# GARCH OPTION PRICING MODEL

*Seminar Paper*
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# 1 Abstract

Duan (1995) proposed a method for option pricing and option delta calculation in which the underlying asset's returns follow the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process. The locally risk-neutral valuation relationship (LRNVR), which only holds under specific combinations of preferences, is notably applied in this context. This seminar paper aim to put this option pricing method into practice. The empirical analyses are conducted on BMW stock price from 2018-12-03 to 2020-12-20. When comparing with the Black-Scholes (B-S) model, the empirical analyses suggest that B-S approach seems to underprice option calls when comparing with GARCH model. The paper also touches empirical implied volatility.

# 2 Introduction

Due to staying-at-home requirement brought about by COVID-19 Pandemic, option trading has been increasing in value to unprecedented quantities. According to Carlson (2020), people got stuck at home having a huge checks and rent to pay without sports to watch or bet on and they went on stock markets. People found that pay-off structures from options trading were the same as sport bets. Until now, the economy has been disturbed by the pandemic but the option markets show no sign of slowing down. In that light, it is high time to take a look at different methodologies to evaluate option price.

Market volatility has been extensively discussed over the years and is known as the key determinant in option valuation. According to Chaudhury and Wei (1996), the sensitivity of option prices are affected by underlying asset return variance, time to maturity and interest rate. Therefore, if one can estimate the accurate value of variance, it will be critical for the option value as well as trading strategies. When Black and Scholes (2019) first developed Black-Scholes model in the early 1970s, it soon became a breakthrough. An important assumptions of this model is that all variables in the equation are not influenced by investor' risk preferences, in particular, variance is set to be constant, which can simplify the option analysis. However, empirical results show that the volatility cannot be constant and varies over time. Variance that has non-constant behavior was first modeled by Engle (1982) using what is called Generalized Autoregressive Conditional Heteroskedasticity, or GARCH. After that, Duan (1995) adapted this idea and developed an approach for pricing options whose continuously compounded asset returns follow GARCH process. Duan (1995) considered Black-Scholes model as a special case of GARCH approach where as the asset return under B-S is assumed to be homoskedastic. This assumption can then be used to explain some well-documented biases associated with B-S model such as underestimating option with short expiration or underpricing out-of-money options.

Unlike previous models, Duan (1995) took into consideration investors' preferences by including the generalized version of risk neutralization, known as Locally Risk Neutral Valuation Relationship (LRNVR), which were first mentioned by Rubinstein (1976) and Brennan (1979). LRNVR states that the conditional variance over one period is invariant when the risk-neutralized measure is changed, which is compatible with the context of GARCH process, the one period ahead conditional or unconditional variance is indifferent to the change of pricing measure. LRNVR holds under some assumptions and preferences. Under locally risk-neutralized pricing measure, conditional variance of the underlying asset will be different from GARCH process. In particular, according to Bollerslev (1986), when the conditional variance follows GARCH process, the innovation is governed by central $\chi^2$-distribution. However, under LRNVR, the innovation is now distributed as noncentral $\chi^2$. This behavior is caused by the existence of risk premium within the past innovation terms of the variance process.

The paper is organized as follows. Immediately after the introduction, description of GARCH option pricing model and the definition of LRNVR are given in Section 2. The assumptions of LRNVR and properties of conditional variance under the risk-neutralized pricing measure are also outlined in this section. In Section 3, we compare the difference between the Black-Scholes formula and GARCH option pricing model. Section 4 is the empirical analysis, which is conducted using Monte Carlo simulation method. Conclusion is given in Section 5.

# 3 The GARCH option pricing model

In this section, we introduce the logarithmic asset returns that follow GARCH process, define the LRNVR and derive the asset returns under the risk-neutralized pricing measure. Moreover, three sufficient assumptions under which LRNVR holds are also mentioned.

Following equations describe one-period ahead rate of return of the asset price $X_t$ at time $t$, with $r$ is constant risk-free rate, $\lambda$ is unit risk premium and $\epsilon_t$ follows GARCH(p,q) process with mean zero and conditional variance $h_t$, that follows (Bollerslev, 1986) set up. In particular, the conditional variance depends on the previous conditional variance and previous squared disturbances. Assume that rate of return is conditionally log-normally distributed under probability measure P. That is,

$$ln\frac{X_t}{X_{t-1}} = r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \epsilon_t, \tag{1}$$

$$\epsilon_t|\phi_{t-1} \sim N(0, h_t) \text{ under measure } P, \tag{2}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_t^2 - i + \sum_{i=1}^{p} \beta_i h_{t-i}, \tag{3}$$

where $\phi_t$ is the information set up to time $t, p \geq 0, q \geq 0, \alpha_0 > 0, \alpha_i \geq 0, i = 1, ..., q, \beta_i \geq 0, i = 1, ..., q$. $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$ is assumed to be less than 1 to ensure the covariance stationarity of $\epsilon_t$ . If $p = 0$ and $q$ =0, this is the special case of GARCH process, which reduces to Black-Scholes model.

