订正：$\{x_n\}_{n=1}^{+\infty}$ independent, $\|x_n\|=1$

E is hamel basis, $\{x_n\}_{n=1}^{+\infty} \subseteq E$

define $\begin{cases} L(x_n)=n \\ L(x)=0 & x\in E\setminus\{x_n\}_{n=1}^{+\infty} \end{cases}$

then $L(\sum_{i=1}^{+\infty} a_i x_i) = \sum_{i=1}^{+\infty} a_i \cdot i$ unbounded

$\therefore L$ not cts

T12.1 Linear map $T:(X,\|\cdot\|_X) \to (Y,\|\cdot\|_Y)$ cts $\Leftrightarrow$ bounded，即说明存在无界泛函

if $\forall$ linear functional bounded,

pick $f, 2f, 3f \cdots nf, \cdots$, Let $g=\sum_{n=1}^{+\infty} nf : (X,\|\cdot\|_X) \to \mathbb{R}^1$

$g(ax+by) = \sum_{n=1}^{+\infty} nf(ax+by) = \sum_{n=1}^{+\infty} naf(x)+nbf(y) = a\sum_{n=1}^{+\infty} nf(x) + b\sum_{n=1}^{+\infty} nf(y) \Rightarrow g$ linear

记 $\|f\|_{X^*}=M$, $\forall N\in\mathbb{R}^+$, $\exists n$ s.t. $(1+2+\cdots+n)M \geq N$, i.e. $\|g\| \geq \|\sum_{i=1}^{n} if\| \geq N$

$\therefore g$ unbounded 这是泛函吗. 没法保证一个 $g=\cdots$, 对应一个 input x

T12.2 Let ϕ_i satisfies: $\begin{cases} \phi_i(e_i)=1 \\ \phi_i(e_j)=0 & j\neq i, \quad i=1,2,\cdots n \end{cases}$

$b_1\phi_1+\cdots b_n\phi_n=0$ let $x=e_i$ $\sum_{i=1}^{n} b_i\phi_i(x)=0 \Rightarrow b_i=0$

$\therefore b_1=b_2=\cdots b_n=0 \Rightarrow$ independent --①

then $f\in X^*$, $f(x)=f(a_1e_1+\cdots a_ne_n)$

$= a_1f(e_1)+a_2f(e_2)+\cdots a_nf(e_n)$

$= a_1f(e_1)\phi_1(e_1) + a_2f(e_2)\phi_2(e_2) + \cdots a_nf(e_n)\phi_n(e_n)$

$= (f(e_1)\phi_1 + f(e_2)\phi_2 + \cdots f(e_n)\phi_n)\underbrace{(a_1e_1+\cdots a_ne_n)}_{=x}$, $\forall x\in X$

$\therefore \forall f\in X^*$, $f=f(e_1)\phi_1+\cdots f(e_n)\phi_n \Rightarrow$ spanning ②

which implies $\{\phi_1,\phi_2,\cdots\phi_n\}$ are basis of X^*, $\dim X^* = \dim X$

T12.3. "only if": if u minimize $F(u)$, then: $F(u) \leq F(u+tv)$ $\forall t\in\mathbb{R}^1$, $v\in H$

$\frac{1}{2}B(u,u)-f(u) \leq \frac{1}{2}B(u+tv,u+tv)-f(u+tv)$

$= \frac{1}{2}B(u,u)+\frac{t^2}{2}B(v,v)+tB(u,v)-f(u)-tf(v)$

$\therefore f(v)\cdot t \leq \frac{t^2}{2}B(v,v)+tB(u,v)$ $\forall t$

$\begin{cases} f(v) \leq \frac{t}{2}B(v,v)+B(u,v) & \text{if } t\geq 0 \\ f(v) \geq \frac{t}{2}B(v,v)+B(u,v) & t<0 \end{cases} \Rightarrow f(v)=\frac{t}{2}B(v,v)+B(u,v) \xRightarrow{\text{Let } t=0} f(v)=B(u,v)$

"if" if $f(v)=B(u,v)$, $F(tv+u) = \frac{1}{2}B(u,u)-f(u)+\frac{t^2}{2}B(v,v)+\cancel{(tB(u,v)-tf(v))}$

$\geq F(u)$

怎么说明 $B(v,v)\geq 0$?