Optimization Models of Airplane Assignment and Crew Scheduling for UC Airlines: A Case Study & Research Report

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Introduction and Background

The global airline industry can be an especially difficult and competitive business, with razor-thin margins and operational constraints such as crew management, plane maintenance, in-flight catering, among a host of other considerations and expenses. A 2021 study conducted by researchers at the Florida International University found that U.S. Airlines have an approximate average annual profit margin of 13.3%, with some airlines having a profit margin as low as only 2.7%[1]. Therefore, as a newly emerging airline company spanning across five UC Campuses, UC Airlines must address key operational research demands in order to ensure a robust and effective service for its customers. In this regard, as operational research consultants, we intend to aid UC Airlines in their company mission to connect UC campuses with the world with mathematical rigor in our industrial optimization.

In particular, we intend to solve two fundamental optimization problems to ensure trustworthy and profit-maximizing operations for UC Airlines. Firstly, we established **airplane assignments** which optimize profit for UC Airlines in the competitive airline industry and provided **crew scheduling** to minimize labor costs by assigning captains and first officers to particular flights, while adhering to FAA regulations and government guidelines.

UC Airlines' operations span across five destinations: UC Berkeley, UCLA, UC Santa Cruz, UC San Diego, and UC Davis. To connect them effectively with flights for customers, UC Airlines also purchased a fleet of 12 airplanes, which vary in model, seat capacity, operating costs, and revenue. To further optimize profits, UC Airlines has also provided the option to rent out unused aircrafts. Each airplane has to have downtime of at least 1 hour to prepare for the next flight after finishing a flight. In terms of crew members, UC Airlines has already employed 16 captains and 16 first officers (FOs), and exactly one first officer and one captain are needed for each flight. After each flight, both the captain and first officer who was on the flight must have an interval at least 1 hour until the next flight. Additionally, FAA regulations limit the amount of time a captain or first officer can fly to be at most 8 hours in any given day.

With these considerations taken into account, our airline assignments optimization model yields the maximum profit of **$93,665**. As for our crew scheduling optimization model, we were able to yield the minimum cost of **$17,140**.

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[1] Profit Margins in U.S. Domestic Airline Routes[1]

Processed Data

Given datasets **flight_schedule.xlsx**, **airplane_data.xlsx**, and **crew_data.xlsx**, we extracted several data for organizing parameters:

- **flights.dat**: containing origin, destination, departure time, and arrival time parameters by flight.
- **revenue.dat**: containing airplane revenue parameters by (flight, airplane) pair.
- **cost.dat**: containing airplane cost parameters by (flight, airplane) pair.
- **airplanes.dat**: containing initial location and leasing revenue parameters by airplane.
- **crew_cost.dat**: containing crew cost parameters by (flight, captain) or (flight, FO) pair.
- **crew_init_loc.dat**: containing crew initial location parameters by captain or FO.

Questions

Q1i) Airplane Assignment Modeling

Sets:

- AIRPLANE = airplanes {1, 2, ..., 12}
- FLIGHT = flights {1, 2, ..., 40}
- PORT = airports {1,2,3,4,5}, which corresponds to {BER, UCL, SCR, SDG, DAV}
- VALID_INIT = valid **initial** flight assignment for each airplane defined as:
  $\{f \in FLIGHT,\ a \in AIRPLANE: initial\_location[a] = origin[f]\}$
- VALID_TRANSIT = valid **transition** for each airplane defined as:
  $\{k \in FLIGHT,\ m \in FLIGHT,\ a \in AIRPLANE:$
  $m > k$
  $\wedge\ (destination[k] = origin[m])$
  $\wedge\ (arrival\_time[k] + downtime \leq depature\_time[m]$
  $\}$

  Note:
  - VALID_INIT covers the constraint for matching each airplane's initial location and origin of each flight.
  - VALID_TRANSIT covers the constraints for matching origin and destination for each flight and downtime.

Decision Variables:

- X[f,a]
  = 1 if airplane a is assigned to flight f, 0 otherwise, where $f \in FLIGHT,\ a \in AIRPLANE$.
- Y[a]
  = 1 if airplane a is leased out, 0 otherwise, where $a \in AIRPLANE$.
- S[f,a]
  = 1 if flight f is the first flight for airplane a, 0 otherwise, where
  $f \in FLIGHT,\ a \in AIRPLANE$.

- Z[k,m,a]
  = 1 if airplane a goes from flight k directly to flight m, 0 otherwise (flight k and m are not necessarily next to each other), where $(k, m) \in VALID\_TRANSIT$.

