$$\frac{1}{2}\Delta V(x) - V(x) + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2} = 0, \quad x \in O = (0,1)^2$$

boundary date

$$V(x) = (x_1 - \tfrac{1}{2})^2 + (x_2 - \tfrac{1}{2})^2, \quad x \notin O$$

1. Show that exact solution is

$$V(x) = (x_1 - \tfrac{1}{2})^2 + (x_2 - \tfrac{1}{2})^2$$

if $x \notin O$, $V(x) = (x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2$ ∴ it satisfies the boundary condition

if $x \in O$, $\frac{\partial V(x)}{\partial x_1 \partial x_1} = 2$, $\frac{\partial V(x)}{\partial x_2 \partial x_2} = 2$

$$\therefore \frac{1}{2}(2+2) - x_1^2 + x_1 - \frac{1}{4} - x_2^2 + x_2 - \frac{1}{4} + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}$$

$$= 2 - \frac{1}{4} - \frac{1}{4} - \frac{3}{2} = 0$$

∴ $(x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2$ is exact Solution

2. Identify γ, ℓ^h, p^h in the CFD solution

$$\frac{1}{2} \sum_{i=1}^{2} \frac{V_i^+ - 2V + V_i^-}{h^2} - V + L = 0$$

$$\cancel{\left(\frac{d}{h^2}+1\right)V = \sum_{i=1}^{2} \frac{1}{2} V_i^+ + \sum_{i=1}^{2} \frac{1}{2} V_i^- + h^2 L}$$

$$\left(\frac{d}{h^2}+1\right)V = \sum_{i=1}^{2} \frac{1}{2h^2} V_i^+ + \sum_{i=1}^{2} \frac{1}{2h^2} V_i^- + L$$

$$(d+h^2)V = \sum_{i=1}^{2} \frac{1}{2} V_i^+ + \sum_{i=1}^{2} \frac{1}{2} V_i^- + h^2 L$$

$$\frac{(d+h^2)}{d} V = \sum_{i=1}^{2} \frac{1}{2d} V_i^+ + \sum_{i=1}^{2} \frac{1}{2d} V_i^- + \frac{h^2}{d} L$$

$$\therefore \gamma = \frac{d}{d+h^2} = \frac{2}{2+h^2}$$

$$\ell^h = \frac{h^2}{d} L = \frac{h^2}{2}\left(x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}\right)$$

$$p^h(x \pm h e_i | x) = \frac{1}{2d} = \frac{1}{4}$$