Example 1: Prove that $\hat{W}$ is an exact sampling

By definition, $W(t_{i+1}) - W(t_i) = \sqrt{t_{i+1} - t_i} \cdot Z_{i+1}$ $\because Z \sim N(0,1)$

$\therefore \sqrt{t_{i+1} - t_i} \cdot Z_{i+1} \sim N(0, t_{i+1} - t_i)$ $\because W(t)$ is Brownian motion

$\therefore W(t_{i+1}) - W(t_i) \perp W(t_{s+1}) - W(t_s)$, $W(t_i) \sim N(0, t_i)$

$\therefore W$ is an exact sampling

Pseudo code. step 1 $W_0 = 0$, $h = 0.1$

step 2 For $i = i{+}{+}$

$Z \leftarrow N(0,1)$

$W_{i+1} = W_i + \sqrt{h} \cdot Z$

step 3 return $(W_0 \cdots W_n)$

Pseudo code for example 3:

step 1: For $i = i{+}{+}$

$W_0 = 0$, $h = 0.1$, $Z \leftarrow N(0,1)$

$W_{i+1} = W_i + \sqrt{h} \cdot Z$

$S_k = S_0 \cdot e^{(r - \frac{1}{2}\sigma^2) t_k + \sigma W_t}$

$A_{rasian} = \frac{1}{n} \sum_{k=1}^{n} S_k$

Price $= e^{-rT} (A_{rasian} - k)^+$

return price

Example 1: Find $\log S_t$ for $S \sim GBM(S, \mu, \sigma^2)$

$\because dS_t = S_t \cdot \mu \cdot dt + S_t \cdot \sigma \cdot dW_t$

By Ito formula. $d(\ln S_t) = \frac{1}{S_t} dS_t + \frac{1}{2}\left(-\frac{1}{S_t^2}\right)(dS_t)^2$

$= \mu \cdot dt + \sigma dW_t - \frac{1}{2}\sigma^2 dt$

$= (\mu - \frac{1}{2}\sigma^2) dt + \sigma dW_t$

$\therefore \ln S_t = S_0 \cdot e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t}$