Parameters:

- revenue[f,a]
  = Revenue generated by assigning airplane a to flight f,
  where $f \in FLIGHT,\ a \in AIRPLANE$.
- cost[f,a]
  = Operational cost of assigning airplane a to flight f, where $f \in FLIGHT,\ a \in AIRPLANE$.
- leasing_revenue[a]
  = Revenue generated by leasing out airplane a, where $a \in AIRPLANE$.
- initial_location[a]
  = Initial airport location of airplane a, where $a \in AIRPLANE$.
- origin[f]
  = Departure airport of flight f, where $f \in FLIGHT$.
- destination[f]
  = Arrival airport of flight f, where $f \in FLIGHT$.
- departure_time[f]
  = Departure time of flight f, where $f \in FLIGHT$.
- arrival_time[f]
  = Arrival time of flight f, where $f \in FLIGHT$.
- downtime
  := 60 minutes (minimum turnaround time)
- M
  := 40 (a big enough (total flight) value for upper limit of assigned-or-leased constraint)

Objective:

- Maximize the total profit generated by both assigning some of the airplanes and leasing out the rest of airplanes.

Objective Function:

$$maximize\ profit = \sum_{f=1}^{40} \sum_{a=1}^{12} (revenue[f,a] - cost[f,a]) \cdot X[f,a] + \sum_{a=1}^{12} (leasing\_revenue[a] \cdot Y[a])$$

Constraints:

1. Each flight must be assigned exactly one airplane:
$$\sum_{a=1}^{12} X[f,a] = 1 \quad \forall f \in FLIGHT$$
2. Each airplane must be either assigned to flights or leased out:
$$1 - Y[a] \leq \sum_{f=1}^{40} X[f,a] \leq M \cdot (1 - Y[a]) \quad \forall a \in AIRPLANE$$
3. Initial flight assignment constraints:

a. Each initial flight assignment (i.e. each S[f,a]) must be upper-limitted by corresponding X[f,a]:

$$S[f,a] \leq X[f,a] \quad \forall (f,a) \in VALID\_INIT$$

b. The total of initial flight assignments for each airplane must be at most 1:

$$\sum_{f=1}^{40} S[f,a] \leq 1 \quad \forall a \in AIRPLANE$$

c. Each assigned initial flight (i.e. each S[f,a]=1) must be the first flight for its airplane:

$$X[k,a] \leq 1 - S[m,a] \quad \forall (m,a) \in VALID\_INIT,\ k \in \{k\colon\ m > k\}$$

d. The sum of all initial flight assignments for each airplane, it should be 0 if it is leased out otherwise should be 1:

$$\sum_{f} S[f,a] = 1 - Y[a] \quad \forall (f,a) \in VALID\_INIT$$

e. Impossible initial flight assignment (i.e. any pair out of VALID_INIT) must be assigned 0:

$$S[f,a] = 0 \quad \forall (f,a) \notin VALID\_INIT$$

4. Transition assignment constraints:
   a. Only if flight k is assigned for an airplane a (i.e. X[k,a]=1), the transition assignment from flight k to any but only one flight m after flight k is allowed for that airplane (i.e. Z[k,m,a]=1 at most one m):

$$\sum_{m} Z[k,m,a] \leq X[k,a] \quad \forall a \in AIRPLANE,\ (k,m) \in VALID\_TRANSIT$$

   b. Only if either:
      i. The transition assignment from flight k to m is valid for a (i.e. one of Z[k,m,a] is 1), or
      ii. The initial flight for airplane a is assigned to flight m (i.e.S[m,a]=1)

   for each flight m, flight assignment for flight m is allowed for that airplane:

$$\sum_{k} Z[k,m,a] + S[m,a] \geq X[m,a] \quad \forall a \in AIRPLANE,\ (k,m) \in VALID\_TRANSIT$$

   c. Enforcing each transition assignment with each assignment of its flight k and m separately:

$Z[k,m,a] \leq X[k,a] \quad \forall (k,m) \in VALID\_TRANSIT,\ a \in AIRPLANE$ and
$Z[k,m,a] \leq X[m,a] \quad \forall (k,m) \in VALID\_TRANSIT,\ a \in AIRPLANE$

Q1ii) Results

Total Daily Profit: $93,665

Airplane Assignments:

- **Assigned Airplanes (8):** Airplane 3, 6, 7, 8, 9, 10, 11, 12
- **Leased Airplanes (4):** Airplane 1, 2, 4, 5

Flight Assignments:

Airplane ID	Enumerated Transition (Transition with Actual Flight IDs)
1	N/A
2	N/A
3	1→12→17→27→36 (1165→3250→4217→6289→8456)
4	N/A
5	N/A
6	7→11→19→26→33→39 (2421→3072→4699→6005→7509→9456)
7	2→10→18→24→32→40 (1432→2972→4461→5531→7398→9785)
8	9→20→28→35 (2828→4858→6522→8324)
9	4→14→22→31 (1843→3571→5172→7023)
10	3→8→15→21→30→37 (1698→2600→3708→5059→6942→8838)
11	6→16→23→29→38 (2135→3984→5307→6759→9122)
12	5→13→25→34 (1987→3386→5715→7961)

Q2i) Modified formulation with Return-to-Initial-Location Constraint

Additional Sets:

- VALID_FINAL
  = valid **final** flight assignments defined as:
  $\{f \in FLIGHT,\ a \in AIRPLANE\colon\ initial\_location[a] = destination[f]\}$

Additional Decision Variables:

- L[f,a]
  = 1 if flight f is the final (L for last) flight for airplane a, 0 otherwise,
  where $f \in FLIGHT,\ a \in AIRPLANE$

Additional Constraints:

1. Assigning the final flight f for airplane a (linking L and X, Z):

$$L[k,a] = X[k,a] - \sum_{m} Z[k,m,a] \quad \forall (k,m) \in VALID\_TRANSIT$$

2. Each assigned airplane must return to the initial location:

$$\sum_{f} L[f, a] = 1 - Y[a] \quad \forall (f, a) \in VALID\_FINAL$$

Q2ii) Results

This optimization problem was feasible. The results are below:

Total Daily Profit: $93,084

Airplane Assignments (no change):
- **Assigned Airplanes (8):** Airplane 3, 6, 7, 8, 9, 10, 11, 12
- **Leased Airplanes (4):** Airplane 1, 2, 4, 5

Flight Assignments:

Airplane ID	Enumerated Transition (Transition with Actual Flight IDs)
1	N/A
2	N/A
3	1→12→17→26→33→39 (1165→3250→4217→6005→7509→9456)
4	N/A
5	N/A
6	7→11→19→27→36 (2421→3072→4699→6289→8456)
7	2→10→18→24→32→40 (1432→2972→4461→5531→7398→9785)
8	9→22→31 (2828→5172→7023)
9	4→14→20→30→38 (1843→3571→4858→6942→9122)
10	3→8→15→21→28→35 (1698→2600→3708→5059→6522→8324)
11	6→16→23→29→37 (2135→3984→5307→6759→8838)
12	5→13→25→34 (1987→3386→5715→7961)

Changes:
1. Optimal Profit Value:
   - The total daily optimal profit in Q2 model decreased by $93,665 - $93,084 = $581 (≈ 0.62%) from the one in Q1 model.
   - This is because adding a constraint generally only decreases or keeps the optimal objective value.

- This decrement implies that Q1 model (partially or entirely) did not satisfy the newly added constraints in Q2 model.

2. Airplane Assignment:
    - For airplane 7 and 12, there was no change in their flight assignments. This means that the flight assignments for these airplanes in Q1 model already satisfied the additional constraints.
    - For other airplanes, there are some changes depending on the airplane mostly found in later flights. Specifically,

Airplane ID	Q1 Model Transition	Q2 Model Transition
1	N/A	N/A
2	N/A	N/A
3	1165→3250→4217→6289→8456	1165→3250→4217→6005→7509→9456
4	N/A	N/A
5	N/A	N/A
6	2421→3072→4699→6005→7509→9456	2421→3072→4699→6289→8456
7	1432→2972→4461→5531→7398→9785	1432→2972→4461→5531→7398→9785
8	2828→4858→6522→8324	2828→5172→7023
9	1843→3571→5172→7023	1843→3571→4858→6942→9122
10	1698→2600→3708→5059→6942→8838	1698→2600→3708→5059→6522→8324
11	2135→3984→5307→6759→9122	2135→3984→5307→6759→8838
12	1987→3386→5715→7961	1987→3386→5715→7961
Note: change transition in Q1 model is in blue, update transition in Q2 model is in red		

- An interesting discovery was that there is no change in the initial flight assignment for each airplane. This implies that the optimal initial flight assignment in Q1 is strongly optimal that the additional constraints in Q2 could not change it. Additionally, the added constraints are all about changing the destination of the final flight -just modifying the final few flights for every airplane should yield to the optimal solution, if possible.

Q3) Address to UC Airlines' Operations President

One common and potential disastrous disruption which could hinder and delay the optimal schedule we calculated is extreme weather and storm conditions. According to data reported by the U.S. Federal Aviation Administration (FAA), adverse weather conditions accounted for nearly three quarters of all U.S. flight delays exceeding one-fourth hour from 2017 until 2023[2]. If any flight encounters dangerous conditions for the flight to be completed safely, such as severe thunderstorms, hailstorms, snowstorms, hurricanes, tornados, then flights will necessarily need to be delayed / cancelled to accommodate safe travel for the crew and passengers[1]. Furthermore, the schedule could also be hindered by maintenance delays - for example, if a plane's refueling is delayed between flights, this can cause a flight to be delayed. In this case, downtime of 1 hour we used in the formulation would not be enough. Additionally, difficulties with managing the availability of crew members can delay a flight's boarding if the airport doesn't have alternate crew members and pilots available to take over.

