mathmatical models from visual inertial slam

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1 abstract

2 short intro

使用camera + IMU的方案来做SLAM/Odometry，一般被称作Visual-Inertial Odometry (VIO)或者Visual-Inertial Navigation System (VINS)。按照David Scaramuzza的分类方法，首先分成filter-based 和 optimization based 的两大类。进一步按照是否把图像特征信息加入状态向量来进行分类，可以分为loosely-couply 和tightly-coupled。几个经典的算法包括:

filter based

msckf msckf没有作者本人的源代码，但是有别人的实现.

rovio Robust Visual Inertial Odometry(有代码).

ssf and msf Vision Based Navigation for Micro Helicopters (有代码).

optimization based

loosely coupled " Inertial aided dense and semi-dense methods for robust direct visual odometry"

tightly coupled okvis " OKVIS: Open Keyframe-based Visual-Inertial SLAM."

tightly coupled orbslam " Visual-Inertial Monocular SLAM with Map Reuse."

2.1 main procedure

流程与实现

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2.2 Notations

2.3 Preliminaries

Rieman manifold The skew symmetric matrices:

$$\omega^{\wedge} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0, -\omega_3, \omega_2 \\ \omega_3, 0, -\omega_1 \\ -\omega_2, \omega_1, 0 \end{bmatrix} \qquad \in so(3) \tag{1}$$

wedge operator converts an vector to its corresponding matrix, and the vee operator convert the skew symmetric matrix to an vector. if we have $S = \omega^{\wedge}$, then

$$S^{\vee} = \omega \tag{2}$$

The exponential map(at identity): 指数映射是skew symmetric matrix 到 rotation matrix 的映射.

$$exp(\phi^{\wedge}) = I + \frac{\sin(\|\phi\|)}{\|\phi\|}\phi^{\wedge} + \frac{1-\cos(\|\phi\|)}{\|\phi\|^2}(\phi^{\wedge})^2 \tag{3}$$

a first order exponential map:

$$exp(\phi^{\wedge}) \approx I + \phi^{\wedge} \tag{4}$$

logarithm map at the identity:

$$\log(R) = \frac{\varphi \cdot (R - R^T)}{2\sin(\varphi)} \qquad with \varphi = \arccos(\frac{tr(R) - 1}{2}) \tag{5}$$

the vectorized versions of the exponential and logarithm map: 向量与SO(3)的映射.

$$Exp : R^3 \to SO(3); \Phi \rightarrowtail exp(\Phi^{wedge}) \tag{6a}$$

$$Log : SO(3) \to R^3; R \rightarrowtail log(R)^{\vee} \tag{6b}$$

Exp 的一阶估计:

$$Exp(\Phi + \delta\Phi) \approx Exp(\Phi)Exp(J_r(\Phi)\delta\Phi) \tag{7}$$

Log 的一阶估计:

$$Log(Exp(\Phi)Exp(\delta\Phi)) \approx \Phi + J_r^{-1}(\Phi)\delta\Phi \tag{8}$$

another useful property of exponential map:

$$RExp(\Phi)R^T = exp(R\Phi^{\wedge}R^T) = Exp(R\Phi) \tag{9a}$$

$$\Leftrightarrow Exp(\Phi)R = RExp(R^T\Phi) \tag{9b}$$

Uncertainty description in SO(3)

$$\tilde{R} = RExp(\epsilon), \quad \epsilon \sim \mathcal{N}(0, \Sigma) \tag{10}$$

the cost function:

$$\mathcal{L}(R) = \frac{1}{2}||Log(R^{-1}\tilde{R})||^2_{\Sigma} + const = \frac{1}{2}||Log(\tilde{R}^{-1})R||^2_{\Sigma} + const \tag{11}$$

this is geodesic uncertainty Σ^{-1}.

Gauss-Newton on Manifold move this paragraph to the supplementes.

