mathmatical models from visual inertial slam

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1 abstract

本文将简单的介绍下visual inertial slam 的概况和使用到得算法。算法主要涉及到IMU的预积分理论和流形的理论。使用camera + IMU的方案来做SLAM/Odometry，一般被称作Visual-Inertial Odometry (VIO)或者Visual-Inertial Navigation System (VINS)，其实只有odometry还不能说是一个slam系统。按照David Scaramuzza的分类方法，首先分成filter-based 和 optimization based 的两大类。Stefan Leutenegger则则把visual inertial slam 分成tightly-coupled 和loosely-coupled，另外他还给了另外一分类方法”batch optimization and recursive filtering”。个人的理解就是按照是否将IMU数据加入到优化框架中为原则进行分类。

state of the art visual inertial slam systems

- manifold preintegration.
- keyframe-based visual-inertial slam using nonlinear opt。
- visual-inertial monocular slam with map reuse.
- A multi-state constraint Kalman filter for vision-aided inertial navigation.
- Direct Visual Inertial Odometry with stereo camera(IMU+LSD-SLAM).
- Robust Visual Inertial Odometry(有代码).
- Vision Based Navigation for Micro Helicopters (有代码).

2 short intro

loosely-coupled systems independently estimate the pose by a vision only algorithm and fuse IMU measurements only in a separate estimation step, limiting computational complexity. Tightly-coupled approaches in contrast include both the measurements from the IMU and the camera into a common problem where all states are jointly estimated, thus considering all correlations amongst them. the later appraoch promize high accuracy, leaving aside computation demands.

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loosely-coupled systems

- large-scale visual odometry for rough terrain.
- factor graph based incremental smoothing in inertialo navigation systems.
- fast 3d pose estimation with out-of-sequence measurements.
- real-time onboard visual-inertial state estimation and self-calibration of mavs in unknown enviroments.

tightly-coupled systems

okvis combines visual and inertial terms in a fully probabilistic manner.At the system level, we developed both the hardware and the algorithms for accurate real-time SLAM, including robust keypoint matching, bootstrapping and outlier rejection using inertial cues.

viorb visual-inertial monocular slam with map reuse.

msckf A multi-state constraint Kalman filter for vision-aided inertial navigation.

filtering based

- robust visual inertial odometry using a direct ekf-based approach.
- robust direct visual inertial odometry via entropy-based relative pose estimation.
- high-precision consistent ekf-based visual-inertial odometry.
- semi-direct ekf-based monocular visual-inertial odometry.

optimization based

- okvis
- IMU preintegration on manifold for efficient visual-inertial map estimation.
- visual-inertial navigation, mapping and localization: a scalable real-time approach.

3 okvis and vi-orb

3.1 vi-orb

个人感觉正如其论文题目”visual-inertial monocular slam with map reuse”, visual inertial orbslam的靓点应该在map reuse部分，在IMU图优化方面只是借鉴了预积分和okvis的算法。visual inertial orb slam 的优化流程：

3.2 okvis

这个算法基本上可以说是imu加入到优化框架中较早的”state of the art”算法，而且开源。

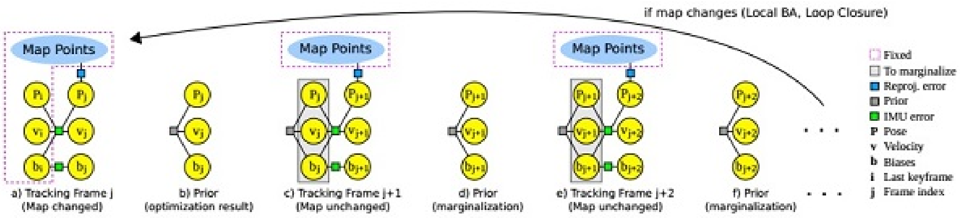

Fig. 2. Evolution of the optimization in the Tracking thread. (a) We start optimizing the frame j linked by an IMU constraint to last keyframe i. (b) The result of the optimization (estimation and Hessian matrix) serves as prior for next optimization. (c) When tracking next frame $j+1$, both frames j and $j+1$ are jointly optimized, being linked by an IMU constraint, and having frame j the prior from previous optimization. (d) At the end of the optimization, the frame j is marginalized out and the result serves as prior for following optimization. (e-f) This process is repeated until there is a map update from the Local Mapping or Loop Closing thread. In such case the optimization links the current frame to last keyframe discarding the prior, which is not valid after the map change.

