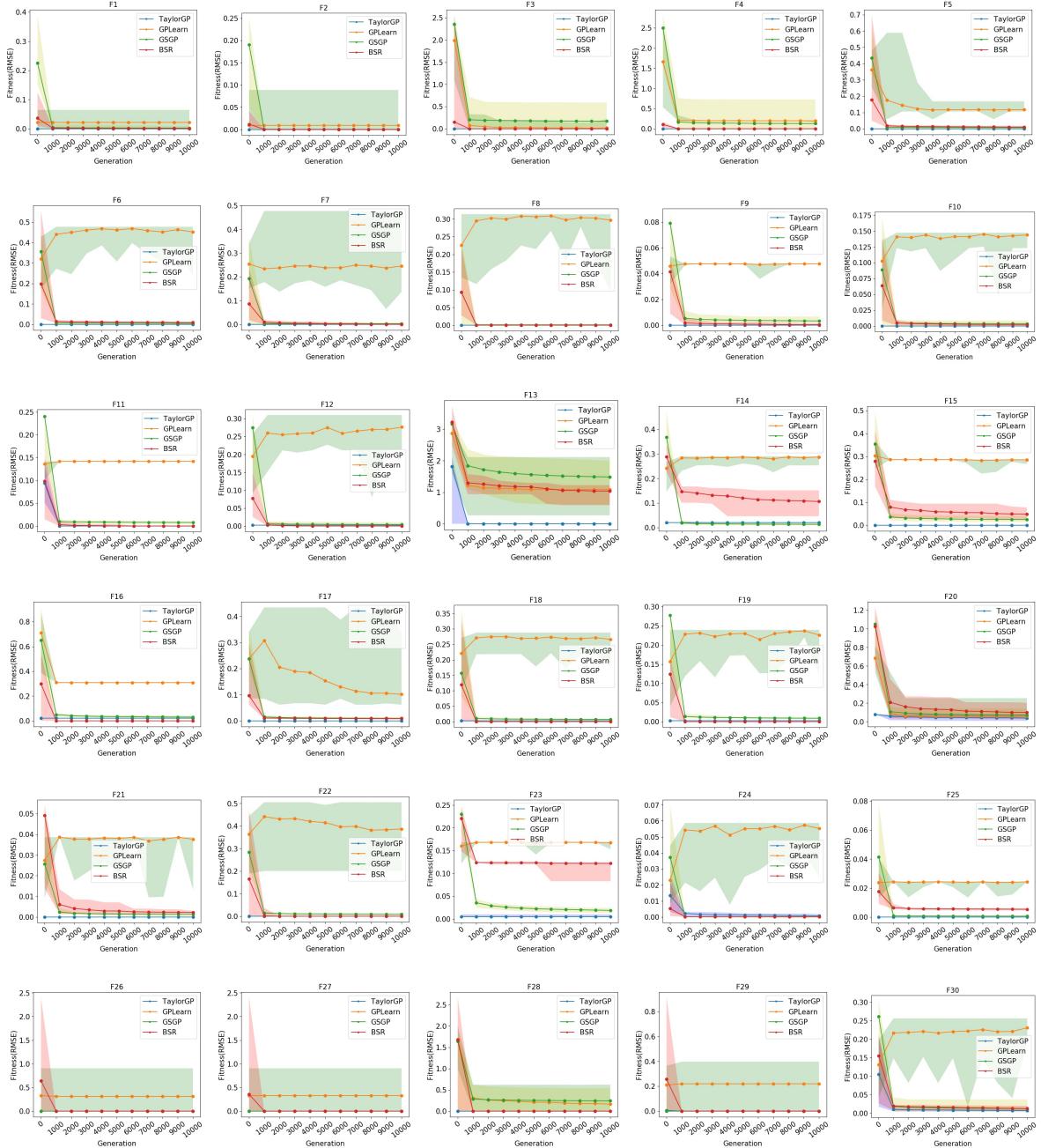


Figure 4: Comparison of convergence. The fitness convergence curve of four algorithms on the benchmarks F1-F30.

