SYSTEM IDENTIFICATION OF COMPLEX AND STRUCTURED SYSTEMS

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Acknowledgements

- Former PhD-students: Kristian Lindqvist, Henrik Jansson, Jonas Mårtensson, Märta Barenthin
- Collaborators: Xavier Bombois, László Gerencsér, Michel Gevers, Roland Hildebrand, Lennart Ljung, Brett Ninness, Cristian Rojas, Bo Wahlberg, James Welsh

What do we mean by complexity?

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- Computer science: Computational complexity (Turing, Church, ...)
- Systems and control:
  - Computational complexity (see survey by Blondel and Tsitsiklis)
  - Feedback control under uncertainty (Zames; Egerstedt and Brockett; Delvenne and Blondel; Zhang and Guo)
- System identification:

???

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- Example: Time complexity of worst-case system identification (Poolla and Tikku)
- Here: *The minimum experimental cost required to achieve a certain performance in the application*

Experiment design for system identification

- Much work in 1970's:
  (Mehra; Goodwin and Payne; Ng, Goodwin and Söderström; Zarrop)
  - Scalar criteria, often not involving the application directly
  - All covariance matrices can be generated by sinusoidal inputs
- Renewed interest in mid 1980's:
  - Use of high order variance expressions (Gevers and Ljung)
  - Design tied to the application (e.g. minimum variance control)
- Revival in 2000's:
  - *Least-costly identification for robust control*
    (Bombois, Scorletti, Van den Hof, Gevers and Hildebrand)
  - Semi-definite programming (Cooley, Lee and Boyd; Lindqvist; Jansson)
  - Robust stability and robust performance criteria (Hildebrand and Gevers; Jansson)
  - Nonlinear systems (Mårtensson; Novara, Vincent and Poolla)
  - Robust input design (Mårtensson; Rojas, Welsh, Goodwin, Feuer)
  - Plant-friendly design (Rivera, Lee, Mittelmann and Braun)

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The water-bed effect (Rojas, Welsh and Agüero):

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{N\Phi_u^{id}(e^{j\omega})}_{\text{Input energy density}} \ \mathsf{Var}[\hat{G}(e^{j\omega})]\mathrm{d}\omega = \underbrace{n}_{\#\text{ parameters}} \ \underbrace{\lambda_e}_{\text{noise variance}}$$

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A fundamental limitation

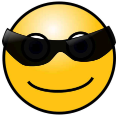
YES, there is a problem!

An appetizer

Please say hello to:

Izzy

and



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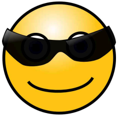

Izzy and Ozzy

Static gain estimate

Model order:	very low	true	very high
Constant input	good	good	good

Impulse response coefficient estimate

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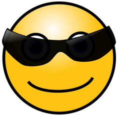

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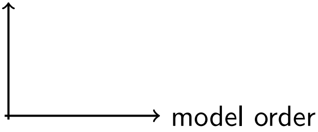

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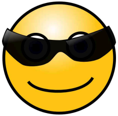

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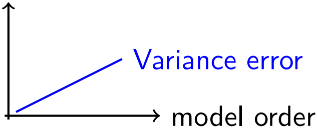

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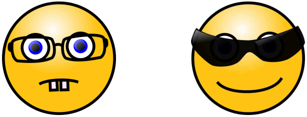

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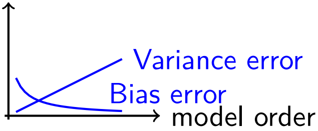

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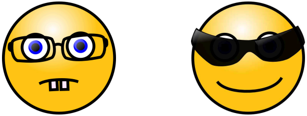

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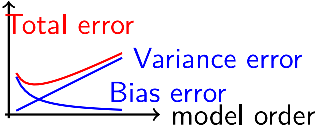

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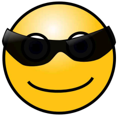

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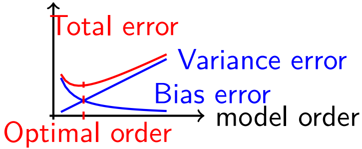

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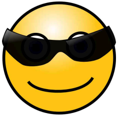

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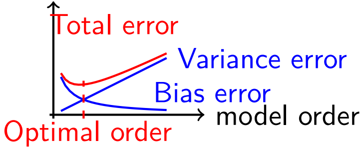


Robustness against choice of model order. Why?

Outline

Cost of complexity

An alternative formulation

Output error models

Some connections to the past

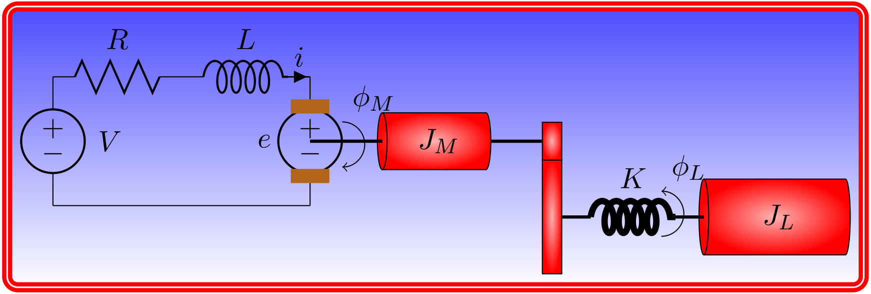
The impact of optimal experiments on the identification problem

Numerical computation of experiment designs

Implementation of experiment designs

Outline

An application example: MPC of a DC-motor

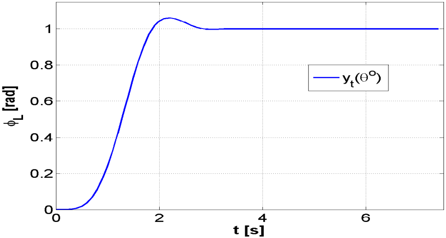


- Input: Voltage V
- Output: Angle ϕ_L
- Model parameters θ: Resistance R, Moment of inertia J_L, Elasticity K, ...
- True parameters: θ^o