For this specific pricing model, the asset return is assumed to be heteroskedastic, hence, the risk-neutral valuation relationship has to be derived. A pricing measure $Q$ satisfies *locally risk neutral valuation relationship* (LRNVR) if measure $Q$ is absolutely mutually continuous with respect to measure $P$ and under measure $Q$, $X_t/X_t - 1$ has to follow log-normal distribution. Particularly, the conditional mean given information set up to time t equals to risk-free rate, which yields that the model is locally independent of preferences. The conditional variance under two measure has to be the same, which means conditional variances can be estimated under measure $P$.

$$E^Q\left(\frac{X_t}{X_{t-1}}|\phi_{t-1}\right) = e^r, \tag{4}$$

$$Var^Q\left(ln\frac{X_t}{X_{t-1}}|\phi_{t-1}\right) = Var^P\left(ln\frac{X_t}{X_{t-1}}|\phi_{t-1}\right) \tag{5}$$

Locally risk neutralization is not sufficient for us to ignore all preference parameters because we can only apply LRNVR for only one-period ahead rate of return. Under the above set up, however, we can eliminate all the preference parameters to the risk premium, $\lambda$. In case of $p = 0$ and $q = 0$, which is called for homoskedastic log-normal process, the conditional variance remains constant and LRNVR becomes conventional risk-neutral valuation relationship.

According to Brennan (1979), in order to make risk-neutral valuation relationship hold, there must be some preferences and assumptions. Based on that, Duan (1995) has assumed three conditions for the validity of LRNVR, in particular, if investors want to maximize their utility expectation and the utility function time additive or separable, then:

1. The utility function is linear,

2. The utility function with constant relative or absolute risk aversion and changes in the logarithmic aggregate consumption are normal distribution with constant mean and variance under measure $P$.

The model can be considerably complicated if the interest rates are stochastic. Therefore, the constant mean and variance assumption certify that interest rates are constant, which allows to compare with Black-Scholes model.

Under measure pricing $Q$, the stock returns of underlying asset is as:

$$\ln \frac{X_t}{X_{t-1}} = r - \frac{1}{2}h_t + \xi_t \tag{6}$$

$$\xi_t|\phi_{t-1} \sim N(0, h_t) \tag{7}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i(\xi_{t-i} - \lambda\sqrt{h_{t-i}})^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{8}$$

As we can see, the conditional variance under locally risk-neutral pricing measure does not follow GARCH process. $h_t$ is expressed as a function of non-central $\chi^2$-distributed random variable with one degree of freedom, while the innovations under GARCH model follow central $\chi^2$-distribution. If we factor out $\sqrt{h_t - i}$, then $\frac{\xi_{t-i}}{\sqrt{h_{t-i}}}$ becomes standard normal random variable and the unit risk premium, $\lambda$, acts as the non-central parameter. Therefore, we can conclude that although the conditional variance is risk-neutralized locally, but globally it is affected by the unit risk premium under pricing measure $Q$. In other words, when the coefficient of conditional variance is 0, local and global risk neutralization is equivalent.

After neutralizing the local risk, some properties of GARCH process will be altered, particularly, the stationary variance of $\xi_t$ under pricing measure $Q$. Under measure $P$, the stationary variance of return under GARCH process is $\alpha_0(1 - \alpha_1 - \beta_1)^{-1}$, which is uncorrelated with the lagged asset return. However, after neutralizing, there is a negative or positive correlation between the conditional variance and the lagged asset return when the risk premium $\lambda$ is positive or vice versa. Local risk neutralization makes the stationary variance become $\alpha_0[(1 - \alpha_1(1+\lambda^2) - \beta_1)^{-1}$, which can conclude that risk premium $\lambda$ induces an increase of stationary variance. Under pricing measure $Q$, if $|\lambda| < \sqrt{(1-\alpha_1-\beta_1)/\alpha_1}$, then

1. The stationary variance of $\xi_t$ equals to $\lambda_0[(1 - \alpha_1(1+\lambda^2) - \beta_1)^{-1}$
2. $\xi_t$ is leptokurtic
3. $Cov^Q(\xi_t/\sqrt{h_t}, h_t + 1) = -2\lambda\alpha_0\alpha_1[(1 - \alpha_1(1+\lambda^2) - \beta_1)^{-1}$