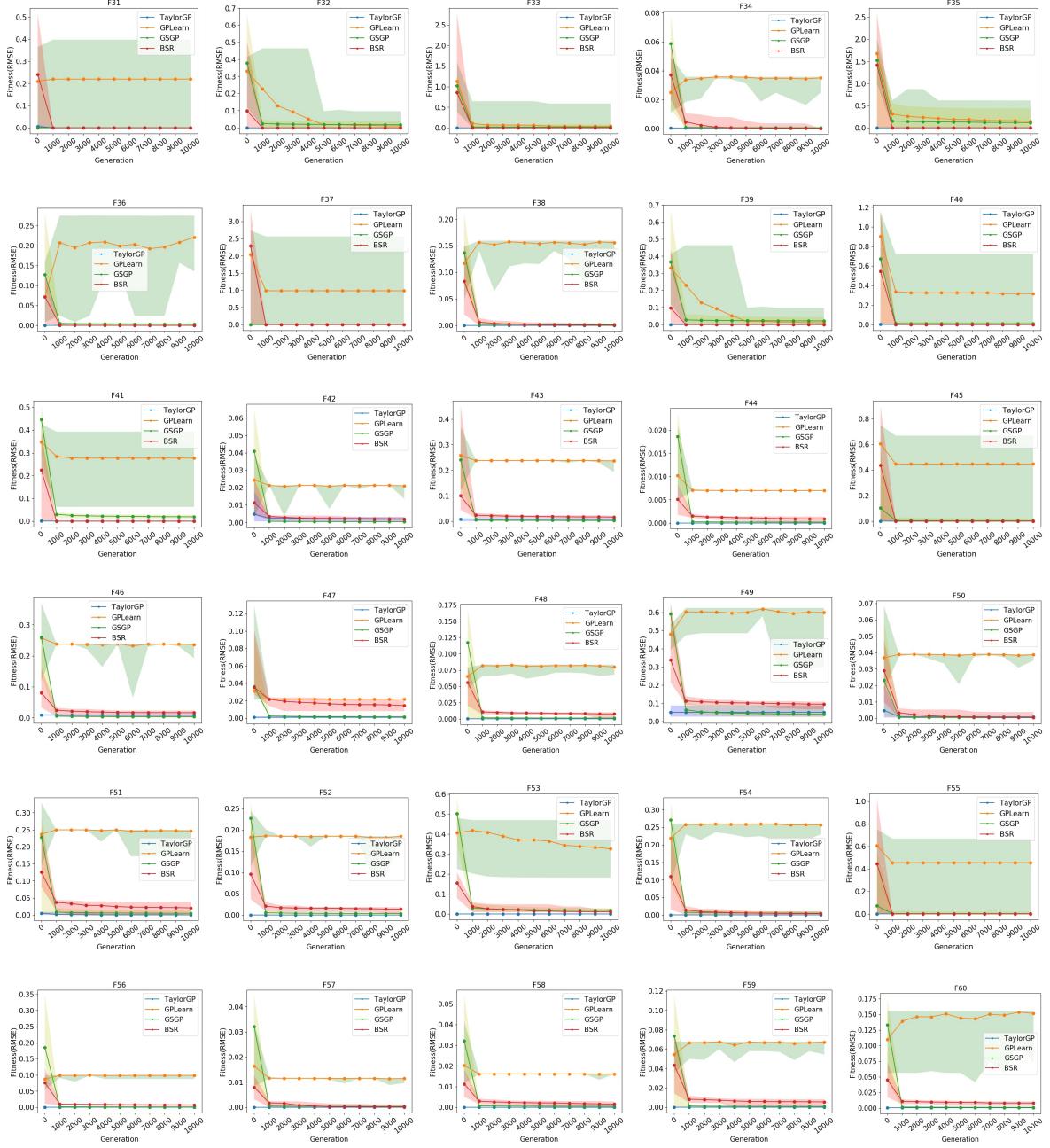
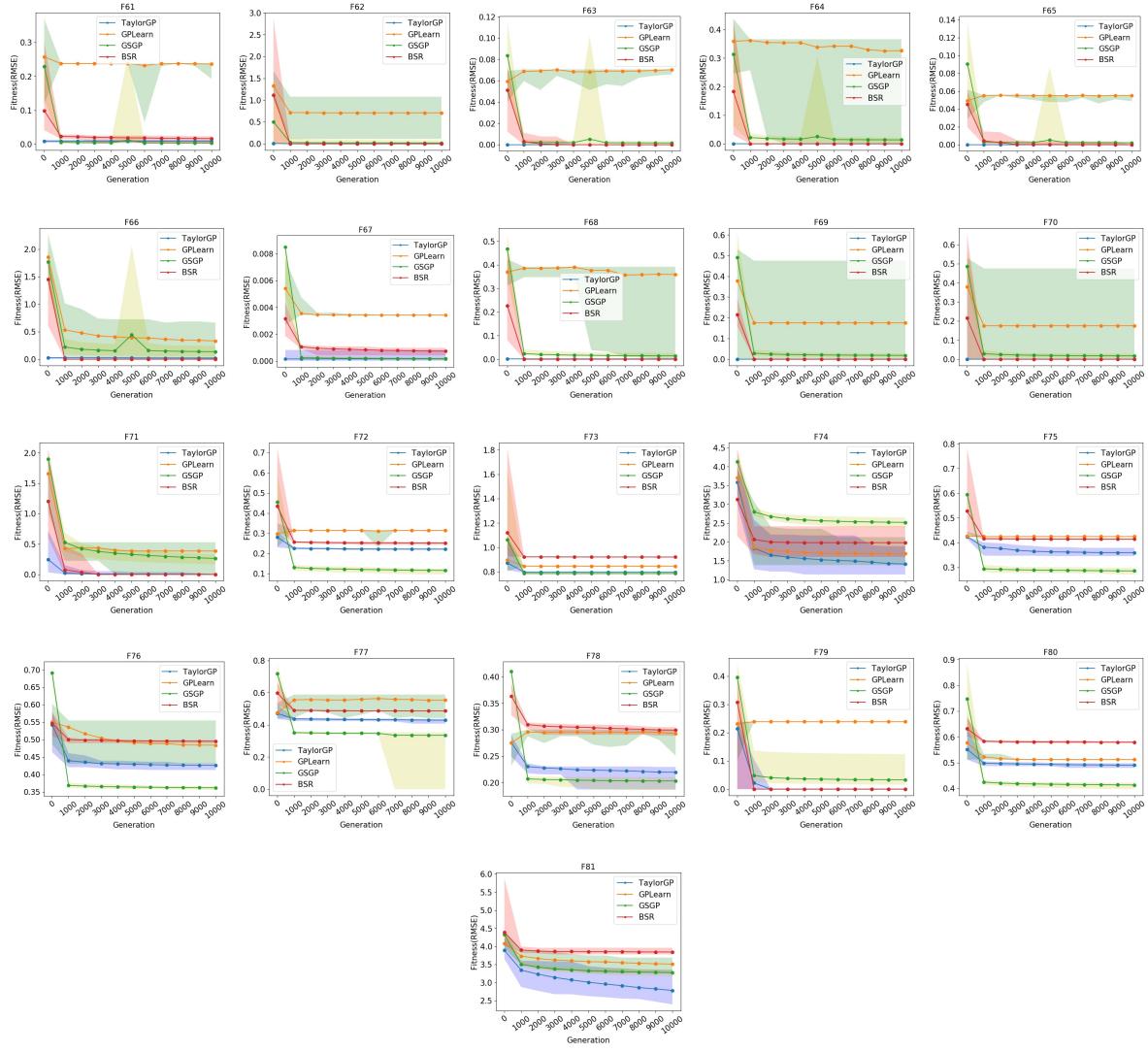


Figure 5: Comparison of convergence. The fitness convergence curve of four algorithms on the benchmarks F31-F60.



**Figure 6: Comparison of convergence. The fitness convergence curve of four algorithms on the benchmarks F61-F81.**