An application example: MPC of a DC-motor

- Ideal response: $y_t(\theta^o)$ - true parameters used in MPC

Constraint: Maximum input move 40



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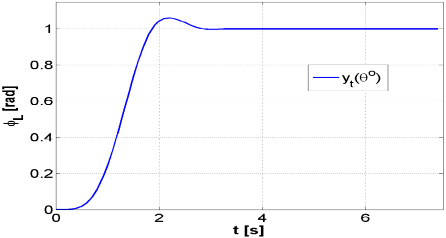

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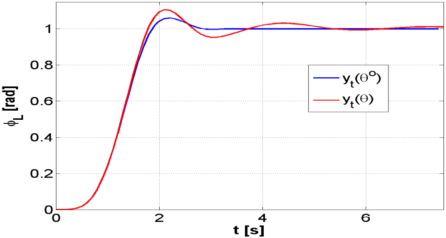

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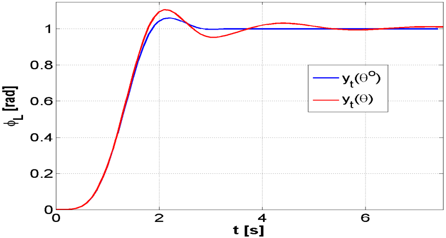

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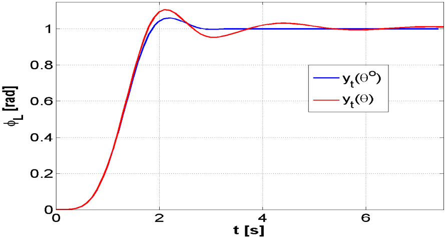

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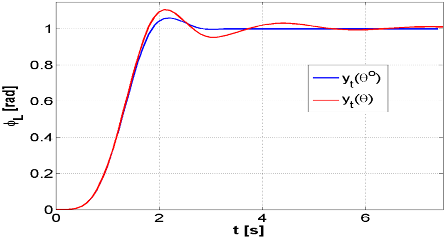

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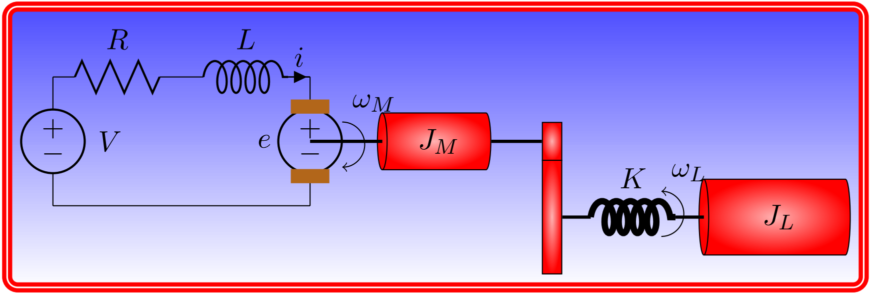

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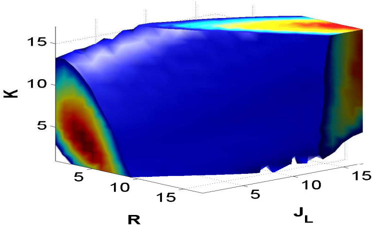

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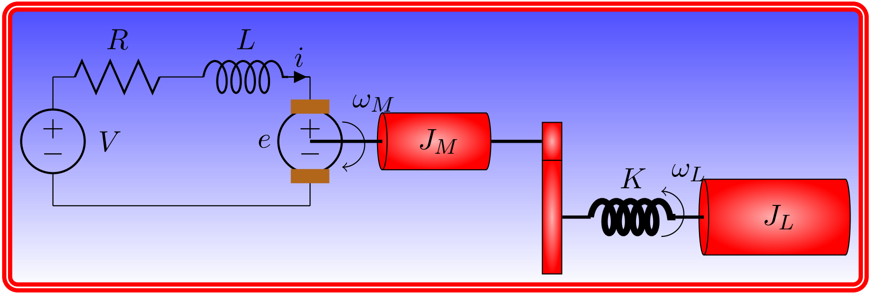

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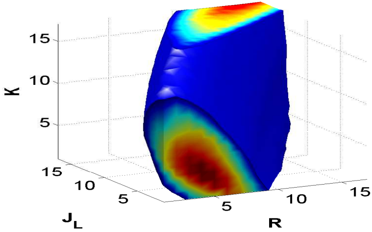

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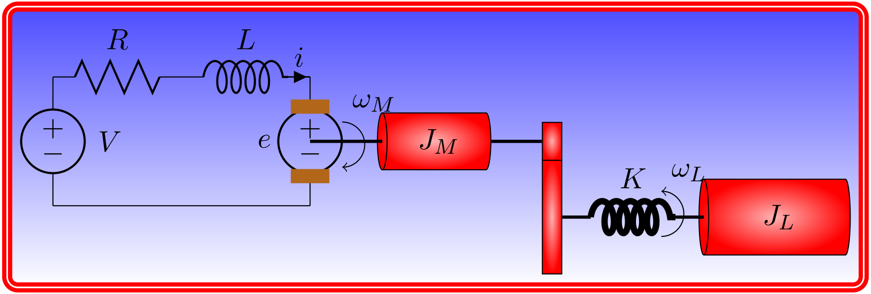

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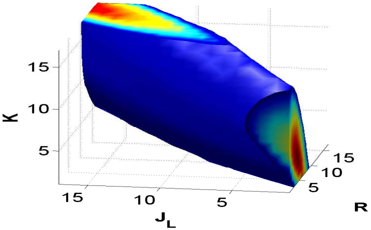