The terminal stock price is obtained by taking the exponential on both sides of function (6). Then under the GARCH$(p, q)$ process, the European call option with exercise price K, time to maturity T is calculated by taking the conditional expectation of terminal price given the information set $\phi_t$ and then discounting at the risk-free rate as below:

$$X_T = X_t exp\left[((T-t)r - \frac{1}{2}\sum_{s=t+1}^{T} h_s + \sum_{s=t+1}^{T} \xi_s\right] \tag{9}$$

$$C_t^{GH} = e^{-(T-t)r} E_P[max(X_T - K, 0)|\phi_t] \tag{10}$$

In Duan (1995) paper, it focuses only on European call option, hence, in order to obtain the put value, we can apply Put-Call parity relationship. Based on option pricing valuation, one can derive the delta hedge of option. Specifically, delta of the option is defined as the first partial derivative of the option price with respect to the underlying asset price.

$$\Delta_t^{GH} = e^{-(T-t)r} E_P \left[ \frac{X_T}{X_t} 1_{X_T \geq K} | \phi_t \right] \tag{11}$$

When the asset return is homoskedastic, we can conclude that the option price and delta under GARCH process will reduce to Black-Scholes model. However, the conditional distribution of more than one period cannot be determined analytically, therefore, the option price and delta can be derived by Monte-Carlo simulation.

# 4 Comparison of Black-Scholes and GARCH(1,1) option pricing model

In this part, the comparison between GARCH(1,1) and Black-Scholes model will be discussed. Since GARCH(1,1) process is the most popular model, we only limit our attention to GARCH(1,1).

Normally, the Black-Scholes formula for a European call option and option delta where the unconditional variance are assumed to be constant over time is:

$$C_t^{BS} = X_t N(d_t) - e^{-(T-t)r} K N(d_t - \sigma\sqrt{T-t}), \tag{12}$$

$$\Delta_t^{BS} = N(d_t) \tag{13}$$

where

$$d_t = \frac{ln(X_t/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \qquad \sigma^2 = \alpha_0(1 - \alpha_1 - \beta_1)^{-1} \tag{14}$$

Under GARCH's perspective, Black-Scholes model is the special case where the variance is homoskedastic, hence, interpreting the Black-Scholes in the GARCH framework is more intricate. Chaudhury and Wei (1996) assumed that if the time to maturity is long then the conditional variance will closely converge to the unconditional variance under measure $Q$. If the terminal stock price under measure $Q$ is a log-normal conditional distribution and there is no correlation between stock return and conditional variance, then unconditional variance under measure $Q$ and variance under original measure $P$ will roughly the same. Consequently, GARCH option price approximately equals to Blask-Scholes price. However, if three assumptions by Chaudhury and Wei (1996) do not hold, the B-S price will be different from GARCH price. In particular, according to Duan (1995), B-S price under GARCH framework is calculated based on an incorrect assumption of homoskedasticity while the real process is heteroskedastic. This wrong assumption requires the asset returns volatility to remain constant with respect to global risk neutralization for the consistency of the model. Therefore, calculating B-S option price when the underlying asset follows GARCH process is to plug the stationary variance of $\xi_t$ as mentioned above in the Black-Scholes formula.

The asset return volatility induces the differences between GARCH(1,1) process and Black-Scholes model. In particular, option price using BS formula assumes the variance is constant and does not depend on time $t$; while the log return of GARCH process is heteroskedastic, hence, the variance of log return is stationary and depends on time $t$. If we set the initial conditional variance the same as unconditional variance at the original pricing measure, then under pricing measure $Q$ the conditional variance will be higher than unconditional variance $\sigma^2$ because of the local risk neutralization.

As mentioned above that the risk-neutralized GARCH process is leptokurtic, so the GARCH option price which is out-of-money will be more likely to finish in-the-money. Specifically, the returns of the underlying asset that follows GARCH process in practice will be leptokurtic, hence, the probability that stock price attains an extremely high value or extremely low is more likely to happen. Therefore, the GARCH option price will be higher than B-S price. The behavior in the opposite direction is also true: in-the-money options are more likely to finish out-of-money. However, this does not imply that GARCH option price in this case will be lower than B-S price. While it is true that out-of-money finishes happen more often under GARCH, the option price in those paths are 0 due to (10). On the other hand, due to the leptokurtosis of returns' innovation's distribution under GARCH, in-the-money finish is likely to have even higher asset prices at maturity in comparison to B-S model where innovation is mesokurtosis. This behavior effectively put higher emphasis on in-the-money values by ruling out the opposite balancing effect of high out-of-money values, possibly skewing GARCH option price to a higher level than B-S option price.

