VIETNAM NATIONAL UNIVERSITY HO CHI MINH CITY
UNIVERSITY OF INFORMATION TECHNOLOGY
FACULTY OF COMPUTER SCIENCE

EXERCISE REPORT
DYNAMIC PROGRAMMING - PART 1

Lecturer: Son Nguyen Thanh McS

Class: CS112.N21.KHTN

Members: Hoang Ha Van - 21520033

Anh Vo Thi Phuong - 21522883

Date: 25/05/2023

Contents

1. Exercise

1.1. Problem: Here

1.2. Solution:

The above problem is converted to the problem of calculating $\sum\limits_{i=0}^{n} \binom{3i}{k}$ for each positive integer k.

We define the array $dp[k]$ of size $3n$, which computes

$$\sum_{i=0}^{n-1} \binom{3i}{k} = \binom{0}{k} + \binom{3}{k} + ... + \binom{3n-3}{k}$$

Under this definition, $ans[k] = (dp[k] + \binom{3n}{k}) \ \ mod \ \ (10^9 + 7)$, where ans is what we want to find.

We known that Hockey Stick Identity is $\binom{0}{k} + \binom{1}{k} + \binom{2}{k} + ... + \binom{3n-1}{k} = \binom{3n}{k+1}$. We also known that $\binom{3i+1}{k} = \binom{3i}{k} + \binom{3i}{k-1}$ and $\binom{3i+2}{k} = \binom{3i}{k} + 2\binom{3i}{k-1} + \binom{3i}{k-2}$. For $k \geq 2$:

$$\begin{aligned}
dp[k] &= \binom{3n}{k+1} - \sum_{i=0}^{n-1} \binom{3i+1}{k} - \sum_{i=0}^{n-1} \binom{3i+2}{k} \\
&= \binom{3n}{k+1} - 2.\sum_{i=0}^{n-1} \binom{3i}{k} - 3.\sum_{i=0}^{n-1} \binom{3i}{k-1} - \sum_{i=0}^{n-1} \binom{3i}{k-2} \\
&= \binom{3n}{k+1} - 2.dp[k] - 3.dp[k-1] - dp[k-2] \\
\iff 3.dp[k] &= \binom{3n}{k+1} - 3.dp[k-1] - dp[k-2] \\
\iff \quad dp[k] &= \frac{\binom{3n}{k+1} - 3.dp[k-1] - dp[k-2]}{3}
\end{aligned}$$

.

The base cases are $dp[0] = n$ and $dp[1] = \sum\limits_{i=0}^{n-1} 3i$.

Thus we can compute ans for all $0 \leq k \leq 3n$ with the above dp array. Each query can now be answered trivially.

The time complexity is $O(n + q)$ with combinatorial precomputation and modular multiplicative inverse, and the memory complexity is $O(n)$.