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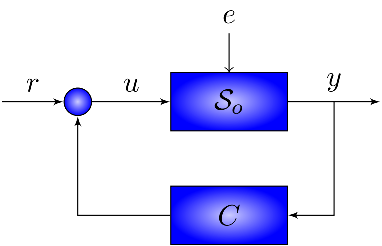
Identification recap



- Prediction error identification:
  - Prediction error: $\varepsilon_t(\theta)$
  - $\hat{\theta}_N = \arg\min \sum_{t=1}^{N} \varepsilon_t^2(\theta)$, $V_{id}(\theta) = \mathrm{E}[\varepsilon_t^2(\theta)] - \lambda_e \geq 0$
- Random noise (innovations (noise) variance λ_e)
- Stationary signals
- True system in the model set: $\mathcal{S}_o \Leftrightarrow \theta^o$ (to be relaxed later)
- High accuracy γ (implies large sample size N)
- $\sqrt{N}\left(\hat{\theta}_N - \theta^o\right) \sim \mathrm{As}\mathcal{N}\left(0, 2\lambda_e V_{id}''(\theta^o)^{-1}\right)$

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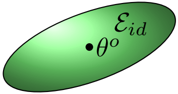

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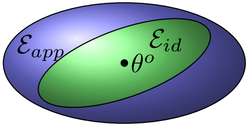

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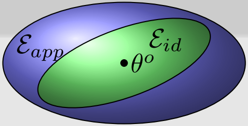

Outline

An alternative expression for the confidence ellipsoid

$$\mathcal{E}_{id} = \left\{ \theta : \ \frac{N}{2} (\theta - \theta^o)^T V''_{id}(\theta^o) (\theta - \theta^o) \leq \lambda_e \, n \right\}$$

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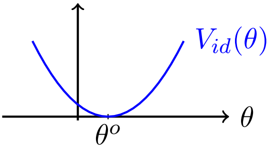

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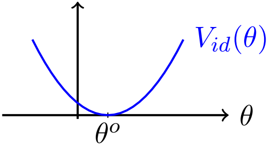

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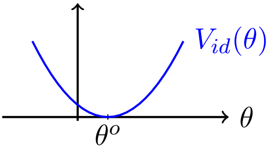

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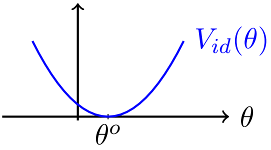

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Confidence ellipsoid = Level set for identification criterion!

An alternative formulation of cost of complexity

Level sets:

$$\mathcal{E}_{app} = \left\{\theta : V_{app}(\theta) \leq \frac{1}{\gamma}\right\}$$

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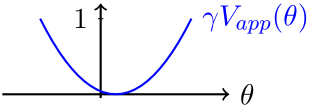

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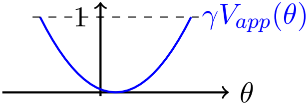

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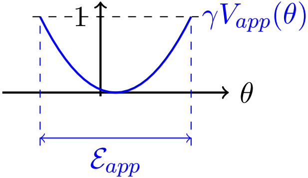

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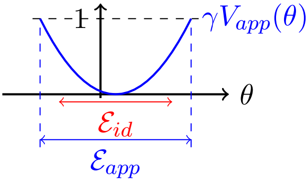

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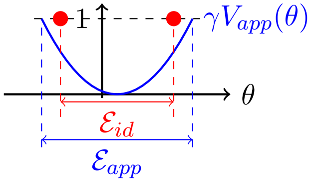

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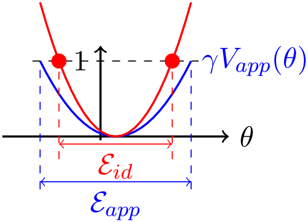

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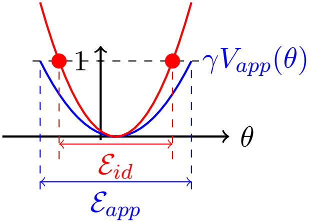

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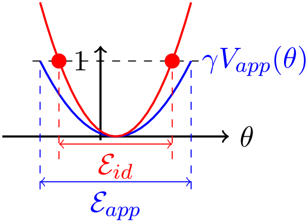

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Cost of complexity

$$Q := \min N\mathrm{E}[u_t^2]$$
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- Model: $y_t = \sum_{k=1}^{n} \theta_k u_{t-k} + e_t$
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- Only one parameter matters!
- but our confidence ellipsoid includes all n parameters
- Use a confidence ellipsoid for that parameter only: $n \Rightarrow 1$

A generalized version

General case:

$n_{app} = \#$ non-singular directions of V_{app} ($=$ rank V''_{app})

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True system: $y_t = G_o(q)u_t + e_t$

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$$\begin{aligned} V_{id}(\theta) &= \mathrm{E}[\varepsilon_t^2(\theta)] - \lambda_e \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_u^{id}(e^{j\omega})\, \left|G(e^{j\omega},\theta) - G_o(e^{j\omega})\right|^2 \,\mathrm{d}\omega \end{aligned}$$

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Output error models

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- Minimization with respect to energy density spectrum $N\Phi_u^{id}$
- Optimization tries to achieve

$$NV_{id}(\theta) = \lambda_e\,\gamma\,n_{app}V_{app}(\theta)$$

Identification cost matched to performance degradation

Output error models: Influence of $\lambda_e \gamma n_{app}$

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- Cost & constraint linear in $N\Phi_u^{id}$

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Optimal energy density: $N\Phi_u^{id} = \lambda_e \gamma n \tilde{\Phi}_u$

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- Normalized problem

The normalized problem - Insights

$$\tilde{Q} := \min \mathrm{E}[u^2(t)]$$
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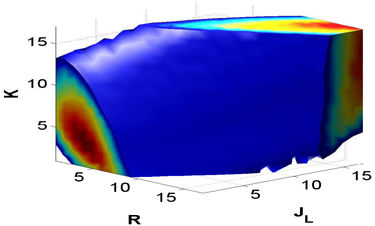
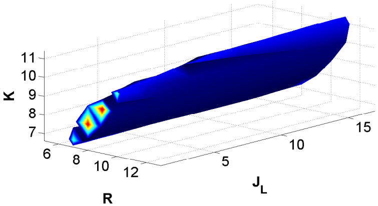
Maximum input move 3