# 5 Empirical analysis and discussion

In this section, we compute the GARCH option prices and deltas using Monte Carlo simulation because it is difficult to derive the distribution of asset return analytically. Parameters $p = 1$, $q = 1$ are chosen for simplicity. Accordingly, we recalculate option prices under Black-Scholes and make a comparison between B-S and GARCH. The results are demonstrated in Table 1 and Table 2. As Duan (1995) mentioned, with the assumptions made in the article, a constant interest rate can only be used for the valuation of individual equity options but not the market portfolio, hence, the market portfolio's return will not follow GARCH process. Therefore, we will consider a equity option, which is option of BMW's stock.

To evaluate the impact of asset price at valuation time, we set can calculate option prices at 3 $s/x$ (moneyness) settings: 0.8 (out-of-money) , 1.0 (at-the-money), and 1.2 (in-the-money). We also analyse the impact of the initial conditional variance on option pricing by setting three levels of initial conditional standard deviations, which equals to $0.8\sigma$ (low initial conditional variance), $\sigma$ (equal to unconditional volatility), $1.2\sigma$ (high initial conditional variance). Effect of option expiration length is also taken into account by 3 settings: $T = 30$, $T = 90$, and $T = 180$. At each possible combination of 3 specified settings, fifty-thousand Monte-Carlo simulations are made to arrive at GARCH option price. Black-Scholes option price is calculated as per the formula given in (12), (13), and (14). Duan (1995) concluded that at any time of pricing the option, the conditional variance of the asset return is high, then all options written on this asset would be approximately more valuable and vice versa.

Firstly, we will consider BMW daily index from December 2018 to December 2020. The estimated parameters are $\hat{\alpha_0} = 0.0001$, $\hat{\alpha_1} = 0,141445$, $\hat{\beta_1} = 0,612669$, $\sqrt{\hat{h_0}} = 0,016939$, while the risk premium $\lambda$ is set at 0.001. We will set the risk free rate at 5% annually and the exercise price is \$100. As we can see from the Table 1, GARCH price increases with the level of initial conditional variance, time to maturity and the level of $s/x$ ratio. Gultekin et al. (1982) mentioned that Black-Scholes model is known to underprice the calls written on low variance and out-of-money options. According to the Table 1, options which are underpriced or overpriced, depend on the level of conditional variance. Duan (1995) concluded that the Black- Scholes always underprices options when they are in deep out-of-money situation. As demonstrated in Table 1, for deep out-of-money option, GARCH prices are always overpriced with respect to B-S price when the conditional variance is 0.8, 1 and 1.2. This underestimation of B-S price with respect to GARCH model seems to be true because BMW is an individual equity options and its return follow GARCH process. We can also conclude that, when time to maturity is short, the underestimation of B-S price is more pronounced.

Table 1: BMW Call Option Pricing Biases as a Percentage of the Black-Scholes Price for Different Maturities, Exercise Prices and Conditional Volatilities

