mathmatical models from visual inertial slam

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May 12, 2017

1 abstract

2 short intro

使用camera + IMU的方案来做SLAM/Odometry，一般被称作Visual-Inertial Odometry (VIO)或者Visual-Inertial Navigation System (VINS)。按照David Scaramuzza的分类方法，首先分成filter-based 和 optimization based 的两大类。进一步按照是否把图像特征信息加入状态向量来进行分类，可以分为loosely-couply 和tightly-coupled。几个经典的算法包括:

filter based

msckf msckf没有作者本人的源代码，但是有别人的实现.

rovio Robust Visual Inertial Odometry(有代码).

ssf and msf Vision Based Navigation for Micro Helicopters (有代码).

optimization based

loosely coupled ” Inertial aided dense and semi-dense methods for robust direct visual odometry”

tightly coupled okvis ” OKVIS: Open Keyframe-based Visual-Inertial SLAM.”

tightly coupled orbslam ” Visual-Inertial Monocular SLAM with Map Reuse.”

2.1 main procedure

流程与实现

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2.2 Notations

2.3 Preliminaries

Rieman manifold The skew symmetric matrices:

$$\omega^\wedge = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 0, -\omega_3, \omega_2 \\ \omega_3, 0, -\omega_1 \\ -\omega_2, \omega_1, 0 \end{bmatrix} \qquad \in so(3) \tag{1}$$

wedge operator converts an vector to its corresponding matrix, and the vee operator convert the skew symmetric matrix to an vector. if we have $S = \omega^\wedge$, then

$$S^\vee = \omega \tag{2}$$

The exponential map(at identity): 指数映射是skew symmetric matrix 到 rotation matrix 的映射.

$$exp(\phi^\wedge) = I + \frac{\sin(\|\phi\|)}{\|\phi\|}\phi^\wedge + \frac{1-\cos(\|\phi\|)}{\|\phi\|^2}(\phi^\wedge)^2 \tag{3}$$

a first order exponential map:

$$exp(\phi^\wedge) \approx I + \phi^\wedge \tag{4}$$

logarithm map at the identity:

$$\log(R) = \frac{\varphi \cdot (R - R^T)}{2\sin(\varphi)} \qquad with \varphi = \arccos(\frac{tr(R)-1}{2}) \tag{5}$$

the vectorized versions of the exponential and logarithm map: 向量与SO(3)的映射.

$$Exp : R^3 \to SO(3); \Phi \rightarrowtail exp(\Phi^{wedge}) \tag{6a}$$

$$Log : SO(3) \to R^3; R \rightarrowtail log(R)^\vee \tag{6b}$$

Exp 的一阶估计:

$$Exp(\Phi + \delta\Phi) \approx Exp(\Phi)Exp(J_r(\Phi)\delta\Phi) \tag{7}$$

Log 的一阶估计:

$$Log(Exp(\Phi)Exp(\delta\Phi)) \approx \Phi + J_r^{-1}(\Phi)\delta\Phi \tag{8}$$

another useful property of exponential map:

$$RExp(\Phi)R^T = exp(R\Phi^\wedge R^T) = Exp(R\Phi) \tag{9a}$$

$$\Leftrightarrow Exp(\Phi)R = RExp(R^T\Phi) \tag{9b}$$

Uncertainty description in SO(3)

$$\tilde{R} = RExp(\epsilon), \quad \epsilon \sim \mathcal{N}(0, \Sigma) \tag{10}$$

the cost function:

$$\mathcal{L}(R) = \frac{1}{2}||Log(R^{-1}\tilde{R})||^2_\Sigma + const = \frac{1}{2}||Log(\tilde{R}^{-1})R||^2_\Sigma + const \tag{11}$$

this is geodesic uncertainty Σ^{-1}.

Gauss-Newton on Manifold move this paragraph to the supplementes.