In order to mitigate the effects of some of the above disruptions, we suggest keeping at least one airplane on standby in case a plane scheduled for a flight experiences unforeseen maintenance issues or delays close to departure. For this reason, we do not recommend leasing all unused airplanes because we recommend keeping at least one airplane as backup to ensure the schedule can be executed as effectively as possible. In this way, if one airplane experiences unforeseen malfunctions, the maintenance delays won't bottleneck the entire flight schedule, but can be replaced with another plane in the case of mechanical failure or maintenance delays. The potential profit applying this strategy to the current formulation is the following:

Airplane to Keep (Airplane ID) (of 1,2,4, and 5, which are leased out in current formulation)	**Profit ($)** (Based on modified model, i.e. $93,084-x)
1	81,049
2	80,473
4	79,157
5	79,828

By only considering the profit, it seems Airplane 1 is the best choice to keep, since the profit that could be generated from leasing it out is maximized. However, we have to account for other circumstances as well. For example, if it is known that SDG has a higher chance of delay, we might want to keep Airplane 4 instead of Airplane 1, since the initial location of Airplane 4 is at SDG airport. Also, if we decide to keep an airplane that originates, for example, at SCR airport, we have to modify the data by removing one of the airplanes from SCR airport (either Airplane 3 or 8), replacing the current assignments. When we rerun the model with this new interest, we could risk making the optimization problem infeasible.

Though keeping planes on stand-by seems like a good plan, we also have to be very careful about it since our goal is to maximize the profit. Even though it is "safer" to keep all the airplanes currently leased out on stand-by instead, this act reduces our profit by more than half, which means the current leasing revenue deeply contributes to our total profit. More precisely, current total revenue received from leasing out Airplane 1,2,4, and 5 is $51,829, which is 55.7% of the

[2] U.S. FAA Data on Weather Delays

total profit (in modified formulation result). Keeping one airplane instead of leasing out will reduce the total profit by 13-15% which is more acceptable. Therefore, we have to understand the trade off and find the sweet spot that can achieve both desired profit and enough backup at the same time.

Q4i) Crew Assignment Modeling

Sets:

- CAPTAIN = captains {1, 2, ..., 16}
- FO = first officers (FOs) {1, 2, ..., 16}
- FLIGHT = flights {1, 2, ..., 40}
- PORT = airports {1,2,3,4,5}, which corresponds to {BER, UCL, SCR, SDG, DAV}
- VALID_CAPTAIN_INIT = valid **initial** flight assignment for each captain defined as:
  $\{f \in FLIGHT,\ c \in CAPTAIN: initial\_location[c] = origin[f]\}$
- VALID_FO_INIT = valid **initial** flight assignment for each FO defined as:
  $\{f \in FLIGHT,\ fo \in FO: initial\_location[fo] = origin[f]\}$
- VALID_CAPTAIN_TRANSIT = valid transition for each captain defined as:
  $\{k \in FLIGHT,\ m \in FLIGHT,\ c \in CAPTAIN:$
  $m > k$
  $\wedge\ (destination[k] = origin[m])$
  $\wedge\ (arrival\_time[k] + downtime \leq depature\_time[m]$
  $\}$
- VALID_FO_TRANSIT = valid transition for each FO defined as:
  $\{k \in FLIGHT,\ m \in FLIGHT,\ fo \in FO:$
  $m > k$
  $\wedge\ (destination[k] = origin[m])$
  $\wedge\ (arrival\_time[k] + downtime \leq depature\_time[m]$
  $\}$

  Note:
  - VALID_CAPTAIN_INIT covers the constraint for matching each captain's initial location and origin of each flight. VALID_FO_INIT does the same for each FO.
  - VALID_CAPTAIN_TRANSIT covers the constraints for matching origin and destination for each flight and downtime for each captain. VALID_FO_TRANSIT does the same for each FO.

Decision Variables:

- X_cap[f,c]
  = 1 if captain c is assigned to flight f, 0 otherwise, where $f \in FLIGHT,\ c \in CAPTAIN$
- X_fo[f,fo]
  = 1 if FO fo is assigned to flight f, 0 otherwise, where $f \in FLIGHT,\ fo \in FO$
- S_cap[f,c]
  = 1 if flight f is the first flight for captain c, 0 otherwise, where $f \in FLIGHT,\ c \in CAPTAIN$

- S_fo[f,fo]
  = 1 if flight f is the first flight for FO fo, 0 otherwise, where $f \in FLIGHT,\ fo \in FO$.
- L_cap[f,c]
  = 1 if flight f is the final flight for captain c, 0 otherwise, where $f \in FLIGHT,\ c \in CAPTAIN$
- L_fo[f,fo]
  = 1 if flight f is the final flight for FO fo, 0 otherwise, where $f \in FLIGHT,\ fo \in FO$.
- Z_cap[k,m,c]
  = 1 if captain c goes from flight k directly to flight m, 0 otherwise (flight k and m are not necessarily next to each other), where $(k, m, c) \in VALID\_CAPTAIN\_TRANSIT$.
- Z_fo[k,m,fo]
  = 1 if FO fo goes from flight k directly to flight m, 0 otherwise (flight k and m are not necessarily next to each other), where $(k, m, fo) \in VALID\_FO\_TRANSIT$.
- ST_cap[c]
  = start time of captain c in a day in minutes, where $c \in CAPTAIN$.
- ST_fo[fo]
  = start time of FO fo in a day in minutes, where $fo \in FO$.
- ET_cap[[c]
  = end time of captain c in a day in minutes, where $c \in CAPTAIN$.
- ET_fo[fo]
  = end time of FO fo in a day in minutes, where $fo \in FO$.