| $\sqrt{h_t}/\sigma$ | | | **0,8** | | | **1** | | | **1,2** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **T** | $S/X$ | **BS** | **GARCH** | **bias mean** | **bias std** | **GARCH** | **bias mean** | **bias std** | **GARCH** | **bias mean** | **bias std** |
| 30 | 0,80 | 0,0875 | 0,0903 | 3,2192 | 10,2918 | 0,1033 | 18,0467 | 11,3829 | 0,1238 | 41,5349 | 12,6384 |
| 30 | 0,90 | 1,0176 | 0,9467 | -6,9667 | 3,1056 | 1,0136 | -0,3932 | 3,2323 | 1,1141 | 9,4829 | 3,4744 |
| 30 | 0,95 | 2,3749 | 2,2964 | -3,3085 | 2,0938 | 2,3732 | -0,0710 | 2,1515 | 2,4433 | 2,8788 | 2,2251 |
| 30 | 1,00 | 4,6034 | 4,4646 | -3,0160 | 1,4942 | 4,5666 | -0,7993 | 1,5321 | 4,7327 | 2,8086 | 1,5983 |
| 30 | 1,05 | 7,7169 | 7,5660 | -1,9544 | 1,1329 | 7,7109 | -0,0769 | 1,1669 | 7,7374 | 0,2658 | 1,1844 |
| 30 | 1,10 | 11,5700 | 11,5087 | -0,5293 | 0,8945 | 11,5661 | -0,0334 | 0,9158 | 11,6248 | 0,4743 | 0,9372 |
| 30 | 1,20 | 20,6373 | 20,7636 | 0,6118 | 0,6081 | 20,6248 | -0,0608 | 0,6221 | 20,7801 | 0,6919 | 0,6402 |
| 90 | 0,80 | 1,1591 | 1,1659 | 0,5874 | 3,9492 | 1,1609 | 0,1522 | 3,9773 | 1,2079 | 4,2086 | 4,0892 |
| 90 | 0,90 | 3,6365 | 3,6191 | -0,4784 | 2,3338 | 3,5143 | -3,3605 | 2,2764 | 3,6781 | 1,1427 | 2,3565 |
| 90 | 0,95 | 5,6484 | 5,5015 | -2,6000 | 1,8594 | 5,5614 | -1,5404 | 1,8767 | 5,6131 | -0,6240 | 1,8911 |
| 90 | 1,00 | 8,2025 | 7,9527 | -3,0448 | 1,5543 | 8,1431 | -0,7240 | 1,5736 | 8,0911 | -1,3573 | 1,5794 |
| 90 | 1,05 | 11,2688 | 11,2976 | 0,2549 | 1,3374 | 11,1610 | -0,9574 | 1,3381 | 11,2976 | 0,2548 | 1,3532 |
| 90 | 1,10 | 14,7895 | 14,6494 | -0,9472 | 1,1533 | 14,7540 | -0,2402 | 1,1645 | 14,8576 | 0,4605 | 1,1723 |
| 90 | 1,20 | 22,8990 | 22,9942 | 0,4158 | 0,9094 | 22,9226 | 0,1030 | 0,9092 | 23,0497 | 0,6584 | 0,9175 |
| 180 | 0,80 | 3,2083 | 3,0968 | -3,4740 | 2,9278 | 3,2435 | 1,0981 | 2,9981 | 3,2771 | 2,1449 | 3,0109 |
| 180 | 0,90 | 6,7524 | 6,7424 | -0,1477 | 2,1190 | 6,7387 | -0,2026 | 2,1473 | 6,8156 | 0,9355 | 2,1428 |
| 180 | 0,95 | 9,1287 | 9,1630 | 0,3759 | 1,8683 | 9,1664 | 0,4133 | 1,8614 | 9,2209 | 1,0109 | 1,8590 |
| 180 | 1,00 | 11,8917 | 11,6721 | -1,8468 | 1,6102 | 11,8396 | -0,4379 | 1,6241 | 11,9954 | 0,8721 | 1,6557 |
| 180 | 1,05 | 15,0133 | 14,9018 | -0,7428 | 1,4384 | 15,0103 | -0,0203 | 1,4675 | 14,8761 | -0,9142 | 1,4650 |
| 180 | 1,10 | 18,4579 | 18,2917 | -0,9006 | 1,3195 | 18,3567 | -0,5486 | 1,3172 | 18,4932 | 0,1914 | 1,3145 |
| 180 | 1,20 | 26,1573 | 26,1983 | 0,1569 | 1,1010 | 26,0055 | -0,5802 | 1,1052 | 26,3370 | 0,6872 | 1,1053 |

Considering option delta, B-S option delta and GARCH counterpart are exhibited in Table 3. As mentioned above, delta is the ratio that compares the price changes of underlying asset corresponding to the change in option price. As we can see, it presents the same pattern as the price comparison above. When time to maturity is short, T = 30, options react much more than long term options and the difference between B-S delta and GARCH delta is more noticeable when options are deep-out-of-money.

Table 2: BMW Call Option Delta Biases as a Percentage of the Black-Scholes Delta for Different Maturities, Exercise Prices and Conditional Volatilities