Maximum input move 40



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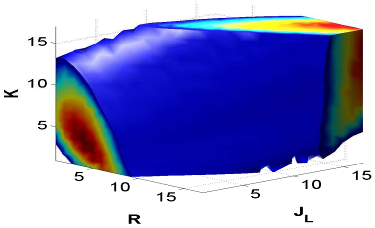

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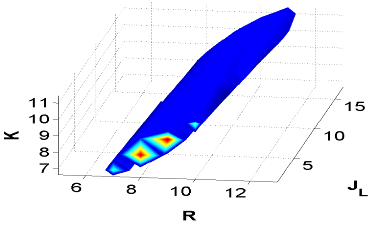

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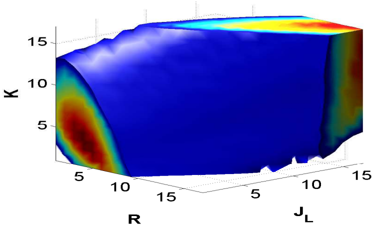

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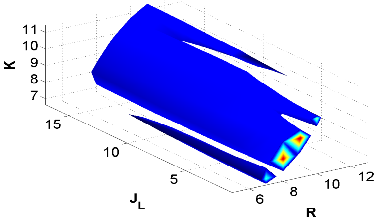
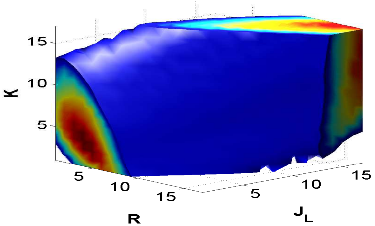


The normalized problem - Insights

$$\tilde{Q} := \min \mathrm{E}[u^2(t)]$$
$$s.t.\ V_{id}(\theta) \geq V_{app}(\theta)$$

Maximum input move 3



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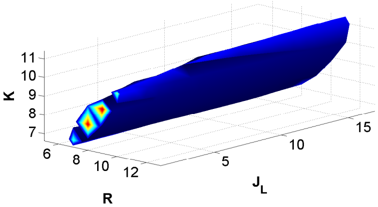

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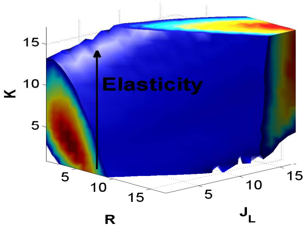

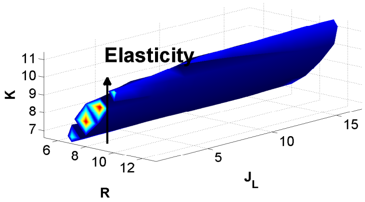

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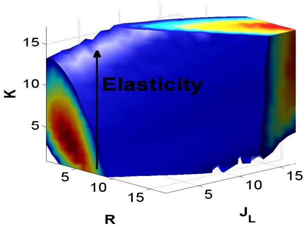

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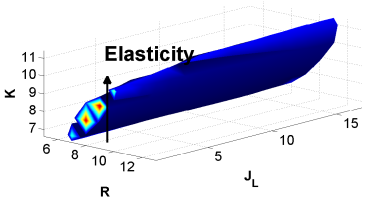

- Performance specifications determine the shape of $V_{app}(\theta)$

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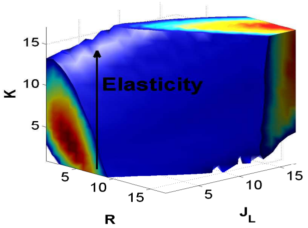

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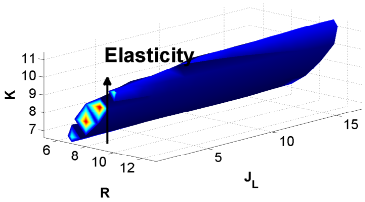

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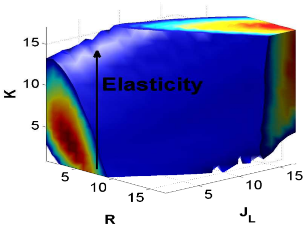

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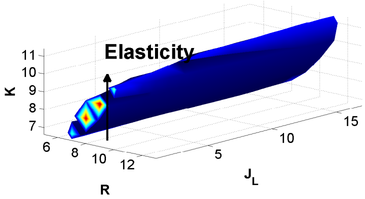

- Performance specifications determine the shape of $V_{app}(\theta)$
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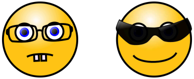
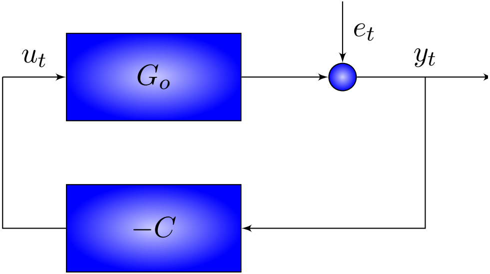
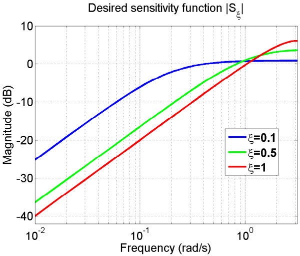
$\Rightarrow \tilde{Q}$ reflects performance specifications in the application
We will use $0 \leq \xi \leq 1$ to indicate specs. i.e. $\tilde{Q}(\xi)$

Izzy and Ozzy goes to MRC (model reference control)