Parameters:
- captain_cost[f,c]
  = Cost of assigning captain c to flight f, where $f \in FLIGHT,\ c \in CAPTAIN$.
- fo_cost[f,fo]
  = Cost of assigning FO fo to flight f, where $f \in FLIGHT,\ fo \in FO$.
- captain_initial_location[c]
  = Initial airport location of captain c, where $c \in CAPTAIN$.
- origin[f]
  = Departure airport of flight f, where $f \in FLIGHT$.
- destination[f]
  = Arrival airport of flight f, where $f \in FLIGHT$.
- departure_time[f]
  = Departure time of flight f, where $f \in FLIGHT$.
- arrival_time[f]
  = Arrival time of flight f, where $f \in FLIGHT$.
- downtime
  := 60 minutes (minimum break time for both captains and FOs)
- faa_regulation_time
  := 480 minutes (maximum working duration for both captains and FOs)

Objective:
- Minimize the total cost of assigning all the crews (both captains and FOs).

Objective Function:

$$minimize\ \ total\_cost\ =\ \sum_{f=1}^{40}\sum_{c=1}^{16}(X\_cap[f,c]\ \cdot\ captain\_cost[c])\ +\ \sum_{f=1}^{40}\sum_{fo=1}^{16}(X\_fo[f,fo]\ \cdot\ fo\_cost[fo])$$

Constraints:

1. Each flight must contain exactly one captain and one FO:

$$\sum_{c=1}^{16} X\_cap[f,c]\ =\ 1\quad \forall f\ \in\ FLIGHT$$

$$\sum_{fo=1}^{16} X\_cap[f,fo]\ =\ 1\quad \forall f\ \in\ FLIGHT$$

2. Initial Flight Constraint
   a. There is exactly one first flight for each captain/fo:

$$\sum_{f=1}^{40} S\_cap[f,c]\ =\ 1\quad \forall c\ \in\ CAPTAIN$$

$$\sum_{f=1}^{40} S\_fo[f,fo]\ =\ 1\quad \forall fo\ \in\ FO$$

   b. Each initial flight assignment must be upper-limited by corresponding flight assignment for both captains and FOs:

$$S\_cap[f,c]\ <=\ X\_cap[f,c]\quad \forall f\ \in\ FLIGHT,\ c\ \in\ CAPTAIN$$

$$S\_fo[f,fo]\ <=\ X\_fo[f,fo]\quad \forall f\ \in\ FLIGHT,\ fo\ \in\ FO$$

   c. Each assigned initial flight must be the first flight for its captain/FO:

$$X\_cap[k,c]\ \le\ 1\ -\ S\_cap[m,c]\quad \forall (m,c)\ \in\ VALID\_CAPTAIN\_INIT,\ k\ \in\ \{k\text{: } m\ >\ k\}$$

$$X\_fo[k,fo]\ \le\ 1\ -\ S\_fo[m,fo]\quad \forall (m,fo)\ \in\ VALID\_FO\_INIT,\ k\ \in\ \{k\text{: } m\ >\ k\}$$

   d. Impossible initial flight assignment (i.e. any pair not in VALID_CAPTAIN_INIT or VALID_FO_INIT) must be assigned 0:

$$S\_cap[f,c]\ =\ 0\quad \forall (f,c)\ \notin\ VALID\_CAPTAIN\_INIT$$

$$S\_fo[f,fo]\ =\ 0\quad \forall (f,fo)\ \notin\ VALID\_FO\_INIT$$

3. Transition Constraint
   a. Only if captain c (FO fo) is assigned to flight k, the transition assignment from flight k to any but only one flight m after k is allowed for that captain (FO):

$$\sum_{m} Z\_cap[k,m,c]\ \le\ X\_cap[k,c]\quad \forall (k,m,c)\ \in\ VALID\_CAPTAIN\_TRANSIT$$

$$\sum_{m} Z\_fo[k,m,c]\ \le\ X\_fo[k,c]\quad \forall (k,m,fo)\ \in\ VALID\_FO\_TRANSIT$$

   b. Only if either:
      i. The transition assignment from flight k to m is valid for captain c (FO fo), or
      ii. The initial flight for captain c (FO fo) is assigned to flight m

for each flight m, flight assignment for flight m is allowed for that captain (FO):

$$\sum_k Z\_cap[k,m,c] + S\_cap[m,c] \geq X\_cap[m,c] \quad \forall (k,m,c) \in VALID\_CAPTAIN\_TRANSIT$$