| $\sqrt{h_t}/\sigma$ | | | **0,8** | | | **1** | | | **1,2** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **T** | **S/X** | **BS** | **GARCH** | **bias mean** | **bias std** | **GARCH** | **bias mean** | **bias std** | **GARCH** | **bias mean** | **bias std** |
| 30 | 0,80 | 0,0269 | 0,0258 | -4,4184 | 6,7351 | 0,0275 | 2,0052 | 6,9656 | 0,0331 | 23,0112 | 7,6318 |
| 30 | 0,90 | 0,1945 | 0,1826 | -6,1023 | 2,1917 | 0,1894 | -2,6367 | 2,2269 | 0,1993 | 2,4888 | 2,2776 |
| 30 | 0,95 | 0,3550 | 0,3495 | -1,5449 | 1,4717 | 0,3543 | -0,2005 | 1,4789 | 0,3546 | -0,0932 | 1,4819 |
| 30 | 1,00 | 0,5368 | 0,5385 | 0,3146 | 1,0160 | 0,5384 | 0,3006 | 1,0184 | 0,5388 | 0,3688 | 1,0221 |
| 30 | 1,05 | 0,7034 | 0,7092 | 0,8309 | 0,7111 | 0,7091 | 0,8140 | 0,7139 | 0,7007 | -0,3853 | 0,7201 |
| 30 | 1,10 | 0,8303 | 0,8429 | 1,5247 | 0,4924 | 0,8347 | 0,5320 | 0,5031 | 0,8285 | -0,2106 | 0,5113 |
| 30 | 1,20 | 0,9593 | 0,9642 | 0,5092 | 0,2444 | 0,9600 | 0,0668 | 0,2527 | 0,9572 | -0,2239 | 0,2637 |
| 90 | 0,80 | 0,1571 | 0,1548 | -1,4716 | 2,7641 | 0,1531 | -2,5646 | 2,7522 | 0,1593 | 1,3607 | 2,8005 |
| 90 | 0,90 | 0,3480 | 0,3464 | -0,4804 | 1,6121 | 0,3415 | -1,8872 | 1,6023 | 0,3496 | 0,4393 | 1,6178 |
| 90 | 0,95 | 0,4570 | 0,4508 | -1,3591 | 1,2783 | 0,4530 | -0,8663 | 1,2807 | 0,4567 | -0,0691 | 1,2831 |
| 90 | 1,00 | 0,5636 | 0,5583 | -0,9388 | 1,0359 | 0,5655 | 0,3419 | 1,0384 | 0,5548 | -1,5590 | 1,0398 |
| 90 | 1,05 | 0,6610 | 0,6672 | 0,9372 | 0,8503 | 0,6595 | -0,2189 | 0,8523 | 0,6601 | -0,1388 | 0,8547 |
| 90 | 1,10 | 0,7448 | 0,7466 | 0,2394 | 0,7033 | 0,7469 | 0,2754 | 0,7051 | 0,7451 | 0,0329 | 0,7083 |
| 90 | 1,20 | 0,8672 | 0,8716 | 0,5173 | 0,4923 | 0,8703 | 0,3649 | 0,4930 | 0,8684 | 0,1417 | 0,4978 |
| 180 | 0,80 | 0,2748 | 0,2707 | -1,4900 | 2,0564 | 0,2760 | 0,4198 | 2,0775 | 0,2772 | 0,8566 | 2,0821 |
| 180 | 0,90 | 0,4353 | 0,4359 | 0,1379 | 1,4432 | 0,4337 | -0,3548 | 1,4436 | 0,4379 | 0,5985 | 1,4469 |
| 180 | 0,95 | 0,5147 | 0,5172 | 0,4776 | 1,2353 | 0,5175 | 0,5404 | 1,2350 | 0,5167 | 0,3846 | 1,2360 |
| 180 | 1,00 | 0,5896 | 0,5846 | -0,8381 | 1,0630 | 0,5904 | 0,1345 | 1,0651 | 0,5918 | 0,3855 | 1,0692 |
| 180 | 1,05 | 0,6579 | 0,6596 | 0,2594 | 0,9272 | 0,6567 | -0,1825 | 0,9323 | 0,6534 | -0,6903 | 0,9318 |
| 180 | 1,10 | 0,7186 | 0,7191 | 0,0731 | 0,8172 | 0,7164 | -0,3063 | 0,8196 | 0,7208 | 0,3108 | 0,8183 |
| 180 | 1,20 | 0,8160 | 0,8174 | 0,1696 | 0,6475 | 0,8139 | -0,2644 | 0,6491 | 0,8206 | 0,5643 | 0,6466 |

Volatility forecasting has been undoubtedly discussed in finance recently, especially on such topics like option pricing, risk management or portfolio management. However, in practice, volatility is too intricate to be predicted precisely. Samsudin and Mohamad (2016) has divided volatility forecasting into two types, namely historical volatility and option-implied volatility. Particularly, historical volatility is simply based on the past observations and does not express information about future volatility. Therefore, the investors can only know the movement of stock price based on past trends but not the prediction of future trends. Nevertheless, Bacha (2017) affirmed that implied volatility can help traders determine the option price as well as the mispricing of options through comparing actual volatility and implied volatility. Hull and White (1987) concluded that during the lifetime of the option contract, we can predict the average of future volatility of its underlying asset based on implied volatility. Therefore, in this paper, we will further examine the relationship among implied volatility and option prices. In Rubinstein (1985), the empirical results showed that with different expiration time and strike prices, using B-S formula to return the value that equals to current market prices of traded options, implied volatilities of said options are obtained. They are exhibited in U-shape or Smile-shape pattern. The graphs of three implied volatilities with respect to three different expiration days are presented in Figure 4.1 and 4.2. All of them are also in U-shape as well. On a side note, it is interesting to observe that deep in-the-money GARCH option prices produce highly variable implied volatility, especially in the case of short expiration. This phenomenon can be attributed to GARCH option prices being skewed significantly towards higher values end, effectively "masking" true implied volatility under GARCH, making our Newton root-finding algorithm arrived at "wrong", highly variant, values. This is discussed briefly at the end of Section 4.