2.4 MAP

Notations

model description the state:

$$x_i \doteq \left\{R_i, p_i, v_i, b_i\right\}. \tag{12}$$

Let $\mathcal{K}$ denote the set of all keyframes up to time k, the state of all keyframes:

$$\mathcal{X}_k \doteq \{x_i\}_{i \in \mathcal{K}_k} \tag{13}$$

the measurements:

$$\mathcal{Z} \doteq \{\mathcal{C}_i, \mathcal{I}_{ij}\}_{(i,j) \in \mathcal{K}_k} \tag{14}$$

factor graphs and map estimation: the psoterior probability of the variables $\mathcal{X}_k$:

$$p(\mathcal{X}_k|\mathcal{Z}_k) \propto p(\mathcal{X}_0)p(\mathcal{Z}_k|\mathcal{X}_k) = p(\mathcal{X}_0) \prod_{(i,j)\in\mathcal{K}_k} p(\mathcal{C}_i, \mathcal{I}_{ij}|\mathcal{X}_k) = p(\mathcal{X}_0) \prod_{(i,j)\in\mathcal{K}_k} p(\mathcal{I}_{ij}|x_i, x_j) \prod_{i\in\mathcal{K}_k} \prod_{l\in\mathcal{C}_i} p(z_{il}|x_i) \tag{15}$$

finally the map estimate:

$$\mathcal{X}_k^* \doteq argmin_{\mathcal{X}_k} - log_e p(\mathcal{X}_k|\mathcal{Z}_k) \tag{16}$$

展开后得到，

$$argmin_{\mathcal{X}_k} ||r_0||^2_{\Sigma_0} + \Sigma_{(i,j)\in\mathcal{K}_k} ||r_{\mathcal{I}_{ij}}||^2_{\Sigma_{ij}} + ||r_{\mathcal{C}_{il}}||^2_{\Sigma_{\mathcal{C}}} \tag{17}$$

2.5 IMU Model and Motion Integration

2.5.1 IMU Model

$$_B\tilde{\omega}_{WB}(t) =_B \omega_{WB}(t) + b^g(t) + \eta^g(t) \tag{18}$$

$$_B\tilde{a}(t) = R^T_{WB}(t)(_W a(t) -_w g) + b^a(t) + \eta^a(t) \tag{19}$$

2.5.2 Kinematic Model

$$\dot{R_{WB}} = R_{WB} \quad _B\omega^{\wedge}_{WB}, \quad _W\dot{v} =_w a, \quad _w\dot{p} =_W v, \tag{20}$$

the discrete form assuming $_Wa$ and $_B\omega_{WB}$ remains constant in the time interval $[t, t+\Delta t]$

$$R_{WB}(t+\Delta t) = R_{WB}(t)Exp(_B\omega_{WB}\Delta t) \tag{21}$$

$$_Wv(t+\Delta t) = _Wv(t) + _wa(t)\Delta t \tag{22}$$

$$_Wp(t+\Delta t) = {}_Wp(t) + {}_Wv(t)\Delta t + \frac{1}{2}{}_Wa(t)\Delta t^2 \tag{23}$$

all in all rewrite equations above, we get,

$$R(t+\Delta t) = R(t)Exp((\tilde{\omega}(t) - b^g(t) - \eta^g(t))\Delta t) \tag{24}$$

$$v(t+\Delta t) = v(t) + g(t)\Delta t + R(t)(\tilde{a}(t) - b^a(t) - \eta^a(t))\Delta t \tag{25}$$

$$p(t+\Delta t) = p(t) + v(t)\Delta t + +\frac{1}{2}g\Delta t^2 + \frac{1}{2}R(t)(\tilde{a}(t) - b^a(t) - \eta^a(t))\Delta t^2 \tag{26}$$

2.6 IMU pre-integration

2.7 Linearizing expression for nonlinear optimization

linearizing rotation matrix

transformation matrix

linearizing homogeneous points

examples on stereo camera

2.8

2.9

2.10

2.11

3 mathematical principles behind the model

3.0.1 *Preliminaries*

3.0.2 *Notions of Riemannian geometry*

3.0.3 *Uncertainty descriptions in SO(3)*

3.0.4 *Gauss-Newton Method on Manifold*

3.1 Maximum a Posteriori visual-inertial state estimation

4 Problems still working on

- right jacobian and left jacobian
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5 supplementaries

5.1 高斯噪声与随机游走

5.2 IMU的工作原理与机制

5.3 gtsam 与isam

5.4 matrix calculus