Figure 1: visual inertial orbslam

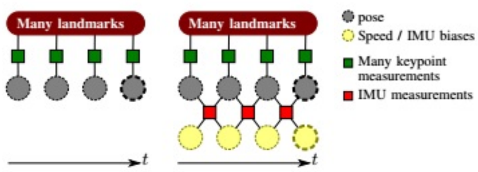

Fig. 3. Graphs of the state variables and measurements involved in the visual SLAM problem (left) versus visual-inertial SLAM (right): incorporating inertial measurements introduces temporal constraints, and necessitates a state augmentation by the robot speed as well as IMU biases.

Figure 2: okvis state graph

3.2.1 okvis graph

okvis 的图优化模型：

3.2.2 initial state for marginalization

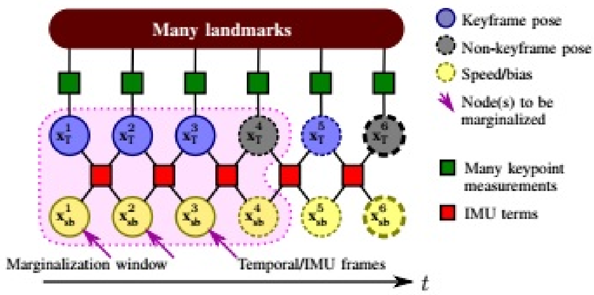

Figure 3: initial state for marginalization

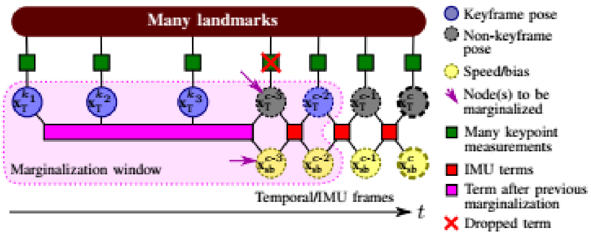

Figure 4: non-key-frame marginalization

3.2.3 marginalization non key frame

3.2.4 marginalization key frame

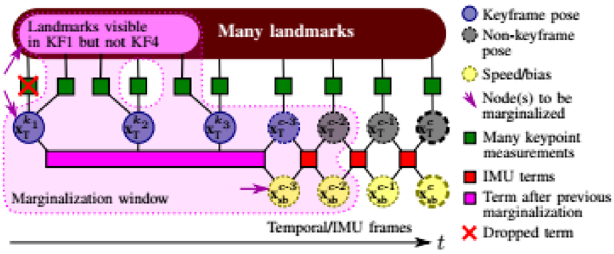

Figure 5: marginalization key-frame

4 visual inertial slam □□□□□□

4.1 Notations

this is todo work.

4.2 Preliminaries

Rieman manifold The skew symmetric matrices:

$$\omega^{\wedge} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0, -\omega_3, \omega_2 \\ \omega_3, 0, -\omega_1 \\ -\omega_2, \omega_1, 0 \end{bmatrix} \qquad \in so(3) \tag{1}$$

wedge operator converts an vector to its corresponding matrix, and the vee operator convert the skew symmetric matrix to an vector. if we have $S = \omega^{\wedge}$, then

$$S^{\vee} = \omega \tag{2}$$

The exponential map(at identity): 指数映射是skew symmetric matrix 到 rotation matrix 的映射.