As we can see from two figures for each expiration days, T = 30, 90, 180 days, the out-of-the money and in-the-money options are having high volatility values while near at-the-money options are having low volatility values. However, in Figure 4.1, generally when conditional volatility is 1.2, 20% above the stationary level, for at-the-money calls, the implied volatility of lowest expirations is the highest. While in Figure 4.2, it is completely opposite. In case of 20% below the stationary level, the implied volatility of lowest maturity is the lowest. These two figures are coincide with Rubinstein (1985)'s findings in his paper. It is said that, when the call options are at-the-money, the longer time to maturity, the higher is the option's implied volatility.

# 6 Conclusion

In this seminar paper, GARCH process with locally risk neutralization is conducted for option pricing. The process possesses some elements that are applicable for empirical performance and correcting the pricing biases such as constant variance assumption of Black-Scholes formula. The use of this model is that; it uses

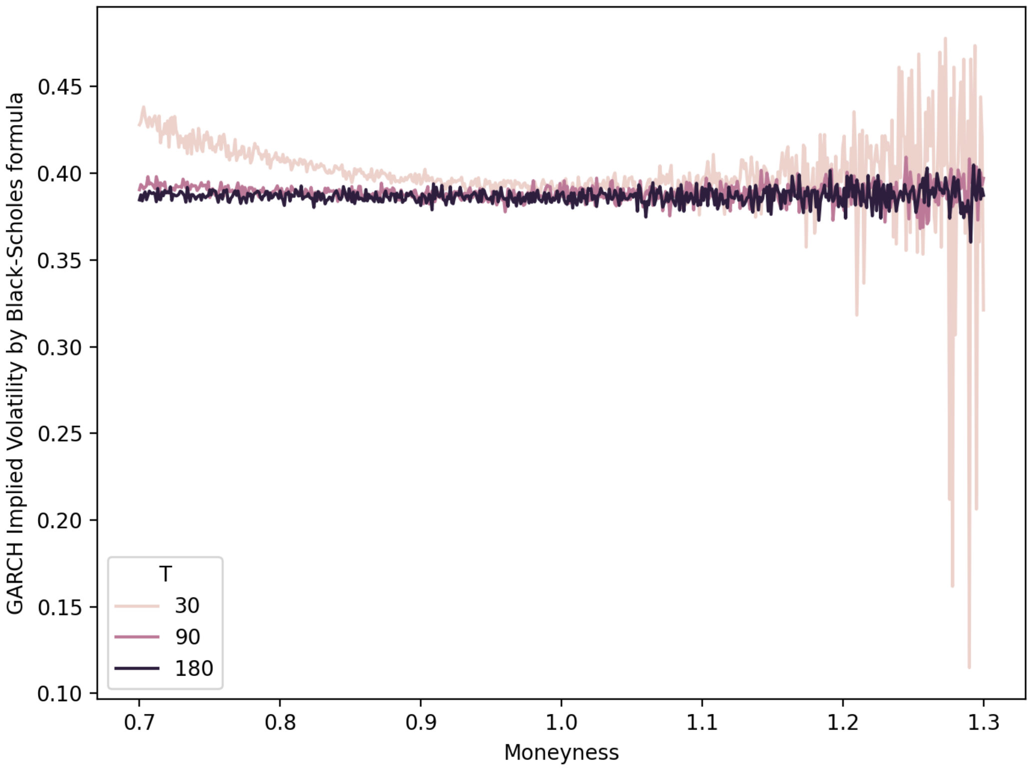


Figure 5.1: High Conditional Volatility at Valuation and its Effect on the Annualized Implied Volatility of the GARCH Option Price

stationary variance under the risk-neutralized GARCH return process to evaluate option price, then compare it with option price under Black-Scholes formula. By using Monte-Carlo simulation, we can easily calculate option price when the underlying asset's returns follow GARCH process.

When we apply this approach, the result is consistent with the conclusion of (Duan, 1995). In particular, when the options are deep out-of-the-money, B-S price are always underpriced corresponding to GARCH price and the shorter the expiration days, the more pronounced the underestimations of Black-Scholes price are. We can also infer the option delta for the delta hedging process, which is now crucial for risk management sector.