- Controller $C = C(G)$, G output error model

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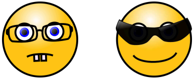

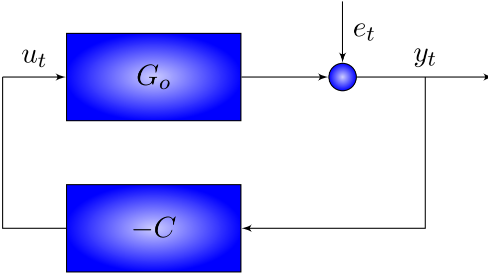

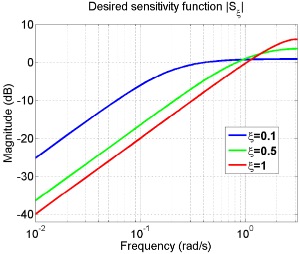

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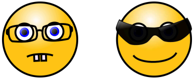

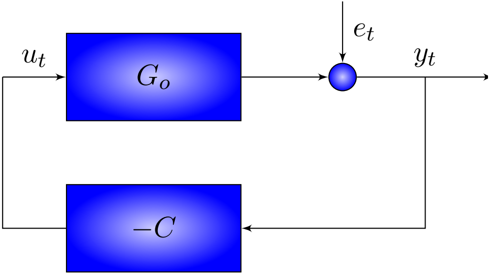

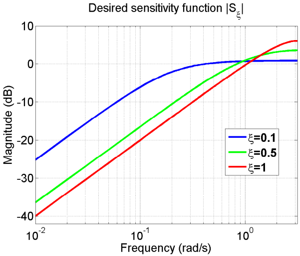

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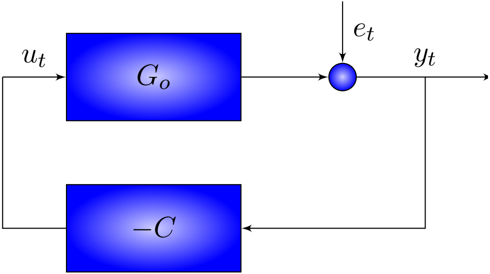

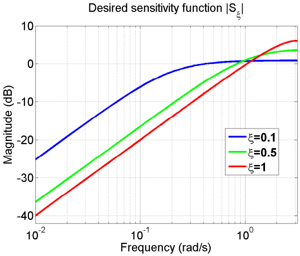

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$\Rightarrow$ Upper bound: $Q \leq \gamma n \|\Phi_u^{desired}\|_1$

MRC: Cost of complexity

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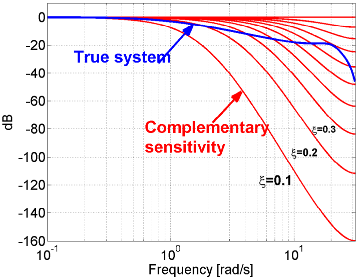

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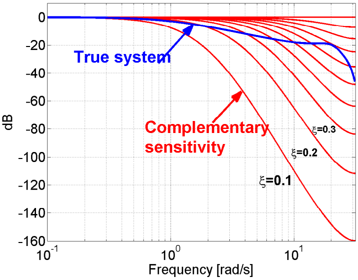

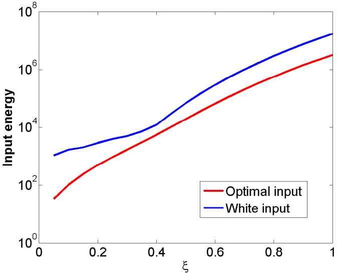

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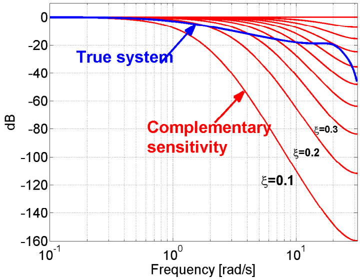

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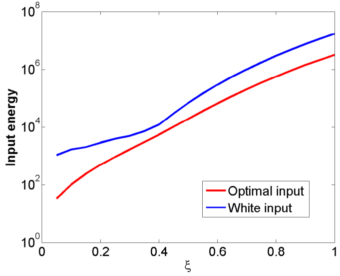

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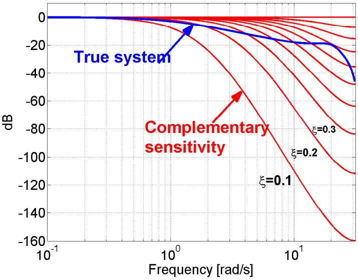

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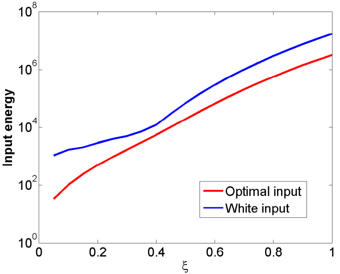

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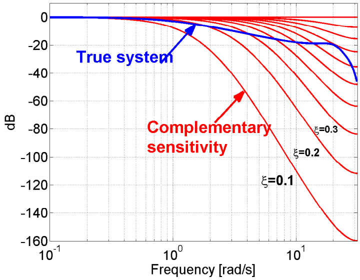
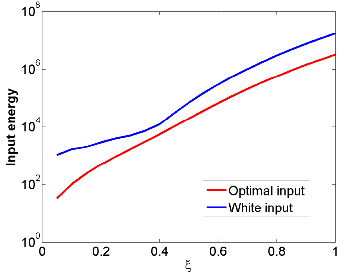
$$\Rightarrow \quad \min\ N = \gamma n \frac{\mathrm{E}[u_{desired}^2]}{\lambda_u^{\max}}$$





Output error models: Cost of complexity - Summary

Cost of complexity

$$Q \approx \lambda_e \, \gamma \, n_{app} \, \tilde{Q}(\xi)$$

- λ_e: noise level
- γ: accuracy
- n_{app}: # of non-singular directions in the parameter space
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- Allows user to make informed trade-offs:
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Outline