$$exp(\phi^\wedge) = I + \frac{\sin(\|\phi\|)}{\|\phi\|}\phi^\wedge + \frac{1-\cos(\|\phi\|)}{\|\phi\|^2}(\phi^\wedge)^2 \tag{3}$$

a first order exponential map:

$$exp(\phi^\wedge) \approx I + \phi^\wedge \tag{4}$$

logarithm map at the identity:

$$\log(R) = \frac{\varphi \cdot (R - R^T)}{2\sin(\varphi)} \qquad with \varphi = \arccos(\frac{tr(R)-1}{2}) \tag{5}$$

the vectorized versions of the exponential and logarithm map: 向量与SO(3)的映射.

$$Exp : R^3 \to SO(3); \Phi \rightarrowtail exp(\Phi^{wedge}) \tag{6a}$$
$$Log : SO(3) \to R^3; R \rightarrowtail log(R)^\vee \tag{6b}$$

Exp 的一阶估计:

$$Exp(\Phi + \delta\Phi) \approx Exp(\Phi)Exp(J_r(\Phi)\delta\Phi) \tag{7}$$

Log 的一阶估计:

$$Log(Exp(\Phi)Exp(\delta\Phi)) \approx \Phi + J_r^{-1}(\Phi)\delta\Phi \tag{8}$$

another useful property of exponential map:

$$RExp(\Phi)R^T = exp(R\Phi^\wedge R^T) = Exp(R\Phi) \tag{9a}$$
$$\Leftrightarrow Exp(\Phi)R = RExp(R^T\Phi) \tag{9b}$$

Uncertainty description in SO(3)

$$\tilde{R} = RExp(\epsilon), \quad \epsilon \sim \mathcal{N}(0, \Sigma) \tag{10}$$

the cost function:

$$\mathcal{L}(R) = \frac{1}{2}||Log(R^{-1}\tilde{R})||^2_\Sigma + const = \frac{1}{2}||Log(\tilde{R}^{-1})R||^2_\Sigma + const \tag{11}$$

this is geodesic uncertainty Σ^{-1}.

Gauss-Newton on Manifold move this paragraph to the supplementes.

4.3 MAP

Notations

model description the state:

$$x_i \doteq \{R_i, p_i, v_i, b_i\}. \tag{12}$$

Let $\mathcal{K}$ denote the set of all keyframes up to time k, the state of all keyframes:

$$\mathcal{X}_k \doteq \{x_i\}_{i \in \mathcal{K}_k} \tag{13}$$

the measurements:

$$\mathcal{Z} \doteq \{\mathcal{C}_i, \mathcal{I}_{ij}\}_{(i,j)\in\mathcal{K}_k} \tag{14}$$

factor graphs and map estimation: the psoterior probability of the variables $\mathcal{X}_k$:

$$p(\mathcal{X}_k|\mathcal{Z}_k) \propto p(\mathcal{X}_0)p(\mathcal{Z}_k|\mathcal{X}_k) = p(\mathcal{X}_0) \prod_{(i,j)\in\mathcal{K}_k} p(\mathcal{C}_i, \mathcal{I}_{ij}|\mathcal{X}_k) = p(\mathcal{X}_0) \prod_{(i,j)\in\mathcal{K}_k} p(\mathcal{I}_{ij}|x_i, x_j) \prod_{i\in\mathcal{K}_k} \prod_{l\in\mathcal{C}_i} p(z_{il}|x_i) \tag{15}$$

finally the map estimate:

$$\mathcal{X}_k^* \doteq argmin_{\mathcal{X}_k} - log_e p(\mathcal{X}_k|\mathcal{Z}_k) \tag{16}$$

展开后得到，

$$argmin_{\mathcal{X}_k} ||r_0||^2_{\Sigma_0} + \Sigma_{(i,j)\in\mathcal{K}_k} ||r_{\mathcal{I}_{ij}}||^2_{\Sigma_{ij}} + ||r_{\mathcal{C}_{il}}||^2_{\Sigma_\mathcal{C}} \tag{17}$$