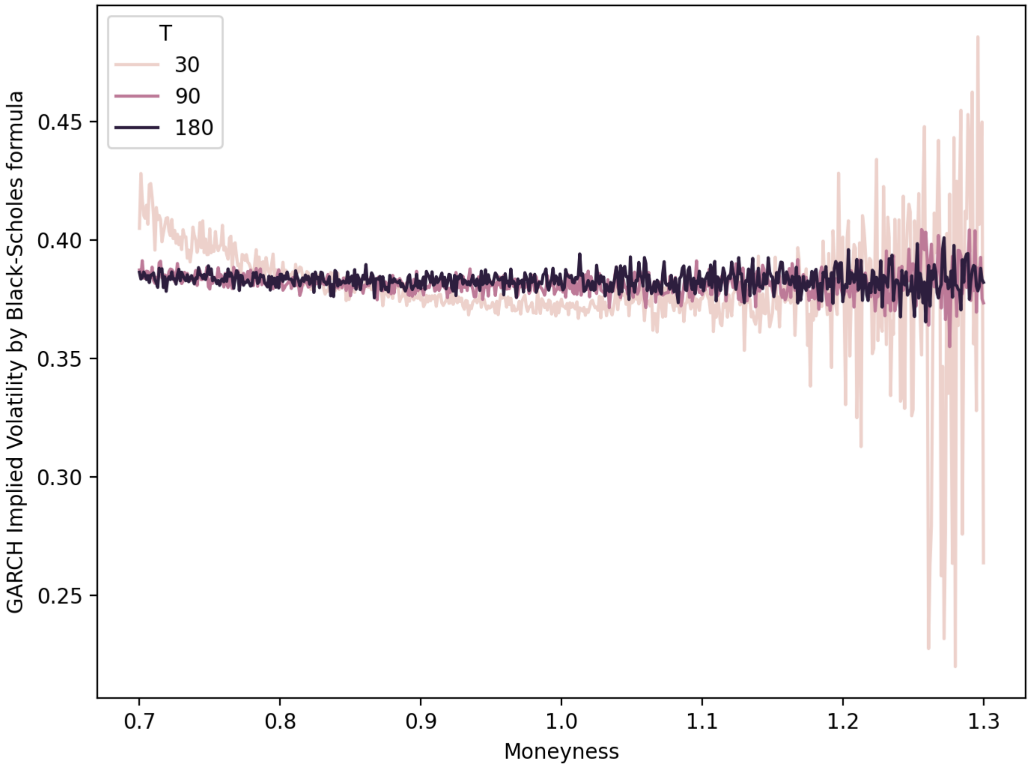


Figure 5.2: Low Conditional Volatility at Valuation and its Effect on the Annualized Implied Volatility of the GARCH Option Price

# References


Bacha, O. I. (2017). *Financial Derivatives: Markets and Applications in Malaysia.* McGraw-Hill Education (Malaysia).

Black, F. and M. Scholes (2019). The pricing of options and corporate liabilities. In *World Scientific Reference on Contingent Claims Analysis in Corporate Finance: Volume 1: Foundations of CCA and Equity Valuation*, pp. 3–21. World Scientific.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics 31*(3), 307–327.

Brennan, M. J. (1979). The pricing of contingent claims in discrete time models. *The journal of finance 34*(1), 53–68.

Carlson, N. (2020). Pandemic-induced options trading craze shows no signs of slowing down. Available online at https://www.cnbc.com/2020/12/04/pandemic-induced-options-trading-craze-shows-no-signs-of-slowing-down.html.

Chaudhury, M. M. and J. Z. Wei (1996). A comparative study of garch (1, 1) and black-scholes option prices. *Desmarais Global Finance Research Centre, McGill University, Working Paper. http://www. mcgill. ca/desautels/dgfc/research/working_papers.[Citations include Journal of Derivatives].*

Duan, J.-C. (1995). The garch option pricing model. *Mathematical finance 5*(1), 13–32.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the econometric society*, 987–1007.

Gultekin, N. B., R. J. Rogalski, and S. M. Tinic (1982). Option pricing model estimates: Some empirical results. *Financial management*, 58–69.

Hull, J. and A. White (1987). The pricing of options on assets with stochastic volatilities. *The journal of finance 42*(2), 281–300.

Rubinstein, M. (1976). The valuation of uncertain income streams and the pricing of options. *The Bell Journal of Economics*, 407–425.

Rubinstein, M. (1985). Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active cboe option classes from august 23, 1976 through august 31, 1978. *The Journal of Finance 40*(2), 455–480.

Samsudin, N. I. M. and A. Mohamad (2016). Implied volatility forecasting in the options market: A survey. *Sains Humanika 8*(2).

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