2.4 MAP

Notations

model description the state:

$$x_i \doteq \left\{R_i, p_i, v_i, b_i\right\}. \tag{12}$$

Let $\mathcal{K}$ denote the set of all keyframes up to time k, the state of all keyframes:

$$\mathcal{X}_k \doteq \{x_i\}_{i \in \mathcal{K}_k} \tag{13}$$

the measurements:

$$\mathcal{Z} \doteq \{\mathcal{C}_i, \mathcal{I}_{ij}\}_{(i,j) \in \mathcal{K}_k} \tag{14}$$

factor graphs and map estimation: the psoterior probability of the variables $\mathcal{X}_k$:

$$p(\mathcal{X}_k|\mathcal{Z}_k) \propto p(\mathcal{X}_0)p(\mathcal{Z}_k|\mathcal{X}_k) = p(\mathcal{X}_0) \prod_{(i,j)\in\mathcal{K}_k} p(\mathcal{C}_i, \mathcal{I}_{ij}|\mathcal{X}_k) = p(\mathcal{X}_0) \prod_{(i,j)\in\mathcal{K}_k} p(\mathcal{I}_{ij}|x_i, x_j) \prod_{i\in\mathcal{K}_k} \prod_{l\in\mathcal{C}_i} p(z_{il}|x_i) \tag{15}$$

finally the map estimate:

$$\mathcal{X}_k^* \doteq argmin_{\mathcal{X}_k} - log_e p(\mathcal{X}_k|\mathcal{Z}_k) \tag{16}$$

展开后得到,

$$argmin_{\mathcal{X}_k} ||r_0||^2_{\Sigma_0} + \Sigma_{(i,j)\in\mathcal{K}_k} ||r_{\mathcal{I}_{ij}}||^2_{\Sigma_{ij}} + ||r_{\mathcal{C}_{il}}||^2_{\Sigma_{\mathcal{C}}} \tag{17}$$

2.5 IMU Model and Motion Integration

2.5.1 IMU Model

$${}_B\tilde{\omega}_{WB}(t) = {}_B\,\omega_{WB}(t) + b^g(t) + \eta^g(t) \tag{18}$$

$${}_B\tilde{a}(t) = R^T_{WB}(t)({}_W a(t) - {}_w\, g) + b^a(t) + \eta^a(t) \tag{19}$$

2.5.2 Kinematic Model

$$\dot{R_{WB}} = R_{WB} \quad {}_B\omega^\wedge_{WB}, \quad {}_W\dot{v} = {}_w\, a, \quad {}_w\dot{p} = {}_W\, v, \tag{20}$$

the discrete form assuming ${}_W a$ and ${}_B\omega_{WB}$ remains constant in the time interval $[t, t + \Delta t]$

$$R_{WB}(t + \Delta t) = R_{WB}(t) Exp({}_B\omega_{WB}\Delta t) \tag{21}$$

$${}_W v(t + \Delta t) = {}_W v(t) + {}_w a(t)\Delta t \tag{22}$$

$$ {}_W p(t+\Delta t) = {}_W p(t) + {}_W v(t)\Delta t + \frac{1}{2} {}_W a(t)\Delta t^2 \tag{23} $$

all in all rewrite equations above, we get,

$$ R(t+\Delta t) = R(t)Exp((\tilde{\omega}(t) - b^g(t) - \eta^g(t))\Delta t) \tag{24} $$

$$ v(t+\Delta t) = v(t) + g(t)\Delta t + R(t)(\tilde{a}(t) - b^a(t) - \eta^a(t))\Delta t \tag{25} $$

$$ p(t+\Delta t) = p(t) + v(t)\Delta t + +\frac{1}{2}g\Delta t^2 + \frac{1}{2}R(t)(\tilde{a}(t) - b^a(t) - \eta^a(t))\Delta t^2 \tag{26} $$

2.6 IMU pre-integration

IMU integration for all Δt between two consecutive frames at times i and j

$$ R_j = R_i \prod_{k=i}^{j-1} Exp((\tilde{\omega} - b_k^g - \eta_k^{gd})\Delta t) \tag{27a} $$

$$ v_j = v_i + g\Delta_{ij} + \Sigma_{k=i}^{j-1} R_k(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t \tag{27b} $$

$$ p_j = p_i + \Sigma_{k=i}^{j-1}[v_k\Delta + \frac{1}{2}g\Delta t^2 + \frac{1}{2}R^k(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t^2] \tag{27c} $$

relative motion increments

$$ \Delta R_{ij} \doteq R_i^T R_j = \prod_{k=i}^{j-1} Exp((\tilde{\omega} - b_k^g - \eta_k^{gd})\Delta t) \tag{28a} $$

$$ \Delta v_{ij} \doteq R_i^T (v_j - v_i - g\Delta t_{ij}) = \Sigma_{k=i}^{j-1} \Delta R_{ik}(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t \tag{28b} $$

$$ \Delta p_{ij} \doteq R_i^T (p_j - p_i - v_i\Delta t_{ij} - \frac{1}{2}\Sigma_{k=i}^{j-1} g\Delta t^2) = \Sigma_{k=i}^{j-1}[\Delta v_{ik}\Delta t + \frac{1}{2}\Delta R_{ik}(\tilde{a}_k - b_k^a - \eta_k^{ad})\Delta t^2] \tag{28c} $$