4.4 IMU Model and Motion Integration

4.4.1 IMU Model

$$_B\tilde{\omega}_{WB}(t) =_B \omega_{WB}(t) + b^g(t) + \eta^g(t) \tag{18}$$

$$_B\tilde{a}(t) = R^T_{WB}(t)(_Wa(t) -_w g) + b^a(t) + \eta^a(t) \tag{19}$$

4.4.2 Kinematic Model

$$\dot{R_{WB}} = R_{WB} \quad _B\omega^{\wedge}_{WB}, \quad _W\dot{v} =_w a, \quad _w\dot{p} =_W v, \tag{20}$$

the discrete form assuming $_Wa$ and $_B\omega_{WB}$ remains constant in the time interval $[t, t+\Delta t]$

$$R_{WB}(t+\Delta t) = R_{WB}(t)Exp(_B\omega_{WB}\Delta t) \tag{21}$$

$$_Wv(t+\Delta t) = _Wv(t) + _wa(t)\Delta t \tag{22}$$

$$_Wp(t+\Delta t) = _Wp(t) + _Wv(t)\Delta t + \frac{1}{2} {_Wa}(t)\Delta t^2 \tag{23}$$

all in all rewrite equations above, we get,

$$R(t+\Delta t) = R(t)Exp((\tilde{\omega}(t) - b^g(t) - \eta^g(t))\Delta t) \tag{24}$$

$$v(t+\Delta t) = v(t) + g(t)\Delta t + R(t)(\tilde{a}(t) - b^a(t) - \eta^a(t))\Delta t \tag{25}$$

$$p(t+\Delta t) = p(t) + v(t)\Delta t + +\frac{1}{2}g\Delta t^2 + \frac{1}{2}R(t)(\tilde{a}(t) - b^a(t) - \eta^a(t))\Delta t^2 \tag{26}$$

4.5 IMU pre-integration

IMU integration for all Δt between two consecutive frames at times i and j

$$R_j = R_i \prod_{k=i}^{j-1} Exp((\tilde{\omega} - b_k^g - \eta_k^{gd})\Delta t) \tag{27a}$$

$$v_j = v_i + g\Delta_{ij} + \Sigma_{k=i}^{j-1} R_k(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t \tag{27b}$$

$$p_j = p_i + \Sigma_{k=i}^{j-1}[v_k\Delta + \frac{1}{2}g\Delta t^2 + \frac{1}{2}R^k(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t^2] \tag{27c}$$

relative motion increments

$$\Delta R_{ij} \doteq R_i^T R_j = \prod_{k=i}^{j-1} Exp((\tilde{\omega} - b_k^g - \eta_k^{gd})\Delta t) \tag{28a}$$

$$\Delta v_{ij} \doteq R_i^T(v_j - v_i - g\Delta t_{ij}) = \Sigma_{k=i}^{j-1}\Delta R_{ik}(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t \tag{28b}$$

$$\Delta p_{ij} \doteq R_i^T(p_j - p_i - v_i\Delta t_{ij} - \frac{1}{2}\Sigma_{k=i}^{j-1} g\Delta t^2) = \Sigma_{k=i}^{j-1}[\Delta v_{ik}\Delta t + \frac{1}{2}\Delta R_{ik}(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t^2] \tag{28c}$$

4.5.1 Preintegrated IMU Measurements

$$\Delta R_{ij} \simeq \prod_{k=i}^{j-1}[Exp((\tilde{\omega}_k - b_i^g)\Delta t) Exp(-J_r^k \eta_k^{gd}\Delta t)] \tag{29a}$$

$$= \Delta\tilde{R}_{ij}\prod_{k=i}^{j-1} Exp(-\Delta\tilde{R}_{k+1j}^T J_r^k \eta_k^{gd}\Delta t) \tag{29b}$$

$$\doteq \Delta\tilde{R}_{ij} Exp(-\Delta\phi_{ij}) \tag{29c}$$

... 类似可得到 Δv_{ij} 和 Δp_{ij}.
接下来就是 “preintegrated measurement model” 。

$$\Delta\tilde{R}_{ij} = R_i^T R_j Exp(\delta\phi_{ij}) \tag{30a}$$

$$\Delta\tilde{v}_{ij} = R_i^T(v_j - v_i - g\Delta t_{ij}) + \delta v_{ij} \tag{30b}$$

$$\Delta p_{ij} = R_i^T(p_j - p_i - v_i\Delta t_{ij} - \frac{1}{2}g\Delta t_{ij}^2) + \delta p_{ij} \tag{30c}$$