$$\sum_k Z\_fo[k,m,fo] + S\_fo[m,fo] \geq X\_fo[m,fo] \quad \forall (k,m,fo) \in VALID\_FO\_TRANSIT$$

c. Enforcing each transition assignment with each assignment of its flight k and m separately for both captains and FOs:

$$Z\_cap[k,m,c] \leq X\_cap[k,c] \quad \forall (k,m,c) \in VALID\_CAPTAIN\_TRANSIT$$
$$Z\_cap[k,m,c] \leq X\_cap[m,c] \quad \forall (k,m,c) \in VALID\_CAPTAIN\_TRANSIT$$
$$Z\_fo[k,m,fo] \leq X\_fo[k,fo] \quad \forall (k,m,fo) \in VALID\_FO\_TRANSIT$$
$$Z\_fo[k,m,fo] \leq X\_fo[m,fo] \quad \forall (k,m,fo) \in VALID\_FO\_TRANSIT$$

4. FAA Regulation Constraint

a. Define a final flight for each captain/FO (if a flight is the final flight for a crew, then there should not be any transition after that flight for that crew):

$$L\_cap[k,c] = X\_cap[k,c] - \sum_m Z\_cap[k,m,c] \quad \forall (k,m,c) \in VALID\_CAPTAIN\_TRANSIT$$

$$L\_fo[k,fo] = X\_fo[k,fo] - \sum_m Z\_fo[k,m,fo] \quad \forall (k,m,fo) \in VALID\_FO\_TRANSIT$$

b. Define start time for each captain/FO:

$$ST\_cap[c] = \sum_f (S\_cap[f,c] \cdot departure\_time[f]) \quad \forall (f,c) \in VALID\_CAPTAIN\_INIT$$

$$ST\_fo[fo] = \sum_f (S\_fo[f,fo] \cdot departure\_time[f]) \quad \forall (f,fo) \in VALID\_FO\_INIT$$

c. Define end time for each captain/FO:

$$ET\_cap[c] = \sum_{f=1}^{40} (L\_cap[f,c] \cdot arrival\_time[f]) \quad \forall c \in CAPTAIN$$

$$ET\_fo[fo] = \sum_{f=1}^{40} (L\_fo[f,fo] \cdot arrival\_time[f]) \quad \forall fo \in FO$$

d. FAA regulation time; each captain/FO must work at most 8h (480 mins):

$$ET\_cap[c] - ST\_cap[c] \leq faa\_regulation\_time \quad \forall c \in CAPTAIN$$
$$ET\_fo[fo] - ST\_fo[fo] \leq faa\_regulation\_time \quad \forall fo \in FO$$

Note:

- ST_cap/ST_fo and ET_cap/ET_fo are still all linear - each of them is defined as a multiplication of constant (parameter) and a variable.

Q4ii) Results

The optimization problem was feasible. The results are below:

Total Crew Assignment Cost: $17,140

Crew Assignment:

- **Captain**

Captain ID	Transition
1	23→36 (5307→8456)
2	14→22→31 (3571→5172→7023)
3	6→17→26 (2135→4217→6005)
4	2→10→18 (1432→2972→4461)
5	33→39 (7509→9456)
6	16→32→40 (3984→7398→9785)
7	5 (1987)
8	7 (2421)
9	13→25→34 (3386→5715→7961)
10	11→19→35 (3072→4699→8324)
11	4→27 (1843→6289)
12	29→38 (6759→9122)
13	3→8→15→21 (1698→2600→3708→5059)
14	9→20→28 (2828→4858→6522)
15	1→12→24 (1165→3250→5531)
16	30→37 (6942→8838)

- **FO Assignment**

FO ID	Transition
1	6 (2135)
2	21→40 (5059→9785)
3	23→29→38 (5307→6759→9122)

4	2→10→18→24 (1432→2972→4461→5531)
5	33→39 (7509→9456)
6	7→11→19→26 (2421→3072→4699→6005)
7	5→13→28 (1987→3386→6522)
8	16→27→36 (3984→6289→8456)
9	31 (7023)
10	12→17→35 (3250→4217→8324)
11	4→14→22 (1843→3571→5172)
12	20→30→37 (4858→6942→8838)
13	3→8 (1698→2600)
14	1 (1165)
15	9→15→32 (2828→3708→7398)
16	25→34 (5715→7961)

Q5i) Modified Formulation with Return-to-Initial-Location Constraint

Additional Sets:

- VALID_CAPTAIN_FINAL
  = valid **final** flight assignments for captains defined as:
  $\{f \in FLIGHT,\ c \in CAPTAIN\colon\ initial\_location[c] = destination[f]\}$
- VALID_FO_FINAL
  = valid **final** flight assignments for FOs defined as:
  $\{f \in FLIGHT,\ fo \in FO\colon\ initial\_location[fo] = destination[f]\}$

Additional Constraints:

- Every captain/fo must return to the initial location:

$$\sum_f L\_cap[f,c] = 1 \quad \forall (f,c) \in VALID\_CAPTAIN\_FINAL$$

$$\sum_f L\_fo[f,fo] = 1 \quad \forall (f,fo) \in VALID\_FO\_FINAL$$

Q5ii) Results and Analysis

The model was **infeasible**. Some intuition and analyses are:

- Constraints of **downtime** and **FAA regulation** are together **overconstraining** the problem. If we relax the downtime to $downtime \leq 34$ and the FAA regulation to $faa\_regulation\_time \geq 750$, we can find feasible solutions (found by experiments).
- **Newly added constraints**, i.e.

$$\sum_f L\_cap[f, c] = 1 \quad \forall(f, c) \in VALID\_CAPTAIN\_FINAL$$

$$\sum_f L\_fo[f, fo] = 1 \quad \forall(f, fo) \in VALID\_FO\_FINAL$$

  are **overconstraining** the problem. We can remove the constraints above and introduce following constraints to relax it:
  - If some captains/fos returns to the initial location, then they have to use the valid final flights:

$$\sum_f L\_cap[f, c] \leq 1 \quad \forall(f, c) \in VALID\_CAPTAIN\_FINAL$$

$$\sum_f L\_fo[f, fo] \leq 1 \quad \forall(f, fo) \in VALID\_FO\_FINAL$$

  - Lower limit the number of captains/fos that return to their initial locations by some number $0 \leq n_1, n_2 \leq 16$:

$$\sum_f \sum_c L\_cap[f, c] \geq n_1 \quad \forall(f, c) \in VALID\_CAPTAIN\_FINAL$$

$$\sum_f \sum_{fo} L\_fo[f, fo] \geq n_2 \quad \forall(f, fo) \in VALID\_FO\_FINAL$$

Note there are $17^2 = 289$ ways to choose a pair of (n_1, n_2), so we experimented only the cases that both returning captains and FOs are lower-limited by n i.e. $n = n_1 = n_2$. Here is the result table:

Lower Limit of Returning Crews (n)	Optimal Crew Assignment Cost ($)	Feasibility
0	17,140	✅
1	17,173	✅
2	17,183	✅
3	17,188	✅
4	17,210	✅
5	17,214	✅
6	17,219	✅
7	17,219	✅

8	17,240	✅
9	17,293	✅
10	17,309	✅
11	17,355	✅
12	17,419	✅
13	17,650	✅
14	N/A	❌
15	N/A	❌
16	N/A	❌

It turned out that this modified crew assignment problem can assign at most 13 crews correctly. This means that with small relaxation to given additional constraints (every crew must return to their initial locations → At least n $(0 \leq n \leq 13)$ captains and n FOs must return to their initial locations), we can correctly implement the model.

Q6) Crew Preference Constraints in Scheduling

As mentioned in the previous analysis, operations have a limited flexibility to adjust crew schedules to meet their demand of "returning home at the end of the day." Airlines have always tried to achieve this demand and other demands such as less work hours, in order to increase crew satisfaction and operation flexibility, yet it is impossible to meet the demands of every single crew member. An alternative solution can be similar to the above scenario in Q5. By modifying the model to include only a selection of crew preferences for both flight routes and total working hours, the schedule can be feasible. These preferences can be modeled through specific constraints and soft penalties. [3] For example, a crew member can submit their desired routes and maximum preferred hours, then the model can include a binary decision variable indicating whether this captain or first officer is assigned to a requested route, or if an upper bound for flight hours is added to this crew member.

On the other hand, when crew members are not assigned to a full day of flying, they are on reserve as standby resources. This can ensure our airline can respond swiftly to disruptions such as sick calls or maintenance delays. Assigning light schedules to certain captains and FOs creates flexibility for substitutes of the flight.[4]

- Maintain a pool of under-scheduled crew for standby
- Prioritize reserving less senior pilots for unassigned roles

[3] Airline Pilot's Schedule: What You Can Expect - Thrust Flight
[4] Airline and Commercial Pilots : Occupational Outlook Handbook

- Use existing crew at base airports to avoid relocation costs

In reality, when preferences of only a portion of the crew can be met, we recommend applying priority rules based on seniority, which is the standard in the industry. Senior pilots often secure preferred routes or layovers, while newer crew receive remaining assignments. If the system still produces infeasibilities, route preferences of more crew members should be relaxed first, followed by the total working hours for new crew members, until feasibility is restored. Total working hours for any crew should not exceed a total of 14 hours though, as stated by the FAA. Some airlines resolve conflicts using a point-based auction or allowing pilots to trade trips post-assignment. [5]

- Prioritize fulfilling preferences for higher-seniority pilots
- Relax route preferences before working hour limits
- Enable trade boards or preference auctions to resolve assignment conflicts

If the model cannot satisfy all operational and preference constraints, we should consider hiring more captains and first officers. We could also offer stipends to crew assigned to less desirable flights or longer shifts. By providing incentives for flexibility, we can improve morale and help cover understaffed shifts without compromising our objectives.

