

$$1. \therefore \mu_k = \frac{1}{k} \sum_{i=1}^k X_i \quad \text{and} \quad \mu_k = \mu_{k-1} + \frac{1}{k}(X_k - \mu_{k-1})$$

$$\therefore b_{k+1} = b_k + \rho_k (X_k - \mu_k), \quad k \in [0, 9999], \quad k \in \mathcal{N}$$

$$\hookrightarrow b_n = \frac{1}{n} \sum_{i=1}^n X_i, \quad X_i \sim \mathcal{N}(b, \sigma^2)$$

$$\hookrightarrow \text{by LCT,} \quad \lim_{n \rightarrow \infty} E[b_n] = \lim_{n \rightarrow \infty} E\left[\frac{1}{n} \sum_{i=1}^n X_i\right] = \lim_{n \rightarrow \infty} \frac{1}{n} E\left[\sum_{i=1}^n X_i\right] = b$$

$$\begin{aligned} 2. \quad \therefore \lim_{n \rightarrow \infty} E[|b_n - b|^2] &= \lim_{n \rightarrow \infty} E[b_n^2 - 2b b_n + b^2] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n^2} \left(\sum_{i=1}^n E[X_i^2] + \sum_{i \neq j} E[X_i] E[X_j] - E[X_i]^2 \right) - b^2 \\ &= \lim_{n \rightarrow \infty} \frac{\sigma^2}{n} = 0 \end{aligned}$$

Identify MRP with CTD in the form of

$$V(x) = \gamma \left\{ \ell^h(x) + \sum_{i=1}^d \tilde{p}^h(x + h e_i | x) V(x + h e_i) + \tilde{p}^h(x - h e_i | x) V(x - h e_i) \right\}$$

$$(3) \quad V(x) = \gamma \cdot \ell^h(x) + \gamma \cdot \sum_{i=1}^2 \tilde{p}^h(x + h e_i | x) V(x + h e_i) + \tilde{p}^h(x - h e_i | x) V(x - h e_i)$$

$$\therefore \gamma = \frac{2}{2+h^2}, \quad \ell^h(x) = \frac{h^2}{2} \left[(x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2 - 2 \right], \quad \tilde{p}^h(x \pm h e_i | x) = \frac{1}{4}$$

\therefore We can get the conclusion