Some connections to the past

Identification experiment = desired closed loop operating conditions:

- Random errors
  - Minimum variance control (Gevers and Ljung 1986, Hjalmarsson, Gevers and De Bruyne 1996, Hildebrand and Solari 2007, Mårtensson, Rojas and Hjalmarsson 2009)

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- Contributions here:
  - Results above different sides of the same coin (matching V_{id} and V_{app})
  - Matching not enough. Sufficient input energy required. $(N\Phi_u^{id} = \lambda_e \gamma n \Phi_u^{desired})$

Outline

The Izzy & Ozzy problems revisited: Static gain estimation

Model order:	low	true	high
Constant input	good	good	good

$$y_t = \sum_{t=1}^{n} \theta_k u_{t-k} + e_t$$

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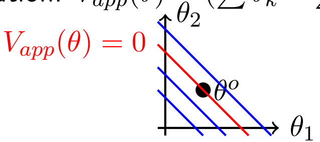

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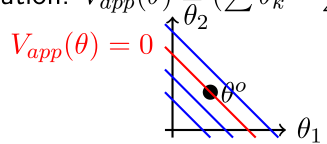

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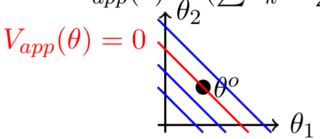

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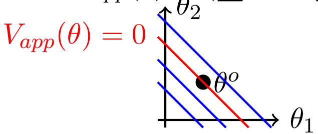

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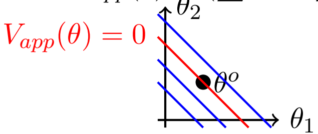

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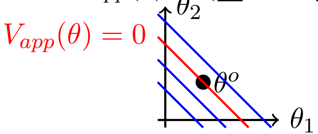

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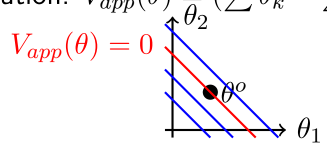

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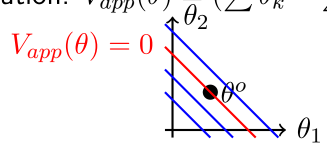

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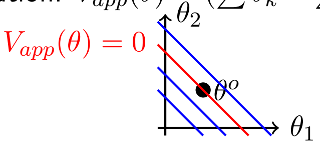

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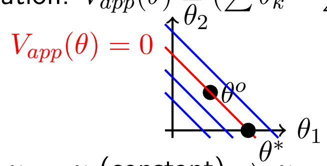

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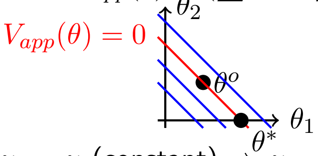

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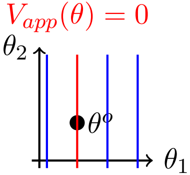

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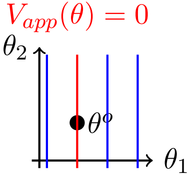

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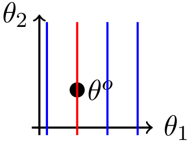

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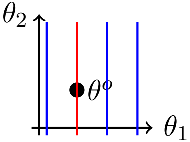

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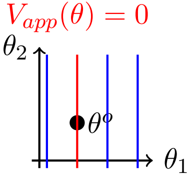

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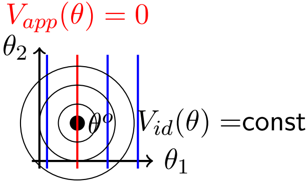

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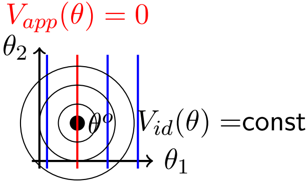

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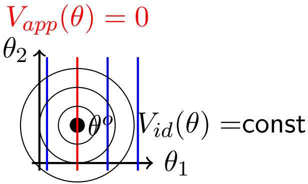

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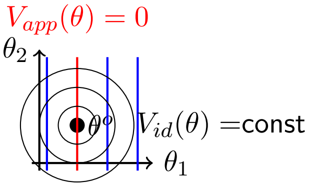

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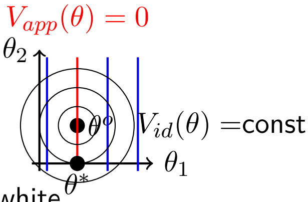

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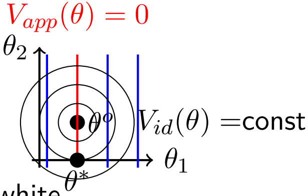

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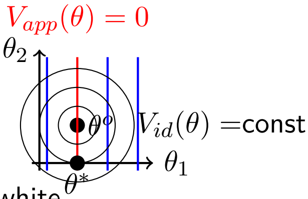

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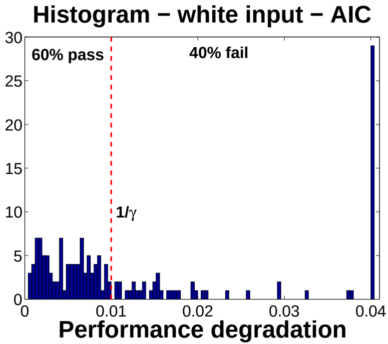

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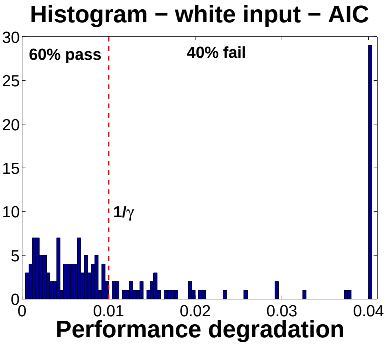

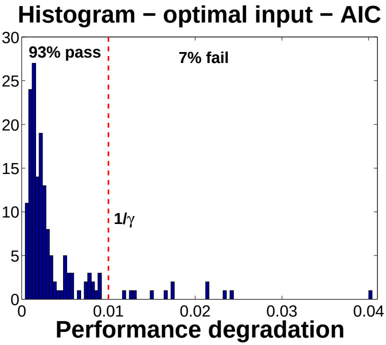

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  - Choice of model structure less critical
  - Advice: Don't use too low order (c.f. impulse response). Use model reduction instead (c.f. the ASYM method by Zhu).