2.6.1 Preintegrated IMU Measurements

$$ \Delta R_{ij} \simeq \prod_{k=i}^{j-1} [Exp((\tilde{\omega}_k - b_i^g)\Delta t) Exp(-J_r^k \eta_k^{gd}\Delta t)] \tag{29a} $$

$$ = \Delta \tilde{R}_{ij} \prod_{k=i}^{j-1} Exp(-\Delta \tilde{R}_{k+1j}^T J_r^k \eta_k^{gd}\Delta t) \tag{29b} $$

$$ \doteq \Delta \tilde{R}_{ij} Exp(-\Delta \phi_{ij}) \tag{29c} $$

... 类似可得到 Δv_{ij} 和 Δp_{ij}.
接下来就是 "preintegrated measurement model"。

$$\Delta\tilde{R}_{ij} = R_i^T R_j Exp(\delta\phi_{ij}) \tag{30a}$$
$$\Delta\tilde{v}_{ij} = R_i^T(v_j - v_i - g\Delta t_{ij}) + \delta v_{ij} \tag{30b}$$
$$\Delta p_{ij} = R_i^T(p_j - p_i - v_i\Delta t_{ij} - \frac{1}{2}g\Delta t_{ij}^2) + \delta p_{ij} \tag{30c}$$

2.6.2 Noise Propagation

最终可以证明ϕ_{ij}), δv_{ij} 和 δp_{ij} 是 η_k^{gd} 的线性函数，服从高斯分布。

2.6.3 Incorporating Bias Updates, given a bias update: $b \leftarrow \bar{b} + \delta b$

$$\Delta\tilde{R}_{ij}(b_i^g) \simeq \Delta\tilde{R}_{ij}(\bar{b}^g)Exp(\frac{\partial\Delta\bar{R}_{ij}}{\partial b^g}\delta b_i^g) \tag{31a}$$
$$\Delta\tilde{v}_{ij}(b_i^g, b_i^a) \simeq \Delta\tilde{v}_{ij}(\bar{b}_i^g, \bar{b}_i^a) + \frac{\partial\Delta\bar{v}_{ij}}{\partial b^g}\delta b_i^g + \frac{\partial\Delta\bar{v}_{ij}}{\partial b^a}\delta b_i^a \tag{31b}$$
$$\Delta\tilde{p}_{ij}(b_i^g, b_i^a) \simeq \Delta\tilde{p}_{ij}(\bar{b}_i^g, \bar{b}_i^a) + \frac{\partial\Delta\bar{p}_{ij}}{\partial b_i^g}\delta b_i^g + \frac{\partial\Delta\bar{p}_{ij}}{\partial b^a}\delta b_i^a \tag{31c}$$
$$\tag{31d}$$

2.6.4 Bias Model

Bias are slowly time-varying quantities, Hence model them with a " Browian Motion" . For more info, see IMU Model

$$\dot{b}^g(t) = \eta^{bg}, \quad \dot{b}^a(t) = \eta ba \tag{32}$$

the discrete form.

$$b_j^g = b_i^g + \eta^{bgd}, \quad b_j^a = b_i^a + \eta^{bad}, \tag{33}$$

covariance of discrete bias.

$$\Sigma^{bgd} \doteq \Delta t_{ij}Cov(\eta^{bg}), \quad \Sigma^{bad} \doteq \Delta t_{ij}Cov(\eta^{ba}), \tag{34}$$

factor graph of bias model.

$$||r_{b_{ij}}||^2 \doteq ||b_j^g - b_i^g||^2_{\Sigma^{bgd}} + ||b_j^a - b_i^a||^2_{\Sigma^{bad}} \tag{35}$$

2.7 Linearizing expression for nonlinear optimization

linearizing rotation matrix

transformation matrix

linearizing homogeneous points

examples on stereo camera

2.8

2.9

2.10

2.11

3 mathematical principles behind the model

3.0.1 Preliminaries

3.0.2 Notions of Riemannian geometry

3.0.3 Uncertainty descriptions in SO(3)

3.0.4 Gauss-Newton Method on Manifold

3.1 Maximum a Posteriori visual-inertial state estimation

4 Problems still working on

- right jacobian and left jacobian
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5 supplementaries

5.1 高斯噪声与随机游走

5.2 IMU的工作原理与机制

5.3 gtsam 与isam

5.4 matrix calculus