Evolvement of Probability Theory

Shuai

Contents

1 Introduction

This is the text at the very beginning of the book *Probability Theory* writen by Loeve. Mathematics could be pure, but every part of science comes from reality. Do not be confused by its abstractness and generality.

Probability theory is concerned with the mathematical analysis of the intuitive notion of "chance" or "randomness", which, like all notions, is born of experience. The quantitative idea of randomness first took form at the gaming tables, and probability theory began, wich Pascal and Fermat(1654), as a theory of games of chance. Since then, the notion of chance has found its way into almost all branches of knowledge. In particular, the discovery that physical "observables", even those which describe the behavior of elementary particles, were to be considered as subject to laws of change made an inverstigation of the notion of chance basic to the whole problem of rational interpretation of nature.[2]

A theory becomes mathematical when it sets up a mathematical model of the phenomena with which it is concerned, that is, when, to describe the phenomena, it uses a collection of well-defined symbols and operations on the symbols. As the number of phenomena, together with their known properties, increases, the mathematical model evolves from early crude notions upon which our intuition was built in the direction of higher generality and abstractness.[2]

In this manner, the inner consistency of the model of random phenomena became doubtful, and this forced a rebuilding of the whole structure in the second quarter of this century, starting with a formulation in terms of axioms and definitions. Thus there appeared a branch of pure mathematics – probability theory – concerned with the construction and inverstigation per se of the mathematical model of randomness.[2]

2 Interpretations of Probability

Intuitively, the probability $P(\alpha)$ of an event α quantifies the degree of confidence that α will occur. If $P(\alpha) = 1$, we are certain that one of the outcomes in α occurs. However, this description does not provide an answer to waht the numbers mean. There are two common interpretations for probabilities.[1]

The frequentist interpretation views probabilities as frequencies of events. More precisely, the probability of an event is the fraction of times the event occurs if we repeat the experiment indefinitely. This interpretation gives probabilities a tangible semantics. When we discuss concrete physical systems(for example, dice, coin, flips, and card games) we can envision how these frequencies are defined.[1]

The frequentist interpretation fails, however, when we consider events such as "It will rain tomorrow afternoon." Although the time span of "Tomorrow afternoon" is somewhat ill defined, we expect it to occur exactly once. It is not clear how we define the frequencies of such events. Several attempts have been made to define the probability for such an event by finding a *reference class* of similar for which frequencies are well defined, but they are not satisfactory.[1]

An alternative interpretation views probabilities as subjective degrees of belief. Under this interpretation, the statement $P(\alpha) = 0.3$ represents a subjective statement about one's own degree of belief that the event α will come about.[1]

What does it mean by saying subjective degrees of belief?

3 Intuitive Background

3.1 Events

The primary notion in the understanding of nature is that of *event* – the occurrence or nonocurrence of a phenomenon. The abstract concept of event pertains only to its occurrence or nonocurrence and not to its nature. This is the concept we intend to analyze.[2]

In science, or, more precisely, in the investigation of "laws of nature," events are classified into conditions and outcomes of an experiment. *Conditions* of an experiment are events which are known or are made to occur. *Outcomes* of an experiment are events which *may* occur when the experiment is performed, that is, when its conditions occur. All(finite) combinations of outcomes by means of "not", "and", "or", are outcomes; in the terminology of sets, the outcomes of an experiment form a *field*(or an "algebra" of sets). The condition of an experiment together with its field of outcomes, constitute a *trial*. Any (finite) number of trials can be combined by "conditioning", as following:

The collective outcomes are combinations by means of "not", "and", "or", of the outcomes of the constituent trials. The conditions are conditions of the first constituent trials together with conditions of the second to which are added the observed outcomes of the first, and so on. Thus, given the observed outcomes the preceding trials, every constituent trial is performed under supplementary conditions: it is conditioned by the observed outcomes. When, for every constituent trial, any outcome occurs if, and only if, it occurs without such conditioning, we say that the trials are *completely independent*. If, moreover, the trials are identical, that is, have the same conditions and the same field of outcomes, we speak of *repeated trials* or equivalently„ *identical and completely independent trials*. The possibility of repeated trials is a basic assumption in science, and in games of chance: *evry trial can be performed again and again, the knowledge of past and present outcomes having no influence upon future ones.*[2]

4 Where I Stop

Initially, I expect to find a answer to the question "What does it mean by saying subjective degrees of belief?" by reading Intuitive Background Chapter of the book[2]. However, after some quick check, no easy question is provided. Thus, this doc stops here for the time being.

References

[1] D. Koller and N. Friedman. *Probabilistic Graphical Models: Principles and Techniques*. Adaptive computation and machine learning. MIT Press, 2009.

[2] Loeve. *Probability Theory I*. F.W.Gehring P.r.Halmos and C.c.Moore. Springer, 1978.