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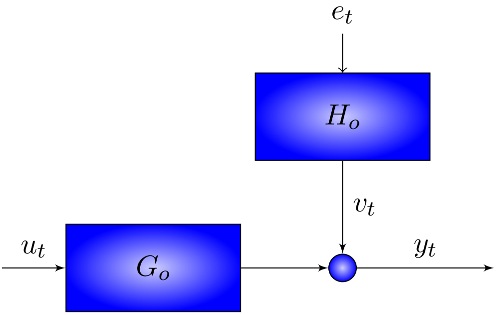

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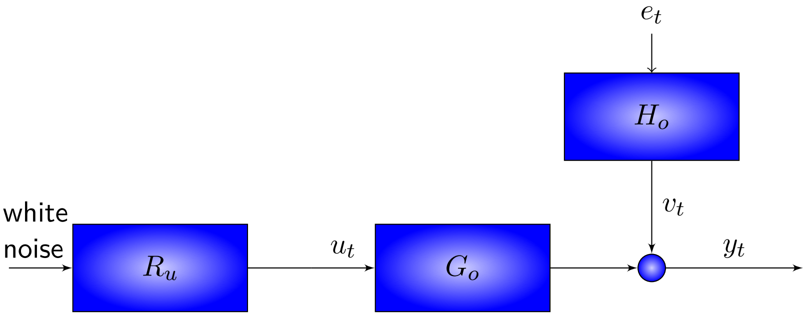

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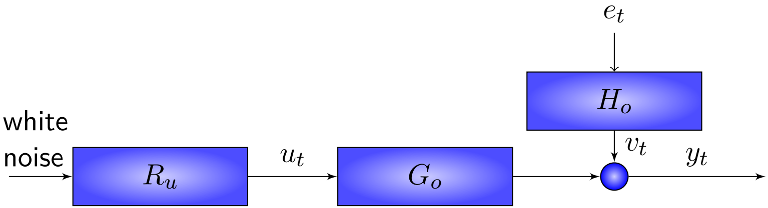
  - Adaptive (sequential) experiment design

Adaptive input design



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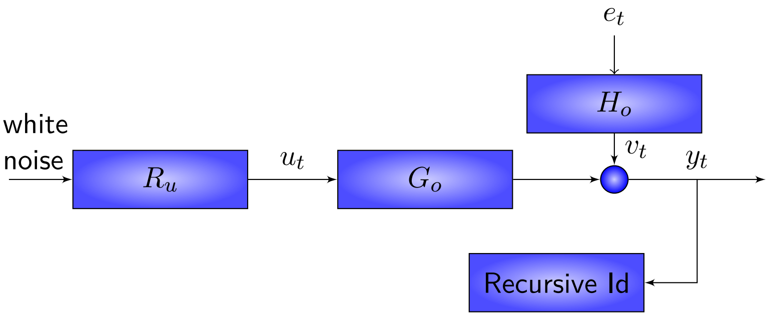

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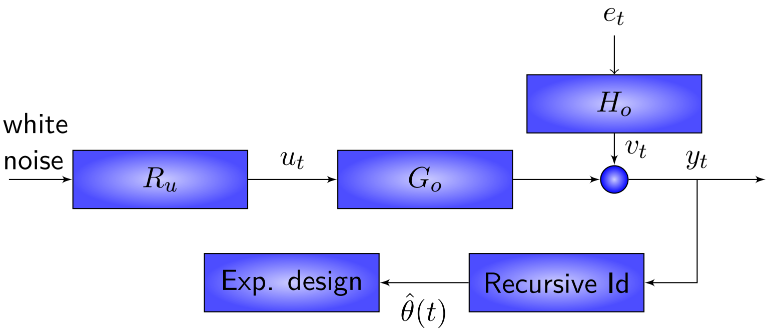

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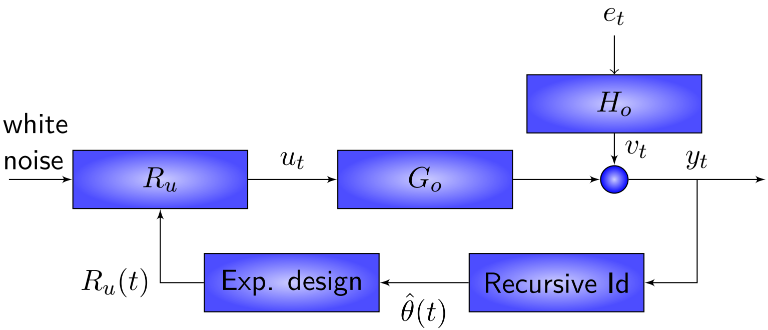