- Hire additional reserve pilots if coverage is inadequate
- Provide bonuses for overnight layovers or longer workdays
- Encourage voluntary shift coverage via internal bidding

Executive Summary of Report

Operating in a demanding business environment characterized by thin profit margins, complex operational restrictions, and strong regulatory control, the global airline industry confronts many challenges. According to a 2021 Florida International University study, some carriers achieve as little as 2.7% annual profit margin, while U.S. airlines average just 13.3%. Recently established regional airline UC Airlines connects five University of California campuses and has to maximize its operations in this competitive environment to ensure both profitability and dependability of service. As operations research consultants, we have developed sophisticated mathematical optimization models to solve two fundamental problems in airline operations: airplane assignment and crew scheduling. Using mixed-integer linear programming techniques, these models maximize profit following all operational constraints and legal requirements.

Our airplane assignment optimization model purposefully arranges UC Airlines's twelve aircraft over forty daily flights to maximize operational profit. The model considers several aircraft specifications, varying revenue, and cost structures by flight, and the possibility of

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[5] Aviation Lifestyle and Work Schedule

leasing idle aircraft to other carriers. Our model satisfies minimum one-hour turnaround times between flights, appropriate matching of aircraft locations, and the operational constraint that every aircraft starts its daily schedule from its starting point. Leased to other carriers are four aircraft (Airplanes 1, 2, 4, and 5), and eight aircraft (Airplanes 3, 6, 7, 8, 9, 10, 11, and 12) assigned to service the flight network produce a daily operational profit of $93,665. We provide thorough routing information for every aircraft that details the flight path each should run across the day to maximize profitability. Under a constraint that helps maintenance scheduling and crew logistics—that is, that all operating aircraft return to their original airports at the end of the day—our model remains feasible with only a marginal decrease in profit to $93,084 (a reduction of just 0.62%). Since our solution can satisfy this extra operational constraint with the least financial impact, it shows great resilience. Amazingly, our research revealed that the ideal starting flight directions stayed the same between the two models, so stressing the quality of our first solution.

We've developed an optimization model for crew scheduling to distribute UC Airlines's 16 captains and 16 first officers to flights in a way that lowers total labor costs while yet satisfying all legal requirements. Every flight is manned with exactly one captain and one first officer; the model ensures that crews have at least one hour between consecutive flights and that no crew member flies more than the FAA-mandated maximum of 8 hours daily. With each crew member assigned thorough tasks, our optimal solution results in a minimum daily crew cost of $17,140. Although a highly desired result for crew satisfaction and operational planning, we found infeasibility when attempting to modify the crew scheduling model to guarantee all personnel return to their original airports at the end of their duty day. Methodical relaxation of constraints revealed that at most 13 crew members could realistically return to their starting points while maintaining all other operational requirements. Our sensitivity study provides a thorough cost analysis showing expenses rising from $17,140 (with no return constraint) to $17,650 (with 13 crew members returning) as the number of returning crew members increases. Keeping at least one aircraft on standby instead of leasing all unneeded aircraft will help mitigate possible schedule interruptions brought on by maintenance issues, weather, or crew absence. This strategic reserve allows operational freedom to respond to unexpected events without compromising the whole system. We propose setting up a reserve pool for unassigned captains and first officers, with assignments based on seniority and preferences, so enabling deployment as required.

For the next developments of the crew scheduling model, we suggest applying a preference-based system including desired paths and working hours. When preferences contradict operational requirements, seniority should guide priorities; more experienced crew members should be assigned preferred tasks. In addition, incentives or stipends for less desirable chores could help to increase crew satisfaction while maintaining operational efficiency.

Our optimization solutions provide UC Airlines with a strong basis for maximizing profitability while ensuring regulatory compliance and consistent service over the University of California campus network using careful analysis of operational constraints and exhaustive mathematical modeling. These models can be developed even as operational data becomes available and function as decision-support tools for daily planning and strategic resource allocation in this demanding but essential transportation service.

Appendices

- Link to the submission folder that contains all folders: IEOR162_project_submission_group18
- Filenames

Part	.mod	.dat	.run	.txt
1	project_part1.mod	airplanes.dat flights.dat revenue.dat cost.dat	project_part1.run	part1_solution.txt
2	project_part2.mod	airplanes.dat flights.dat revenue.dat cost.dat	project_part2.run	part2_solution.txt
4	project_part4.mod	flights.dat crew_cost.dat crew_init_loc.dat	project_part4.run	part4_solution.txt
5	project_part5.mod	flights.dat crew_cost.dat crew_init_loc.dat	project_part5.run	part5_solution.txt

Appendix A: AMPL Model Files

Link to the folder containing our models (.mod files): model

Appendix B: Data Files

Link to the folder containing our processed data (.dat files): data

Appendix C: Run Files

Link to the folder containing our run files (.run files): run

Appendix D: Solution Files:

Link to the folder containing all solutions (.txt files): organized_solution

Appendix E: References

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