4.5.2 Noise Propagation

最终可以证明ϕ_{ij})， δv_{ij} 和 δp_{ij} 是 η_k^{gd} 的线性函数，服从高斯分布。

4.5.3 *Incorporating Bias Updates, given a bias update:* $b \leftarrow \bar{b} + \delta b$

$$\Delta\tilde{R}_{ij}(b_i^g) \simeq \Delta\tilde{R}_{ij}(\bar{b}^g)Exp(\frac{\partial\Delta\bar{R}_{ij}}{\partial b^g}\delta b_i^g) \tag{31a}$$

$$\Delta\tilde{v}_{ij}(b_i^g, b_i^a) \simeq \Delta\tilde{v}_{ij}(\bar{b}_i^g, \bar{b}_i^a) + \frac{\partial\Delta\bar{v}_{ij}}{\partial b^g}\delta b_i^g + \frac{\partial\Delta\bar{v}_{ij}}{\partial b^a}\delta b_i^a \tag{31b}$$

$$\Delta\tilde{p}_{ij}(b_i^g, b_i^a) \simeq \Delta\tilde{p}_{ij}(\bar{b}_i^g, \bar{b}_i^a) + \frac{\partial\Delta\bar{p}_{ij}}{\partial b_i^g}\delta b_i^g + \frac{\partial\Delta\bar{p}_{ij}}{\partial b^a}\delta b_i^a \tag{31c}$$

(31d)

4.5.4 *Bias Model*

Bias are slowly time-varying quantities, Hence model them with a " Browian Motion" . For more info, see IMU Model

$$\dot{b}^g(t) = \eta^{bg}, \quad \dot{b}^a(t) = \eta ba \tag{32}$$

`okvis` uses a different bias model for acc(bounded random walk).

$$\dot{b}_g(t) = w_{bg}, \quad \dot{b}_a(t) = -\frac{1}{\tau}b_a + w_{ba} \tag{33}$$

the discrete form.

$$b_j^g = b_i^g + \eta^{bgd}, \quad b_j^a = b_i^a + \eta^{bad}, \tag{34}$$

covariance of discrete bias.

$$\Sigma^{bgd} \doteq \Delta t_{ij} Cov(\eta^{bg}), \quad \Sigma^{bad} \doteq \Delta t_{ij} Cov(\eta^{ba}), \tag{35}$$

factor graph of bias model.

$$||r_{b_{ij}}||^2 \doteq ||b_j^g - b_i^g||^2_{\Sigma^{bgd}} + ||b_j^a - b_i^a||^2_{\Sigma^{bad}} \tag{36}$$

4.6 Linearizing expression for nonlinear optimization

`linearizing rotation matrix`

`transformation matrix`

`linearizing homogeneous points`

`examples on stereo camera`

5 Problems still working on

- right jacobian and left jacobian
-

6 supplementaries

6.1 高斯噪声与随机游走

6.1.1 Bias Model

Bias are slowly time-varying quantities, Hence model them with a ” Browian Motion” . For more info, see IMU Model

$$\dot{b}^g(t) = \eta^{bg}, \quad \dot{b}^a(t) = \eta ba \tag{37}$$

`okvis` uses a different bias model for acc(bounded random walk).

$$\dot{b}_g(t) = w_{bg}, \quad \dot{b}_a(t) = -\frac{1}{\tau} b_a + w_{ba} \tag{38}$$

6.2 IMU的工作原理与机制

6.3 gtsam 与isam

6.4 matrix calculus