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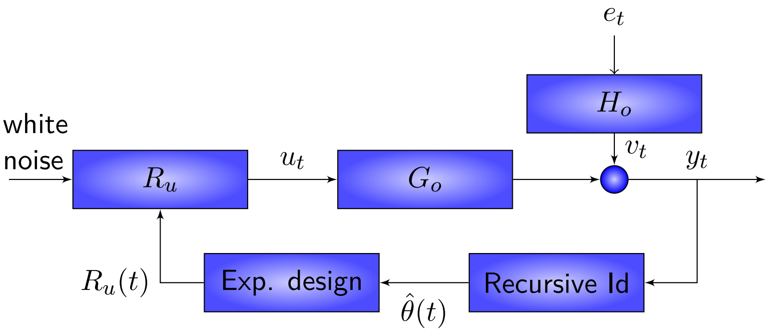
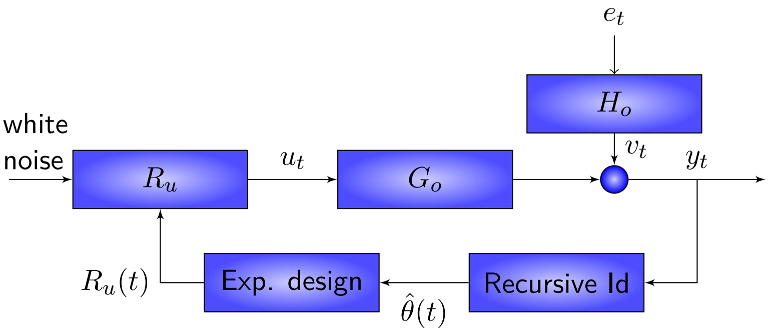


- An adaptive feedback system

Adaptive input design



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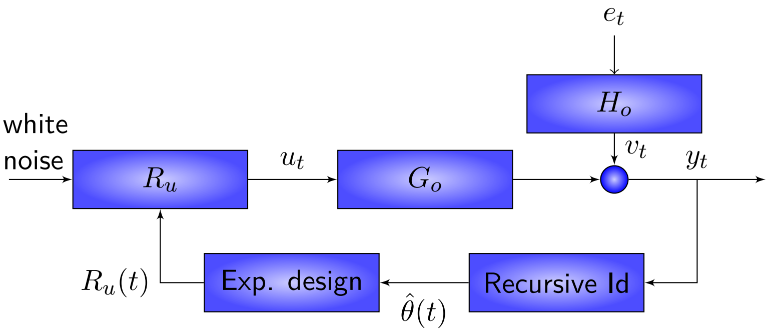

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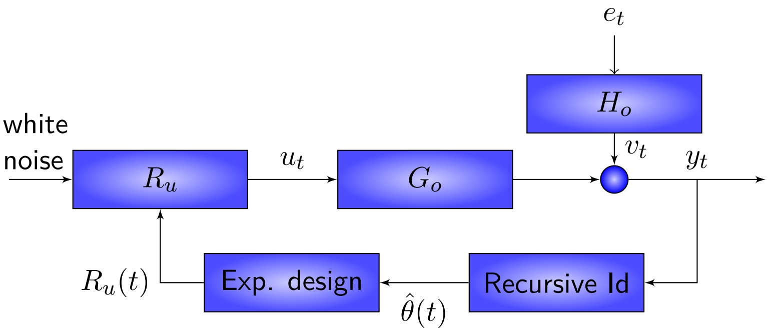

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Key questions:

- Convergence?
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Adaptive input design

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Theorem (Gerencsér's free lunch theorem for ARX-models)

- *True system in the model set*
- *System stable*

$\Rightarrow$ *Optimality when sample size grows*

Gerencsér

Adaptive input design

What happens when true system is not in the model set?

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Example

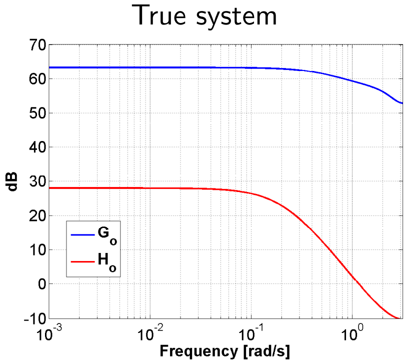
NMP-zero estimation

- Quantity of interest: z_o: $G_o(z_o) = 0$, $|z_o| > 1$
- Optimal input: $u_t = \frac{c}{z^{-1} - z_o} w_t$
- V_{id} and V_{app} not matched (c.f. impulse response problem)
- Still $y_t = \theta_1 u_t + \theta_2 u_{t-1}$ $\Rightarrow$ consistent estimate

Example: Non-minimum phase zero estimation

$$\text{True system: } y_t = \frac{(q-3)(q-0.1)(q-0.2)(q+0.3)}{q^4(q-0.5)} u_t + \frac{q}{q-0.8} e_t^o$$

$$\text{Model: } y_t = \frac{\theta_1 q + \theta_2}{q^2} u_t + e_t$$



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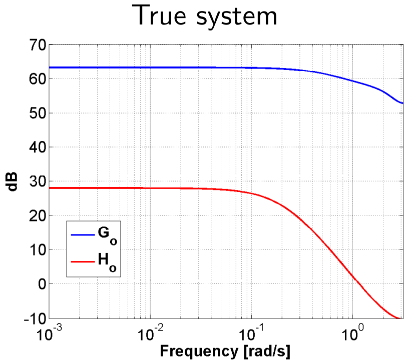

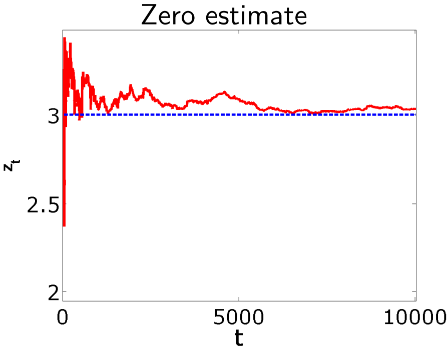

Example: Non-minimum phase zero estimation

Theorem (Rojas and Gerencsér)

$$\textit{True system: } y_t = G_o(q)u_t + H_o(q)e_t^o$$

with G_o *and* H_o *stable and rational.*

$$\hat{z}_t \longrightarrow \textit{ largest NMP-zero of system w.p.1}$$

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Future directions:

- Nonlinear systems
- Structured systems (e.g. decentralized and networked)
- Communication systems
- Adaptive control

A lot of exciting problems remain!!!