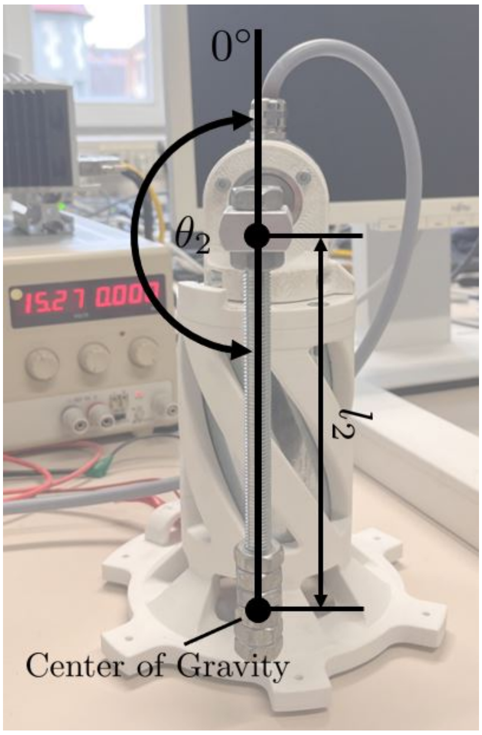
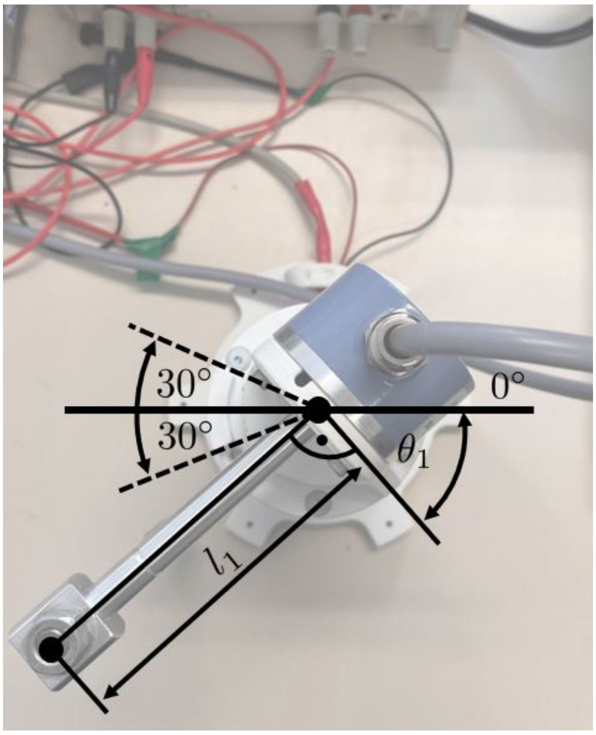
Exercise 1 - LQR Furuta Pendulum



(a) Front View



(b) Top View

Figure 1: Photography of the Furuta pendulum with a selection of states and model parameters

The control of complex systems leads to enormous progress in diverse areas of society. The task is to stabilise a Furuta pendulum around its unstable rest position using a state controller. This is shown in Figure 1. In the following, the task of controlling the pendulum is broken down into smaller tasks, each of which can be assigned to a part of the lecture.

1. Watch the video of the Furuta pendulum swinging up:
   `https://www.youtube.com/shorts/oJYyD5beMqM/`

2. Get an overview of the modelled system dynamics. Use the following paper: `https://onlinelibrary.wiley.com/doi/10.1155/2011/528341`. Answer the following questions in writing: What system states are there? How are they related? What parameters are there? What inputs does the system have? Which effects are not modelled by the nonlinear model?

3. The `MATLAB` script contains the nonlinear equations of motion $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u)$ of the Furuta pendulum with the parameters of the real system. Use these to simulate the behaviour of the pendulum with the explicit 4th order Runge-Kutta method (RK4). Use a step size of $h = 15$ ms and a simulation time of $T = 3$ s. You can visualize the current state during the simulation using the embedded function
   `visu_furuta(Theta_z,Theta_x)`. What do you notice?

4. Linearize the system around the unstable rest position $\boldsymbol{x}_R = \mathbf{0}$. Due to the complex dynamics, it is permitted to use the finite difference method $\frac{\partial f_i}{\partial x_1} \approx \frac{f_i([0+\epsilon,0,0,0],0)-f_i([0-\epsilon,0,0,0],0)}{2\epsilon}$ and so on.

5. Is the linearized system controllable?

6. Is the linearized system observable if $\boldsymbol{c}^\top = (0 \quad 1 \quad 0 \quad 0)$ would apply? Which state is measured here?

7. Synthesise a state feedback controller so that the controlled system has the pole positions $p_1 = -10$, $p_2 = -20$, $p_3 = -30$ and $p_4 = -40$. Calculate the control value u by applying the resulting controller law.

8. Check the system behaviour in the simulation for different initial states $\boldsymbol{x}_0$ in the immediate and further vicinity of the rest position $\boldsymbol{x}_R$.

9. Is the control value limit of the system $M_{max} < 0.45$ Nm adhered to? If this is not the case, what could be changed to comply with the manipulated variable limit?

10. Design a LQ regulator using the parameterization $Q = \text{diag}(50, 500, 1, 1)$ and $R = 1000$ and the `MATLAB` function $lqr()$.

11. How does the LQ regulator affect the system? Vary the input cost R. What is noticeable?